Physics

Single correct answer type:

1. A conducting metal circular – wire – loop of radius r is placed perpendicular to a magnetic field which varies with time as $B=B_0 e^{\frac{-t}{\tau}}$, where $B_0 \wedge \tau$ are constants, at time t = 0. If the resistance of the loop is R then the heat generated in the loop after a long time $(t \rightarrow \infty)$ is:

(A) $\frac{\pi^2 r^4 B_0^4}{2\tau R}$

(B) $\frac{\pi^2 r^4 B_0^2}{2\tau R}$

(C) $\frac{\pi^2 r^4 B_0^2 R}{\tau}$

(D) $\frac{\pi^2 r^4 B_0^2}{\tau R}$

Solution: (B)

$$\phi = B_0 \pi r^2 e^{\frac{-t}{p}}$$

$$\varepsilon = \frac{d\phi}{dt} = \frac{B_0 \pi r^2}{T} e^{\frac{-t}{r}}$$

Heat $¿\int_0^\infty \frac{\varepsilon^2}{R} = \frac{\pi^2 r^4 B_0^2}{2\pi R}$

2. Within a spherical charge distribution of charge density $\rho(r)$, N equipotential surfaces of potential $V_0, V_0+\Delta V, V_0+2\Delta V, \ldots V_0+N\Delta V (\Delta V>0)$, are drawn and have increasing radii $r_0, r_1, r_2, \ldots r_N$, respectively. If the difference in the radii of the surfaces is constant for all values of $V_0 \wedge \Delta V$ then:

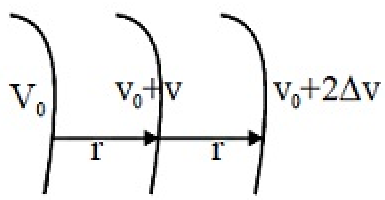
(A) $\rho(r)=$¿ constant

(B) $\rho(r) \alpha \frac{1}{r^2}$

(C) $\rho(r) \alpha \frac{1}{r}$

(D) $\rho(r) \alpha r$

Solution: (C)



$$E=\frac{-dv}{dr}$$

$\because$ dv and dr is same

E = constant

$$E=\frac{K\phi}{r^2}=c$$

$$\Rightarrow \phi \propto r^2$$

$$\phi = \int_0^r \rho 4\pi r^2 dr$$

$$\Rightarrow \rho \propto \frac{1}{r}$$

3. A thin 1m long rod has a radius of 5mm. A force of $50\pi kN$ is applied at one end to determine its Young's modulus. Assume that the force is exactly known. If the least count in the measurement of all lengths is 0.01 mm, which of the following statements is false?

(A) The maximum value of Y that can be determined is $10^{14}\frac{N}{m^2}$

(B) $\frac{\Delta Y}{Y}$ gets minimum contribution from the uncertainty in the length

(C) $\frac{\Delta Y}{Y}$ gets its maximum contribution from the uncertainty in strain

(D) The figure of merit is the largest for the length of the rod

Solution: (A)

$$Y = \frac{Fl}{\pi r^2 \Delta l}$$

$$¿ 2 \times 10^{14}\frac{N}{m^2}$$

4. Concrete mixture is made by mixing cement, stone and sand in a rotating cylindrical drum. If the drum rotates too fast, the ingredients remain stuck to the wall of the drum and proper mixing of ingredients remain stuck to the wall of the drum and proper mixing of ingredients does not take place. The maximum rotational

speed of the drum in revolutions per minute (rpm) to ensure proper mixing is close to:

(Take the radius of the drum to be 1.25m and its axle to be horizontal):

(A) 27.0

(B) 0.4

(C) 1.3

(D) 8.0

Solution: (A)

For just complete rotation

$v=\sqrt{Rg}$ at top point

$$\Rightarrow w=\frac{v}{R}=\sqrt{\frac{g}{R}}=\sqrt{\frac{10}{1.25}}$$

$$w(rpm)=\frac{60}{2\pi}\sqrt{\frac{10}{1.25}}=27$$

5. The ratio (R) of output resistance r_0, and the input resistance r_i in measurements of input and output characteristics of a transistor is typically in the range:

(A) $R\ \ 10^2-10^3$

(B) $R\ \ 1-10$

(C) $R\ \ 0.1-1.0$

(D) $R\ \ 0.1-0.01$

Solution: (A)

Conceptual

6. Consider an electromagnetic wave propagating in vacuum. Choose the correct statement:

(A) For an electromagnetic wave propagating in $+y$ direction the electric field is $\vec{E}=\frac{1}{\sqrt{2}}E_{yz}(x,t)\hat{z}$ and the magnetic field is $\vec{B}=\frac{1}{\sqrt{2}}B_z(x,t)\hat{y}$

(B) For an electromagnetic wave propagating in + y direction the electric field is $\vec{E}=\frac{1}{\sqrt{2}}B_{yz}(x,t)\hat{z}$

(C) For an electromagnetic wave propagating in + x direction the electric field is $\vec{E}=\frac{1}{\sqrt{2}}E_{yz}(y,z,t)(\hat{y}+\hat{z})$

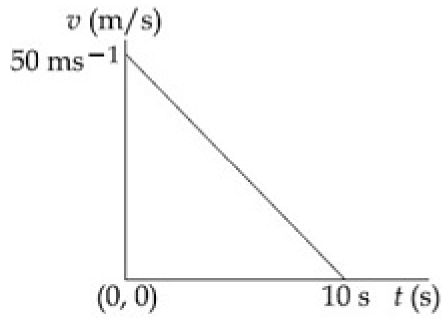
(D) For an electromagnetic wave propagating in + x direction the electric filed is $\vec{E}=\frac{1}{\sqrt{2}}E_{yz}(x,t)(\hat{y}-\hat{z})$ and the magnetic filed is $\vec{B}=\frac{1}{\sqrt{2}}B_{yz}(x,t)(\hat{y}+\hat{z})$

Solution: (D)

Wave in X – direction means E and B should be function of x and t

$\hat{y}-\hat{z}\perp\hat{y}+\hat{z}$

7. Velocity time graph for a body of mass 10kg is shown in figure. Work done on the body in first two seconds of the motion is:



(A) $-9300\,J$

(B) $12000\,J$

(C) $-4500\,J$

(D) $-12000\,J$

Solution: (C)

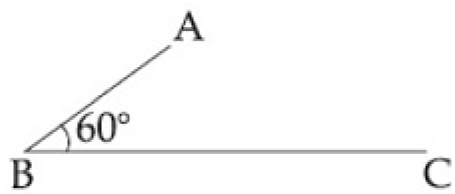
$v=-5t+50=0$

$v_{at\ t=2}=40$

$\Delta K.E.=W.D.=\frac{1}{2}\left(40^2-50^2\right)\times 10$

¿ -4500

8. In the figure shown ABC is a uniform wire. If centre of mass of wire lies vertically below point A, then $\frac{BC}{AB}$ is close to:



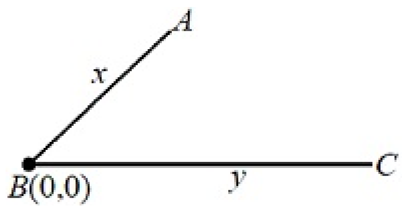
(A) 1.85

(B) 1.5

(C) 1.37

(D) 3

Solution: (C)



$$x_{cm} = \frac{x}{2} \frac{(\rho x)\left(\frac{x}{2}\right)\frac{1}{2} + \rho\, y^{\frac{y}{2}}}{\rho(x+y)}$$

$$\Rightarrow \frac{1}{2} + \frac{y}{x} = \frac{y^2}{x^2}$$

$$\Rightarrow \frac{y}{x} = \frac{1+\sqrt{3}}{2} = 1.37$$

9. A particle of mass m is acted upon by a force F given by the empirical law $F = \frac{R}{t^2} v(t)$. If this law is to be tested experimentally by observing the motion starting from rest, the best way is to plot:

(A) $\log v(t) \, aginst \, \frac{1}{t}$

(B) $v(t) \, agians \, t \, t^2$

(C) $\log v(t) \, against \, \frac{1}{t^2}$

(D) $\log v(t) \, against \, t$

Solution: (A)

$$F=\frac{R}{t^2}v(t)\Rightarrow m\frac{dv}{dt}=\frac{R}{t^2}v(t)$$

$$\int\frac{dv}{v}=\int\frac{Rdt}{mt^2}$$

In $v=\frac{-R}{mt}$

$$▯\in v\propto\frac{1}{t}\Rightarrow(A)$$

10. To determine refractive index of glass slab using a travelling microscope, minimum number of readings required are:

(A) Two

(B) Four

(C) Three

(D) Five

Solution: (C)

Reading one $\Rightarrow$ without slab

Reading two $\Rightarrow$ with slab

Reading three $\Rightarrow$ with saw dust

11. A fighter plane of length 20m, wing span (distance from tip of one wing to the tip of the other wing) of 15m and height 5m is flying towards east over Delhi. Its speed is $240\,m\,s^{-1}$. The earth's magnetic field over Delhi is $5\times10^{-5}T$ with the

declination angle 0^o and dip of θ such that $\sin\theta = \frac{2}{3}$. If the voltage developed is V_B between the lower and upper side of the plane and V_W between the tips of the wings then $V_B \wedge V_W$ are close to:

