CHEMISTRY

1. A stream of electrons from a heat filament was passed between two charge plates kept at a potential difference V esu. If e and m are charge and mass of an electron, respectively, then the value of $\frac{h}{\lambda}$ (where λ is wavelength associated with electron wave) is given by:

(1) $2\ \mathrm{me}\ V$

(2) $\sqrt{\mathrm{meV}}$

(3) $\sqrt{2\ \mathrm{meV}}$

(4) $\mathrm{me}\ V$

Solution: (3)

Given stream of electron from heated filament was passed between two charge plates at potential difference V

e, m are charge and mass of electron

$V = \frac{E}{e}$

$eV = \frac{1}{2} \times m \times V^2$

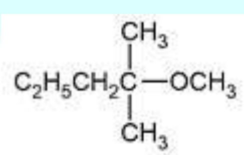
$\lambda = \frac{h}{mv} \quad V = \frac{h}{m\lambda}$

$eV = \frac{1}{2} \times m \times \left[\frac{h}{m\lambda}\right]^2$

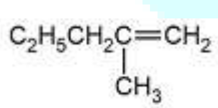
$\frac{h}{\lambda} = \sqrt{2\ \mathrm{meV}}$

2. 2 – chloro – 2 – methylpentane on reaction with sodium methoxide in methanol yields:

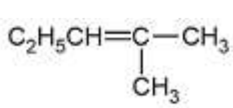
(i)



(ii)



(iii)



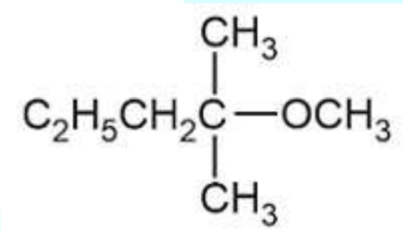
(1) i and iii

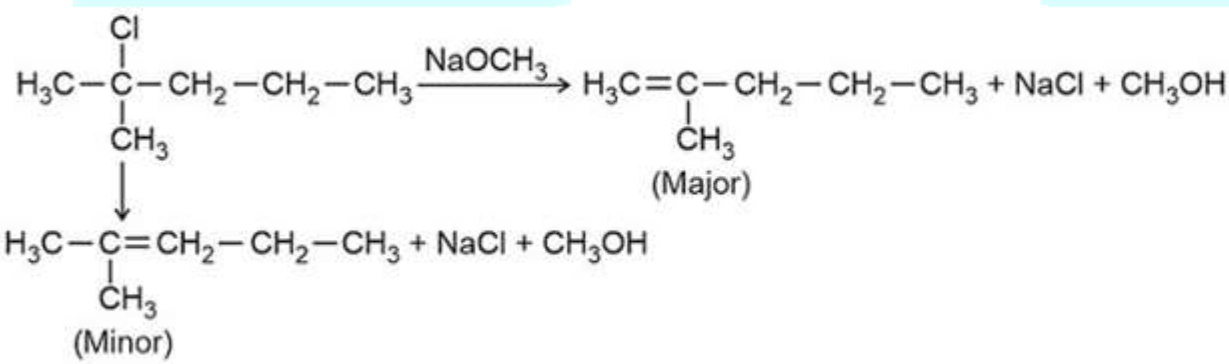
(2) iii only

(3) i and ii

(4) All of these

Solution: (4)





3. Which of the following compounds is metallic and ferromagnetic?

(1) CrO_2

(2) VO_2

(3) MnO_2

(4) TiO_2

Solution: (1)

d – block elements are metals.

MnO_2 exhibit strong attraction to magnetic fields and are able to retain their magnetic properties.

So, it exhibits metallic character and it’s ferromagnetic.

4. Which of the following statements about low density polythene is FALSE?

(1) It is a poor conductor of electricity

(2) Its synthesis required dioxygen or a peroxide initiator as a catalyst

(3) It is used in the manufacture of buckets, dust – bins etc.

(4) Its synthesis requires high pressure

Solution: (3)

Low density polythene: It is obtained by the polymerization of ethane high pressure of 1000-2000 atm. at a temp. of 350 K to 570 K in the pressure of traces of dioxygen or a peroxide initiator (cont).

Low density polythene is chemically inert and poor conductor of electricity. It is used for manufacture squeeze bottles. Toys and flexible pipes.

5. For a linear plot of log $\left(\frac{x}{m}\right)$ versus log p in a Freundlich adsorption isotherm, which of the following statements is correct? (k and n are constants)

(1) $\frac{1}{n}$ appears as the intercept

(2) Only $\frac{1}{n}$ appears as the slope

(3) $\log\left(\frac{1}{n}\right)$ appears as the intercept

(4) Both k and $\frac{1}{n}$ appear in the slope term

Solution: (2)

According to Freundlich isotherm

$$\frac{x}{m} = k.p^{\frac{1}{n}}$$

$$\log\frac{x}{m} = \log k + \frac{1}{n}\log P$$

So intercept is log k and slope is $\frac{1}{n}$

6. The heats of combustion of carbon and carbon monoxide are -393.5 and $-283.5\ \mathrm{kJ\ mol^{-1}}$, respectively. The heat of formation (in kJ) of carbon monoxide per mole is:

(1) 676.5

(2) -676.5

(3) -110.5

(4) 110.5

Solution: (3)

Given heat of combustion of carbon and carbon monoxide are -393.5 and $-283.5\ \mathrm{kJ\ mol^{-1}}$, respectively

(i) $C + O_2 \rightarrow CO_2 \quad \Delta H_1 = -393.5\ \mathrm{kJ\ mol^{-1}}$

(ii) $CO + \frac{1}{2}O_2 \rightarrow CO_2 \quad \Delta H_2 = -283.5\ \mathrm{kJ\ mol^{-1}}$

To find heat of formation of CO per mole.

i.e. $C + \frac{1}{2}O_2 \rightarrow CO$

(ii) × (2)

$2CO + O_2 \rightarrow 2CO_2 \quad \Delta H_3 = -283.5 \times 2 \text{ kJ mol}^{-1}$

$2CO_2 \rightarrow 2CO + O_2 \quad \Delta H_4 = 567 \text{ kJ mol}^{-1}$

(i) × (2)

$2C + 2O_2 \rightarrow 2CO_2 \quad \Delta H_5 = -393.5 \times 2 \text{ kJ mol}^{-1}$

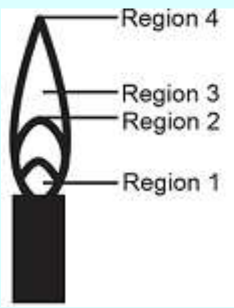
$= -787 \text{ kJ mol}^{-1}$

$2C + O_2 \rightarrow 2CO \quad \Delta H_6 = -220 \text{ kJ mol}^{-1}$

For one mole of CO,

$\Delta H = \frac{-220}{2} = -110 \text{ kJ mol}^{-1}$

7. The hottest region of Bunsen flame shown in the figure below is:



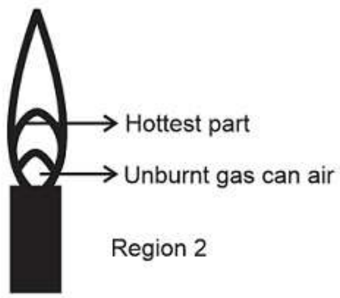
(1) Region 2

(2) Region 3

(3) Region 4

(4) Region 1

Solution: (1)



8. Which of the following is an anionic detergent?

(1) Sodium lauryl sulphate

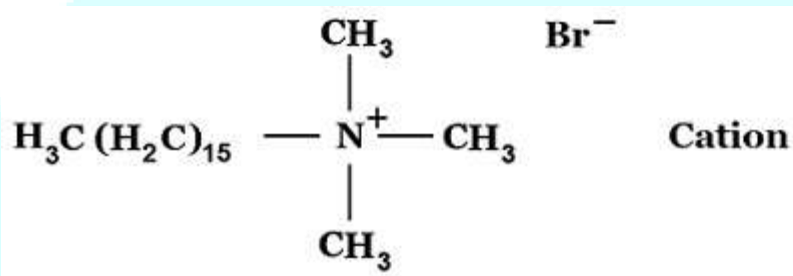
(2) Cetyltrimethyl ammonium bromide

(3) Glyceryl oleate

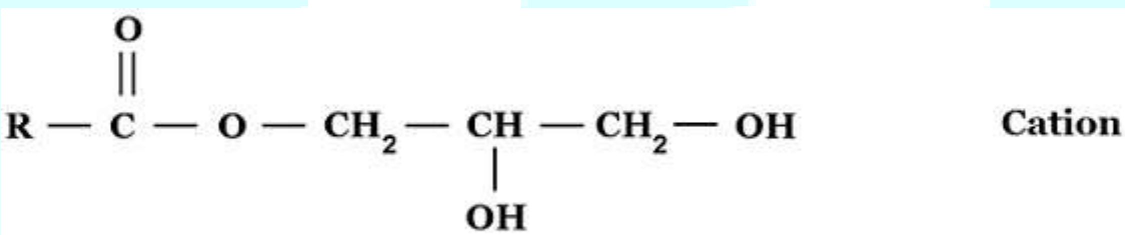
(4) Sodium stearate

Solution: (1)

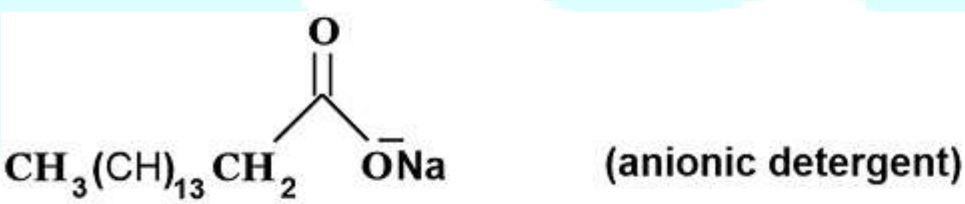
(i) Cetyltrimethyl ammonium bromide



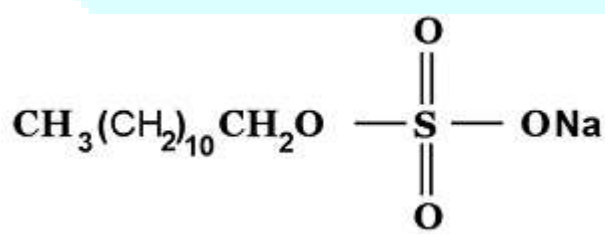
(ii) Glyceryl oleate



(iii) Sodium stearate



(iv) Anionic surfactant



9. 18 g glucose $(C_6H_{12}O_6)$ is added to 178.2 g water. The vapor pressure of water (in torr) for this aqueous solution is:

(1) 76.0

(2) 752.4

(3) 759.0

(4) 7.6

Solution: (2)

$\frac{\Delta P}{P^o}$ = mol. Fraction of glucose

$$\frac{760-P_{Soln}}{760} = \frac{\frac{W_1}{Mwt_1}}{\frac{W_1}{M.wt_1}+\frac{W_2}{M.wt_2}} = \frac{\frac{18}{180}}{\frac{18}{180}+\frac{178.2}{18}} = \frac{0.1}{0.1+9.9} = \frac{1}{100}$$

$$760 - P_{Soln} = \frac{760}{100}$$

$$P_{Sol} = 752.4$$

10. The distillation technique most suited for separating glycerol from spent – lye in the soap industry is:

(1) Fractional distillation

(2) Steam distillation

(3) Distillation under reduced pressure

(4) Simple distillation

Solution: (3)

Glycerol (B.P. 290^oC) is separated from spent – lye in the soap industry by distillation under reduced pressure, as for simple distillation very high temperature is required which might decompose the component.

