MATHEMATICS

1. A value of θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is :

(1) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (2) $\frac{\pi}{3}$

(3) $\frac{\pi}{6}$ (4) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

Solution: (1)

Let $z = \frac{2+3i\sin\theta}{1-2i\sin\theta}$

Rationalizing the complex number.

$$\frac{(2+3i\sin\theta)\,(1+2i\sin\theta)}{1+4\sin^2\theta}$$

$$= \frac{(2-6\sin^2\theta)+i\,(7\sin\theta)}{1+4\sin^2\theta}$$

To make it purely imaginary. It real part should be 'O'.

Hence $2 = 6\sin^2\theta$

$$\sin\theta = \frac{1}{\sqrt{3}}$$

$$\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

2. The system of linear equations

$x + \lambda y - z = 0$

$\lambda x - y - z = 0$

$x + y - \lambda z = 0$

Has a non –trivial solution for :

(1) Exactly three values of λ

(2) Infinitely many values of λ

(3) Exactly one value of λ

(4) Exactly two values of λ

Solution: (1)

For non – trivial solution.

$$\Delta = \begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$1(\lambda + 1) - \lambda(-\lambda^2 + 1) - 1(\lambda + 1) = 0$

$(\lambda + 1) - \lambda(1 - \lambda)\,(1 + \lambda) - (\lambda + 1) = 0$

$\lambda(1 - \lambda)\,(1 + \lambda) = 0$

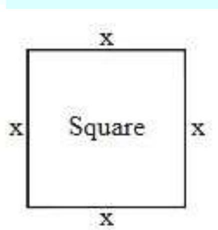
$\lambda = 0, \lambda = 1, \lambda = -1$

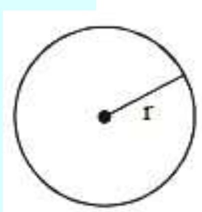
Exactly there value of λ.

3. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side $= x$ units and a circle of radius = r units. If the sum of the areas of the square and the circle so formed is minimum, then:

(1) $2x = r$ (2) $2x = (\pi + 4)r$ (3) $(4 - \pi)x = \pi r$ (4) $x = 2r$

Solution: (4)





Given that $4x + 2\pi r = 2$

i.e., $2x + \pi r = 1$

$\therefore \quad r = \frac{1-2x}{\pi}$ …….(i)

Area $A = x^2 + \pi x^2$

$= x^2 + \frac{1}{\pi}(2x - 1)^2$

For min vale of area A

$\frac{dA}{dx} = 0$ given $x = \frac{2}{\pi+4}$ ....(ii)

From (i) and (ii)

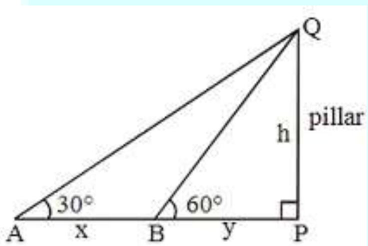
$r = \frac{1}{\pi+4}$ ....(iii)

$\therefore \quad x = 2r$

4. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30^o. After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60^o. Then the time taken (in minutes) by him, from B to reach the pillar, is :

(1) 5 (2) 6 (3) 10 (4) 20

Solution: (1)



$\Delta QPA : \frac{h}{x+y} = \tan 30^o \;\Rightarrow\; \sqrt{3}h = x + y$ ......(i)

$\Delta QPB : \frac{h}{y} = \tan 60^o \;\Rightarrow\; h = \sqrt{3}y$ ......(ii)

By (i) and (ii) : $3y = x + y \;\Rightarrow\; y = \frac{x}{2}$

$\because$ Speed is uniform

Distance x in 10 mins

$\Rightarrow$ Distance $\frac{x}{2}$ in 5 mins.

5. Let two fair six – faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statement is NOT true ?

(1) E_1, E_2 and E_3 are independent

(2) E_1 and E_2 are independent

(3) E_2 and E_3 are independent

(4) E_1 and E_3 are independent

Solution: (1)

$E_1 \rightarrow A$ show up 4

$E_2 \rightarrow B$ shows up 2

$E_3 \rightarrow$ Sum is odd (i.e., even + odd or odd + even)

$$P(E_1) = \frac{6}{6.6} = \frac{1}{6}$$

$$P(E_2) = \frac{6}{6.6} = \frac{1}{6}$$

$$P(E_3) = \frac{3 \times 3 \times 2}{6.6} = \frac{1}{2}$$

$$P(E_1 \cap E_2) = \frac{1}{6.6} = P(E_1)\,.\,P(E_2)$$

$\Rightarrow \quad E_1 \; and \; E_2$ are independent

$$P(E_1 \cap E_3) = \frac{1.3}{6.6} = P(E_1)\,.\,P(E_3)$$

$\Rightarrow \quad E_1 \; and \; E_2$ are independent

$$P(E_2 \cap E_3) = \frac{1.3}{6.6} = \frac{1}{12} = P(E_2)\,.\,P(E_3)$$

$\Rightarrow \quad E_2 and \; E_3$ are independent

$P(E_1 \cap E_2 \cap E_3) = 0$ i.e., impossible event.

6. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true ?

(1) $3a^2 - 23a + 44 = 0$

(2) $3a^2 - 26a + 55 = 0$

(3) $3a^2 - 32a + 84 = 0$

(4) $3a^2 - 34a + 91 = 0$

Solution: (3)

x	x^2
2	4
3	9
a	a^2
11	121
$16+a$	$134 + a^2$

$$\sqrt{\frac{\sum x^2}{4} - \left(\frac{\sum x_i}{4}\right)^2}$$

$$\sqrt{\frac{134+a^2}{4} - \left(\frac{16+a}{4}\right)^2} = \frac{35}{10}$$

$$\frac{1}{2}\sqrt{134 + a^2 - \frac{(16+a)^2}{4}} = \frac{7}{2}$$

$$\sqrt{536 + 4a^2 - 256 - a^2 - 32a} = 7$$

7. For $x \in R, f(x) = |\log 2 - \sin x|$ and $g(x) = f\big(f(x)\big)$, then:

(1) g is differentiable at $x = 0$ and $g'(0) = -\sin(\log 2)$

(2) g is not differentiable at $x = 0$

(3) $g'(0) = \cos(\log 2)$

(4) $g'(0) = -\cos(\log 2)$

Solution: (3)

In the neighborhood of $x = 0, f(x) = \log 2 - \sin x$

∴ $g(x) = f(f(x) = \log 2 - \sin\big(f(x)\big)$

$= \log 2 - son\ (\log 2 - \sin x)$

It is differentiable at $x = 0$, so

∴ $g'(x) = -\cos(\log 2 - \sin x)\ (-\cos x)$

∴ $g'(0) = \cos(\log 2)$

8. The distance of the point $(1, -5, 9)$ from the plane $x - y + z = 5$ measured along the line $x = y = z$ is :

(1) $\frac{20}{3}$

(2) $3\sqrt{10}$

(3) $10\sqrt{3}$

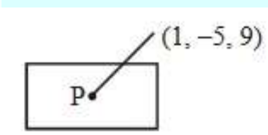
(4) $\frac{10}{\sqrt{3}}$

Solution: (3)

Equation of line parallel to $x = y = z$ through

$(1, -5, 9)$ is $\frac{x-1}{1} = \frac{y+5}{1} = \frac{z-9}{1} = \lambda$

If $P(\lambda + 1, \lambda - 5, \lambda + 9)$ be point of intersection of line and plane.



⇒ $\lambda + 1 - \lambda + 5 + \lambda + 9 = 5$

⇒ $\lambda = -10$

⇒ Coordinates point are $(-9, -15, -1)$

⇒ Required distance $= 10\sqrt{3}$

9. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is :

(1) $\sqrt{3}$

(2) $\frac{4}{3}$

(3) $\frac{4}{\sqrt{3}}$

(4) $\frac{2}{\sqrt{3}}$

Solution: (4)

$\frac{2b^2}{a} = 8$ ...(i)

$2b = \frac{1}{2}\, 2ae$ ....(ii)

$\frac{b}{a} = \frac{e}{2}$

From (ii)

Now, $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{e^2}{4}}$

$e^2 = 1 + \frac{e^2}{4}$

$\frac{3}{4}e^2 = 1$

$e^2 = \frac{4}{3} = 1$

$e = \frac{2}{\sqrt{3}}$

10. Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle, $x^2 + (y+6)^2 = 1$. Then the equation of the circle, passing through C and having its centre at P is:

(1) $x^2 + y^2 - 4x + 9y + 18 = 0$

(2) $x^2 + y^2 - 4x + 8y + 12 = 0$

(3) $x^2 + y^2 - x + 4y - 12 = 0$

(4) $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$

Solution: (2)

$y^2 = 8x$ is the equation of the given parabola. If P is a point at minimum distance from (0, -6) then it should be normal to the parabola at P.