(A) $V_B = 40\,mV\,; V_W = 135\,mV$ with left side of pilot at higher voltage

(B) $V_B = 45\,mV\,; V_W = 120\,mV$ with right side of pilot at higher voltage

(C) $V_B = 40\,mV\,; V_W = 135\,mV$ with right side of pilot at high voltage

(D) $V_B = 45\,mV\,; V_W = 120\,mV$ with left side of pilot at higher voltage

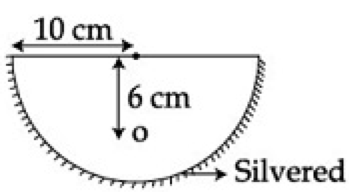
Solution: (D)

$$V_B = V\,B_H\,l = 240 \times 5 \times 10^{-5} \cos(\theta) \times 5$$

$$¿44.7\,mv$$

By right hand rule, the change moves to the left of pilot

12. A hemispherical glass body of radius 10 cm and refractive index 1.5 is silvered on its curved surface. A small air bubble is 6 cm below the flat surface inside it along the axis. The position of the image of the air bubble made by the mirror is seen:



(A) 14cm below flat surface

(B) 20cm below flat surface

(C) 16cm below flat surface

(D) 30cm below flat surface

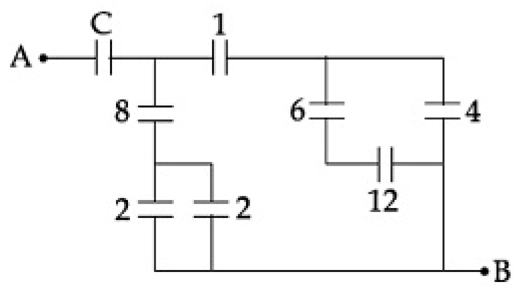
Solution: (B)

Mirror image

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{v}+\frac{1}{-4}=\frac{1}{-5}$$

$$h_a=h_r\frac{\mu_1}{\mu_2}=30\times\frac{1}{1.5}=20$$

13. Figure shows a network of capacitors where the number indicates capacitances in micro Farad. The value of capacitance C if the equivalent capacitance between point A and B is to be $1\,\mu F$ is:



(A) $\frac{32}{23}\mu F$

(B) $\frac{31}{23}\mu F$

(C) $\frac{33}{23}\mu F$

(D) $\frac{34}{23}\mu F$

Solution: (A)

C_{eq} of circuit $¿\frac{32}{9}$

$$With\, C\, \frac{1}{C_{eq}} = \frac{1}{C} + \frac{9}{32} = 1$$

$$\Rightarrow C = \frac{32}{23}$$

14. A particle of mass M is moving in a circle of fixed radius R in such a way that its centripetal acceleration at time t is given by $n^2 R t^2$ where n is a constant. The power delivered to the particle by the force acting on it, is:

(A) $\frac{1}{2} M n^2 R^2 t^2$

(B) $M n^2 R^2 t$

(C) $Mn R^2 t^2$

(D) $Mn R^2 t$

Solution: (B)

$$a_c = \frac{v^2}{R} = n^2 R t^2$$

$$v = nRT$$

$$a_t = \frac{dv}{dt} = nR$$

Power ¿ $m a_t v$

¿ $m nR\, nRt$

¿ $M n^2 R^2 t$

15. A, B, C and D are four different physical quantities having different dimensions. None of them is dimensionless. But we know that the equation AD = C In (BD) holds true. Then which of the combination is not a meaningful quantity?

(A) $\frac{C}{BD}-\frac{AD^2}{C}$

(B) $A^2-B^2C^2$

(C) $\frac{A}{B}-C$

(D) $\frac{(A-C)}{D}$

Solution: (D)

dim (A) $\neq$ dim (C)

Hence A – C is not possible

16. A modulated signal $C_m(t)$ has the form $C_m(t)=30\sin 300\pi t+10(\cos 200\pi t-\cos 400\pi t)$. The carrier frequency f_c the modulating frequency (message frequency) f_ω and the modulation index μ are respectively given by:

(A) $f_c=200\,Hz;\,f_\omega=50\,Hz;\,\mu=\frac{1}{2}$

(B) $f_c=150\,Hz;\,f_\omega=50\,Hz;\,\mu=\frac{2}{3}$

(C) $f_c=150\,Hz;\,f_\omega=30\,Hz;\,\mu=\frac{1}{3}$

(D) $f_c = 200\,Hz\,; f_\omega = 30\,Hz\,; \mu = \frac{1}{2}$

Solution: (B)

$$\mu \frac{A_c}{2} = 10 \Rightarrow \mu = \frac{2}{3}$$

$$A_c = 30$$

$$W_c - W_m = 200\,\pi$$

$$W_c + W_m = 400\,\pi$$

$$\Rightarrow f_c = 150, f_w = 50\,Hz$$

17. In an engine the piston undergoes vertical simple harmonic motion with amplitude 7cm. A washer rests on top of the piston and moves with it. The motor speed is slowly increased. The frequency of the piston at which the washer no longer stays in contact with the piston, is close it:

(A) 0.7 Hz

(B) 1.9 Hz

(C) 1.2 Hz

(D) 0.1 Hz

Solution: (B)

No contact $\Rightarrow N = 0$

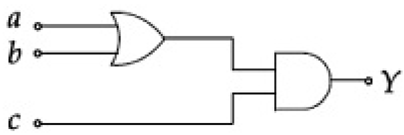
$$\Rightarrow a_{max} = g = w^2 A$$

$$w = \sqrt{\frac{g}{A}}$$

$$¿\sqrt{\frac{10}{0.07}}$$

$$f=\frac{w}{2\pi}=\sqrt{\frac{1000}{7}}\frac{1}{2\pi}=1.9$$

18. To get an output of 1 from the circuit shown in figure the input must be:



(A) $a=0, b=0, c=1$

(B) $a=1, b=0, c=0$

(C) $a=1, b=0, c=1$

(D) $a=0, b=1, c=0$

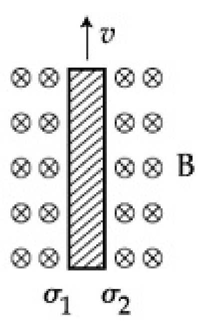
Solution: (C)

Output of OR gate must be 1 and C = 1

So, $a=1, b=0$

¿ $a=0, b=1$

19. Consider a thin metallic sheet perpendicular to the plane of the paper moving with speed 'v' in a uniform magnetic field B going into the plane of the paper (See figure). If charge densities $\sigma_1 \wedge \sigma_2$ are induced on the left and right surfaces, respectively, of the sheet then (ignore fringe effects):



(A) $\sigma_1 = \frac{-\epsilon_0 vB}{2}, \sigma_2 = \frac{\epsilon_0 vB}{2}$

(B) $\sigma_1 = \epsilon_0 v B, \sigma_2 = -\epsilon_0 vB$

(C) $\sigma_1 = \frac{\epsilon_0 vB}{2}, \sigma_2 = \frac{-\epsilon_0 vB}{2}$

(D) $\sigma_1 = \sigma_2 = \epsilon_0 vB$

Solution: (B)

$\sigma_1 = -\sigma_2$ [due to magnetic force]

$E = \frac{\sigma}{\varepsilon_0} = vB$

$\sigma = \varepsilon_0 vB$

20. A Carnot freezer takes heat from water at $0^o C$ inside it and rejects it to the room at a temperature of $27^o C$. The latent heat of ice is $336 \times 10^3 Jkg^{-1}$. If 5kg of water at $0^o C$ is converted into ice at $0^o C$ by the freezer, then the energy consumed by the freezer is close to:

(A) $1.51 \times 10^5 J$

(B) $1.68 \times 10^6 J$

(C) $1.71 \times 10^7 J$

(D) $1.67 \times 10^5 J$

Solution: (D)

$\Delta \theta = mL$

$¿5 \times 336 \times 10^3 = Q_{sink}$

$$\frac{Q_{sink}}{Q_{source}} = \frac{T_{sink}}{T_{source}}$$

$$▯Q_{source} = \frac{T_{source}}{T_{sink}} \propto Q_{sink}$$

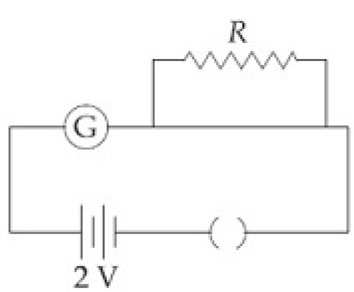
$$▯w.o. = Q_{source} - Q_{sink}$$

$$¿Q_{sink}\left(\frac{T_{source}}{T_{sink}} - 1\right)$$

$$¿5 \times 336 \times 10^3\left(\frac{300}{273} - 1\right)$$

$$¿1.67 \times 10^5 J$$

21. A galvanometer has a 50 division scale. Battery has no internal resistance. It is found that there is deflection of 40 divisions when $R = 2400\,\Omega.$ Then we can conclude:



(A) Current sensitivity of galvanometer is $20\frac{\mu A}{division}$

(B) Resistance of galvanometer is $200\,\Omega.$

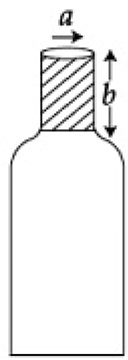
(C) Resistance required on R.B. for a deflection of 10 divisions is $9800\,\Omega$.

(D) Full scale deflection current is 2mA.