11. The species in which the N atom is a state of sp hybridization is:

(1) NO_2^-

(2) NO_3^-

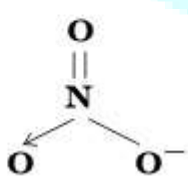
(3) NO_2

(4) NO_2^+

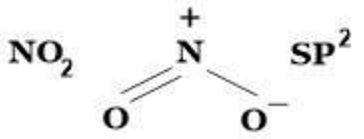
Solution: (4)

N SP

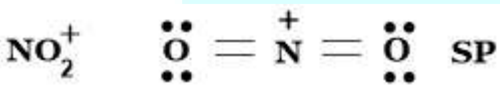
(i)

NO_3^-  SP^2

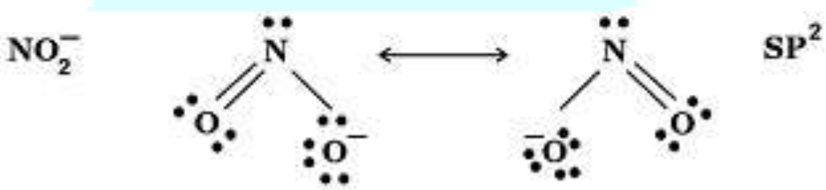
(ii)



(iii)



(iv)



12. Decomposition of H_2O_2 follows a first order reaction. In fifty minutes the concentration of H_2O_2 decreases from 0.5 to 0.125 M in one such decomposition. When the concentration of H_2O_2 reaches 0.05 M, the rate of formation of O_2 will be:

(1) 6.93×10^{-4} mol min^{-1}

(2) 2.66 L min^{-1} at STP

(3) 1.34×10^{-2} mol min^{-1}

(4) 6.93×10^{-2} mol min^{-1}

Solution: (1)

$2H_2O_2 \rightarrow 2H_2O + O_2$

$[O_2] = \frac{[H_2O_2]}{2} = \frac{[H_2O]}{2}$

For, $t_{\frac{1}{2}}$, H_2O_2 decreases to 0.125M from 0.5M

So, $2 \times t_{\frac{1}{2}} = 50$

$t_{\frac{1}{2}} = 25$

$t_{\frac{1}{2}} = \frac{0.69314}{K}$

$K = \frac{0.69314}{25}$

$[O_2] = \frac{1}{2} \times \frac{0.69314}{25}$

$= 6.93 \times 10^{-4}$ mol min^{-1}

13. The pair having the same magnetic moment is:

[At. No.: Cr = 24, Mn = 25, Fe = 26, Co = 27]

(1) $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$

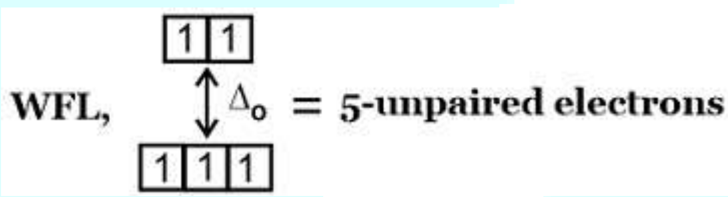
(2) $[Mn(H_2O)_6]^{2+}$ and $[Cr(H_2O)_6]^{2+}$

(3) $[CoCl_4]^{2-}$ and $[Fe(H_2O)_6]^{2+}$

(4) $[Cr(H_2O)_6]^{2+}$ and $[CoCl_4]^{2-}$

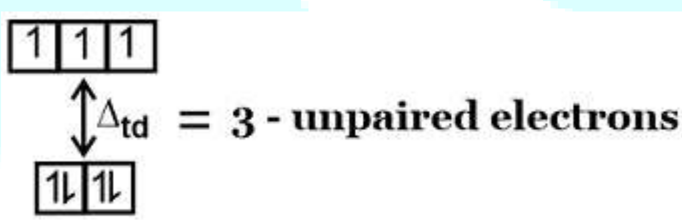
Solution: (1)

In option A: $[Mn(H_2O)_6]^{2+}(3d^5)$ with

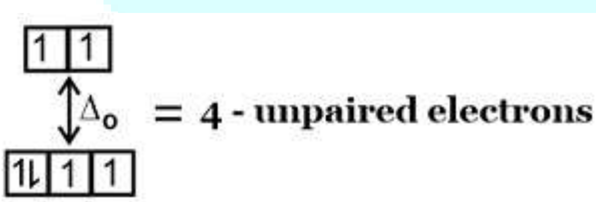


& $[Cr(H_2O)_6]^{2+}, Cr^{2+}(3d^4)$ with W.F.L.,

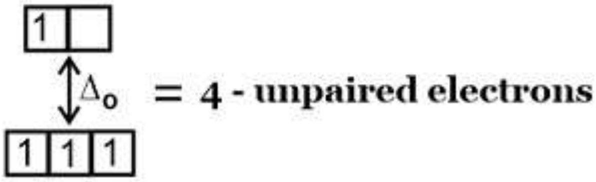
In option B: $[CoCl_4]^{2-}, Co^{2+}(3d^7)$ with W.F.L.,



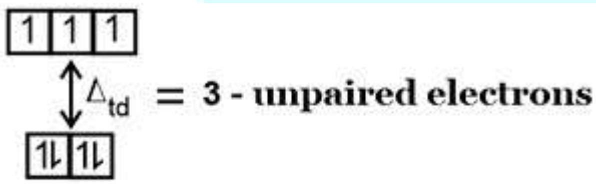
& $[Fe(H_2O)_6]^{2+}, Fe^{2+}(3d^6)$ with W.F.L.,



In option C: $[Cr(H_2O)_6]^{2+}, Cr^2 + (3d^4)$ with W.F.L.,

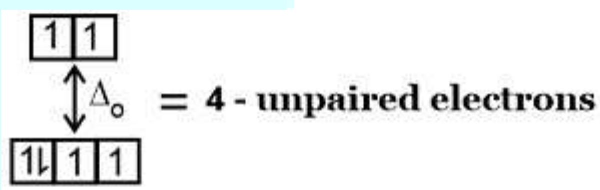


& $[CoCl_4]^{2-}$, Co^{2+} $(3d^7)$ with W.F.L.,



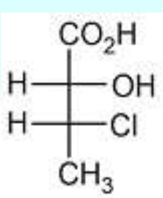
In option D: $[Cr(H_2O)_6]^{2+}, Cr^{2+}(3d^4)$ with W.F.L.,

& $[Fe(H_2O)_6]^{2+}, Fe^{2+}(3d^6)$ with W.F.L.,



Here both complexes have same unpaired electrons i.e. = 4

14. The absolute configuration of



is:

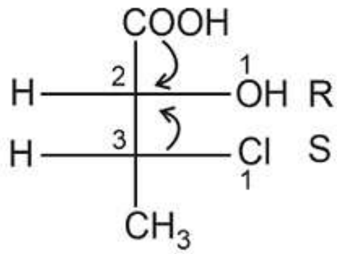
(1) $(2S, 3R)$

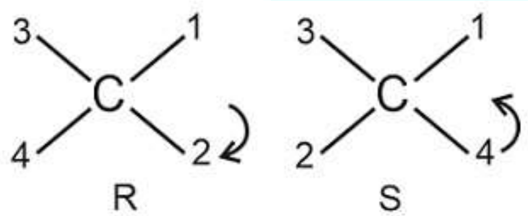
(2) $(2S, 3S)$

(3) $(2R, 3R)$

(4) $(2R, 3S)$

Solution: (1)





∴ Configuration is 2S, 3R.

15. The equilibrium constant at 298 K for a reaction $\mathrm{A + B \rightleftharpoons C + D}$ is 100, If the initial concentration of all the four species were 1 M each, then equilibrium concentration of D (in mol $\mathrm{L^{-1}}$) will be:

(1) 0.818

(2) 1.818

(3) 1.182

(4) 0.182

Solution: (2)

K at 298 K for the reaction

$\mathrm{A + B \rightleftharpoons C + D}$ is 100

Given initial concentration of all four species is 1M

At 1 = 0,

∴ $\underset{1}{\mathrm{A}} + \underset{1}{\mathrm{B}} \rightleftharpoons \underset{1}{\mathrm{C}} + \underset{1}{\mathrm{D}}$

At equilibrium,

$$\underset{1-x\quad 1-x}{\mathrm{A + B}} \rightleftharpoons \underset{1+x\quad 1+x}{\mathrm{C + D}}$$

$$K = \frac{(1+x)\,(1+x)}{(1-x)\,(1-x)} = 100$$

$$\frac{(1-x)}{1-x} = 10$$

$$x = \frac{9}{11} = 0.818$$

$$[D] = 1 + x = 1 + 0.818 = 1.818$$

16. Which one of the following ores is best concentrated by froth floatation method?

(1) Siderite

(2) Galena

(3) Malachite

(4) Magnetite

Solution: (2)

Froth floatation method is used for concentration of sulphide ores.

↓ Galena → Pbs

17. At 300 K and 1 atm, 15mL of a gaseous hydrocarbon requires 375 mL air containing 20% O_2 by volume for complete combustion. After combustion the gases occupy 330 mL. Assuming that the water formed is in liquid form and the volumes were measured at the same temperature and pressure, the formula of the hydrocarbon is:

(1) C_3H_8

(2) C_4H_8

(1) C_4H_{10}

(2) C_3H_6

Solution: (Bonus or 1)

Volume of N_2 in air $= 375 \times 0.8 = 300\ ml$

Volume of O_2 in air $= 375 \times 0.2 = 75\ ml$

$$\begin{array}{ccccccc} C_xH_y & + & \left(x+\frac{y}{4}\right)O_2 & \rightarrow & xCO_2(g) & + & \frac{y}{2}H_2O(\ell) \\ 15ml & & 15\left(x+\frac{y}{4}\right) & & 15x & & - \\ 0 & & 0 & & & & \end{array}$$

After combustion total volume

$330 = V_{N_2} + V_{CO_2}$

$330 = 300 + 15x$

$x = 2$

Volume of O_2 used

$15\left(x+\frac{y}{4}\right) = 5$

$x + \frac{y}{4} = 5$

$y = 12$

So hydrocarbon is $= C_2H_{12}$

None of the option matches it therefore it is a BONUS.

Alternatively

$$C_xH_y \quad + \quad \left(x+\frac{y}{4}\right)O_2 \rightarrow \quad xCO_2(g) \quad + \quad \frac{y}{2}H_2O(\ell)$$

$$15ml \qquad 15\left(x+\frac{y}{4}\right) \qquad 15x \qquad -$$

$$0 \qquad 0$$

Volume of O_2 used

$15\left(x+\frac{y}{4}\right) = 75$

$x+\frac{y}{4} = 5$

If further information (i.e., 330 ml) is neglected, option (C_3H_8) only satisfy the above equation.