Slope of tangent

$\frac{2dy}{dx} . y = 8$

$\frac{dy}{dx} = \frac{4}{y}.$

$\therefore$ Slope of normal = $\left(-\frac{y}{4}\right)$

Any point on the parabola would be $\left(\frac{y^2}{8}, y\right)$, and hence slope of the normal would be

$\frac{(y+6)}{\frac{y^2}{8}} = -\frac{y}{4}$

$(y+6) = -\frac{y^3}{32}$

At $y = -4$, LHS = RHS

$(2) = +\frac{64}{32} = 2$

At $y = -4\ x = 2$

So point $P(2, -4)$.

Radius of the desired circle would be $\sqrt{2^2 + 2^2} = 2\sqrt{2}$,

So equation of the circle would be $\sqrt{(x-2)^2 + (y+4)^2} = 2\sqrt{2}$

$x^2 - 4x + 4 + y^2 + 8y + 16 = 8$

$x^2 + y^2 - 4x + 8y + 12 = 0$

11. If$A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and A adj $A = A\,A^T$, then $5a + b$ is equal to :

(1) 13

(2) -1

(3) 5

(4) 4

Solution: (3)

$A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A^T = \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$

$AA^T = \begin{bmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{bmatrix}$

Now, A adj $A = |A| I_2 = \begin{bmatrix} 10a + 3b & 0 \\ 0 & 10a + 3b \end{bmatrix}$

Given $AA^T = A. adj\ A$

$15a - 2b = 0$ ……(i)

$10a + 3b = 13$ …….(ii)

Solving we get

$5a = 2$ and $b = 3$

∴ $5a + b = 5$

12. Consider $f(x) = \tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right), x \in \left(0, \frac{\pi}{2}\right)$. A normal to $y = f(x)$ at $x = \frac{\pi}{6}$ also passes through the point :

(1) $\left(\frac{\pi}{4}, 0\right)$

(2) $(0, 0)$

(3) $\left(0, \frac{2\pi}{3}\right)$

(4) $\left(\frac{\pi}{6}, 0\right)$

Solution: (3)

$$f(x) = \tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right), x \in \left(0, \frac{\pi}{2}\right)$$

$$f'(x) = \frac{1}{1+\left(\frac{1+\sin x}{1-\sin x}\right)} \cdot \frac{1}{2}\left(\frac{1+\sin x}{1-\sin x}\right)^{-\frac{1}{2}} \cdot \left(\frac{\cos x(1-\sin x)+\cos(1+\sin x)}{(1-\sin x)^2}\right) \text{ at } x = \frac{\pi}{2}$$

$$f'(x) = \frac{\frac{1}{2}}{2} \cdot \frac{1}{2}\left(\frac{1}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{\frac{1}{4}}\right) = \frac{1}{2}$$

Slope of tangent $= \frac{1}{2}$

So slope of normal = -2

Also at $x = \frac{\pi}{6}$ $y = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$.

So equation of the tangent would be $\left(y - \frac{\pi}{3}\right) = -2\left(x - \frac{\pi}{6}\right)$

It passes through $\left(0, \frac{2\pi}{3}\right)$

13. Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. If its diagonals intersect at $(-1, -2)$, then which one of the following is a vertex of this rhombus ?

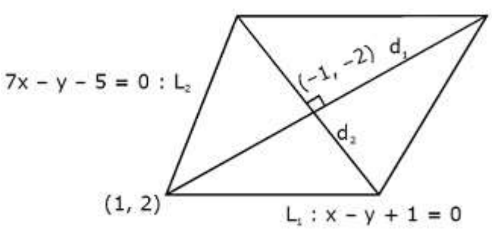
(1) $\left(-\frac{10}{3}, -\frac{7}{3}\right)$

(2) $(-3, -9)$

(3) $(-3, -8)$

(4) $\left(\frac{1}{3}, -\frac{8}{3}\right)$

Solution: (4)



$d_1 : y - 2 = \dfrac{-2-2}{-1-1}(x - 1)$

$y - 2 = 2(x - 1)$

$y - 2x = 0$

$d_2 \perp d_1 \Rightarrow \quad 2y + x = k\ p$ on $(-1, -2)$

$2y + x + 5 = 0$

Non P.O.I. of d_2 and L_1

$x - y + 1 = 0$

$x + 2y + 5 = 0$

$-3y-4=0$

$y=-\frac{4}{3}$

And P.O.I. of d_2 and L_2

$x+2y+5=0$

$14x-2y-10=0$

And $y=-\frac{8}{3}$

$15x-5=0$

$\Rightarrow \quad x=\frac{1}{3}$

14. If a curve $y=f(x)$ passes through the point (1, -1) and satisfies the differential equation, $y\,(1+xy)dx=x\,dy$, then $f\left(-\frac{1}{2}\right)$ is equal to :

(1) $\frac{4}{5}$

(2) $-\frac{2}{5}$

(3) $-\frac{4}{5}$

(4) $\frac{2}{5}$

Solution: (1)

Given differential equation

$ydx+xy^2dx=xdy$

$\Rightarrow \quad \frac{xdy-ydx}{y^2}=xdx$

$\Rightarrow \quad -d\left(\frac{x}{y}\right)=d\left(\frac{x^2}{2}\right)$

Integrating we get

$=\frac{x}{y}=\frac{x^2}{2}+C$

$\because$ It is passes through (1, -1)

$\therefore$ $1 = \frac{1}{2} + C \Rightarrow C = \frac{1}{2}$

$\therefore$ $x^2 + 1 + \frac{2x}{y} = 0 \Rightarrow y = \frac{-2x}{x^2+1}$

$\therefore$ $f\left(-\frac{1}{2}\right) = \frac{4}{5}$

15. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is :

(1) 58^{th}

(2) 46^{th}

(3) 59^{th}

(4) 52^{nd}

Solution: (1)

SMALL

Total number of words formed would be $\frac{\lfloor 5}{\lfloor 2} = 60$

When arranged as per dictionary. The words starting from A

$A - - - - \frac{\lfloor 4}{\lfloor 2} = 12$

The words standing from L

$L - - - - = \lfloor 4 = 24$

The words starting form M

$M - - - - = \frac{\lfloor 4}{\lfloor 2} = 12$

The words starting from

$S\,A - - - = \frac{\lfloor 3}{\lfloor 2} = 3$

$S\,L - - - \lfloor 3 = 6$

S M A L L = 1

Rank would be 12 + 24 + 12 + 3 + 6 + 1 = 58

16. If the $2^{nd}, 5^{th}$ and 9^{th} term of a non – constant A.P. are in G.P., then the common ratio of this G.P. is:

(1) $\frac{7}{4}$

(2) $\frac{8}{5}$

(3) $\frac{4}{3}$

(4) 1

Solution: (3)

Let the A.P. be a, a + d, a + 2d,…..

Given $(a+d).(a+8d) = (a+4d)^2$

$a^2 + 9ad + 8d^2 = a^2 + 8ad + 16d^2$

$8d^2 - ad = 0$

$d[8d - a] = 0$

$\because \quad d \neq 0$

$d = \frac{a}{8}$

So, 2^{nd} term 5^{th} term 9^{th} term

Would be $\left(a + \frac{a}{8}\right)$ $\left(a + \frac{a}{2}\right)$ $(a + a)$

$\frac{9a}{8}$ $\frac{3a}{2}$ $2a$

Common Ratio : $\frac{2^{nd}\ term}{1^{st}\ term} = \frac{3a}{2.9a}.8 = \frac{4}{3}$

17. If the number of terms in the expansion of $\left(a - \frac{2}{x} + \frac{4}{x^2}\right)^n, x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is :

(1) 729

(2) 64

(3) 2187

(4) 243

Solution: (1 or Bonus)

$\left(a-\frac{2}{x}+\frac{4}{x^2}\right)^n$ as the question is having three variables the total number of terms would be

$\frac{(n+1)(n+2)}{1.2}$ which is equal to 28

$\therefore \quad (n+1)(n+2\,) = 56$

Which gives $n = 6$, and sum of coefficients would be $(1-2+4)^6 = 3^6 = 729$.