Solution: (A)

Question is not clear, diagram is wrong

22. A bottle has an opening of radius a and length b. A cork of length b and radius $(a+\Delta a)\, where\, (\Delta a \ll a)$ is compressed to fit into the opening completely (See figure). If the bulk modulus of cork is B and frictional coefficient between the bottle and cork is μ then the force needed to push the cork into the bottle is:



(A) $(\pi\mu Bb)a$

(B) $(2\pi\mu Bb)\Delta a$

(C) $(\pi\mu Bb)\Delta a$

(D) $(4\pi\mu Bb)\Delta a$

Solution: (D)

$$p=\frac{N}{A}=\frac{N}{(2\pi a)b}$$

$$\Rightarrow stress=B\,strain$$

$$\frac{N}{(2\pi a)b} = B\frac{2\pi a\,\Delta a \times b}{\pi a^2 b}$$

$$f = \mu N = \mu\, 4\pi b\,\Delta a B$$

23. A toy – car, blowing its horn, is moving with a steady speed of $5\frac{m}{s}$, away from a wall. An observer, towards whom the toy car is moving, is able to hear 5 beats per second. If the velocity of sound in air is $340\frac{m}{s}$, the frequency of the horn of the toy car is close to:

(A) 680 Hz

(B) 510 Hz

(C) 340 Hz

(D) 170 Hz

Solution: (D)

$$f(direct) = f\left(\frac{340}{340-5}\right) = f,$$

$$f(by\,wall) = f\left(\frac{340}{340+5}\right)$$

$$\text{¿}f_{\text{¿}wall} = f_2$$

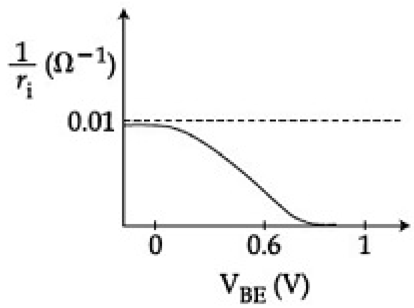
Beats $\text{¿}(f_1 - f_2)$

$$5 = f\left(\frac{340}{340-5} - \frac{340}{340+5}\right)$$

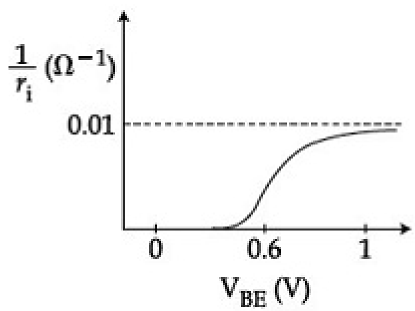
$$\Rightarrow f = 170$$

24. A realistic graph depicting the variation of the reciprocal of input resistance in an input characteristics measurement in a common emitter transistor configuration is:

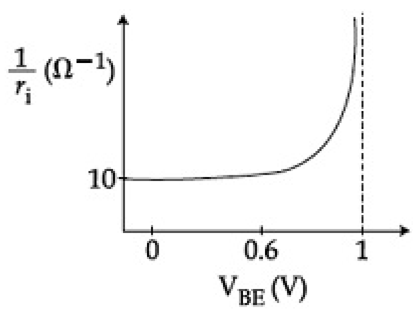
(A)



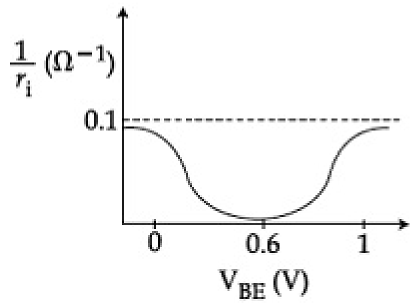
(B)



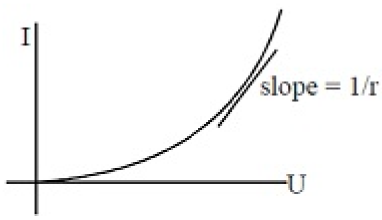
(C)

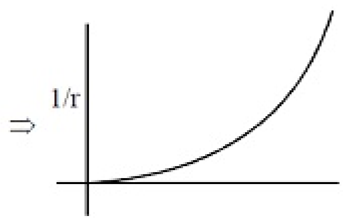


(D)



Solution: (B)





25. A neutron moving with a speed 'v' makes a head on collision with a stationary hydrogen atom in ground state. The minimum kinetic energy of the neutron for which inelastic collision will take place is:

(A) 20.4 eV

(B) 10.2 eV

(C) 12.1 eV

(D) 16.8 eV

Solution: (A)

$n \rightarrow v(H)$ Before

$(n)(H) \to \frac{v}{2}$ After

Loss in K.E. $¿\frac{1}{2}mv^2 - \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2$

$¿\frac{1}{4}mv^2$

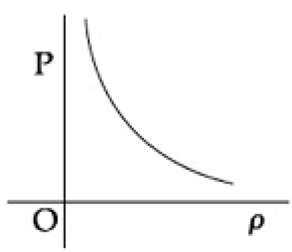
K.E. lost is used to jump from 1^{st} orbit to 2^{nd} orbit $\Delta K.E. = 10.2\,ev$

$\Rightarrow \frac{1}{4}mv^2 = 10.2$

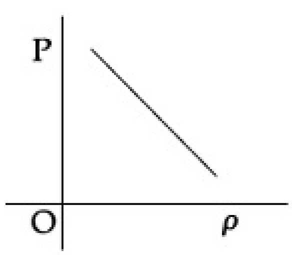
$\frac{1}{2}mv^2 = 2 \times 10.2 = 20.4\,eV$

26. Which of the following shows the correct relationship between the pressure 'P' and density ρ of an ideal gas at constant temperature?

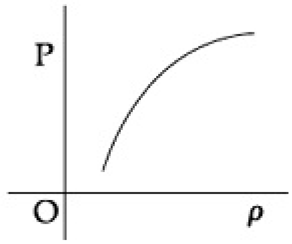
(A)



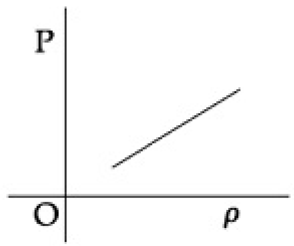
(B)



(C)



(D)



Solution: (D)

$$p = \frac{\rho RT}{M}$$

27. The resistance of an electrical toaster has a temperature dependence given by $R(T) = R_0[1 + \alpha(T - T_0)]$ in its range of operation. At $T_0 = 300\,K, R = 100\,\Omega \wedge at\; T = 500\,K, R = 120\,\Omega$. The toaster is connected to a voltage source at 200 V and its temperature is raised at a constant rate from 300 to 500 K in 30s. The total work done in raising the temperature is:

(A) $400 \in \frac{5}{6} J$

(B) $200 \in \frac{2}{3} J$

(C) $300\,J$

(D) $400 \in \frac{1.5}{1.3} J$

Solution: (A)

Option not matching

28. Two stars are 10 light years away from the earth. They are seen through a telescope of objective diameter 30 cm. the wavelength of light is 600nm. To see the stars just resolved by the telescope, the minimum distance between them should be $\left(1\, light\ year = 9.46 \times 10^{15} m\right)$ of the order of:

(A) $10^{8} km$

(B) $10^{10} km$

(C) $10^{11} km$

(D) $10^{6} km$

Solution: (A)

$$\Delta \theta = \frac{0.61 \lambda}{4} = \frac{1}{R}$$

$$l = \frac{R}{9} 0.61 \times \lambda$$

$$¿ \frac{9.46 \times 10^{15} \times 10 \times 0.61 \times 600 \times 10^{-9}}{0.3}$$

$$¿ 1.15 \times 10^{11} m$$

$$\Rightarrow 1.115 \times 10^{8} km$$

29. An astronaut of mass m is working on a satellite orbiting the earth at a distance h from the earth's surface. The radius of the earth is R, while its mass is M. The gravitational pull F_G on the astronaut is:

(A) Zero since astronaut feels weightless

(B) $\frac{GMm}{(R+h)^2} < F_G < \frac{GMm}{R^2}$

(C) $F_G = \frac{GMm}{(R+h)^2}$

(D) $0 < F_G < \frac{GMm}{R^2}$

Solution: (C)

Gravitational force ¿ $\frac{G m_1 m_2}{(r+h)^2}$

30. A photoelectric surface is illuminated successively by monochromatic light of wavelengths $\frac{\lambda \wedge \lambda}{2}$. If the maximum kinetic energy of the emitted photoelectrons in the second case is 3 times that in the first case, the work function of the surface is:

(A) $\frac{hc}{2\lambda}$

(B) $\frac{hc}{\lambda}$

(C) $\frac{hc}{3\lambda}$

(D) $\frac{3hc}{\lambda}$

Solution: (A)

$$K.E_{\lambda}=\frac{hc}{\lambda}-\phi$$

$$K.E_{\frac{\lambda}{2}}=\frac{hc}{\frac{\lambda}{2}}-\phi$$

$$E.E_{\frac{\lambda}{2}}=3\left(K.E_{\lambda}\right)$$

$$\Rightarrow\phi=\frac{hc}{2\lambda}$$

Chemistry

Single Correct Answer Type

1. Which of the following polymers is synthesized using a fee radical polymerization technique?
(A) Terylene
(B) Melamine polymer
(C) Nylon 6,6
(D) Teflon

Solution: (D) Terylene, Melamine and Nylon – 6, 6 are condensation polymers. Teflon is addition polymer as monomer is alkene derivative $\left(CF_2=CF_2\right)$

2. The volume of 0.1 N dibasic acid sufficient to neutralize 1 g of a base that furnishes 0.04 mole of $\overset{-¿}{OH^{¿}}$ in aqueous solution is:

(A) 400 mL
(B) 600 mL
(C) 200 mL
(D) 800 mL

Solution: (A) Law of equivalence (Apply)

equivalence of acid = equivalence of base

$0.1 \times v = 0.04 \times 1$

$v = 0.4\,L$

$¿ 0.4 \times 1000 = 400\,ml$

3. Aqueous solution of which salt will not contain ions with the electronic configuration $1s^2 2s^2 2p^6 3s^2 3p^6$?

(A) NaF
(B) KBr
(C) NaCl
(D) CaI_2

Solution: (A) $\underset{Na^{¿}}{+¿} \rightarrow [He]2s^2 2p^6$

$\underset{F^{¿}}{-¿} \rightarrow [He]2s^2 2p^6$

Which is not matching with $1s^2 2s^2 2p^6 3s^2 3p^6$

4. Fluorination of an aromatic ring is easily accomplished by treating a diazonium salt with HBF_4. Which of the following conditions is correct about this reaction?