18. The pair in which phosphorous atoms have a formal oxidation state of + 3 is:

(1) Pyrophosphorous and hypophosphoric acids

(2) Orthophosphorous and hypophospheric acids

(3) Pyrophosphorous and pyrophosphoric acids

(4) Orthophosphorous and pyrophosphorous acids

Solution: (4)

Acid	Formula	Formal oxidation state of phosphorous
Pyrophosphorous acid	$H_4P_2O_5$	$+3$
Pyrophosphoric acid	$H_4P_2O_7$	$+5$
Orthophosphorous acid	H_3PO_3	$+3$
Hypophosphoric acid	$H_4P_2O_6$	$+4$

Both pyrophosphorous and orthophosphorous acid have $+3$ formal oxidation state

19. Which one of the following complexes shows optical isomerism?

(1) cis $[Co(en)_2\,Cl_2]Cl$

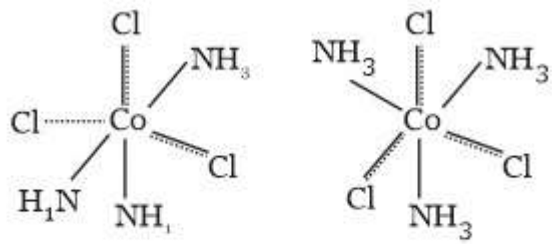
(2) trans $[Co(en)_2\,Cl_2]\,Cl$

(3) $[Co(NH_3)_4\,Cl_2]\,Cl$

(4) $[Co\,(NH_3)_3\,Cl_3]$

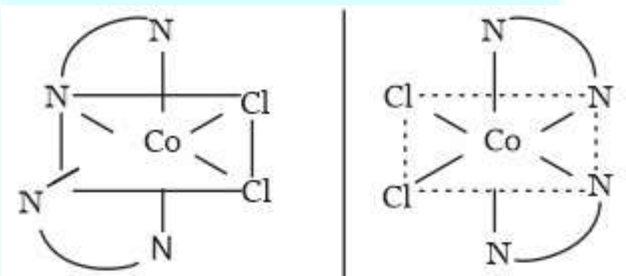
Solution: (1)

Geometrical isomers



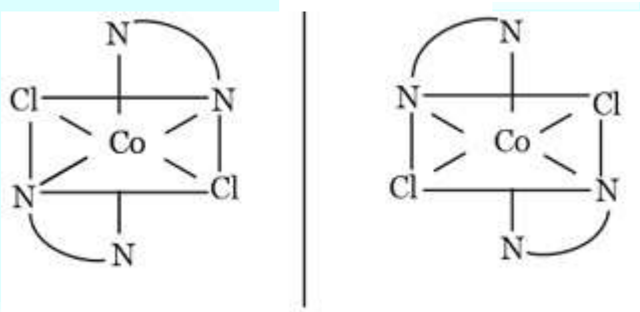
facial ineridinal

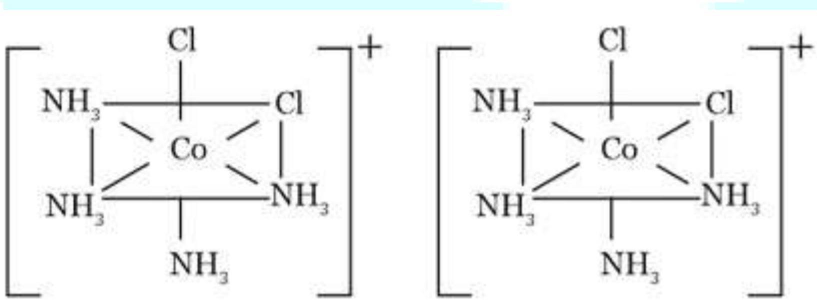
cis $[Co(en)_2Cl_2]Cl$



Non-Super imposable Mirror images

trans $[Co(en)_2Cl_2]Cl$





Cis trans

20. The reaction of zinc with dilute and concentrated nitric acid, respectively, produces:

(1) NO_2 and NO

(2) NO and N_2O

(3) NO_2 and N_2O

(4) N_2O and NO_2

Solution: (4)

Zn on reaction with HNO_3

$4Zn + 10HNO_3 \rightarrow 4Zn(NO_3)_2 + N_2O + 5H_2O$

$Zn + 4HNO_{3(con)} \rightarrow Zn(NO_3)_2 + 2NO_2 + 2H_2O$

21. Which one of the following statements about water is FALSE?

(1) Water can act both as an acid and as a base

(2) There is extensive intramolecular hydrogen boding in the condensed phase

(3) Ice formed by heavy water sinks in normal water

(4) Water is oxidized to oxygen during photosynthesis

Solution: (2)

(i) Water can show amphiprotic nature and hence water can act both as an acid a base.

(ii) There is extensive intermolecular hydrogen bonding in the condensed phase instead of intramolecular H – bonding.

(iii) Ice formed by heavy water sinks in normal water due to higher density of D_2O than normal water.

(iv)

$$6CO_2 + 6H_2O \xrightarrow[\text{chlorophyll}]{h\nu} C_6H_{12}O_6 + 6O_2$$

22. The concentration of fluoride, lead, nitrate and iron in a water sample from an underground lake was found to be 1000 ppb, 40 ppb, 100 ppm and 0.2 ppm, respectively. This water is unsuitable for drinking due to high concentration of:

(1) Lead

(2) Nitrate

(3) Iron

(4) Fluoride

Solution: (2)

Concentration of fluoride = 1000 PPb

= 1 PPm

Concentration of lead = 40 PPb

= 0.04 PPm

Concentration of nitrate = 100 PPm

Concentration of iron = 0.2 PPm

High concentration of nitrate

23. The main oxides formed on combustion of Li, Na and K in excess of air are, respectively:

(1) LiO_2, Na_2O_2 and K_2O

(2) Li_2O_2, Na_2O_2 and KO_2

(3) Li_2O, Na_2O_2 and KO_2

(4) Li_2O, Na_2O and KO_2

Solution: (3)

In 1A group

Li on $r \times n$ with excess air

$4Li + O_2 \rightarrow 2Li_2O$

Na on $r \times n$ with excess air

$2Na + O_2 \rightarrow Na_2O_2$

K on $r \times n$ with excess air

$K + O_2 \rightarrow KO_2$

24. Thiol group is present in:

(1) Cystine

(2) Cysteine

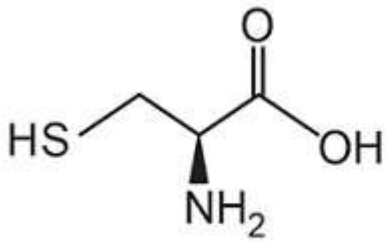
(3) Methionine

(4) Cytosine

Solution: (2)

Thiol group $(-SH)$

Cysteine



25. Galvanization is applying a coating of:

(1) Cr

(2) Cu

(3) Zn

(4) Pb

Solution: (3)

Galvanization is the process of applying zinc coating to steel (or) iron, to prevent rusting

26. Which of the following atoms has the highest first ionization energy?

(1) Na

(2) K

(3) Sc

(4) Rb

Solution: (3)

Na is the smallest element in the IA group elements and it has highest IE among K, Rb

Sc has lowest effective nuclear charge

Effective nuclear charge $\times \frac{1}{\text{I.E}}$

Sc has low effective nuclear charge than Na.

So it has I.E. among given elements.

27. In the Hofmann bromamide degradation reaction, the number of moles of NaOH and $\mathrm{Br_2}$ used per mole of amine produced are:

(1) Four moles of NaOH and two moles of $\mathrm{Br_2}$

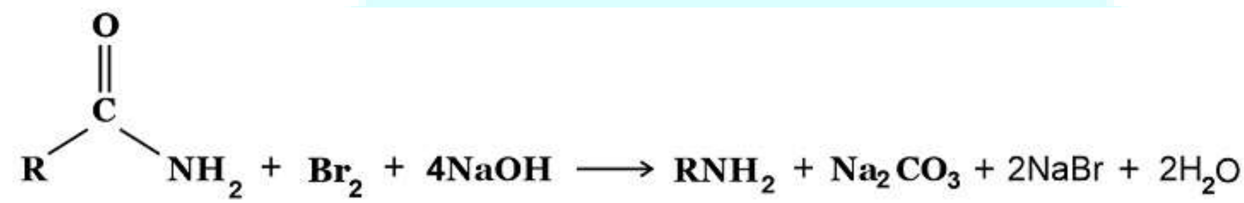
(2) Two moles of NaOH and two moles of $\mathrm{Br_2}$

(3) Four moles of NaOH and one mole of Br_2

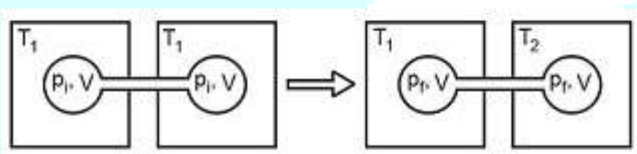
(4) One mole of NaOH and one mole of Br_2

Solution: (3)

To find number of moles of NaOH and Br_2 used per mole of amine produced.



28. Two closed bulbs of equal volume (V) containing an ideal gas initially at pressure p_i and temperature T_1 are connected through a narrow tube of negligible volume as shown in the figure below. The temperature of one of the bulbs is then raised to T_2. The final pressure P_f is:



(1) $2p_i\left(\frac{T_1}{T_1+T_2}\right)$

(2) $2p_i\left(\frac{T_2}{T_1+T_2}\right)$

(3) $2p_i\left(\frac{T_1T_2}{T_1+T_2}\right)$

(4) $p_i\left(\frac{T_1T_2}{T_1+T_2}\right)$

Solution: (2)

Given two closed bulbs of equal volume (v) containing ideal gas initially of pressure p_i and temperature T_1 which are connected by narrow tube of negligible volume.

To find final pressure P_f when one raised to T_2

No. of moles of gas doesn't change

$(n_T)_i = (n_T)_f$

$$\frac{P_iV}{RT_1}+\frac{P_iV}{RT_1}=\frac{P_fV}{RT_1}+\frac{P_fV}{RT_2}$$

$$2\frac{P_1}{T_1}=\frac{P_f}{T_1}+\frac{P_f}{T_2}$$

29. The reaction of propene with HOCl $(cl_2 + H_2O)$ proceeds through the intermediate:

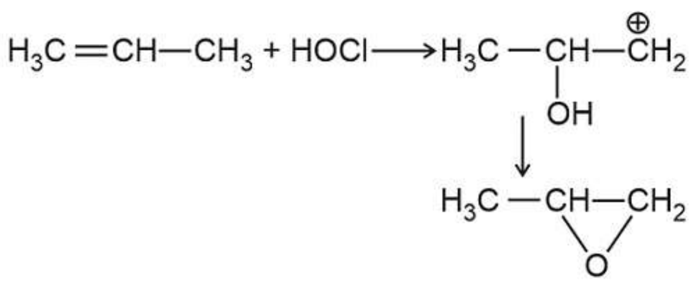
(1) $CH_3 - CH^+ - CH_2 - Cl$

(2) $CH_3 - CH(OH) - CH_2^+$

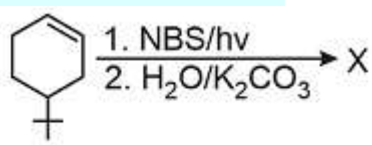
(3) $CH_3 - CHCl - CH_2^+$

(4) $CH_3 - CH^+ - CH_2 - OH$

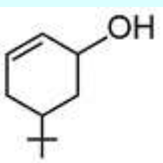
Solution: (1)



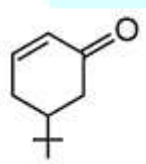
30. The product of the reaction give below is:



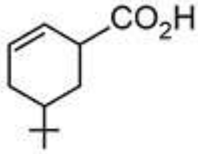
(1)



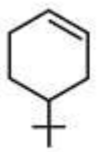
(2)



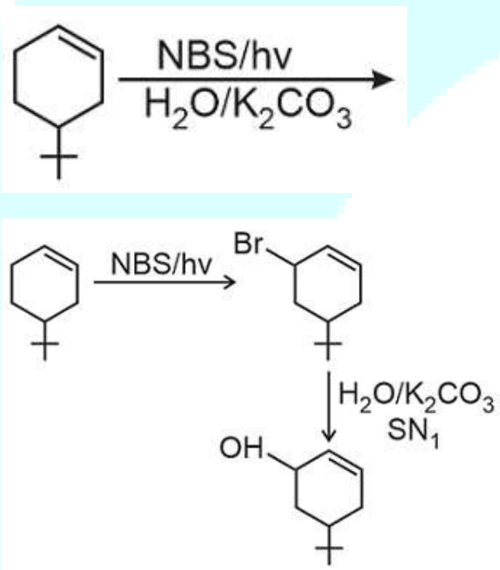
(3)



(4)



Solution: (1)



MATHEMATICS

31. Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. If its diagonals intersect at $(-1, -2)$, then which one of the following is a vertex of this rhombus ?