18. If the sum of the first ten terms of the series $\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + \cdots$., is $\frac{16}{5}m$, then m is equal to :

(1) 99

(2) 102

(3) 101

(4) 100

Solution: (3)

$\left(\frac{8}{5}\right)^2 + \left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2$……..10 terms

$$T_n = \left(\frac{4n+4}{5}\right)^2$$

$$T_n = 16\left(\frac{n^2+2n+1}{25}\right)$$

$$T_n = \frac{16}{25}(4^2+2n+1)$$

$$T_n = S_n = \left(\frac{16}{25}\right)\left(\frac{n(n+1)\,(2n+1)}{6} + \frac{2(n)\,(n+1)+4}{2}\right)$$

Put $n = 10$

$$\frac{16}{25}\left(\frac{10\,.\,11.\,21}{6}\right) + \frac{2\,.\,10\,.\,11}{2} + 10$$

$= \frac{16}{25}(385 + 110 + 10) = \frac{16}{25}.505$

$= \frac{16}{5}\ 101.$

Hence m = 101

19. If the line, $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+4}{3}$ lies in the plane, $lx + my - z = 9$, then $l^2 + m^2$ is equal to :

(1) 2

(2) 26

(3) 18

(4) 5

Solution: (1)

$\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+4}{3}$ lies in $lx + my - z = 9$

The point (3, -2, -4) lies as the plane. So it should satisfy the equation of the plane.

$3l - 2m + 4 = 9$

$3l - 2m = 5$ …….(i)

The direction ratio 2, -1, 3 should be perpendicular to the line

$2(l) - 1.(m) - 3 = 0$

$2l - m = 3$ …….(ii)

$l = 1$ and $m = -1$

∴ $l^2 + m^2 = 1 + 1 = 2$

Hence Option [2] is correct

20. The Boolean Expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ is equivalent to :

(1) $p \vee \sim q$

(2) $\sim p \wedge q$

(3) $p \wedge q$

(4) $p \vee q$

Solution: (4)

$(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$

Set equivalent

$= (A \cap \bar{B}) \cup (\bar{A} \cap B) \cup B$

$= ((A \cup B) - (A \cap B) \cup B$

$= A \cup B$

Hence answer is $p \vee q$.

21. The integral $\int \frac{2x^{12}+5x^9}{(x^5+x^3+1)^3} dx$ is equal to:

where C is an arbitrary constant.

(1) $\frac{-x^{10}}{2(x^5+x^3+1)^2} + C$

(2) $\frac{-x^5}{(x^5+x^3+1)^2} + C$

(3) $\frac{x^{10}}{2(x^5+x^3+1)^2} + C$

(4) $\frac{x^5}{2(x^5+x^3+1)^2} + C$

Solution: (3)

$$\int \frac{2x^{12} + 5x^9}{(x^5 + x^3 + 1)^3} dx$$

$$\int \frac{(2x^{12} + 5 + 9)\, dx}{x^{15}\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^3}$$

$$\int \frac{\left(\frac{2}{x^3} + \frac{5}{x^6}\right) dx}{\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^3}$$

Let $1 + \frac{1}{x^2} + \frac{1}{x^5} = t$

$$\left(-\frac{2}{x^3}-\frac{5}{x^6}\right)dx = dt$$

$$\left(\frac{2}{x^3}+\frac{5}{x^6}\right)dx = -dt$$

$$-\int \frac{dt}{t^3} = -\left(\frac{1}{(-2)t^2}\right) = \frac{1}{2\,.\,t^2}$$

$$= \frac{1}{2}\frac{1}{\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^2} + C = \frac{1}{2}\frac{x^{10}}{(x^5+x^3+1)^2} + C$$

$$= \frac{x^{10}}{2(x^5+x^3+1)^2} + C$$

22. If one of the diameters of the circle, given by the equation, $x^2+y^2-4x+6y-12=0$, is a chord of a circle S, whose centre is at (-3, 2), then the radius of S is:

(1) 10

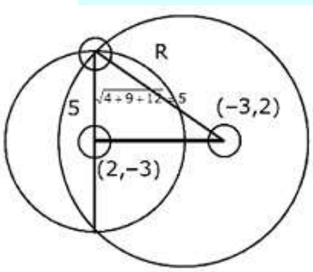
(2) $5\sqrt{2}$

(3) $5\sqrt{3}$

(4) 5

Solution: (3)

The centre of the given circle $x^2+y^2-4x+6y-12=0$ is (2, -3) and the radius is 5.



The distance between the centres $5\sqrt{2}$ and radius is 5. The triangle OPQ is a right angled triangle

$$OQ = \sqrt{\left(5\sqrt{2}\right)^2 + 5^2} = \sqrt{\left(5\sqrt{3}\right)^2} = 5\sqrt{3}$$

Hence answer is $5\sqrt{3}$

23. $\lim_{n\to\infty}\left(\frac{(n+1)\ (n+2)....3n}{n^{2n}}\right)^{\frac{1}{n}}$ is equal to :

(1) $\frac{18}{e^4}$

(2) $3\log 3-2$

(3) $\frac{27}{e^2}$

(4) $\frac{9}{e^2}$

Solution: (3)

$$\lim_{n\to\infty}\left(\frac{(n+1)\ (n+2)\dots.3n}{n^{2n}}\right)^{\frac{1}{n}}$$

Let $y=\left(\frac{(n+1)\ (n+2).....2n+1)}{n^{2n}}\right)^{\frac{1}{n}}$

$$y=\left(\frac{(n+1)}{n}.\frac{(n+2)}{n}\dots\dots\frac{(2n+n)}{n}\right)^{\frac{1}{n}}$$

$$\log y=\frac{1}{n}\left[\log\left(1+\frac{1}{n}\right)+\log\left(1+\frac{2}{n}\right)+\cdots..\log(1+2)\right]$$

As $n\to\ \infty$

$$\log y=\int_0^2\log(1+x)\ dx$$

Integrating by parts

$$\log y=\int_0^2 1.\log(1+x)\ dx$$

$$=(x\,.\log(1+x))_0^2-\int_0^2\frac{1}{1+x}\ x$$

$= (x \log(1+x))_0^2 - \int_0^2 \left(\frac{1+x}{1+x}\right) + \int_0^2 \frac{1}{1+x}$

$= \ (x\log(1+x))_0^2 - (x)_0^2 + (\log(1+x))_0^2$

$= \ (2\log 3 - 0) - (2-0) + (\log 3 - \log 1)$

$= \ 3\log 3 - 2$

Since $\log y \ = 3\log 3 - 2$

$= \ y = \frac{e^{\log 27}}{e^2} = \frac{27}{e^2}$

$= \frac{27}{e^2}$

24. The centres of those circles which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$, externally and also touch the x – axis, lie on :

(1) A parabola

(2) A circle

(3) An ellipse which is not a circle

(4) A hyperbola

Solution: (1)

$x^2 + y^2 - 8x - 8y - 4 = 0$

has centre (4, 4) and radius 6.

Let (h, k) be the centre of the circle which is touching the circle externally

Then

$\sqrt{(h-4)^2 + (k-4)^2} = 6 + k$

$h^2 - 8h + 16 + k^2 - 8k + 16 = 36 + 12k + k^2$

$h^2 - 8h - 20k - 4 = 0,$

Replacing h by x and k by y

$x^2 - 8x - 20y - 4 = 0$

Equation of parabola.

25. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is :

(1) $\frac{5\pi}{6}$

(2) $\frac{3\pi}{4}$

(3) $\frac{\pi}{2}$

(4) $\frac{2\pi}{3}$

Solution: (1)

$$\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$$

$$(\vec{a} \,.\, \vec{c})\, \vec{b} - (\vec{a} \,.\, \vec{b})\, \vec{c} = \frac{\sqrt{3}}{2}\, \vec{b} + \frac{\sqrt{3}}{2}\vec{c}$$

On comparing the coefficient of $\vec{c}$

On both the sides.

$$\vec{a} \,.\, \vec{b} = -\frac{\sqrt{3}}{2}$$

$$|\vec{a}| \,.\, |\vec{b}| \cos\theta = -\frac{\sqrt{3}}{2}$$

$$\cos\theta = -\frac{\sqrt{3}}{2}$$

$$\theta = \frac{5\pi}{6}$$

26. Let $p = \lim_{x \to 0+} (1 + \tan^2 \sqrt{x})^{\frac{1}{2x}}$, then log p is equal to :

(1) $\frac{1}{4}$

(2) 2

(3) 1

(4) $\frac{1}{2}$

Solution: (4)

Let p = $\ln\left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$.