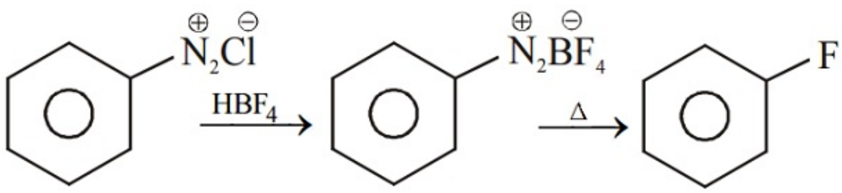
(A) NaF/Cu

(B) Cu_2O/H_2O

(C) Only Heat

(D) $NaNO_2/Cu$

Solution: (C)



5. Gold number of some colloids are: Gelatin: 0.005 – 0.01, Gum Arabic: 0.15 – 0.25; Oleate: 0.04 – 1.0; Starch: 15 – 25. Which among these is a better protective colloid?

(A) Gelatin

(B) Starch

(C) Oleate

(D) Gum Arabic

Solution: (A) $Gold\,number \propto \frac{1}{protective\ power}$

6. Sodium extract is heated with concentrated HNO_3 before testing for halogens because:

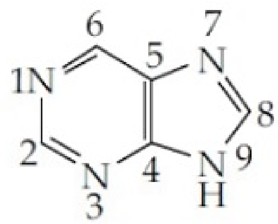
(A) Ag_2S and AgCN are soluble in acidic medium.

(B) Silver halides are totally insoluble in nitric acid

(C) S^{2-} and CN^{-}, if present, are decomposed by conc. HNO_3 and hence do not interfere in the test
(D) Ag reacts faster with halides in acidic medium

Solution: (C) S^{2-} and CN^{-} ions if present are decomposed by conc. HNO_3 and hence do not interfere in the test.

7. The "N" which does not contribute to the basicity for the compound is:



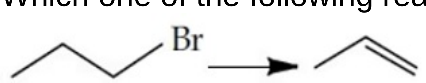
(A) N 9
(B) N 3
(C) N 1
(D) N 7

Solution: (A) N_9 lone pair is stabilized by resonance so it is not basic in nature.

8. The commercial name for calcium oxide is:
(A) Quick lime
(B) Milk of lime
(C) Slaked lime
(D) Limestone

Solution: (A) Quick lime is commercial name of CaO (Quick lime).

9. Which one of the following reagents is not suitable for the elimination reaction?



(A) NaI

(B) $NaOEt/EtOH$

(C) $NaOH/H_2O$

(D) $NaOH/H_2O-EtOH$

Solution: (A)

With NaI substitution takes place.

10. Which one of the following substances used in dry cleaning is a better strategy to control environmental pollution?
(A) Sulphur dioxide
(B) Carbon dioxide
(C) Nitrogen dioxide
(D) Tetrachloroethylene

Solution: (D) Dry cleaning is any cleaning process for clothing and textiles using a chemical solvent other than water. The solvent used is typically toxic tetrachloroethylene or technically "PERC".

11. The transition metal ions responsible for color in ruby and emerald are, respectively:

(A) Co^{3+} and Cr^{3+}

(B) Co^{3+} and Co^{3+}

(C) Cr^{3+} and Cr^{3+}

(D) Cr^{3+} and Co^{3+}

Solution: (C) Ruby is Al_2O_3 in which Red colour is obtained when Cr^{+3} is replacing Al^{+3} ions in octahedral sites. Emerald is $Be_3Al_2(SiO_3)_6$ in which green colour is obtained when Cr^{+3} is replacing Al^{+3} in octahedral site.

12. The bond angle H-X-H is the greatest in the compound:
(A) PH_3
(B) CH_4
(C) NH_3
(D) H_2O

Solution: (B)

	Molecule	Hybridization	Bond Angle
1.	PH_3	sp^3	98^o
2.	CH_4	sp^3	$109^o 28'$
3.	NH_3	sp^3	107^o
4.	H_2O	sp^3	104.5^o

13. An aqueous solution of a salt MX_2 at certain temperature has a van't Hoff factor of 2. The degree of dissociation for this solution of the salt is:
(A) 0.50
(B) 0.33
(C) 0.67
(D) 0.80

Solution: (A) $\alpha = \frac{i-1}{k-1}$

$¿\frac{2-1}{3-1}=\frac{1}{2}=50$

14. Oxidation of succinate ion produces ethylene and carbon dioxide gases. On passing 0.2 Faraday electricity through an aqueous solution of potassium succinate, the total volume of gases (at both cathode and anode) at STP (1 atm and 273 K) is:
(A) 8.96 L
(B) 4.48 L
(C) 6.72 L
(D) 2.24 L

Solution: (A)

$$\begin{matrix} CH_2 + COOK \\ CH_2\,COOK \end{matrix} + 2H_2O \longrightarrow \underbrace{\begin{matrix} CH_2 \\ \| \\ CH_2 \end{matrix}}_{\text{anode}} + \underbrace{2CO_2 + H_2}_{\text{cathode}} + 2KOH$$

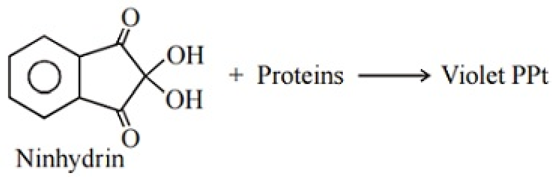
Total equivalents of $C_2H_4 + CO_2 + H_2 = 0.2 + 0.2 + 0.2 = 0.6$

Total moles of gases $¿\frac{0.2}{2}+\frac{0.2}{1}+\frac{0.2}{2}=0.4$

$$v=\frac{nRT}{P}=\frac{0.4\times 0.0821\times 273}{1}=8.96\,L$$

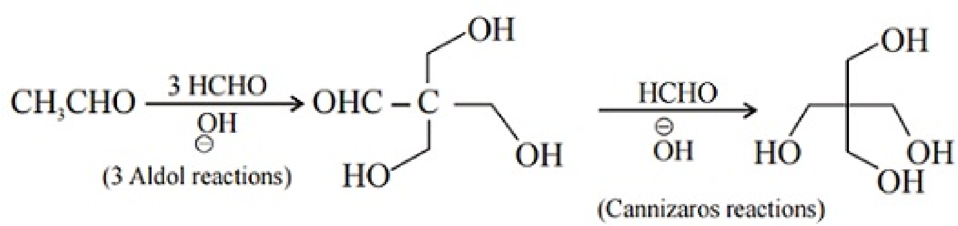
15. Observation of “Rhumann’s purple” is a confirmatory test for the presence of:
(A) Starch
(B) Reducing sugar
(C) Protein
(D) Cupric ion

Solution: (C) Rhumann’s putple is ninhydrin.



16. The correct statement about the synthesis of erythritol $\left(C\left(CH_2OH\right)_4\right)$ used in the preparation of PETN is:
(A) The synthesis requires three aldol condensations and one cannizzaro reaction
(B) Alpha hydrogen of ethanol and methanol are involved in this reaction
(C) The synthesis requires two aldol condensation and two cannizzaro reactions
(D) The synthesis requires four aldol condensations between methanol and ethanol.

Solution: (A)



17. Which of the following is a bactericidal antibiotic?
(A) Ofloxacin
(B) Tetracycline
(C) Chloramphenicol
(D) Erythromycin

Solution: (A) Bactericidal antibiotics are antibiotics which can kill bacteria and Bacteriastatic are which can inhibit growth of bacteria ofloxacin is bactericidal antibiotic.

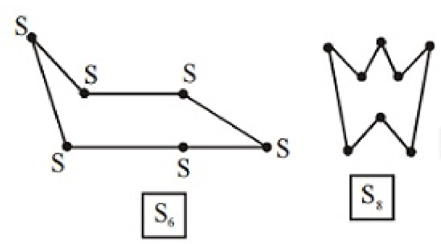
18. Identify the incorrect statement:
(A) The S – S – S bond angles in the S_8 and S_6 rings are the same

(B) Rhombic and monoclinic Sulphur have S_8 molecules

(C) S_2 is paramagnetic like oxygen

(D) S_8 ring has a crown shape

Solution: (A) Sulphur have puckered S_8 rings with crown conformation, e-sulphur also known as Engel's sulphur contain S_6 rings arranged in a chair conformation.



19. Extraction of copper by smelting uses silica as an additive to remove:

(A) Cu_2O

(B) FeS

(C) FeO

(D) Cu_2S

Solution: (C) FeO is gangue and SiO_2 is flux to form slag $FeSiO_3$.

$$[FeO + SiO_2 \rightarrow FeSiO_3]$$

20. Identify the reaction which does not liberate hydrogen:

(A) Reaction of lithium hydride with B_2H_6

(B) Electrolysis of acidified water using Pt electrodes

(C) Reaction of zinc with aqueous alkali

(D) Allowing a solution of sodium in liquid ammonia to stand

Solution: (A) Sodium is added to liquid ammonia, producing a solution containing solvated electrons and used as reducing agent.

$$2\,LiH + B_2H_6 \rightarrow 2\,Li[B\,H_4]$$

Complex and do not liberate H_2 gas.

21. The rate law for the reaction below is given by the expression $k[A][B]$

$$A + B \rightarrow Product$$

If the concentration of B is increased from 0.1 to 0.3 mole, keeping the value of A at 0.1 mole, the rate constant will be:

(A) 3k
(B) 9k
(C) k/3
(D) k

Solution: (C) Rate constant is independent of concentration.