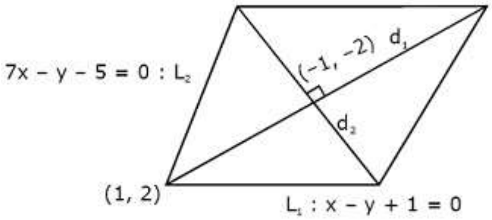
(1) $(-3, -8)$

(2) $\left(\frac{1}{3}, -\frac{8}{3}\right)$

(3) $\left(-\frac{10}{3}, -\frac{7}{3}\right)$

(4) $(-3, -9)$

Solution: (2)



$d_1 : y - 2 = \frac{-2-2}{-1-1}(x-1)$

$y - 2 = 2(x-1)$

$y - 2x = 0$

$d_2 \perp d_1 \Rightarrow \quad 2y + x = k\ p$ on $(-1,-2)$

$2y + x + 5 = 0$

Non P.O.I. of d_2 and L_1

$x - y + 1 = 0$

$x + 2y + 5 = 0$

$-3y - 4 = 0$

$y = -\frac{4}{3}$

And P.O.I. of d_2 and L_2

$x + 2y + 5 = 0$

$14x - 2y - 10 = 0$

And $y = -\frac{8}{3}$

$15x - 5 = 0$

$\Rightarrow \qquad x = \frac{1}{3}$

32. If the $2^{nd}, 5^{th}$ and 9^{th} term of a non – constant A.P. are in G.P., then the common ratio of this G.P. is:

(1) $\frac{4}{3}$

(2) 1

(3) $\frac{7}{4}$

(4) $\frac{8}{5}$

Solution: (1)

Let the A.P. be a, a + d, a + 2d,…..

Given $(a+d).(a+8d) = (a+4d)^2$

$a^2 + 9ad + 8d^2 = a^2 + 8ad + 16d^2$

$8d^2 - ad = 0$

$d[8d - a] = 0$

$\because \quad d \neq 0$

$d = \frac{a}{8}$

So,	2^{nd} term	5^{th} term	9^{th} term
Would be	$\left(a+\frac{a}{8}\right)$	$\left(a+\frac{a}{2}\right)$	$(a+a)$
	$\frac{9a}{8}$	$\frac{3a}{2}$	$2a$

Common Ratio : $\frac{2^{nd}\ term}{1^{st}\ term} = \frac{3a}{2.9a}.8 = \frac{4}{3}$

33. Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle, $x^2 + (y+6)^2 = 1$. Then the equation of the circle, passing through C and having its centre at P is:

(1) $x^2 + y^2 - x + 4y - 12 = 0$

(2) $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$

(3) $x^2 + y^2 - 4x + 9y + 18 = 0$

(4) $x^2 + y^2 - 4x + 8y + 12 = 0$

Solution: (4)

$y^2 = 8x$ is the equation of the given parabola. If P is a point at minimum distance from (0, -6) then it should be normal to the parabola at P.

Slope of tangent

$\frac{2dy}{dx}.y = 8$

$\frac{dy}{dx} = \frac{4}{y}.$

$\therefore \quad$ Slope of normal $= \left(-\frac{y}{4}\right)$

Any point on the parabola would be $\left(\frac{y^2}{8}, y\right)$, and hence slope of the normal would be

$$\frac{(y+6)}{\frac{y^2}{8}} = -\frac{y}{4}$$

$$(y+6) = -\frac{y^3}{32}$$

At $y = -4$, LHS = RHS

$$(2) = +\frac{64}{32} = 2$$

At $y = -4$ $x = 2$

So point $P(2, -4)$.

Radius of the desired circle would be $\sqrt{2^2 + 2^2} = 2\sqrt{2}$,

So equation of the circle would be $\sqrt{(x-2)^2 + (y+4)^2} = 2\sqrt{2}$

$$x^2 - 4x + 4 + y^2 + 8y + 16 = 8$$

$$x^2 + y^2 - 4x + 8y + 12 = 0$$

34. The system of linear equations

$$x + \lambda y - z = 0$$

$$\lambda x - y - z = 0$$

$$x + y - \lambda z = 0$$

Has a non –trivial solution for :

(1) Exactly one value of λ

(2) Exactly two values of λ

(3) Exactly three values of λ

(4) Infinitely many values of λ

Solution: (3)

For non – trivial solution.

$$\Delta = \begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$1(\lambda+1)-\lambda(-\lambda^2+1)-1(\lambda+1)=0$

$(\lambda+1)-\lambda(1-\lambda)\,(1+\lambda)-(\lambda+1)=0$

$\lambda(1-\lambda)\,(1+\lambda)=0$

$\lambda=0,\lambda=1,\lambda=\ -1$

Exactly there value of λ.

35. If $f(x)+2f\left(\frac{1}{x}\right)=3x, x\neq 0$, and $S=\{x\ \in R : f(x)=f(-x)\}$; then S :

(1) Contains exactly one element

(2) Contains exactly two elements

(3) Contains more than two elements

(4) Is an empty set

Solution: (2)

$f(x)+2\,.\,f\left(\frac{1}{x}\right)=3x$ …….(i)

Replace x by $\frac{1}{x}$

$f\left(\frac{1}{x}\right)+2f(x)=\frac{3}{x}$ ……..(ii)

$3f(x)=\frac{6}{x}-3x$

$f(x)=\frac{2}{x}-x$

$\because \quad f(x)=f(-x)$

Therefore $\frac{2}{x}-x=\ -\frac{2}{x}+x$

$\frac{4}{x}=2x$

$2=x^2$

$x=\ \pm\sqrt{2}$

Contains exactly two elements

36. Let $p=\ \lim_{x\to 0+}\left(1+\tan^2\sqrt{x}\,\right)^{\frac{1}{2x}}$, then log p is equal to :

(1) 1

(2) $\frac{1}{2}$

(3) $\frac{1}{4}$

(4) 2

Solution: (2)

Let p = $\ln\left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$.

The limit is of the form $(1+0)^{\propto} = e^{0\,.\propto}$

$e^{\frac{\tan\sqrt{x}}{\sqrt{x}}.\frac{\tan\sqrt{x}}{\sqrt{x}}.\frac{1}{2}}$

$\lim_{x\to 0+} p = e^{\frac{1}{2}}$

$\log p = \log e^{\frac{1}{2}} = \frac{1}{2}$

$= \frac{1}{2}$

37. A value of θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is :

(1) $\frac{\pi}{6}$

(2) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

(3) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(4) $\frac{\pi}{3}$

Solution: (3)

Let $z = \frac{2+3i\sin\theta}{1-2i\sin\theta}$

Rationalizing the complex number.

$$\frac{(2+3i\sin\theta)\,(1+2i\sin\theta)}{1+4\sin^2\theta}$$

$$= \frac{(2-6\sin^2\theta)+i\,(7\sin\theta)}{1+4\sin^2\theta}$$

To make it purely imaginary. It real part should be ‘O’.

Hence $2 = 6\sin^2\theta$

$\sin\theta = \frac{1}{\sqrt{3}}$

$\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

38. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is :

(1) $\frac{4}{\sqrt{3}}$

(2) $\frac{2}{\sqrt{3}}$

(3) $\sqrt{3}$

(4) $\frac{4}{3}$

Solution: (2)

$\frac{2b^2}{a} = 8$ …(i)

$2b = \frac{1}{2}\ 2ae$ ….(ii)

$\frac{b}{a} = \frac{e}{2}$

From (ii)

Now, $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{e^2}{4}}$

$e^2 = 1 + \frac{e^2}{4}$

$\frac{3}{4}e^2 = 1$

$e^2 = \frac{4}{3} = 1$

$e = \frac{2}{\sqrt{3}}$

39. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true ?

(1) $3a^2 - 32a + 84 = 0$

(2) $3a^2 - 34a + 91 = 0$

(3) $3a^2 - 23a + 44 = 0$

(4) $3a^2 - 26a + 55 = 0$

Solution: (1)

x	x^2
2	4
3	9
a	a^2
11	121
$16 + a$	$134 + a^2$

$$\sqrt{\frac{\sum x^2}{4} - \left(\frac{\sum x_i}{4}\right)^2}$$

$$\sqrt{\frac{134 + a^2}{4} - \left(\frac{16 + a}{4}\right)^2} = \frac{35}{10}$$

$$\frac{1}{2}\sqrt{134 + a^2 - \frac{(16 + a)^2}{4}} = \frac{7}{2}$$

$$\sqrt{536 + 4a^2 - 256 - a^2 - 32a} = 7$$

40. The integral $\int \frac{2x^{12}+5x^9}{(x^5+x^3+1)^3} dx$ is equal to:

where C is an arbitrary constant.

(1) $\frac{x^{10}}{2(x^5+x^3+1)^2} + C$

(2) $\frac{x^5}{2(x^5+x^3+1)^2} + C$

(3) $\frac{-x^{10}}{2(x^5+x^3+1)^2} + C$

(4) $\frac{-x^5}{(x^5+x^3+1)^2} + C$

Solution: (1)

$$\int \frac{2x^{12} + 5x^9}{(x^5 + x^3 + 1)^3} dx$$

$$\int \frac{(2x^{12} + 5 + 9)\, dx}{x^{15}\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^3}$$

$$\int \frac{\left(\frac{2}{x^3}+\frac{5}{x^6}\right)dx}{\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^3}$$

Let $1+\frac{1}{x^2}+\frac{1}{x^5}=t$

$$\left(-\frac{2}{x^3}-\frac{5}{x^6}\right)dx = dt$$

$$\left(\frac{2}{x^3}+\frac{5}{x^6}\right)dx = -dt$$

$$-\int \frac{dt}{t^3} = -\left(\frac{1}{(-2)t^2}\right) = \frac{1}{2\,.\,t^2}$$

$$= \frac{1}{2}\frac{1}{\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^2} + C = \frac{1}{2}\frac{x^{10}}{(x^5+x^3+1)^2} + C$$

$$= \frac{x^{10}}{2(x^5+x^3+1)^2} + C$$

41. If the line, $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z+4}{3}$ lies in the plane, $lx+my-z=9$, then l^2+m^2 is equal to :

(1) 18

(2) 5

(3) 2

(4) 26

Solution: (3)

Solution: (1)

$\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z+4}{3}$ lies in $lx+my-z=9$

The point (3, -2, -4) lies as the plane. So it should satisfy the equation of the plane.

$3l-2m+4=9$

$3l-2m=5$ …….(i)

The direction ratio 2, -1, 3 should be perpendicular to the line

$2(l)-1.(m)-3=0$

$2l-m=3$ …….(ii)

$l=1$ and $m=-1$

$\therefore \quad l^2 + m^2 = 1 + 1 = 2$

Hence Option [3] is correct

42. If $0 \leq x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$, is :

(1) 5

(2) 7

(3) 9

(4) 3

Solution: (2)

$2\cos 2x \cos x + 2\cos 3x \cos x = 0$

$\Rightarrow \quad 2\cos x\,(\cos 2x + \cos 3x) = 0$

$2\cos x \, 2\cos\frac{5x}{2}\cos\frac{x}{2} = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}, \pi, \frac{\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{9\pi}{5}$

7 Solutions.

43. The area (in sq. units) of the region $\{(x, y): y^2 \geq 2x$ and $x^2 + y^2 \leq 4x, x \geq 0, y \geq 0\}$ is:

(1) $\pi - \frac{8}{3}$

(2) $\pi - \frac{4\sqrt{2}}{3}$

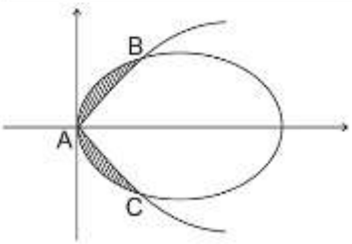
(3) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$

(4) $\pi - \frac{4}{3}$

Solution: (1)

$y^2 = 2x$ (Area out side the parabola)

$x^2 + y^2 \leq 4x$ (Area inside the circle)



First finding point of intersection of the curves

$x^2 + y^2 = 4x$ and $y^2 = 2x$

$x^2 + 2x = 4x$

$x^2 = 2x$

$x = 0, x = 2$

If $x = 0$, then $y = 0$ *and if* $x = 2$, then $y = \pm 2$.