The limit is of the form $(1+0)^{\infty} = e^{0\,.\infty}$

$e^{\frac{\tan\sqrt{x}}{\sqrt{x}}.\frac{\tan\sqrt{x}}{\sqrt{x}}.\frac{1}{2}}$

$\lim_{x\to 0+} p = e^{\frac{1}{2}}$

$\log p = \log e^{\frac{1}{2}} = \frac{1}{2}$

$= \frac{1}{2}$

27. If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$, is :

(1) 9

(2) 3

(3) 5

(4) 7

Solution: (4)

$2\cos 2x \cos x + 2\cos 3x \cos x = 0$

$\Rightarrow \quad 2\cos x\,(\cos 2x + \cos 3x) = 0$

$2\cos x\, 2\cos\frac{5x}{2}\cos\frac{x}{2} = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}, \pi, \frac{\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{9\pi}{5}$

7 Solutions.

28. The sum of all real values of x satisfying the equation $(x^2-5x+5)^{x^2+4x-60}=1$ is:

(1) 5

(2) 3

(3) - 4

(4) 6

Solution: (2)

$(x^2-5x+5)^{x^2+4x-60}=1$

$(x^2-5x+5)^{(x+10)\,(x-6)}=1$

$x=-10$ and $x=6$ will make L.H.S = 1.

Also at $x=1;\ (1)^{11.(-5)}=1$

And at $x=4;\ (1)^{14.(-2)}=1$

We should also considered the case when $x^2-5x+5=-1$, and it has even power

$x^2-5x+6=0$

$(x-2)(x-3)=0$

So $x=2$ will give = $(-1)^{even}$

At $x=2$

$(-1)^{12(-4)}=1$

So sum would be

$-10+6+1+4+2=3$

Hence answer is 3

29. The area (in sq. units) of the region $\{(x,y): y^2\geq 2x$ and $x^2+y^2\leq 4x, x\geq 0, y\geq 0\}$ is:

(1) $\frac{\pi}{2}-\frac{2\sqrt{2}}{3}$

(2) $\pi-\frac{4}{3}$

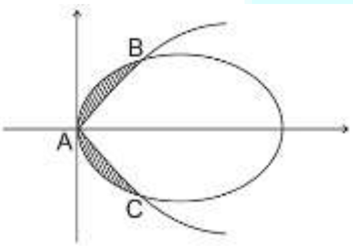
(3) $\pi-\frac{8}{3}$

(4) $\pi - \frac{4\sqrt{2}}{3}$

Solution: (3)

$y^2 = 2x$ (Area out side the parabola)

$x^2 + y^2 \leq 4x$ (Area inside the circle)



First finding point of intersection of the curves

$x^2 + y^2 = 4x$ and $y^2 = 2x$

$x^2 + 2x = 4x$

$x^2 = 2x$

$x = 0, x = 2$

If $x = 0$, then $y = 0$ *and if* $x = 2$, then $y = \pm 2$.

Co – ordinates of $A(0,0)$ and $B(2,2)$

As $x \geq 0$ $y \geq 0$ only area above $x - axis$ would be considered

$$= \left[\int_0^2 \sqrt{4x - x^2}\, dx - \sqrt{2}\int_0^2 \sqrt{x}\, dx\right]$$

$$= \left[\int_0^2 \sqrt{4 - (x-2)^2}\, dx - \sqrt{2}\int_0^2 \sqrt{x}\, dx\right]$$

$$= \left[\left(\frac{2-x}{2}\right)\sqrt{4x - x^2} + \frac{4}{2}.\sin^{-1}\left(\frac{x-2}{2}\right)\right]_0^2 - \sqrt{2}\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^2$$

$$= [0 - 2\sin^{-1}(-1)] - \sqrt{2}.\frac{2}{3}2\sqrt{2}]$$

$\left[\pi - \frac{8}{3}\right]$

30. If $f(x) + 2f\left(\frac{1}{x}\right) = 3x, x \neq 0$, and $S = \{x \in R : f(x) = f(-x)\}$; then S :

(1) Contains more than two elements

(2) Is an empty set

(3) Contains exactly one element

(4) Contains exactly two elements

Solution: (4)

$f(x) + 2 . f\left(\frac{1}{x}\right) = 3x$ ....(i)

Replace x by $\frac{1}{x}$

$f\left(\frac{1}{x}\right) + 2f(x) = \frac{3}{x}$ ....(ii)

$3f(x) = \frac{6}{x} - 3x$

$f(x) = \frac{2}{x} - x$

$\because \quad f(x) = f(-x)$

Therefore $\frac{2}{x} - x = -\frac{2}{x} + x$

$\frac{4}{x} = 2x$

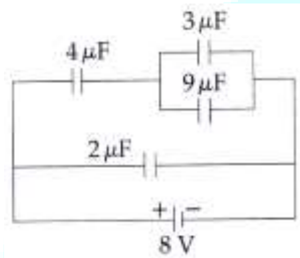
$2 = x^2$

$x = \pm\sqrt{2}$

Contains exactly two elements.

PHYSICS

31. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the $4\ \mu F$ and $9\ \mu F$ capacitors), at a point distant 30 m from it, would equal:



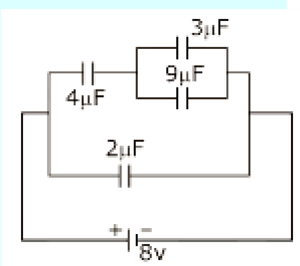
(1) 480 N/C

(2) 240 N/C

(3) 360 N/C

(4) 420 N/C

Solution: (4)



Potential at $4\mu F = 6$ volt

∴ charge $q_1 = 24\mu C$

Potential at $9\mu F = 2$ volt

∴ charge $q_2 = 18\ \mu C$

Total $q = 42\ \mu C$

$E = \frac{kq}{r^2} = \frac{9\times10^9\times42\times10^{-6}}{900} = 420$ N/C

32. An observer looks at a distant tree of height 10m with a telescope of magnifying power of 20. To the observer the tree appears:

(1) 20 times nearer.

(2) 10 times taller.

(3) 10 times nearer.

(4) 20 times taller.

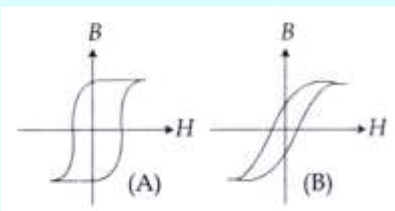
Solution: (4)

$$\theta = \frac{10}{x}$$

$$\theta_1 = \frac{10}{x}(20)$$

Now 20 times taller.

33. Hysteresis loops for two magnetic materials A and B are given below:



These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then it is proper to use:

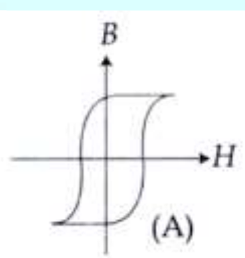
(1) B for electromagnets and transformers.

(2) A for electric generators and transformers.

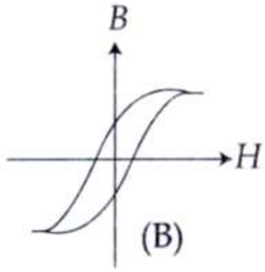
(3) A for electromagnets and B for electric generators.

(4) A for transformers and B for electric generators.

Solution: (1)



Graph A is hard ferromagnetic material substance.



The graph of B is graph of soft ferromagnetic material which is we use to consist of electromagnets and transformers.

34. Half-lives of two radioactive elements A and B are 20 minutes and 40 minutes, respectively. Initially, the samples have equal number of nuclei. After 80 minutes, the ratio of decayed numbers of A and B nuclei will be:

(1) 5 : 4

(2) 1 : 16

(3) 4 : 1

(4) 1 : 4

Solution: (1)

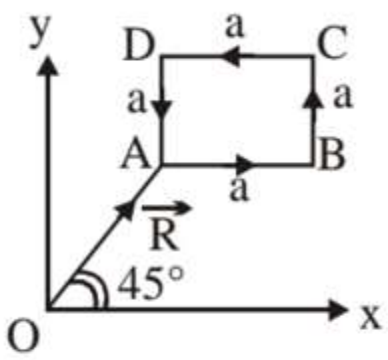
$t = 80 \text{ min} = 4\, T_A = 2T_B$

no. of nuclei of A decayed $= N_0 - \frac{N_0}{2^4} = \frac{15N_0}{16}$

no. of nuclei of B decayed $= N_0 - \frac{N_0}{2^2} = \frac{3N_0}{4}$

required ratio $= \frac{5}{4}$

35. A particle of mass m is moving along the side of a square of side 'a', with a uniform speed υ in the x-y plane as shown in the figure:



Which of the following statements is false for the angular momentum $\vec{L}$ about the origin?