22. The following statement concern elements in the periodic table. Which of the following is true?

(A) For group 15 elements, the stability of +5 oxidation state increases down the group
(B) Elements of group 16 have lower ionization enthalpy values compared to those of group 15 in the corresponding periods
(C) The group 13 elements are all metals
(D) All the elements in group 17 are gases

Solution: (B) Group 16 elements have less IP than Group 15 elements due to half filled configurations of Group 15 elements.

23. A solid XY kept in an evacuated sealed container undergoes decomposition to form a mixture of gases X and Y at temperature T. The equilibrium pressure is 10 bar in this vessel. K_p for this reaction is:

(A) 25
(B) 100
(C) 10
(D) 5

Solution: (B) $XY ¿ (S) \rightleftharpoons X ¿(g) ¿ +¿ Y (g)$

$K_p = (P_x)(P_y)$

$¿ 10 \times 10$

$¿ 100$

24. If 100 mole of H_2O_2 decompose at 1 bar and 300 K, the work done (kJ) by one mole of $O_2(g)$ as it expands against 1 bar pressure is:

$2H_2O_2(l) \rightleftharpoons 2H_2O(l) + O_2(g)$

$(R = 8.3\, J\, K^{-1} mol^{-1})$

(A) 124.50
(B) 249.00
(C) 498.00
(D) 62.25

Solution: (B) $w = P\Delta v = nRT$

$¿ 100 \times 8.3 \times 300$

$¿ 249000\, J$

¿249 *KJ*

25. Which of the following is an example of homoleptic complex?

(A) $[Co(NH_3)_6]Cl_3$

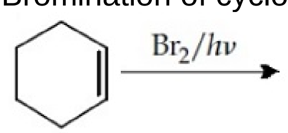
(B) $[Pt(NH_3)_2Cl_2]$

(C) $[Co(NH_3)_4Cl_2]$

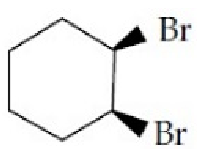
(D) $[Co(NH_3)_5Cl]Cl_2$

Solution: (A) Homoleptic complexes have only one type of ligands.

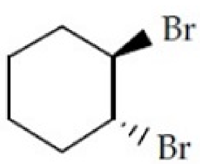
26. Bromination of cyclohexene under conditions given below yields:



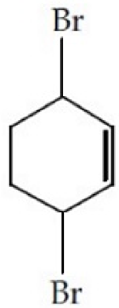
(A)



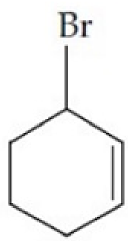
(B)



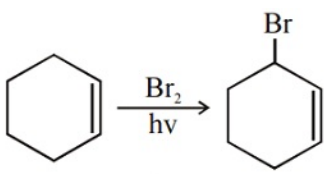
(C)



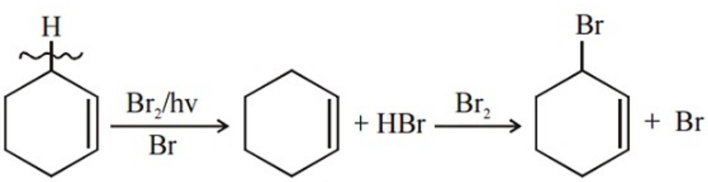
(D)



Solution: (D)



In presence of u.v. light allylic C – H bond undergoes bromination.



27. Assertion: Among the carbon allotropes, diamond is an insulator, whereas, graphite is a good conductor of electricity

Reason: Hybridization of carbon in diamond and graphite are sp^3 and sp^2 respectively

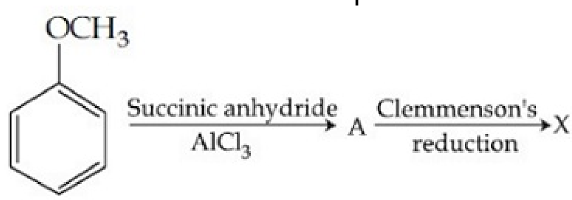
(A) Both assertion and reason are correct, but the reason is not the correct explanation for the assertion

(B) Both assertion and reason are correct, but the reason is the correct explanation for the assertion

(C) Both assertion and reason are incorrect
(D) Assertion is incorrect statement, but the reason is correct

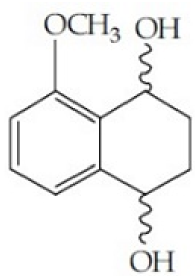
Solution: (B) Diamond has strong 3-Dimensional network structure as every carbon is linked with 4 other carbons tetrahedrally via sp^3 hybridization. In graphite every carbon is linked to 3 other carbon to forms sheet like hexagonal structure.

28. Consider the reaction sequence below:

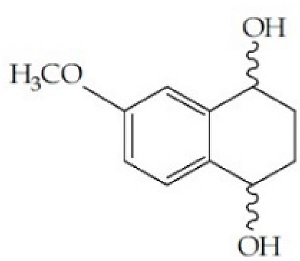


is:

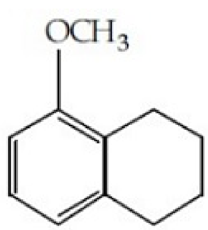
(A)



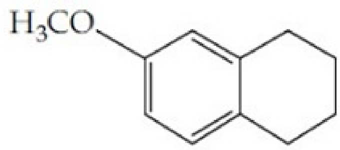
(B)



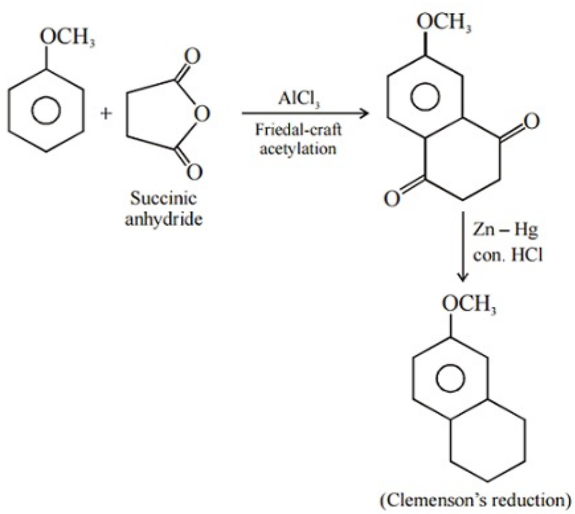
(C)



(D)



Solution: (D)



29. Initially, the root mean square (rms) velocity of N_2 molecules at certain temperature is u. If this temperature is doubled and all the nitrogen molecules dissociate into nitrogen atoms, then the new rms velocity will be:

(A) 2u
(B) 14u
(C) 4u
(D) u/2

Solution: (A) $u = \sqrt{\frac{3RT}{2\frac{n}{0}}}$

$$u' = \sqrt{\frac{3R \times 2T}{14}}$$

$$\frac{u}{u'}=\sqrt{\frac{1}{4}}$$

$$\frac{u}{u'}=\frac{1}{2}$$

$$u'=2u$$

30. Identify the correct statement:
(A) Corrosion of iron can be minimized by forming a contact with another metal with a higher reduction potential
(B) Iron corrodes in oxygen-free water
(C) Corrosion of iron can be minimized by forming an impermeable barrier at its surface
(D) Iron corrodes more rapidly in salt water because its electrochemical potential is higher

Solution: (C) Corrosion of Iron can be minimized by forming an impermeable barrier at its surface.

Mathematics

Single Correct Answer Type:

1. For $x \in R, x \neq 0,$ if $y(x)$ is a differentiable function such that $x\int_1^x y(t)dt=(x+1)\int_1^x ty(t)dt,$ then $y(x)$ equals:

(where C is a constant.)

(A) $c x^3 e^{\frac{1}{x}}$

(B) $\frac{C}{x^2} e^{\frac{-1}{x}}$

(C) $\frac{C}{x} e^{\frac{-1}{x}}$

(D) $\frac{C}{x^3} e^{\frac{-1}{x}}$

Solution: (D)

$$\int_1^x y(t)dt = x\int_1^x ty(t)dt + \int_1^x ty(t)dt$$

Differentiate w.r.t. to x.x

$$\int_1^x y(t)dt + x[(x) - y(1)] = \int_1^x ty(t)dt + x[xy(x) - y(1)] + xy(x) - y(1)$$

$$\int_1^x y(t)dt = \int_0^1 t(y)(t)dt + x^2 y(x) - y(1)$$

Differentiate again w.r.t. to x

$$y(x) - y(1) = xy(x) - y(1) + 2xy(x) + x^2 y^1(x)$$

$$(1-3x)y(x) = x^2 y^1(x)$$

$$\frac{y^1(x)}{y(x)} = \frac{1-3x}{x^2}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1-3x}{x^2} \Rightarrow \in y = \frac{-1}{x} - 3\in x$$

$$¿(yx^3) = \frac{-1}{x}$$

$$y x^3 = -e^{\frac{-1}{x}}$$

$$y = \frac{e^{\frac{-1}{x}}}{x^3}$$

Or $$y = \frac{c\,e^{\frac{-1}{x}}}{x^3}$$

2. The sum $\sum_{r=1}^{10} (r^2+1) \times (r!)$ Is equal to:

(A) $11 \times (11!)$

(B) $10 \times (11!)$

(C) $(11)!$

(D) $101 \times (10!)$

Solution: (B)

$$\sum_{r=1}^{10} (r^2+1) \llcorner r$$

$$T_r = (r^2+1+r-r) \llcorner r = (r^2+r) \llcorner r - (r-1) \llcorner r$$

$$T_r = r \llcorner r+1 = -(r-1) \llcorner r$$

$$T_1 = 1 \llcorner 2 - 0$$

$$T_2 = 2 \llcorner 3 - 1 . \llcorner 2$$

$$T_3 = 3 \llcorner 4 - 2 . \llcorner 3$$

$$\underline{T_{10} = 10 \llcorner 11 - 9 \llcorner 10}$$

$$\sum_{r=1}^{10}\left(r^2+1\right) \llcorner r = 10 \llcorner 11$$

3. Let $a, b \in R, (a \neq 0)$. If the function f defined as

$$f(x)=\begin{cases} \dfrac{2x^2}{a}, & 0 \le x < 1 \\ a, & 1 \le x < \sqrt{2} \\ \dfrac{2b^2-4b}{x^3}, & \sqrt{2} \le x < \infty \end{cases}$$