Co – ordinates of $A(0,0)$ and $B(2,2)$

As $x \geq 0$ $y \geq 0$ only area above $x - axis$ would be considered

$$= \left[\int_0^2 \sqrt{4x - x^2}\, dx - \sqrt{2}\int_0^2 \sqrt{x}\, dx\right]$$

$$= \left[\int_0^2 \sqrt{4 - (x-2)^2}\, dx - \sqrt{2}\int_0^2 \sqrt{x}\, dx\right]$$

$$= \left[\left(\frac{2-x}{2}\right)\sqrt{4x - x^2} + \frac{4}{2}.\sin^{-1}\left(\frac{x-2}{2}\right)\right]_0^2 - \sqrt{2}\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^2$$

$$= [0 - 2\sin^{-1}(-1)] - \sqrt{2}.\frac{2}{3}2\sqrt{2}]$$

$$\left[\pi - \frac{8}{3}\right]$$

44. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is :

(1) $\frac{\pi}{2}$

(2) $\frac{2\pi}{3}$

(3) $\frac{5\pi}{6}$

(4) $\frac{3\pi}{4}$

Solution: (3)

$$\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$$

$$(\vec{a}\,.\,\vec{c})\,\vec{b} - (\vec{a}\,.\,\vec{b})\,\vec{c} = \frac{\sqrt{3}}{2}\,\vec{b} + \frac{\sqrt{3}}{2}\vec{c}$$

On comparing the coefficient of $\vec{c}$

On both the sides.

$$\vec{a}\,.\,\vec{b} = -\frac{\sqrt{3}}{2}$$

$$|\vec{a}|\,.\,|\vec{b}|\cos\theta = -\frac{\sqrt{3}}{2}$$

$$\cos\theta = -\frac{\sqrt{3}}{2}$$

$$\theta = \frac{5\pi}{6}$$

45. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side = x units and a circle of radius = r units. If the sum of the areas of the square and the circle so formed is minimum, then:

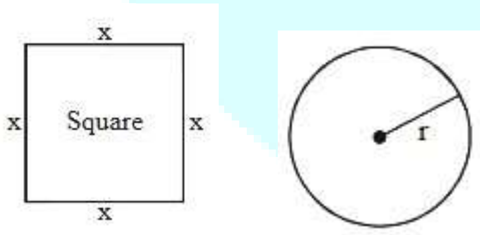
(1) $(4-\pi)x = \pi r$

(2) $x = 2r$

(3) $2x = r$

(4) $2x = (\pi + 4)r$

Solution: (2)



Given that $4x + 2\pi r = 2$

i.e., $2x + \pi r = 1$

$\therefore \qquad r = \frac{1-2x}{\pi} \qquad$ …….(i)

Area $A = x^2 + \pi x^2$

$= x^2 + \frac{1}{\pi}(2x-1)^2$

For min vale of area A

$\frac{dA}{dx} = 0$ given $x = \frac{2}{\pi+4}$ ....(ii)

From (i) and (ii)

$r = \frac{1}{\pi+4}$ ....(iii)

$\therefore$ $x = 2r$

46. The distance of the point $(1, -5, 9)$ from the plane $x - y + z = 5$ measured along the line $x = y = z$ is :

(1) $10\sqrt{3}$

(2) $\frac{10}{\sqrt{3}}$

(3) $\frac{20}{3}$

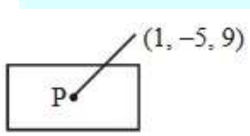
(4) $3\sqrt{10}$

Solution: (1)

Equation of line parallel to $x = y = z$ through

$(1, -5, 9)$ is $\frac{x-1}{1} = \frac{y+5}{1} = \frac{z-9}{1} = \lambda$

If $P(\lambda + 1, \lambda - 5, \lambda + 9)$ be point of intersection of line and plane.



$\Rightarrow$ $\lambda + 1 - \lambda + 5 + \lambda + 9 = 5$

$\Rightarrow$ $\lambda = -10$

$\Rightarrow$ Coordinates point are $(-9, -15, -1)$

$\Rightarrow$ Required distance $= 10\sqrt{3}$

47. If a curve $y = f(x)$ passes through the point (1, -1) and satisfies the differential equation, $y\,(1 + xy)dx = x\,dy$, then $f\left(-\frac{1}{2}\right)$ is equal to :

(1) $-\frac{4}{5}$

(2) $\frac{2}{5}$

(3) $\frac{4}{5}$

(4) $-\frac{2}{5}$

Solution: (3)

Given differential equation

$ydx + xy^2dx = xdy$

$\Rightarrow \quad \frac{xdy - ydx}{y^2} = xdx$

$\Rightarrow \quad -d\left(\frac{x}{y}\right) = d\left(\frac{x^2}{2}\right)$

Integrating we get

$= \frac{x}{y} = \frac{x^2}{2} + C$

$\because \quad$ It is passes through (1, -1)

$\therefore \quad 1 = \frac{1}{2} + C \Rightarrow C = \frac{1}{2}$

$\therefore \quad x^2 + 1 + \frac{2x}{y} = 0 \Rightarrow y = \frac{-2x}{x^2+1}$

$\therefore \quad f\left(-\frac{1}{2}\right) = \frac{4}{5}$

48. If the number of terms in the expansion of $\left(a - \frac{2}{x} + \frac{4}{x^2}\right)^n, x \neq 0,$ is 28, then the sum of the coefficients of all the terms in this expansion, is :

(1) 2187

(2) 243

(3) 729

(4) 64

Solution: (3 or Bonus)

$\left(\mathrm{a} - \frac{2}{\mathrm{x}} + \frac{4}{x^2}\right)^{\mathrm{n}}$ as the question is having three variables the total number of terms would be

$\frac{(\mathrm{n}+1)(\mathrm{n}+2)}{1.2}$ which is equal to 28

$\therefore \quad (\mathrm{n}+1)(\mathrm{n}+2) = 56$

Which gives $\mathrm{n} = 6$, and sum of coefficients would be $(1-2+4)^6 = 3^6 = 729$.

49. Consider $f(x) = \tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right), x \in \left(0, \frac{\pi}{2}\right)$. A normal to $y = f(x)$ at $x = \frac{\pi}{6}$ also passes through the point :

(1) $\left(0, \frac{2\pi}{3}\right)$

(2) $\left(\frac{\pi}{6}, 0\right)$

(3) $\left(\frac{\pi}{4}, 0\right)$

(4) $(0, 0)$

Solution: (1)

$$f(x) = \tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right), x \in \left(0, \frac{\pi}{2}\right)$$

$$f'(x) = \frac{1}{1+\left(\frac{1+\sin x}{1-\sin x}\right)} \cdot \frac{1}{2}\left(\frac{1+\sin x}{1-\sin x}\right)^{-\frac{1}{2}} \cdot \left(\frac{\cos x(1-\sin x)+\cos(1+\sin x)}{(1-\sin x)^2}\right) \text{ at } x = \frac{\pi}{2}$$

$$f'(x) = \frac{\frac{1}{2}}{2} \cdot \frac{1}{2}\left(\frac{1}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{\frac{1}{4}}\right) = \frac{1}{2}$$

Slope of tangent $= \frac{1}{2}$

So slope of normal $= -2$

Also at $x = \frac{\pi}{6}$ $y = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$.

So equation of the tangent would be $\left(y - \frac{\pi}{3}\right) = -2\left(x - \frac{\pi}{6}\right)$

It passes through $\left(0, \frac{2\pi}{3}\right)$

50. For $x \in R, f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then:

(1) $g'(0) = \cos(\log 2)$

(2) $g'(0) = -\cos(\log 2)$

(3) g is differentiable at $x = 0$ and $g'(0) = -\sin(\log 2)$

(4) g is not differentiable at $x = 0$

Solution: (1)

In the neighborhood of $x = 0, f(x) = \log 2 - \sin x$

$\therefore \quad g(x) = f(f(x) = \log 2 - \sin\big(f(x)\big)$

$= \log 2 - son\ (\log 2 - \sin x)$

It is differentiable at $x = 0$, so

$\therefore \quad g'(x) = -\cos(\log 2 - \sin x)\ (-\cos x)$

$\therefore \quad g'(0) = \cos(\log 2)$

51. Let two fair six – faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statement is NOT true ?

(1) E_2 and E_3 are independent

(2) E_1 and E_3 are independent

(3) E_1, E_2 and E_3 are independent

(4) E_1 and E_2 are independent

Solution: (3)

$E_1 \rightarrow A$ show up 4

$E_2 \rightarrow B$ shows up 2

$E_3 \rightarrow$ Sum is odd (i.e., even + odd or odd + even)

$$P(E_1) = \frac{6}{6.6} = \frac{1}{6}$$

$$P(E_2) = \frac{6}{6.6} = \frac{1}{6}$$

$$P(E_3) = \frac{3 \times 3 \times 2}{6.6} = \frac{1}{2}$$

$$P(E_1 \cap E_2) = \frac{1}{6.6} = P(E_1)\,.\,P(E_2)$$

$\Rightarrow \quad E_1\ and\ E_2$ are independent

$$P(E_1 \cap E_3) = \frac{1.3}{6.6} = P(E_1)\,.\,P(E_3)$$

$\Rightarrow \quad E_1\ and\ E_2$ are independent

$$P(E_2 \cap E_3) = \frac{1.3}{6.6} = \frac{1}{12} = P(E_2)\,.\,P(E_3)$$

$\Rightarrow \quad E_2 and\ E_3$ are independent

$P(E_1 \cap E_2 \cap E_3) = 0$ i.e., impossible event.

52. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and A adj $A = A\,A^T$, then $5a + b$ is equal to :

(1) 5

(2) 4

(3) 13

(4) -1

Solution: (1)

$A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A^T = \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$

$AA^T = \begin{bmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{bmatrix}$

Now, A adj $A = |A| I_2 = \begin{bmatrix} 10a + 3b & 0 \\ 0 & 10a + 3b \end{bmatrix}$

Given $AA^T = A. adj\ A$

$15a - 2b = 0$ ……(i)

$10a + 3b = 13$ …….(ii)

Solving we get

$5a = 2$ and $b = 3$

$\therefore \quad 5a + b = 5$

53. The Boolean Expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ is equivalent to :

(1) $p \wedge q$

(2) $p \vee q$

(3) $p \vee \sim q$

(4) $\sim p \wedge q$

Solution: (2)

$(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$

Set equivalent

$= (A \cap \bar{B}) \cup (\bar{A} \cap B) \cup B$

$= ((A \cup B) - (A \cap B) \cup B$

$= A \cup B$

Hence answer is $p \vee q$.

54. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2+4x-60} = 1$ is:

(1) - 4

(2) 6

(3) 5

(4) 3

Solution: (4)

$(x^2 - 5x + 5)^{x^2+4x-60} = 1$

$(x^2 - 5x + 5)^{(x+10)\,(x-6)} = 1$

$x = -10$ and $x = 6$ will make L.H.S = 1.

Also at $x = 1;\ (1)^{11.(-5)} = 1$

And at $x = 4;\ (1)^{14.(-2)} = 1$

We should also considered the case when $x^2 - 5x + 5 = -1$, and it has even power

$x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 0$

So $x = 2$ will give = $(-1)^{even}$

At $x = 2$

$(-1)^{12(-4)} = 1$

So sum would be

$-10 + 6 + 1 + 4 + 2 = 3$

Hence answer is 3

55. The centres of those circles which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$, externally and also touch the x – axis, lie on :

(1) An ellipse which is not a circle

(2) A hyperbola

(3) A parabola

(4) A circle

Solution: (3)

$x^2 + y^2 - 8x - 8y - 4 = 0$

has centre (4, 4) and radius 6.