(1) $\vec{L} = \frac{m\upsilon}{\sqrt{2}} R\hat{k}$ when the particle is moving from D to A.

(2) $\vec{L} = -\frac{m\upsilon}{\sqrt{2}} R\hat{k}$ when the particle is moving from A to B.

(3) $\vec{L} = m\upsilon\left[\frac{R}{\sqrt{2}} - a\right]\hat{k}$ when the particle is moving from C to D.

(4) $\vec{L} = m\upsilon\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$ when the particle is moving from B to C.

Solution: (1, 3)

$\vec{L} = \vec{r} \times \vec{P}$ or $\vec{L} = rp\sin\theta\,\hat{n}$

Or $\vec{L} = r_{\perp}(P)\hat{n}$

For D to A, $\vec{L} = \frac{R}{\sqrt{2}} mV(-\hat{k})$

For A to B, $\vec{L} = \frac{R}{\sqrt{2}} mV(-\hat{k})$

For C to D, $\vec{L} = \left(\frac{R}{\sqrt{2}} + a\right) mV(\hat{k})$

For B to C, $\vec{L} = \left(\frac{R}{\sqrt{2}} + a\right) mV(\hat{k})$

36. Choose the correct statement:

(1) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the frequency of the audio signal.

(2) In amplitude modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

(3) In amplitude modulation the frequency of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

(4) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

Solution: (2)

As per properties of A.M. in amplitude modulation the amplitude of high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

37. In an experiment for determination of refractive index of glass of a prism by $i = \delta$, plot, it was found that a ray incident at angle 35^o, suffers a deviation of 40^o and that it emerges at angle 79^o. In that case which of the following is closest to the maximum possible value of the refractive index?

(1) 1.8

(2) 1.5

(3) 1.6

(4) 1.7

Solution: (2)

$i = 35^o, \delta = 40^o, e = 79^o$

$\delta = i + e - A$

$40^o = 35^o + 79^o - A$

$A = 74^o$

And $r_1 + r_2 = A = 74^o$

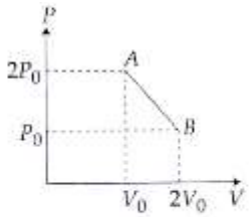
Solving these, we get $\mu = 1.5$

Since $\delta_{min} < 40^o$

$$\mu < \frac{\sin\left(\frac{74+40}{2}\right)}{\sin 37}$$

$\mu_{max} = 1.44$

38. 'n' moles of an ideal gas undergoes a process $A \rightarrow B$ as shown in the figure. The maximum temperature of the gas during the process will be:



(1) $\frac{9\,P_0V_0}{nR}$

(2) $\frac{9\,P_0V_0}{4nR}$

(3) $\frac{3\,P_0V_0}{2nR}$

(4) $\frac{9\,P_0V_0}{2nR}$

Solution: (2)

T_{max} at mid point

$$T = \frac{pv}{nR} = \frac{\left(\frac{3}{2}P_0\right)\left(\frac{3V_0}{2}\right)}{nR}$$

$$= \frac{9}{4}\left(\frac{P_0V_0}{nR}\right)$$

39. Two identical wires A and B, each of length 'l', carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side 'a'. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is :

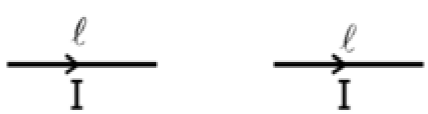
(1) $\frac{\pi^2}{8\sqrt{2}}$

(2) $\frac{\pi^2}{8}$

(3) $\frac{\pi^2}{16\sqrt{2}}$

(4) $\frac{\pi^2}{16}$

Solution: (1)



$2\pi R = 4a$

$\frac{a}{R} = \frac{2\pi}{4}\frac{a}{R} = \frac{\pi}{2}$

$$B_A = \frac{\mu_0 i}{2R}$$

$$B_B = \frac{\mu_0 i}{\pi a}(2\sqrt{2})$$

$$\frac{B_A}{B_B} = \frac{\mu_0 i}{2R} \times \frac{\pi a}{2\sqrt{2}\,\mu_0 i}$$

$$= \frac{\pi a}{4\sqrt{2}R} = \frac{\pi}{4\sqrt{2}}\left(\frac{\pi}{2}\right) = \frac{\pi^2}{8\sqrt{2}}$$

40. A screw gauge with a pitch of 0.5 mm and a circular scale with 50 divisions is used to measure the thickness of a thin sheet of Aluminium. Before starting the measurement, it is found that when the two jaws of the screw gauge are brought in contact, the 45^{th} division coincides with the main scale line and that the zero of the main scale is barely visible. What is the thickness of the sheet if the main scale reading is 0.5 mm and the 25^{th} division coincides with the main scale line?

(1) 0.50 mm

(2) 0.75 mm

(3) 0.80 mm

(4) 0.70 mm

Solution: (3)

$$LC = \frac{0.5}{50} = 0.01\text{ mm}$$

Zero error $= 0.50 - 0.45 = -0.05$

Thickness $= (0.5 + 25 \times 0.01) + 0.05$

$= 0.5 + 0.25 + 0.05$

$= 0.8\text{ mm}$

41. For a common emitter configuration, if α and β have their usual meanings, the incorrect relationship between α and β is:

(1) $\alpha = \frac{\beta^2}{1+\beta^2}$

(2) $\frac{1}{\alpha} = \frac{1}{\beta} + 1$

(3) $\alpha = \frac{\beta}{1-\beta}$

(4) $\alpha = \frac{\beta}{1+\beta}$

Solution: (1, 3)

$I_E = I_C + I_B;$

$\frac{I_E}{I_C} = 1 + \frac{I_B}{I_C}$

$\frac{1}{\propto} = 1 + \frac{1}{\beta}; \frac{1}{\propto} = \frac{\beta+1}{\beta}$

$\propto = \frac{\beta}{1+\beta}$

42. The box of a pin hole camera, of length L has a hole of radius a. It is assumed that when the hole is illuminated by a parallel beam of light of wavelength λ the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size (say b_{min}) when:

(1) $a = \frac{\lambda^2}{L}$ and $b_{min} = \sqrt{4\lambda L}$

(2) $a = \frac{\lambda^2}{L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$

(3) $a = \sqrt{\lambda L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$

(4) $a = \sqrt{\lambda L}$ and $b_{min} = \sqrt{4\lambda L}$

Solution: (4)

The diffraction angle λa cause a spreading of $\frac{L\lambda}{a}$ in the size of the spot. These become large when a (Radius) is small.

So adding of two kind of spreading (for simplicity) we get spot size is

$a + \frac{La}{a}$.

Hence to find out minimum value of this

We can write it as $\sqrt{\left(a-\frac{L\lambda}{a}\right)^2+4L\lambda}$

∵ The minimum value is when $a=\frac{La}{a}$ i.e. the geometric and diffraction broadening are equal $\sqrt{4L\lambda}$

∴ When $a=\sqrt{\lambda L}$ and $b_{min}=\sqrt{4\lambda L}$

43. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take $g=9.8\ ms^{-2}$:

(1) 12.89×10^{-3} kg

(2) 2.45×10^{-3} kg

(3) 6.45×10^{-3} kg

(4) 9.89×10^{-3} kg

Solution: (1)

$m=10kg, h=1m, 1000$ times

$PE=98\ J\times1000=98000\ J=98\ kJ$

$=9.8\times10^4\ J$

Fat burn $=3.8\times10^7\ J\times0.2$

$=7.6\times10^6$ J per kg

$$m=\frac{9.8\times10^4}{7.6\times10^6}=1.289\times10^{-2}$$

$=12.89\times10^{-3}$ kg

44. Arrange the following electromagnetic radiations per quantum in the order of increasing energy:

A : Blue light

B: Yellow light

C: X-ray

D: Radiowave

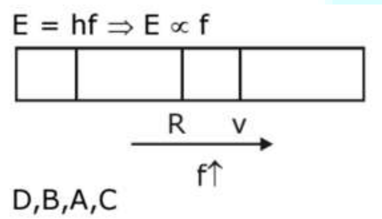
(1) B, A, D, C

(2) D, B, A, C

(3) A, B, D, C

(4) C, A, B, D

Solution: (2)



45. An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by PV^n = constant, then n is given by (Here C_P and C_V are molar specific heat at constant pressure and constant volume, respectively) :

(1) $n = \frac{C-C_V}{C-C_P}$

(2) $n = \frac{C_P}{C_V}$

(3) $n = \frac{C-C_P}{C-C_V}$

(4) $n = \frac{C_P-C}{C-C_V}$

Solution: (3)

$PV^n = k$

$C = C_v + \frac{R}{1-n}; C - C_v = \frac{R}{1-n}$

$1 - n = \frac{R}{C-C_v}; n = 1 - \frac{R}{C-C_v}$

$n = \frac{C-C_v-R}{C-C_v}; n = \frac{C-C_v-(C_p-C_v)}{C-C_v}$

$n = \frac{C-C_v-C_p+C_v}{C-C_v}; n = \frac{C-C_p}{C-C_v}$

46. A satellite is revolving in a circular orbit at a height ‘h’ from the earth’s surface (radius of earth R; h<<R). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth’s gravitational field, is close to: (Neglect the effect of atmosphere.)