Is continuous in the interval $[0, \infty)$, then an ordered pair (a, b) is :

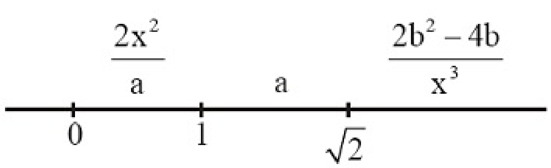
(A) $\left(-\sqrt{2}, 1-\sqrt{3}\right)$

(B) $\left(\sqrt{2}, -1+\sqrt{3}\right)$

(C) $\left(\sqrt{2}, 1-\sqrt{3}\right)$

(D) $\left(-\sqrt{2}, 1+\sqrt{3}\right)$

Solution: (C)



Continuity at $x=1$

$$\frac{2}{a}=a \Rightarrow a=\pm\sqrt{2}$$

Continuity at $x=\sqrt{2}$

$$a=\frac{2b^2}{2\sqrt{2}}$$

Put $a=\sqrt{2}$

$$2=b^2-2b \Rightarrow b^2-2b-2=0$$

$$b=\frac{1\pm\sqrt{4+4.2}}{2}=1\pm\sqrt{3}$$

$$\left(\sqrt{2},1-\sqrt{3}\right)$$

4. The angle of elevation of the top of a vertical tower from a point A, due east of it is 45^o. The angle of elevation of the top of the same tower from a point B, due south of A is 30^o. If the distance between A and B is $54\sqrt{2}m$, then the height of the tower (in metres), is:

(A) 108

(B) $36\sqrt{3}$

(C) $54\sqrt{3}$

(D) 54

Solution: (D)

Let $AP=x$

$$BP=y$$

$$\tan 45^o=\frac{H}{x}\Rightarrow H=x$$

$$\tan 30^o=\frac{H}{y}\Rightarrow y=\sqrt{3}H$$

$$x^2+\left(54\sqrt{2}\right)^2=y^2$$

$$H^2+\left(54\sqrt{2}\right)^2=3H^2$$

$\left(54\sqrt{2}\right)^2=2H^2$

$54\sqrt{2}=\sqrt{2}H$

$54=H$

5. P and Q are two distinct points on the parabola, $y^2=4x,$ with parameters t and t_1 respectively. If the normal at P passes through Q, then the minimum value of t_1^2 is:

(A) 8
(B) 4
(C) 6
(D) 2

Solution: (A)

$t_1=-t-\frac{2}{t}$

$t_1^2=t^2+\frac{4}{t^2}+4$

$t^2+\frac{4}{t^2}\geq 2\sqrt{t^2.\frac{4}{t^2}}=4$

Minimum value at $t_1^2=8$

6. ABC is a triangle in a plane with vertices $A(2,3,5), B(-1,3,2)$ and $C(\lambda,5,\mu)$. If the median through A is equally inclined to the coordinate axes, then the value of $(\lambda^3+\mu^3+5)$ is:

(A) 1130
(B) 1348
(C) 1077

(D) 676

Solution: (B)

DR's of AD are $\frac{\lambda-1}{2}-2, 4-3, \frac{\mu+2}{2}-5$

i.e., $\frac{\lambda-5}{2}, 1, \frac{\mu-8}{2}$

$\because$ This median is making equal angles with coordinate axes, therefore,

$\frac{\lambda-5}{2}=1=\frac{\mu-8}{2}$

$\Rightarrow \quad \lambda=7 \wedge \mu=10$

$\therefore \quad \lambda^3+\mu^3+5=1348$

7. The solution of the differential equation $\frac{dy}{dx}+\frac{y}{2}\sec x=\frac{\tan x}{2y}$, where $0 \le x < \frac{\pi}{2}$, and $y(0)=1$, is given by :

(A) $y^2=1+\frac{x}{\sec x+\tan x}$

(B) $y=1+\frac{x}{\sec x+\tan x}$

(C) $y=1-\frac{x}{\sec x+\tan x}$

(D) $y^2=1-\frac{x}{\sec x+\tan x}$

Solution: (D)

$\frac{dy}{dx}+\frac{y}{2}\sec x=\frac{\tan x}{2y}$

$$2y\frac{dy}{dx}+y^2 sec\,x=\tan x$$

Put $y^2=t \Rightarrow 2y\frac{dy}{dx}=\frac{dt}{dx}$

$$\frac{dt}{dx}+t\,sec\,x=\tan x$$

If ¿ $e^{\int sec\,x dx}=e^{¿(sec\,x+\tan x)}=sec\,x+\tan x$

$$\frac{dt}{dx}(sec\,x+\tan x)+t\,sec\,x\,(sec\,x+\tan x)=\tan x\,(sec\,x+\tan x)$$

$$\int d\left(t\,(sec\,x+\tan x)\right)=\tan x\,(sec\,x+\tan x)\,dx$$

$$t\,(sec\,x+\tan x)=sec\,x+\tan x-x$$

$$t=1-\frac{x}{sec\,x+\tan x}\; y^2=1-\frac{x}{sec\,x+\tan x}$$

8. The value of the integral $\int_4^{10}\frac{[x^2]dx}{[x^2-28x+196]+[x^2]}$, where $[x]$ denotes the greatest integer less than or equal to x, is:

(A) $\frac{1}{3}$

(B) 6

(C) 7

(D) 3

Solution: (D)

$$I=\int_4^{10} \frac{[x^2]}{[x^2-28x+196]+[x^2]}dx \qquad \text{........(i)}$$

Use $\int_a^b f(a+b-x)dx=\int_a^b f(x)dx$

$$I=\int_4^{10} \frac{[(x-14)^2]}{[x^2]+[(x-14)^2]}dx \qquad \text{......(ii)}$$

(i) + (ii)

$$2I=\int_4^{10} \frac{[(x-14)^2]+[x^2]}{[x^2]+[(x-14)^2]}dx$$

$$2I=\int_4^{10} dx$$

$$2I=6$$

$$I=3$$

9. Let A be a 3×3 matrix such that $A^2-5A+7I=O$.

Statement – I : $A^{-1}=\frac{1}{7}(5I-A)$.

Statement – II : The polynomial reduced to $5(A-4I)$. Then :

(A) Both the statements are true
(B) Both the statements are false
(C) Statement – I is true, but Statement – II is false
(D) Statement – I is false, but Statement – II is true

Solution: (A)

$A^2=5A=-7I$

$AAA^{-1}-5AA^{-1}=-7IA^{-1}$

$AI=5I=-7A^{-1}$

$AI-SI=-7A^{-1}$

$A^{-1}=\frac{1}{7}(5I-A)$

A^3-2A^2-3A+I

¿ $A(5A-7I)-2A^2-3A+I$

¿ $5A^2-7A-2A^2-3A+I$

¿ $3A^2-10A+I$

¿ $3(5A-7I)-10A+I$

¿ $5A-20I$

¿ $5(A-4I)$

10. Let $a_1, a_2, a_3, \ldots a_n, \ldots$ be in A.P. If $a_3+a_7+a_{11}+a_{15}=72,$ then the sum of its first 17 terms is equal to :

(A) 306

(B) 204

(C) 153

(D) 612

Solution: (A)

$a_3+a_7+a_{11}+a_{15}=72$

$(a_3+a_{15})+(a_7+a_{11})=72$

$a_3+a_{15}=a_7+a_{11}=a_1+a_{17}$

$a_1+a_{17}=36$

$S_{17}=\frac{17}{2}[a_1+a_{17}]=17\times18=306$

11.If $A=\begin{bmatrix}-4 & -1\\ 3 & 1\end{bmatrix}$, then the determinant of the matrix $(A^{2016}-2A^{2015}-A^{2014})$ is :

(A) -175

(B) 2014

(C) 2016

(D) -25

Solution: (D)

$\begin{bmatrix}-4 & -1\\ 3 & 1\end{bmatrix} \Rightarrow A^2=\begin{bmatrix}-4 & -1\\ 3 & 1\end{bmatrix}\begin{bmatrix}-4 & -1\\ 3 & 1\end{bmatrix}=\begin{bmatrix}13 & 3\\ -9 & -2\end{bmatrix}$ and $|A|=1$

Now, $a^{2016}-2A^{2015}-A^{2014}=A^{2014}(A^2-2A-I)$

$\Rightarrow$ $|A^{2016}-2A^{2015}-A^{2014}|=|A^{2014}||A^2-2A-I|$

$¿|A|^{2014}\begin{vmatrix}20 & 5\\ -15 & -5\end{vmatrix}$

$¿-25$

12.Let ABC be a triangle whose circumcenter is at P. If the position vectors of A, B, C and P are $\vec{a}, \vec{b}, \vec{c}$ and $\frac{\vec{a}+\vec{b}+\vec{c}}{4}$ respectively, then the position vector of the orthocentre of this triangle, is :