Let (h, k) be the centre of the circle which is touching the circle externally

Then

$\sqrt{(h-4)^2 + (k-4)^2} = 6 + k$

$h^2 - 8h + 16 + k^2 - 8k + 16 = 36 + 12k + k^2$

$h^2 - 8h - 20k - 4 = 0,$

Replacing h by x and k by y

$x^2 - 8x - 20y - 4 = 0$

Equation of parabola.

56. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is :

(1) 59^{th}

(2) 52^{nd}

(3) 58^{th}

(4) 46^{th}

Solution: (3)

SMALL

Total number of words formed would be $\frac{\llcorner 5}{\llcorner 2} = 60$

When arranged as per dictionary. The words starting from A

$A - - - - \frac{\llcorner 4}{\llcorner 2} = 12$

The words standing from L

$L - - - - = \llcorner 4 = 24$

The words starting form M

$M - - - - = \frac{\llcorner 4}{\llcorner 2} = 12$

The words starting from

$S\ A - - - = \frac{\llcorner 3}{\llcorner 2} = 3$

$S\ L - - - \llcorner 3 = 6$

S M A L L = 1

Rank would be 12 + 24 + 12 + 3 + 6 + 1 = 58

57. $\lim_{n\to\infty}\left(\frac{(n+1)\ (n+2)....3n}{n^{2n}}\right)^{\frac{1}{n}}$ is equal to :

(1) $\frac{27}{e^2}$

(2) $\frac{9}{e^2}$

(3) $3\log 3 - 2$

(4) $\frac{18}{e^4}$

Solution: (1)

$$\lim_{n\to\infty}\left(\frac{(n+1)\ (n+2)\ldots .3n}{n^{2n}}\right)^{\frac{1}{n}}$$

Let $y = \left(\frac{(n+1)\,(n+2).....2n+1)}{n^{2n}}\right)^{\frac{1}{n}}$

$$y = \left(\frac{(n+1)}{n}\ .\frac{(n+2)}{n}\ldots\ldots\frac{(2n+n)}{n}\right)^{\frac{1}{n}}$$

$$\log y = \frac{1}{n}\left[\log\left(1+\frac{1}{n}\right) + \log\left(1+\frac{2}{n}\right) + \cdots ..\log(1+2)\right]$$

As $n \to \infty$

$$\log y = \int_0^2 \log(1+x)\ dx$$

Integrating by parts

$$\log y = \int_0^2 1.\log(1+x)\ dx$$

$= (x\,.\log(1+x))_0^2 - \int_0^2 \frac{1}{1+x}\,x$

$= (x\log(1+x))_0^2 - \int_0^2 \left(\frac{1+x}{1+x}\right) + \int_0^2 \frac{1}{1+x}$

$= \ (x\log(1+x))_0^2 - (x)_0^2 + (\log(1+x))_0^2$

$= \ (2\log 3 - 0) - (2-0) + (\log 3 - \log 1)$

$= \ 3\log 3 - 2$

Since $\log y \ = 3\log 3 - 2$

$= \ y = \frac{e^{\log 27}}{e^2} = \frac{27}{e^2}$

$= \frac{27}{e^2}$

58. If the sum of the first ten terms of the series $\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + \cdots$., is $\frac{16}{5}m$, then m is equal to :

(1) 101

(2) 100

(3) 99

(4) 102

Solution: (1)

$\left(\frac{8}{5}\right)^2 + \left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2$……..10 terms

$T_n = \left(\frac{4n+4}{5}\right)^2$

$T_n = 16\left(\frac{n^2+2n+1}{25}\right)$

$T_n = \frac{16}{25}(4^2 + 2n + 1)$

$T_n = S_n = \left(\frac{16}{25}\right)\left(\frac{n(n+1)\,(2n+1)}{6} + \frac{2(n)\,(n+1)+4}{2}\right)$

Put $n = 10$

$$\frac{16}{25}\left(\frac{10\,.\,11.\,21}{6}\right) + \frac{2\,.\,10\,.\,11}{2} + 10$$

$$= \frac{16}{25}\,(385 + 110 + 10) = \frac{16}{25}\,.\,505$$

$$= \frac{16}{5}\,101.$$

Hence m = 101

59. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is at (-3, 2), then the radius of S is:

(1) $5\sqrt{3}$

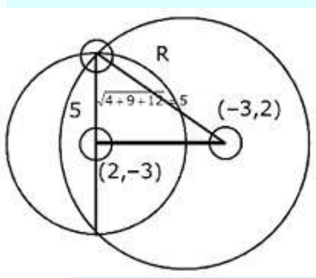
(2) 5

(3) 10

(4) $5\sqrt{2}$

Solution: (1)

The centre of the given circle $x^2 + y^2 - 4x + 6y - 12 = 0$ is (2, -3) and the radius is 5.



The distance between the centres $5\sqrt{2}$ and radius is 5. The triangle OPQ is a right angled triangle

$$OQ = \sqrt{\left(5\sqrt{2}\right)^2 + 5^2} = \sqrt{\left(5\sqrt{3}\right)^2} = 5\sqrt{3}$$

Hence answer is $5\sqrt{3}$

60. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30^o. After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60^o. Then the time taken (in minutes) by him, from B to reach the pillar, is :

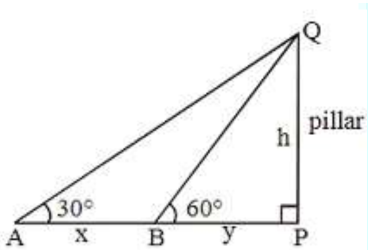
(1) 10

(2) 20

(3) 5

(4) 6

Solution: (3)



$\Delta QPA : \frac{h}{x+y} = \tan 30^o \Rightarrow \sqrt{3}h = x + y$ ……(i)

$\Delta QPB : \frac{h}{y} = \tan 60^o \Rightarrow h = \sqrt{3}y$ ……(ii)

By (i) and (ii) : $3y = x + y \Rightarrow y = \frac{x}{2}$

∵ Speed is uniform

Distance x in 10 mins

⇒ Distance $\frac{x}{2}$ in 5 mins.

PHYSICS

61. A uniform string of length 20m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is:

(take $\mathrm{g} = 10\ \mathrm{ms}^{-2}$)

(1) $2s$
(2) $2\sqrt{2}\ s$
(3) $\sqrt{2}\ s$
(4) $2\pi\sqrt{2}\ s$

Solution: (2)

$t = 2\sqrt{\frac{l}{g}} = 2\sqrt{2}$ second.

62. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up

considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take $g = 9.8\ ms^{-2}$:

(1) 6.45×10^{-3} kg
(2) 9.89×10^{-3} kg
(3) 12.89×10^{-3} kg
(4) 2.45×10^{-3} kg

Solution: (3)

$m = 10kg, h = 1m, 1000$ times

$PE = 98\ J \times 1000 = 98000\ J = 98\ kJ$

$= 9.8 \times 10^4\ J$

Fat burn $= 3.8 \times 10^7\ J \times 0.2$

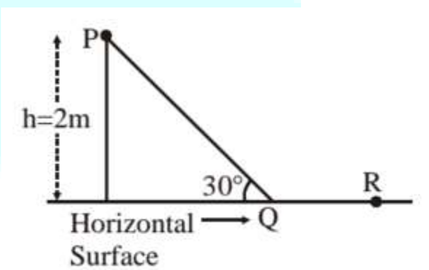
$= 7.6 \times 10^6\ J$ per kg

$$m = \frac{9.8 \times 10^4}{7.6 \times 10^6} = 1.289 \times 10^{-2}$$

$$= 12.89 \times 10^{-3}\ kg$$

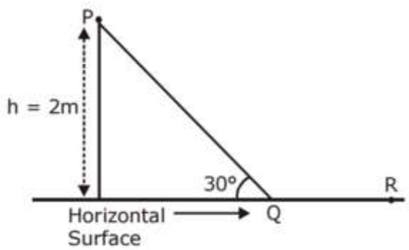
63. A point particle of mass m, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, from rest, from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR.

The values of the coefficient of friction μ and the distance $x = (QR)$, are respectively close to:



(1) 0.2 and 3.5 m
(2) 0.29 and 3.5 m
(3) 0.29 and 6.5 m
(4) 0.2 and 6.5 m

Solution: (2)



$\tan 30^o = \frac{h}{l}$

$l = h\sqrt{3} = 2\sqrt{3}\ m$

$W_f = -\mu mgl$ or $W_f = -\mu mgx$

$\mu mgl = \mu mgx;\ x = l$

$x = 2\sqrt{3}\ m;\ W_{all} = \Delta K$

$mgh - \mu mgl - \mu mgx = 0$

$h - \mu l - \mu x = 0$

$$2 = \mu(l + x) \Rightarrow \mu = \frac{2}{l + x} = \frac{2}{4\sqrt{3}} = \frac{1}{2\sqrt{3}}$$

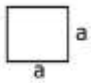
64. Two identical wires A and B, each of length 'l', carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side 'a'. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is :

(1) $\frac{\pi^2}{16\sqrt{2}}$
(2) $\frac{\pi^2}{16}$
(3) $\frac{\pi^2}{8\sqrt{2}}$
(4) $\frac{\pi^2}{8}$

Solution: (3)

$2\pi R = 4a$

$\frac{a}{R} = \frac{2\pi}{4}\frac{a}{R} = \frac{\pi}{2}$



$$B_A = \frac{\mu_0 i}{2R}$$

$$B_B = \frac{\mu_0 i}{\pi a}(2\sqrt{2})$$

$$\frac{B_A}{B_B} = \frac{\mu_0 i}{2R} \times \frac{\pi a}{2\sqrt{2}\,\mu_0 i}$$

$$= \frac{\pi a}{4\sqrt{2}R} = \frac{\pi}{4\sqrt{2}}\left(\frac{\pi}{2}\right) = \frac{\pi^2}{8\sqrt{2}}$$

65. A galvanometer having a coil resistance of $100\ \Omega$ gives a full scale deflection, when a current of 1 mA is passed through it. The value of the resistance, which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10 A, is:

(1) $2\ \Omega$

(2) $0.1\ \Omega$

(3) $3\ \Omega$

(4) $0.01\ \Omega$

Solution: (4)

We know that

$$R_S = \frac{I_G}{1 - I_G} R_G$$

$$= \frac{1 \times 10^{-3}}{10} \times 100$$

$$= 10^{-2}$$

$$= 0.01\ \Omega$$

66. An observer looks at a distant tree of height 10m with a telescope of magnifying power of 20. To the observer the tree appears:

(1) 10 times nearer.

(2) 20 times taller.

(3) 20 times nearer.

(4) 10 times taller.

Solution: (2)

$$\theta = \frac{10}{x}$$

$$\theta_1 = \frac{10}{x}(20)$$

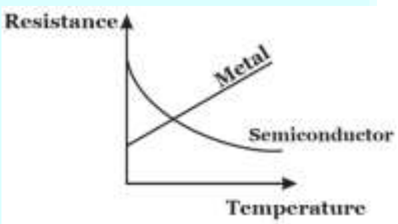
Now 20 times Taller.

67. The temperature dependence of resistance of Cu and undoped Si in the temperature range 300-400 K, is best described by:

(1) Linear increase for Cu, exponential increase for Si.
(2) Linear increase for Cu, exponential decrease for Si.
(3) Linear decrease for Cu, linear decrease for Si
(4) Linear increase for Cu, linear increase for Si.

Solution: (2)

Resistance variation with temperature: Cu-metal, undoped Silicon-Semi Conductor resistance of metal increases with increase in temperature linearly resistance of semi Conductor decreases exponentially with increase in temperature.



68. Choose the correct statement:

(1) In amplitude modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

(2) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

(3) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the frequency of the audio signal.