(1) $\sqrt{gR}\left(\sqrt{2}-1\right)$

(2) $\sqrt{2\,gR}$

(3) $\sqrt{gR}$

(4) $\sqrt{\frac{gR}{2}}$

Solution: (1)

Since h<<R

$V_0 = \sqrt{2gR}$

& $V_e = \sqrt{gR}$

∴ min velocity required

$V_0 - V_e = \sqrt{2gR} - \sqrt{gR}$

$= (\sqrt{2} - 1)\sqrt{gR}$

47. A galvanometer having a coil resistance of $100\ \Omega$ gives a full scale deflection, when a current of 1 mA is passed through it. The value of the resistance, which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10 A, is:

(1) $3\ \Omega$

(2) $0.01\ \Omega$

(3) $2\ \Omega$

(4) $0.1\ \Omega$

Solution: (2)

We know that

$R_S = \frac{I_G}{1 - I_G} R_G$

$= \frac{1 \times 10^{-3}}{10} \times 100$

$= 10^{-2}$

$= 0.01\ \Omega$

48. Radiation of wavelength λ, is incident on a photocell. The fastest emitted electron has speed v. If the wavelength is changed to $\frac{3\lambda}{4}$, the speed of the fastest emitted electron will be:

(1) $= v\left(\frac{3}{4}\right)^{\frac{1}{2}}$

(2) $> v\left(\frac{4}{3}\right)^{\frac{1}{2}}$

(3) $< v\left(\frac{4}{3}\right)^{\frac{1}{2}}$

(4) $= v\left(\frac{4}{3}\right)^{\frac{1}{2}}$

Solution: (2)

$\frac{1}{2}mv^2 = \frac{hc}{\lambda} - \phi \quad \ldots$(i)

$\frac{1}{2}mv'^2 = \frac{4}{3}\frac{hc}{\lambda} - \phi \quad \ldots$(ii)

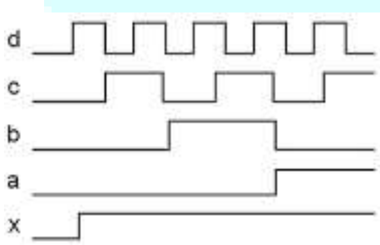
From eqn. (i) $\frac{hc}{\lambda} = \frac{1}{2}mv^2 + \phi$

On putting this equ. (ii)

$$\frac{1}{2}mv'^2 = \frac{4}{3}\left(\frac{1}{2}mv^2 + \phi\right) - \phi$$

$$v' > v\sqrt{\frac{4}{3}}$$

49. If a, b, c are inputs to a gate and x is its output, then, as per the following time graph, the gate is:



(1) NAND

(2) NOT

(3) AND

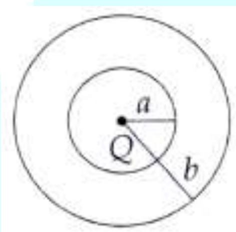
(4) OR

Solution: (4)

a	b	c	d	X
0	0	0	0	0
0	0	0	1	1

When any input is one output is one hence the gate is 'OR' gate.

50. The region between two concentric spheres of radii 'a' and 'b', respectively (see figure), has volume charge density $\rho = \frac{A}{r}$, where A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant, is:



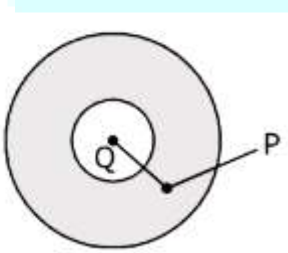
(1) $\frac{2Q}{\pi a^2}$

(2) $\frac{Q}{2\pi a^2}$

(3) $\frac{Q}{2\pi(b^2-a^2)}$

(4) $\frac{2Q}{\pi(a^2-b^2)}$

Solution: (2)



$E(r^2) = \frac{1}{\epsilon_0}\int_a^r \rho r^2 dr;$

$$E(r^2) = \frac{A}{\epsilon_0}\int_a^r r dr$$

$$= \frac{A}{\epsilon_0}\left[\frac{r^2}{2}\right]_a^r$$

$$E(r^2) = \frac{A}{2\epsilon_0}\,[r^2 - a^2];$$

$$E = \frac{A}{2\,\epsilon_0\,r^2}(r^2 - a^2) + \frac{kQ}{r^2}$$

$$E = \frac{A}{\epsilon_0}\left[1 - \frac{a^2}{r^2}\right] + \frac{kQ}{r^2}$$

$$\frac{dE}{dr} = 0;$$

$$\frac{a^2 A}{2\,\epsilon_0} = KQ$$

$$A = \frac{2KQ\epsilon_0}{a^2};$$

$$= \frac{2}{4\pi\varepsilon_0}\frac{Q\varepsilon_0}{a^2} = \frac{Q}{2\pi a^2}$$

51. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90s, 91s, 95s and 92s. If the minimum division in the measuring clock is 1s, then the reported mean time should be:

(1) 92 ± 3 s

(2) 92 ± 2 s

(3) 92 ± 5.0 s

(4) 92 ± 1.8 s

Solution: (2)

T	T_s	$T_i - T$	$(T_i - T)^2$
t_1	90	-2	4
t_2	91	-1	1
t_3	95	3	9
t_4	92	0	0
T_i	92	$\frac{\Sigma T_i - T}{N} = 0$	$\frac{\Sigma (T_i - T)^2}{N} = 3.5$

$$T_r = T \pm \sqrt{\frac{\Sigma (T_i - T)^2}{N}}$$

$T_r = 92 \pm \sqrt{3.5}$

$T_r = 92 \pm \sqrt{1.8}$

$T_r = 92 \pm 2$

Because least count of clock is 1s.

52. The temperature dependence of resistance of Cu and undoped Si in the temperature range 300-400 K, is best described by:

(1) Linear decrease for Cu, linear decrease for Si.

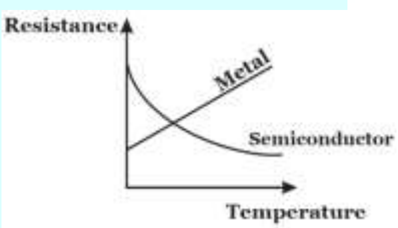
(2) Linear increase for Cu, linear increase for Si.

(3) Linear increase for Cu, exponential increase for Si.

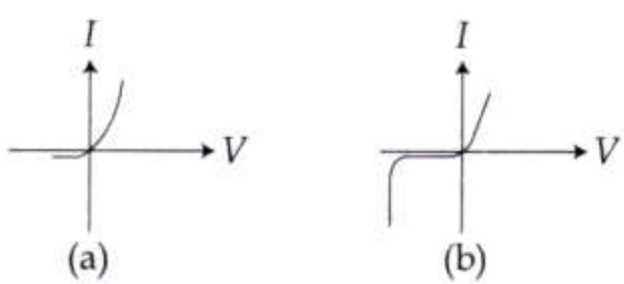
(4) Linear increase for Cu, exponential decrease for Si.

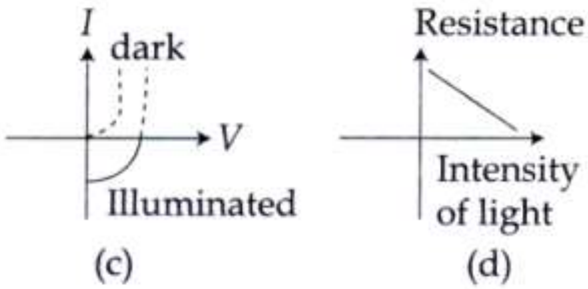
Solution: (4)

Resistance variation with temperature: Cu-metal, undoped Silicon-Semi Conductor resistance of metal increases with increase in temperature linearly resistance of semi Conductor decreases exponentially with increase in temperature.