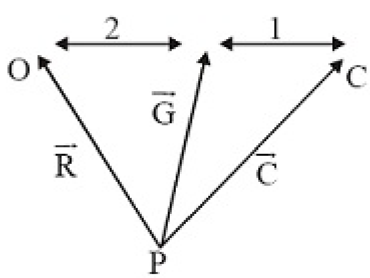
(A) $-\left(\frac{\vec{a}+\vec{b}+\vec{c}}{2}\right)$

(B) $\vec{a}+\vec{b}+\vec{c}$

(C) $\frac{(\vec{a}+\vec{b}+\vec{c})}{2}$

(D) $\vec{0}$

Solution: (C)



Position vector of centroid $\vec{G}=\frac{\vec{a}+\vec{b}+\vec{c}}{3}$

Position vector of circum center $\vec{C}=\frac{\vec{a}+\vec{b}+\vec{c}}{4}$

$$\vec{G}=\frac{2\vec{C}+\vec{r}}{3}$$

$$3\vec{G}=2\vec{C}+\vec{r}$$

$$\vec{r}=3\vec{G}-2\vec{C}=(\vec{a}+\vec{b}+\vec{c})-2\left(\frac{\vec{a}+\vec{b}+\vec{c}}{4}\right)$$

¿ $\frac{\vec{a}+\vec{b}+\vec{c}}{2}$

13. Let $z=1+ai$ be a complex number, $a>0$, such that z^3 is a real number.

Then the sum $1+z+z^2+....+z^{11}$ is equal to :

(A) $1365\sqrt{3}i$

(B) $-1365\sqrt{3}i$

(C) $-1250\sqrt{3}i$

(D) $1250\sqrt{3}i$

Solution: (B)

$z=1+ai$

$z^2=1-a^2+2ai$

$z^2.z=\{(1-a^2)+2ai\}\{1+ai\}$

¿ $(1-a^2)+2ai+(1-a^2)ai-2a^2$

$\because$ z^3 is real $\Rightarrow 2a+(1-a^2)a=0$

$a(3-a^2)=0 \Rightarrow a=\sqrt{3}$ $(a>0)$

$$1+z+z^2.....z^{11}=\frac{z^{12}-1}{z-1}=\frac{(1+\sqrt{3}i)^{12}-1}{1+\sqrt{3}i-1}=\frac{(1+\sqrt{3}i)^{12}-1}{\sqrt{3}i}$$

$$(1+\sqrt{3}i)^{12}=2^{12}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^{12}=2^{12}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^{12}=2^{12}(\cos 4\pi+i\sin 4\pi)=2^{12}$$

$$\Rightarrow \quad \frac{2^{12}-1}{\sqrt{3}i}=\frac{4095}{\sqrt{3}i}=\frac{-4095}{3}\sqrt{3}i=-1365\sqrt{3}i$$

14. Equation of the tangent to the circle, at the point (1, -1), whose centre is the point of intersection of the straight lines $x-y=1$ and $2x+y=3$ is:

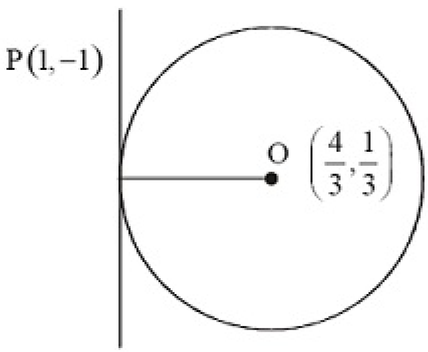
(A) $x+4y+3=0$

(B) $3x-y-4=0$

(C) $x-3y-4=0$

(D) $4x+y-3=0$

Solution: (A)



Paint of intersection of lines

$x-y=1$ and $2x+y=3$

O is o $\left(\frac{4}{3},\frac{1}{3}\right)$

Slope of OP ¿ $\frac{\frac{1}{3}+1}{\frac{4}{3}-1}=\frac{\frac{4}{3}}{\frac{1}{3}}=4$

Slope of tangent ¿ $-\frac{1}{4}$

Slope of tangent $y+1=\frac{-1}{4}(x-1)$

$4y+4=-x+1$

$x+4y+3=0$

15. A straight line through origin O meets the lines $3y=10-4x$ and $8x+6y+5=0$ at points A and B respectively. Then O divides the segment AB in the ratio:

(A) 2 : 3
(B) 1 : 2
(C) 4 : 1
(D) 3 : 4

Solution: (C)

Length of $\perp$ an $4x+3y=10$

$p_1=\frac{10}{5}=2$

Length of $\perp$ an $8x+6y+5=0$

$p_2=\frac{5}{10}=\frac{1}{2}$

$\because$ Lines are parallel to each other $\Rightarrow$ ratio will be $4:1$ or $1:4$

16. The integral $\int \frac{dx}{(1+\sqrt{x})\sqrt{x-x^2}}$ is equal to :

(where C is a constant of integration.)

(A) $-2\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}}+C$

(B) $-\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+C$

(C) $-2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+C$

(D) $\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}}+C$

Solution: (C)

$$I=\int \frac{dx}{\left(1+\sqrt{x}\right).\sqrt{x}\sqrt{1-x}}$$

Put $1+\sqrt{x}=t \Rightarrow \frac{1}{2\sqrt{x}}dx=dt$

$$I=\int \frac{2\,dt}{t\sqrt{2t-t^2}}$$

Again put $t=\frac{1}{Z} \Rightarrow dt=\frac{-1}{2^2}dz$

$$\Rightarrow \qquad I=2\int \frac{\frac{-1}{z^2}dz}{\frac{1}{z}\sqrt{\frac{2}{z}-\frac{1}{z^2}}}=2\int \frac{-dz}{\sqrt{2z-1}}=-2\sqrt{2z-1}+c$$

¿ $-2\sqrt{\frac{2}{t}-1}+c$

¿ $-2\sqrt{\frac{2-t}{t}}+c$

¿ $-2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+c$

17.If $A>0, B>0$ and $A+B=\frac{\pi}{6}$, then the minimum value of $\tan A+\tan B$ is :

(A) $\sqrt{3}-\sqrt{2}$

(B) $4-2\sqrt{3}$

(C) $\frac{2}{\sqrt{3}}$

(D) $2-\sqrt{3}$

Solution: (B)

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

$\Rightarrow$ $\frac{1}{\sqrt{3}}=\frac{y}{1-\tan A\tan B}$ where $y=\tan A+\tan B$

$\Rightarrow$ $\tan A+\tan B=1-\sqrt{3}\,y$

Also $AM\geq GM$

$\Rightarrow$ $\frac{\tan A+\tan B}{2}\geq\sqrt{\tan A\tan B}$

$\Rightarrow$ $y\geq 2\sqrt{1-\sqrt{3}\,y}$

$\Rightarrow$ $y^2\geq 4-4\sqrt{3}\,y-4\geq 0$

$\Rightarrow$ $y\leq -2\sqrt{3}-4$ or $y\geq -2\sqrt{3}+4$

$\left(y\leq -2\sqrt{3}-4 \text{ } is\, not\ possible\ as \tan A;\tan B>0\right)$

18.A ray of light is incident along a line which meets another line, $7x-y+1=0$, at the point (0, 1) . The ray is then reflected from this point along the line, $y+2x=1$. Then the equation of the line of incidence of the ray of light is :

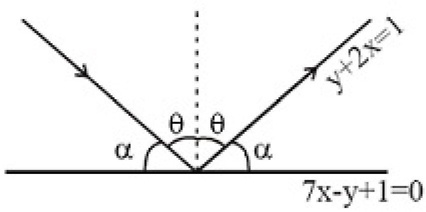
(A) $41x-25y+25=0$

(B) $41x+25y-25=0$

(C) $41x-38y+38=0$

(D) $41x+38y-38=0$

Solution: (C)



Let slope of incident ray be m.

$\therefore$ angle of incidence = angle of reflection

$$\therefore \quad \left|\frac{m-7}{1+7m}\right|=\left|\frac{-2-7}{1-14}\right|=\frac{9}{13}$$

$$\Rightarrow \quad \frac{m-7}{1+7m}=\frac{9}{13} \vee \frac{m-7}{1+7m}=\frac{9}{13}$$

$$\Rightarrow \quad 13m-91=9+63m \vee 13m-91=-9-63m$$

$$\Rightarrow \quad m=\frac{-1}{2} \qquad m=\frac{41}{38}$$

$$\Rightarrow \quad y-1=\frac{-1}{2}(x-0) \qquad \Rightarrow y-1=\frac{41}{38}(x-0)$$

i.e., $x+2y-2=0$ $\Rightarrow$ $38y-38-41x=0$

19. A hyperbola whose transverse axis is along the major axis of the conic, $\frac{x^2}{3}+\frac{y^2}{4}=4$ and has vertices at the foci of this conic. If the eccentricity of the hyperbola is $\frac{3}{2}$, then which of the following points does NOT lie on it ?