(4) In amplitude modulation the frequency of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

Solution: (1)

As per properties of A.M. in amplitude modulation the amplitude of high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

69. Half-lives of two radioactive elements A and B are 20 minutes and 40 minutes, respectively. Initially, the samples have equal number of nuclei. After 80 minutes, the ratio of decayed numbers of A and B nuclei will be:

(1) 4 : 1

(2) 1 : 4

(3) 5 : 4

(d) 1 : 16

Solution: (3)

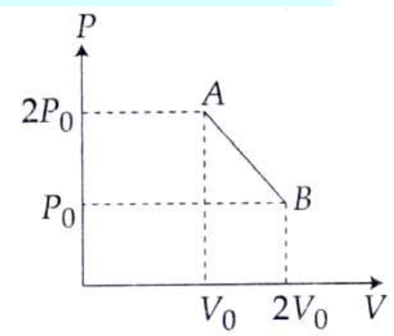
$t = 80 \text{ min} = 4\, T_A = 2T_B$

no. of nuclei of A decayed $= N_0 - \frac{N_0}{2^4} = \frac{15N_0}{16}$

no. of nuclei of B decayed $= N_0 - \frac{N_0}{2^2} = \frac{3N_0}{4}$

required ratio $= \frac{5}{4}$

70. 'n' moles of an ideal gas undergoes a process $A \rightarrow B$ as shown in the figure. The maximum temperature of the gas during the process will be:



(1) $\frac{3\,P_0V_0}{2nR}$
(2) $\frac{9\,P_0V_0}{2nR}$
(3) $\frac{9\,P_0V_0}{nR}$
(4) $\frac{9\,P_0V_0}{4nR}$

Solution: (4)

T_{max} at mid point

$$T = \frac{pv}{nR} = \frac{\left(\frac{3}{2}P_0\right)\left(\frac{3V_0}{2}\right)}{nR}$$

$= \frac{9}{4}\left(\frac{P_0 V_0}{nR}\right)$

71. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to:

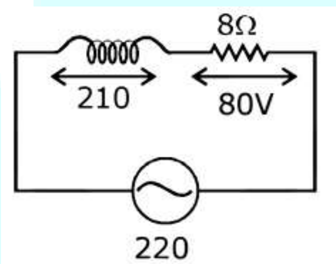
(1) 0.08 H

(2) 0.044 H

(3) 0.065 H

(4) 80 H

Solution: (3)



$V_L^2 + 6400 = 220 \times 220$

$IR = 80$

$V_L = \sqrt{48400 - 6400}$

$I = \frac{80}{8} = 10 = \sqrt{42000} = 210$

$I X_L = 210$

$X_L = 2\pi f L = 210$

$L = \frac{210}{10 \times 100\,\pi} = 0.065\ H$

72. A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now:

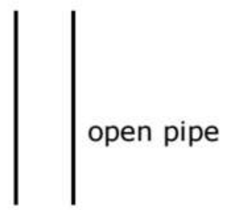
(1) $\frac{3f}{4}$

(2) $2f$

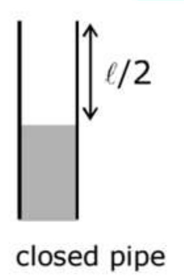
(3) f

(4) $\frac{f}{2}$

Solution: (3)



$f = \frac{v}{2l}$



$f' = \frac{v}{4l'} = \frac{v}{4\left(\frac{l}{2}\right)} = f$

73. The box of a pin hole camera, of length L has a hole of radius a. It is assumed that when the hole is illuminated by a parallel beam of light of wavelength λ the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size (say b_{min}) when:

(1) $a = \sqrt{\lambda L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$
(2) $a = \sqrt{\lambda L}$ and $b_{min} = \sqrt{4\lambda L}$
(3) $a = \frac{\lambda^2}{L}$ and $b_{min} = \sqrt{4\lambda L}$
(4) $a = \frac{\lambda^2}{L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$

Solution: (2)

The diffraction angle λa cause a spreading of $\frac{L\lambda}{a}$ in the size of the spot. These become large when a (Radius) is small.

So adding of two kind of spreading (for simplicity) we get spot size is

$a + \frac{La}{a}$.

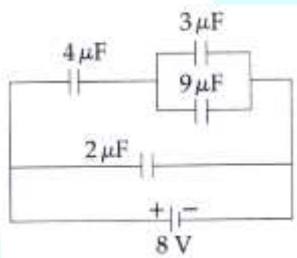
Hence to find out minimum value of this

We can write it as $\sqrt{\left(a - \frac{L\lambda}{a}\right)^2 + 4L\lambda}$

$\because$ The minimum value is when $a = \frac{La}{a}$ i.e. the geometric and diffraction broadening are equal $\sqrt{4L\lambda}$

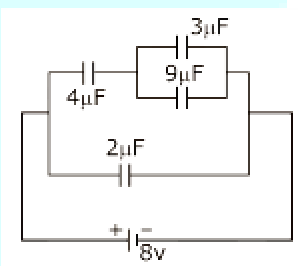
$\therefore$ When $a = \sqrt{\lambda L}$ and $b_{min} = \sqrt{4\lambda L}$

74. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the $4\ \mu F$ and $9\ \mu F$ capacitors), at a point distant 30 m from it, would equal:



(1) 360 N/C
(2) 420 N/C
(3) 480 N/C
(4) 240 N/C

Solution: (2)



Potential at $4\mu F = 6$ volt

$\therefore$ charge $q_1 = 24\mu C$

Potential at $9\mu F = 2$ volt

$\therefore$ charge $q_2 = 18\ \mu C$

Total $q = 42\ \mu C$

$E = \frac{kq}{r^2} = \frac{9\times10^9\times42\times10^{-6}}{900} = 420$ N/C

75. Arrange the following electromagnetic radiations per quantum in the order of increasing energy:

A: Blue light

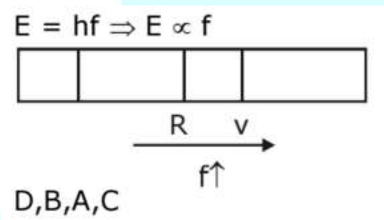
B: Yellow light

C: X-ray

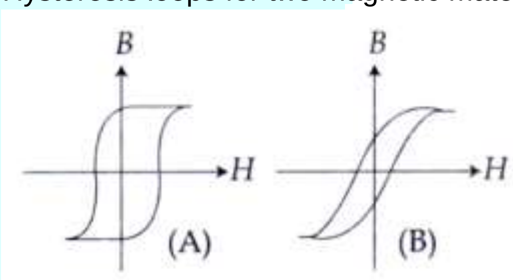
D: Radiowave

(1) A, B, D, C
(2) C, A, B, D
(3) B, A, D, C
(4) D, B, A, C

Solution: (4)



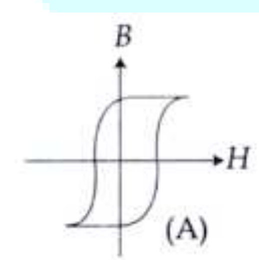
76. Hysteresis loops for two magnetic materials A and B are given below:



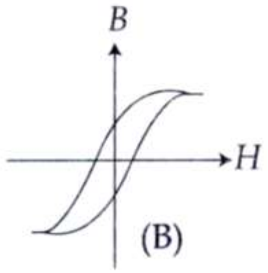
These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then it is proper to use:

(1) A for electromagnets and B for electric generators.
(2) A for transformers and B for electric generators.
(3) B for electromagnets and transformers.
(4) A for electric generators and transformers.

Solution: (3)



Graph A is hard ferromagnetic material substance.



The graph of B is graph of soft ferromagnetic material which is we use to consist of electromagnets and transformers.

77. A pendulum clock loses 12s a day if the temperature is 40^o C and gains 4s a day if the temperature is 20^o C. The temperature at which the clock will show correct time, and the co-efficient of linear expansion (α) of the metal of the pendulum shaft are respectively:

(1) $60^o C; \alpha = 1.85 \times 10^{-4}/ \ ^oC$
(2) $30^o C; \alpha = 1.85 \times 10^{-3}/ \ ^oC$
(3) $55^o C; \alpha = 1.85 \times 10^{-2}/ \ ^oC$
(4) $25^o C; \alpha = 1.85 \times 10^{-5}/ \ ^oC$

Solution: (4)

$$\Delta T \propto \Delta\theta$$

$$\frac{12}{4} = \frac{40-\theta}{\theta-20}$$

$$3\theta - 60 = 40 - \theta$$

$$4\theta = 100$$

$$\theta = 25^o C$$

$$\Delta T = \frac{1}{2}\alpha\,\Delta\theta \times T$$

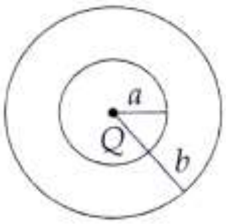
$$4 = \frac{1}{2}\alpha\, 5 \times 86400;$$

$$\frac{8\times10^5}{5\times86400} = \alpha;$$

$$\frac{8000}{4320} = \alpha$$
$$\alpha = 1.05 \times 10^{-5}/\,^oC$$

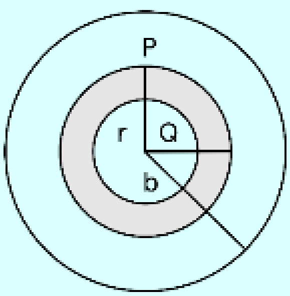
78. The region between two concentric spheres of radii ‘a’ and ‘b’, respectively (see figure), has volume charge density $\rho = \frac{A}{r}$, where A is a constant and r is the distance from the centre. At the centre of the spheres

is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant, is:



(1) $\frac{Q}{2\pi(b^2-a^2)}$
(2) $\frac{2Q}{\pi(a^2-b^2)}$
(3) $\frac{2Q}{\pi a^2}$
(4) $\frac{Q}{2\pi a^2}$

Solution: (4)



$$E = \frac{K\left[Q + \int_a^r 4\pi x^2 dx \frac{A}{x}\right]}{r^2}$$

$$E = K\left[\frac{Q}{r^2} + 4\mu A\left\{\frac{r^2}{2r^2} - \frac{a^2}{2r^2}\right\}\right]$$

$$\frac{dE}{dr} = 0$$

$$\frac{Q}{r^2} + \frac{4\pi A a^2}{2r^2}$$

$$A = \frac{Q}{2\pi a^2}$$

79. In an experiment for determination of refractive index of glass of a prism by $i = \delta$, plot, it was found that a ray incident at angle 35^o, suffers a deviation of 40^o and that it emerges at angle 79^o. In that case which of the following is closest to the maximum possible value of the refractive index?