53. Identify the semiconductor devices whose characteristics are given below, in the order (a), (b), (c), (d):





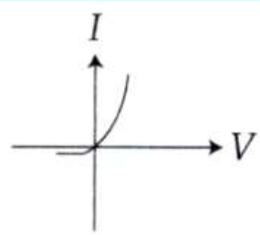
(1) Zener diode, Solar cell, Simple diode Light dependent resistance

(2) Simple diode, Zener diode, Solar cell, Light dependent resistance

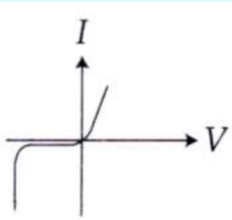
(3) Zener diode, Simple diode, Light dependent resistance, Solar cell

(4) Solar cell, Light dependent resistance, Zener diode, Simple diode

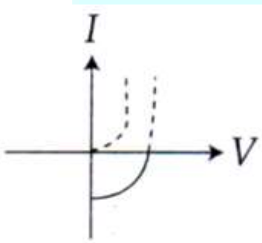
Solution: (2)



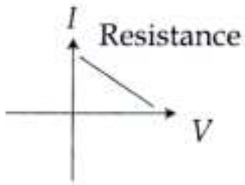
Its V-I characteristics of simple diode.



Its V-I characteristic of Zener diode.

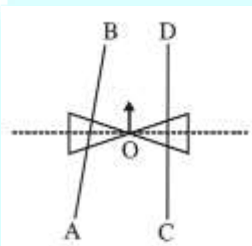


V-I characteristics of Solar cell



V-I characteristics of light dependence resistance.

54. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to:



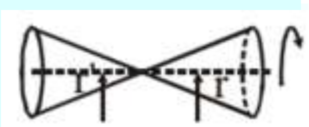
(1) turn left right alternately.

(2) turn left.

(3) turn right.

(4) go straight.

Solution: (2)



Say the distance of central line from instantaneous axis of rotation is r.

Then r from the point on left becomes lesser than that for right.

So V_{left} point $= \omega r' < \omega r = v_{right}$ point

So the roller will turn to left.

55. A pendulum clock loses 12s a day if the temperature is 40^{o} C and gains 4s a day if the temperature is 20^{o} C. The temperature at which the clock will show correct time, and the co-efficient of linear expansion (α) of the metal of the pendulum shaft are respectively:

(1) $55^{o}C; \alpha = 1.85 \times 10^{-2}/ \ {}^{o}C$

(2) $25^oC; \alpha = 1.85 \times 10^{-5}/ \ ^oC$

(3) $60^oC; \alpha = 1.85 \times 10^{-4}/ \ ^oC$

(4) $30^oC; \alpha = 1.85 \times 10^{-3}/ \ ^oC$

Solution: (2)

$\Delta T \propto \Delta\theta$

$$\frac{12}{4} = \frac{40-\theta}{\theta-20}$$

$3\theta - 60 = 40 - \theta$

$4\theta = 100$

$\theta = 25^oC$

$$\Delta T = \frac{1}{2}\alpha\,\Delta\theta \times T$$

$4 = \frac{1}{2}\alpha\, 5 \times 86400;$

$\frac{8\times10^5}{5\times86400} = \alpha;$

$$\frac{8000}{4320} = \alpha$$
$\alpha = 1.05 \times 10^{-5}/ \ ^oC$

56. A uniform string of length 20m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is:

(take $g = 10\ ms^{-2}$)

(1) $\sqrt{2}\ s$

(2) $2\pi\sqrt{2}\ s$

(3) $2s$

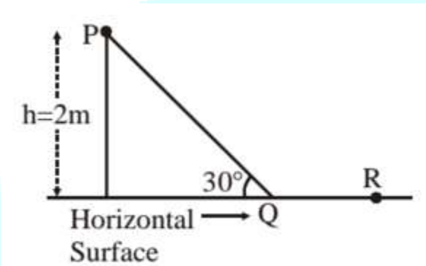
(4) $2\sqrt{2}\ s$

Solution: (4)

$t = 2\sqrt{\frac{l}{g}} = 2\sqrt{2}$ second.

57. A point particle of mass m, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, from rest, from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR.

The values of the coefficient of friction μ and the distance $x = (QR)$, are respectively close to:



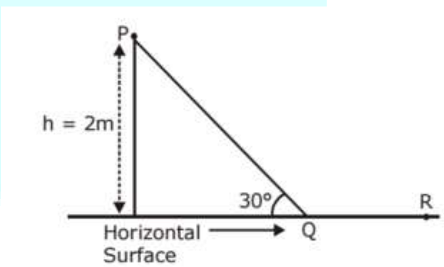
(1) 0.29 and 6.5 m

(2) 0.2 and 6.5 m

(3) 0.2 and 3.5 m

(4) 0.29 and 3.5 m

Solution: (4)



$$\tan 30^o = \frac{h}{l}$$

$l = h\sqrt{3} = 2\sqrt{3}\text{ m}$

$W_f = -\mu mgl$ or $W_f = -\mu mgx$

$\mu mgl = \mu mgx; \; x = l$

$x = 2\sqrt{3}\text{ m}; \; W_{all} = \Delta K$

$mgh - \mu mgl - \mu mgx = 0$

$h - \mu l - \mu x = 0$

$2 = \mu(l + x) \Rightarrow \mu = \frac{2}{l + x} = \frac{2}{4\sqrt{3}} = \frac{1}{2\sqrt{3}}$

58. A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now:

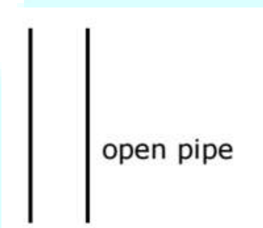
(1) f

(2) $\frac{f}{2}$

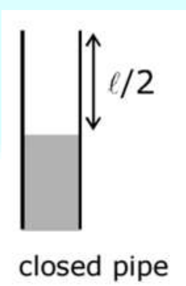
(3) $\frac{3f}{4}$

(4) $2f$

Solution: (1)



$f = \frac{v}{2l}$



$f' = \frac{v}{4l'} = \frac{v}{4\left(\frac{l}{2}\right)} = f$

59. A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is:

(1) $\frac{7A}{3}$

(2) $\frac{A}{3}\sqrt{41}$

(3) $3A$

(4) $A\sqrt{3}$

Solution: (1)

$v = \omega\sqrt{A^2 - X^2}$;

$$v = \omega\sqrt{A^2 - \frac{4A^2}{9}}$$

$$= \frac{\omega\sqrt{5}A}{3}$$

New SHM will be,

$3v - \omega\sqrt{A_N^2 - X_n^2}$;

$$\frac{3\omega\sqrt{5}A}{3} = \omega\sqrt{A_N^2 - \frac{4A^2}{9}}$$

$$5A^2 = A_N^2 - \frac{4A^2}{9}$$

$$A_N^2 = \frac{49A^2}{9}$$

$$A_N = \frac{7A}{3}$$

60. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to:

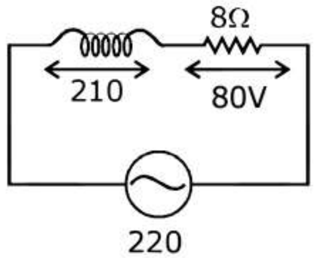
(1) 0.065 H

(2) 80 H

(3) 0.08 H

(4) 0.044 H

Solution: (1)



$V_L^2 + 6400 = 220 \times 220$

$IR = 80$

$V_L = \sqrt{48400 - 6400}$

$I = \frac{80}{8} = 10 = \sqrt{42000} = 210$

$I\,X_L = 210$

$X_L = 2\pi fL = 210$

$L = \frac{210}{10 \times 100\,\pi} = 0.065\ H$

CHEMISTRY

61. Which of the following statements about water is FALSE?

(1) Ice formed by heavy water sinks in normal water

(2) Water is oxidized to oxygen during photosynthesis

(3) Water can act both as an acid and as a base

(4) There is extensive intramolecular hydrogen boding in the condensed phase

Solution: (4)

(i) Ice formed by heavy water sinks in normal water due to higher density of D_2O than normal water.

(ii)

$$6CO_2 + 6H_2O \xrightarrow[\text{chlorophyll}]{h\nu} C_6H_{12}O_6 + 6O_2$$

(iii) Water can show amphiprotic nature and hence water can act both as an acid a base.

(iv) There is extensive intermolecular hydrogen bonding in the condensed phase instead of intramolecular H – bonding.