(A) $\left(\sqrt{5}, 2\sqrt{2}\right)$

(B) $(0, 2)$

(C) $\left(5, 2\sqrt{3}\right)$

(D) $\left(\sqrt{10}, 2\sqrt{3}\right)$

Solution: (C)

$$\frac{x^2}{12}+\frac{y^2}{16}=1$$

$$e=\sqrt{1-\frac{12}{16}}=\frac{1}{2}$$

Foci (0, 2) and $(0,-2)$

So, transverse axis of hyperbola $¿2b=4$

$\Rightarrow \quad b=2$

And $a^2=1^2\left(e^2-1\right)$

$\Rightarrow \quad a^2=4\left(\frac{9}{4}-1\right)$

$\Rightarrow \quad a^2=5$

$\therefore$ It's equation is $\frac{x^2}{5}-\frac{y^2}{4}=-1$

20. An experiment succeeds twice as often as it fails. The probability of at least 5 successes in the six trials of this experiment is :

(A) $\frac{496}{729}$

(B) $\frac{192}{729}$

(C) $\frac{240}{729}$

(D) $\frac{256}{729}$

Solution: (D)

Let p be the probability of successes and q be the probability of failure

$p=2q$ and $p+q=1$

Gives $p=\frac{2}{3}$ and $q=\frac{1}{3}$

For having at least 5 successes in 6 trial

$$b=\square\,{}^{6}C_{0}\left(\frac{2}{3}\right)^{6}\left(\frac{1}{3}\right)^{0}6_{9}\left(\frac{2}{3}\right)^{5}\left(\frac{1}{3}\right)^{9}$$

$$\frac{64+6.2^{5}}{3^{6}}=\frac{256}{729}$$

21. The contrapositive of the following statement, “If the side of a square doubles, then its area increases four times”, is:

(A) If the area of a square increases four times, then its side is not doubled

(B) If the area of a square increases four times, then its side is doubled

(C) If the area of a square does not increase four times, then its side is not doubled

(D) If the side of a square is not doubled, then its area does not increase four times

Solution: (C)

Contrapositive of $p \to q$ is given by $q \to \ p$

So, (iii) is the right option.

22.Let $P=\{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q=\{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then:

(A) $P \subset Q$ and $Q - P \neq \phi$

(B) $Q \not\subset P$

(C) $P = Q$

(D) $P \not\subset Q$

Solution: (C)

$\sin\theta - \cos\theta = \sqrt{2}\cos\theta$

$\Rightarrow \quad \sin\theta = \cos\theta + \sqrt{2}\cos\theta$

¿$(\sqrt{2}+1)\cos\theta$ ¿$\left(\dfrac{2-1}{\sqrt{2}-1}\right)\cos\theta$

$\Rightarrow \quad (\sqrt{2}-1)\sin\theta = \cos\theta \Rightarrow \sin\theta + \cos\theta = \sqrt{2}\sin\theta$

$\therefore \quad P = Q$

23.If $\dfrac{{}^{n+2}C_6}{{}^{n-2}P_2} = 11,$ then n satisfies the equation:

(A) $n^2 + n - 110 = 0$

(B) $n^2 + 2n - 80 = 0$

(C) $n^2 + 3n - 108 = 0$

(D) $n^2 + 5n - 84 = 0$

Solution: (C)

$$\frac{{}^{n+2}C_6}{{}^{n-2}P_2}=11$$

$\Rightarrow \quad \dfrac{\dfrac{(n+2)(n+1)n(n-1)(n-2)(n-3)}{6.5.4.3.2.1}}{(n-2)(n-3)}=11$

$\Rightarrow \quad (n+2)(n+1)n(n-1)=11.10.9.8$

$\Rightarrow \quad n=9$

24. If x is a solution of the equation, $\sqrt{2x+1}-\sqrt{2x-1}=1, \left(x\geq\frac{1}{2}\right)$, then $\sqrt{4x^2-1}$ is equal to :

(A) $\frac{3}{4}$

(B) $\frac{1}{2}$

(C) $2\sqrt{2}$

(D) 2

Solution: (A)

$\sqrt{2x+1}-\sqrt{2x-1}=1$ ....(i)

$\Rightarrow \quad 2x+1+2x-1-2\sqrt{4x^2-1}=1$

$\Rightarrow \quad 4x-1=2\sqrt{4x^2-1}$

$\Rightarrow \quad 16x^2-8x+1=16x^2-4$

$\Rightarrow \quad 8x=5$

$\Rightarrow \quad x=\frac{5}{8}$ which satisfies equation (i)

So, $\sqrt{4x^2-1}=\frac{3}{4}$

25.Let C be a curve given by $y(x)=1+\sqrt{4x-3}, x>\frac{3}{4}$. If P is a point on C, such that the tangent at P has slope $\frac{2}{3}$, then a point through which the normal at P passes, is :

(A) $(1,7)$

(B) $(3,-4)$

(C) $(4,-3)$

(D) $(2,3)$

Solution: (A)

$\frac{dy}{dx}=\frac{1}{2\sqrt{4x-3}}\times 4=\frac{2}{3}$

$\Rightarrow \quad 4x-3=9$

$\Rightarrow \quad x=3$

So, $y=4$

Equation of normal at P(3, 4) is $y-4=\frac{-3}{2}(x-3)$

i.e., $2y-8=-3x+9$

$\Rightarrow \quad 3x+2y-17=0$

26. The number of distinct real values of λ for which the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{\lambda^2}$ and $\frac{x-3}{1}=\frac{y-2}{\lambda^2}=\frac{z-1}{2}$ are coplanar is :

(A) 2

(B) 4

(C) 3

(D) 1

Solution: (C)

$\because$ lines are coplanar

$$\therefore \begin{vmatrix} 3-1 & 2-2 & 1-(-3) \\ 1 & 2 & \lambda^2 \\ 1 & \lambda^2 & 2 \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} 2 & 0 & 4 \\ 1 & 2 & \lambda^2 \\ 1 & \lambda^2 & 2 \end{vmatrix}=0$$

$$\Rightarrow 2(4-\lambda^4)+4(\lambda^2-2)=0$$

$$\Rightarrow 4-\lambda^4+2\lambda^2-4=0$$

$$\Rightarrow \lambda^2(\lambda^2-2)=0$$

$$\Rightarrow \lambda=0,\sqrt{2},-\sqrt{2}$$

27. The mean of 5 observations is 5 and their variance is 124. If three of the observations are 1, 2 and 6; then the mean deviation from the mean of the data is :

(A) 2.5

(B) 2.6

(C) 2.8

(D) 2.4

Solution: (C)

Or Bonus

28. Let $f(x)=\sin^4 x+\cos^4 x$. Then f is an increasing function in the interval:

(A) $\left[\frac{5\pi}{8},\frac{3\pi}{4}\right]$

(B) $\left[\frac{\pi}{2},\frac{5\pi}{8}\right]$

(C) $\left[\frac{\pi}{4},\frac{\pi}{2}\right]$

(D) $\left[0,\frac{\pi}{4}\right]$

Solution: (C)

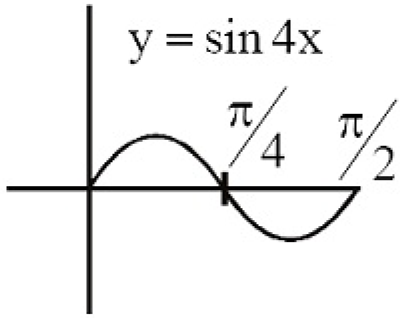
$f(x)=\sin^4 x+\cos^4 x$

$f'(x)=4\sin^3 x\cos x+4\cos^3 x(-\sin x)$

¿$4\sin x\cos x(\sin^2 x-\cos^2 x)$

¿$-2\sin 2x\cos 2x$

¿$-\sin 4x$



$f(x)$ is increasing when $f'(x)>0$

$\Rightarrow \quad -\sin 4x>0$

$\Rightarrow \quad \sin 4x<0$

$\Rightarrow \quad x\in\left(\frac{\pi}{4},\frac{\pi}{2}\right)$

29. $\lim_{x\to 0}\frac{(1-\cos 2x)^2}{2x\tan x-x\tan 2x}$ is:

(A) 2

(B) $\frac{-1}{2}$

(C) -2

(D) $\frac{1}{2}$

Solution: (C)

$$\lim_{x\to 0}\frac{(1-\cos 2x)^2}{2x\tan x-x\tan 2x}$$

$$¿\lim_{x\to 0}\frac{(2\sin^2 x)^2}{2x\left(x+\frac{x^3}{3}+\frac{2x^5}{15}+\ldots\right)-x\left(2x+\frac{2^3x^3}{3}+2\frac{2^5x^5}{15}+\ldots\right)}$$

$$¿\lim_{x\to 0}\frac{4\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots.\right)^4}{x^4\left(\frac{2}{3}-\frac{8}{3}\right)+x^6\left(\frac{4}{15}-\frac{64}{15}\right)}$$ (dividing numerator and denominator

by x^4)

$$¿\lim_{x\to 0}\frac{4\left(1-\frac{x^2}{3!}+\frac{x^4}{5!}-\ldots.\right)^4}{-2+x^2\left(\frac{-60}{15}\right)+\ldots}$$

$$¿-2$$

30.If the coefficients of x^{-2} and x^{-4} in the expansion of $\left(x^{\frac{1}{3}}+\frac{1}{2x^{\frac{1}{3}}}\right)^{18},(x>0)$,

are m and n respectively, then $\frac{m}{n}$ is equal to:

(A) 27

(B) 182

(C) $\frac{5}{4}$

(D) $\frac{4}{5}$

Solution: (B)

$$T_{r+1} = {}^{18}C_r \left(x^{\frac{1}{3}}\right)^{18-r} \left(\frac{1}{2x^{\frac{1}{3}}}\right)^r = {}^{18}C_r \, x^{6-\frac{2r}{3}} \frac{1}{2^r}$$

$$\left.\begin{array}{l} 6-\frac{2r}{3}=-2 \quad \Rightarrow \quad r=12 \\ ¿\,6-\frac{2r}{3}=-4 \quad \Rightarrow \quad r=15 \end{array}\right\} \Rightarrow \frac{\text{coefficient of } x^{-2}}{\text{coefficient of } x^{-4}} = \frac{{}^{18}C_{12}\frac{1}{2^{12}}}{{}^{18}C_5\frac{1}{2^{15}}} = 182$$