(1) 1.6
(2) 1.7
(3) 1.8
(4) 1.5

Solution: (4)

$i = 35^o, \delta = 40^o, e = 79^o$

$\delta = i + e - A$

$40^o = 35^o + 79^o - A$

$A = 74^o$

And $r_1 + r_2 = A = 74^o$

Solving these, we get $\mu = 1.5$

Since $\delta_{min} < 40^o$

$$\mu < \frac{\sin\left(\frac{74+40}{2}\right)}{\sin 37}$$

$\mu_{max} = 1.44$

80. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90s, 91s, 95s and 92s. If the minimum division in the measuring clock is 1s, then the reported mean time should be:

(1) $92 \pm 5.0s$
(2) $92 \pm 1.8s$
(3) $92 \pm 3s$
(4) $92 \pm 2s$

Solution: (4)

T	T_s	$T_i - T$	$(T_i - T)^2$
t_1	90	-2	4
t_2	91	-1	1
t_3	95	3	9
t_4	92	0	0
T_i	92	$\frac{\Sigma T_i - T}{N} = 0$	$\frac{\Sigma (T_i - T)^2}{N} = 3.5$

$$T_r = T \pm \sqrt{\frac{\Sigma (T_i - T)^2}{N}}$$

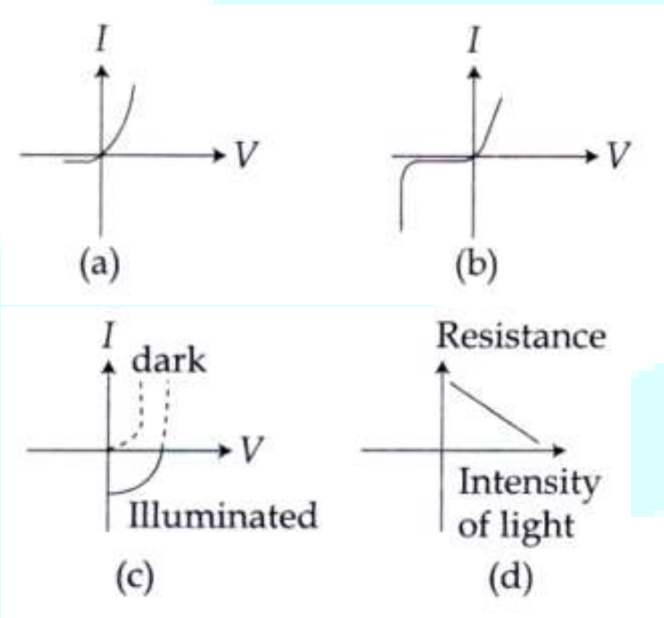
$T_r = 92 \pm \sqrt{3.5}$

$T_r = 92 \pm 1.8$

$T_r = 92 \pm 2$

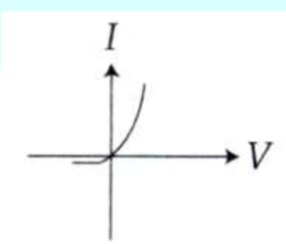
Because least count of clock is 1s.

81. Identify the semiconductor devices whose characteristics are given below, in the order $(a), (b), (c), (d)$:

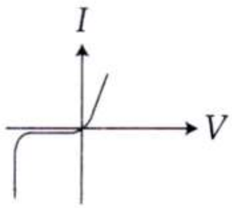


(1) Zener diode, Simple diode, Light dependent resistance, Solar cell
(2) Solar cell, Light dependent resistance, Zener diode, Simple diode
(3) Zener diode, Solar cell, Simple diode Light dependent resistance
(4) Simple diode, Zener diode, Solar cell, Light dependent resistance

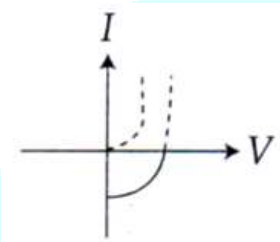
Solution: (4)



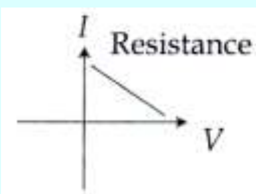
Its V-I characteristics of simple diode.



Its V-I characteristic of Zener diode.



V-I characteristics of Solar cell



V-I characteristics of light dependence resistance.

82. Radiation of wavelength λ, is incident on a photocell. The fastest emitted electron has speed υ. If the wavelength is changed to $\frac{3\lambda}{4}$, the speed of the fastest emitted electron will be:

(1) $< \upsilon\left(\frac{4}{3}\right)^{\frac{1}{2}}$

(2) $= \upsilon\left(\frac{4}{3}\right)^{\frac{1}{2}}$

(3) $= \upsilon\left(\frac{3}{4}\right)^{\frac{1}{2}}$

(4) $> \upsilon\left(\frac{4}{3}\right)^{\frac{1}{2}}$

Solution: (4)

$\frac{1}{2}mv^2 = \frac{hc}{\lambda} - \phi$ ...(i)

$\frac{1}{2}mv'^2 = \frac{4}{3}\frac{hc}{\lambda} - \phi$ ...(ii)

From eqn. (i) $\frac{hc}{\lambda} = \frac{1}{2}mv^2 + \phi$

On putting this equ. (ii)

$$\frac{1}{2}mv'^2 = \frac{4}{3}\left(\frac{1}{2}mv^2 + \phi\right) - \phi$$

$$v' > v\sqrt{\frac{4}{3}}$$

83. A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is:

(1) $3A$
(2) $A\sqrt{3}$
(3) $\frac{7A}{3}$
(4) $\frac{A}{3}\sqrt{41}$

Solution: (3)

$\mathrm{v} = \omega\sqrt{\mathrm{A}^2 - \mathrm{X}^2}$;

$$\mathrm{v} = \omega\sqrt{\mathrm{A}^2 - \frac{4\mathrm{A}^2}{9}}$$

$$= \frac{\omega\sqrt{5}\mathrm{A}}{3}$$

New SHM will be,

$$3\mathrm{v} - \omega\sqrt{\mathrm{A}_\mathrm{N}^2 - \mathrm{X}_\mathrm{n}^2};$$

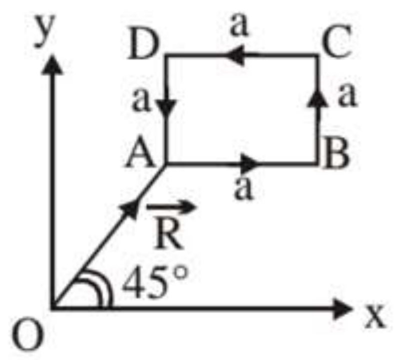
$$\frac{3\omega\sqrt{5}\mathrm{A}}{3} = \omega\sqrt{\mathrm{A}_\mathrm{N}^2 - \frac{4\mathrm{A}^2}{9}}$$

$$5\mathrm{A}^2 = \mathrm{A}_\mathrm{N}^2 - \frac{4\mathrm{A}^2}{9}$$

$$\mathrm{A}_\mathrm{N}^2 = \frac{49\mathrm{A}^2}{9}$$

$$\mathrm{A}_\mathrm{N} = \frac{7\mathrm{A}}{3}$$

84. A particle of mass m is moving along the side of a square of side 'a', with a uniform speed υ in the x-y plane as shown in the figure:



Which of the following statements is false for the angular momentum $\vec{L}$ about the origin?

(1) $\vec{L} = m\upsilon\left[\frac{R}{\sqrt{2}} - a\right]\hat{k}$ when the particle is moving from C to D.
(2) $\vec{L} = m\upsilon\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$ when the particle is moving from B to C.
(3) $\vec{L} = \frac{m\upsilon}{\sqrt{2}}R\hat{k}$ when the particle is moving from D to A.
(4) $\vec{L} = -\frac{m\upsilon}{\sqrt{2}}R\hat{k}$ when the particle is moving from A to B.

Solution: (1, 3)

$\vec{L} = \vec{r} \times \vec{P}$ or $\vec{L} = rp\sin\theta\,\hat{n}$

Or $\vec{L} = r_{\perp}(P)\hat{n}$

For D to A, $\vec{L} = \frac{R}{\sqrt{2}}mV(-\hat{k})$

For A to B, $\vec{L} = \frac{R}{\sqrt{2}}mV(-\hat{k})$

For C to D, $\vec{L} = \left(\frac{R}{\sqrt{2}} + a\right)mV(\hat{k})$

For B to C, $\vec{L} = \left(\frac{R}{\sqrt{2}} + a\right)mV(\hat{k})$

85. An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by PV^n = constant, then n is given by (Here C_P and C_V are molar specific heat at constant pressure and constant volume, respectively) :

(1) $n = \frac{C-C_P}{C-C_V}$

(2) $n = \frac{C_P - C}{C - C_V}$
(3) $n = \frac{C - C_V}{C - C_P}$
(4) $n = \frac{C_P}{C_V}$

Solution: (1)

$PV^n = k$

$C = C_v + \frac{R}{1-n}; C - C_v = \frac{R}{1-n}$

$1 - n = \frac{R}{C - C_v}; n = 1 - \frac{R}{C - C_v}$

$n = \frac{C - C_v - R}{C - C_v}; n = \frac{C - C_v - (C_p - C_v)}{C - C_v}$

$n = \frac{C - C_v - C_p + C_v}{C - C_v}; n = \frac{C - C_p}{C - C_v}$

86. A screw gauge with a pitch of 0.5 mm and a circular scale with 50 divisions is used to measure the thickness of a thin sheet of Aluminium. Before starting the measurement, it is found that when the two jaws of the screw gauge are brought in contact, the 45^{th} division coincides with the main scale line and that the zero of the main scale is barely visible. What is the thickness of the sheet if the main scale reading is 0.5 mm and the 25^{th} division coincides with the main scale line?

(1) 0.80 mm
(2) 0.70 mm
(3) 0.50 mm
(4) 0.75 mm

Solution: (1)

$$LC = \frac{0.5}{50} = 0.01 \text{ mm}$$

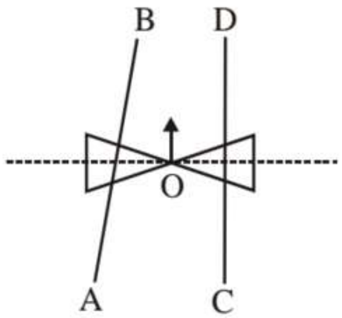
Zero error $= 0.50 - 0.45 = -0.05$

Thickness $= (0.5 + 25 \times 0.01) + 0.05$

$= 0.5 + 0.25 + 0.05$

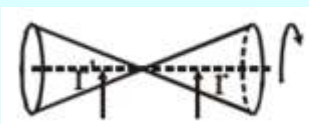
$= 0.8 \text{ mm}$

87. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to:



(1) to right.
(2) go straight.
(3) turn left and right alternately.
(4) turn left.

Solution: (4)



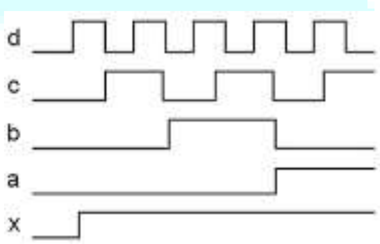
Say the distance of central line from instantaneous axis of rotation is r.

Then r from the point on left becomes lesser than that for right.

So V_{left} point $= \omega r' < \omega r = v_{\text{right}}$ point

So the roller will turn to left.

88. If a, b, c are inputs to a gate and x is its output, then, as per the following time graph, the gate is:



(1) AND
(2) OR
(3) NAND
(4) NOT

Solution: (2)

a	b	c	d	X
0	0	0	0	0

0	0	0	1	1

When any input is one output is one hence the gate is 'OR' gate.

89. For a common emitter configuration, if α and β have their usual meanings, the incorrect relationship between α and β is:

(1) $\alpha = \frac{\beta}{1-\beta}$
(2) $\alpha = \frac{\beta}{1+\beta}$
(3) $\alpha = \frac{\beta^2}{1+\beta^2}$
(4) $\frac{1}{\alpha} = \frac{1}{\beta} + 1$

Solution: (1, 3)

$I_E = I_C + I_B;$

$\frac{I_E}{I_C} = 1 + \frac{I_B}{I_C}$

$\frac{1}{\propto} = 1 + \frac{1}{\beta}; \frac{1}{\propto} = \frac{\beta+1}{\beta}$

$\propto = \frac{\beta}{1+\beta}$

90. A satellite is revolving in a circular orbit at a height 'h' from the earth's surface (radius of earth R; $h << R$). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to: (Neglect the effect of atmosphere.)

(1) $\sqrt{gR}$
(2) $\sqrt{\frac{gR}{2}}$
(3) $\sqrt{gR}\left(\sqrt{2}-1\right)$
(4) $\sqrt{2gR}$

Solution: (3)

Since $h << R$

$V_0 = \sqrt{2gR}$

& $V_e = \sqrt{gR}$

∴ min velocity required

$V_0 - V_e = \sqrt{2gR} - \sqrt{gR}$

$= \left(\sqrt{2}-1\right)\sqrt{gR}$