62. The concentration of fluoride, lead, nitrate and iron in a water sample from an underground lake was found to be 1000 ppb, 40 ppb, 100 ppm and 0.2 ppm, respectively. This water is unsuitable for drinking due to high concentration of:

(1) Iron

(2) Fluoride

(3) Lead

(4) Nitrate

Solution: (4)

Concentration of fluoride = 1000 PPb

= 1 PPm

Concentration of lead = 40 PPb

= 0.04 PPm

Concentration of nitrate = 100 PPm

Concentration of iron = 0.2 PPm

High concentration of nitrate

63. Galvanization is applying a coating of:

(1) Zn

(2) Pb

(3) Cr

(4) Cu

Solution: (1)

Galvanization is the process of applying zinc coating to steel (or) iron, to prevent rusting

64. Which one of the following complexes shows optical isomerism?

(1) $[Co(NH_3)_4 Cl_2]$ Cl

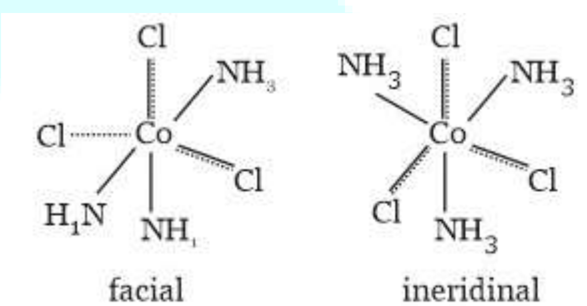
(2) $[Co (NH_3)_3 Cl_3]$

(3) cis $[Co(en)_2 Cl_2]Cl$

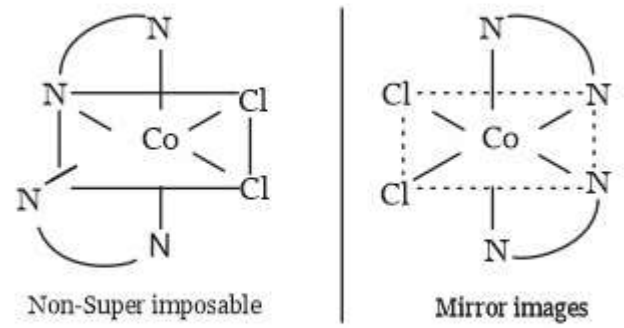
(4) trans $[Co(en)_2 Cl_2]$ Cl

Solution: (3)

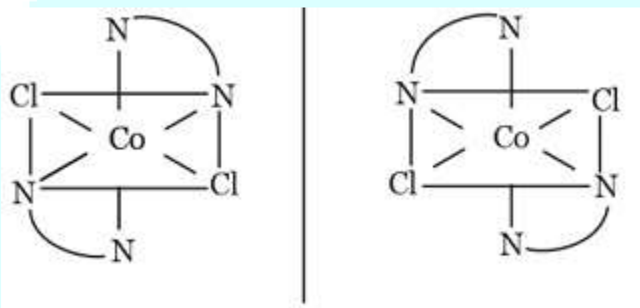
Geometrical isomers

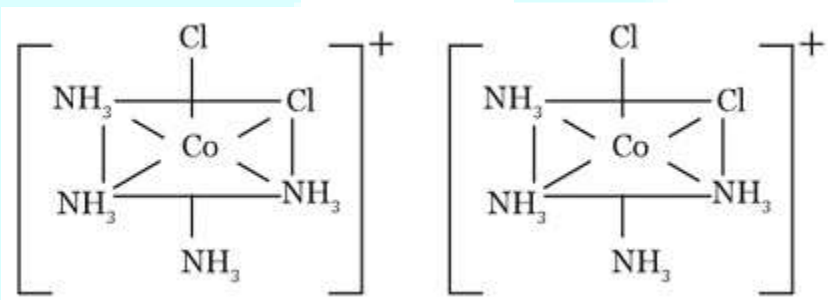


cis $[Co(en)_2Cl_2]Cl$



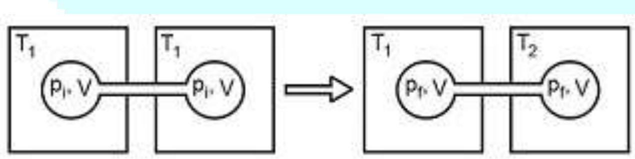
trans $[Co(en)_2Cl_2]Cl$





Cis trans

65. Two closed bulbs of equal volume (V) containing an ideal gas initially at pressure p_i and temperature T_1 are connected through a narrow tube of negligible volume as shown in the figure below. The temperature of one of the bulbs is then raised to T_2. The final pressure P_f is:



(1) $2p_i\left(\frac{T_1T_2}{T_1+T_2}\right)$

(2) $p_i\left(\frac{T_1T_2}{T_1+T_2}\right)$

(3) $2p_i\left(\frac{T_1}{T_1+T_2}\right)$

(4) $2p_i\left(\frac{T_2}{T_1+T_2}\right)$

Solution: (4)

Given two closed bulbs of equal volume (v) containing ideal gas initially of pressure p_i and temperature T_1 which are connected by narrow tube of negligible volume.

To find final pressure P_f when one raised to T_2

No. of moles of gas doesn't change

$(n_T)_i = (n_T)_f$

$$\frac{P_iV}{RT_1}+\frac{P_iV}{RT_1}=\frac{P_fV}{RT_1}+\frac{P_fV}{RT_2}$$

$$2\frac{P_1}{T_1}=\frac{P_f}{T_1}+\frac{P_f}{T_2}$$

66. The heats of combustion of carbon and carbon monoxide are -393.5 and -283.5 kJ mol^{-1}, respectively. The heat of formation (in kJ) of carbon monoxide per mole is:

(1) -110.5

(2) 110.5

(3) 676.5

(4) -676.5

Solution: (1)

Given heat of combustion of carbon and carbon monoxide are -393.5 and -283.5 kJ mol^{-1}, respectively

(i) $C + O_2 \rightarrow CO_2 \quad \Delta H_1 = -393.5\text{ kJ mol}^{-1}$

(ii) $CO + \frac{1}{2}O_2 \rightarrow CO_2 \quad \Delta H_2 = -283.5\text{ kJ mol}^{-1}$

To find heat of formation of CO per mole.

i.e. $C + \frac{1}{2}O_2 \rightarrow CO$

$(ii) \times (2)$

$2CO + O_2 \rightarrow 2CO_2 \quad \Delta H_3 = -283.5 \times 2 \text{ kJ mol}^{-1}$

$2CO_2 \rightarrow 2CO + O_2 \quad \Delta H_4 = 567 \text{ kJ mol}^{-1}$

(i) × (2)

$2C + 2O_2 \rightarrow 2CO_2 \quad \Delta H_5 = -393.5 \times 2 \text{ kJ mol}^{-1}$

$= -787 \text{ kJ mol}^{-1}$

$2C + O_2 \rightarrow 2CO \quad \Delta H_6 = -220 \text{ kJ mol}^{-1}$

For one mole of CO,

$\Delta H = \frac{-220}{2} = -110 \text{ kJ mol}^{-1}$

67. At 300 K and 1 atm, 15mL of a gaseous hydrocarbon requires 375 mL air containing 20% O_2 by volume for complete combustion. After combustion the gases occupy 330 mL. Assuming that the water formed is in liquid form and the volumes were measured at the same temperature and pressure, the formula of the hydrocarbon is:

(1) C_4H_{10}

(2) C_3H_6

(3) C_3H_8

(4) C_4H_8

Solution: (Bonus or 3)

Volume of N_2 in air $= 375 \times 0.8 = 300\, ml$

Volume of O_2 in air $= 375 \times 0.2 = 75\, ml$

$$\begin{array}{ccccc} C_xH_y & + \left(x+\frac{y}{4}\right)O_2 & \rightarrow & xCO_2(g) & + \frac{y}{2}H_2O(\ell) \\ 15ml & 15\left(x+\frac{y}{4}\right) & & 15x & - \\ 0 & 0 & & & \end{array}$$

After combustion total volume

$330 = V_{N_2} + V_{CO_2}$

$330 = 300 + 15x$

$x = 2$

Volume of O_2 used

$15\left(x+\frac{y}{4}\right)=5$

$x+\frac{y}{4}=5$

$y=12$

So hydrocarbon is $= C_2H_{12}$

None of the option matches it therefore it is a BONUS.

Alternatively

$$\begin{array}{lllll} C_xH_y & + & \left(x+\frac{y}{4}\right)O_2 & \rightarrow & xCO_2(g) & + & \frac{y}{2}H_2O(\ell) \\ 15ml & & 15\left(x+\frac{y}{4}\right) & & 15x & & - \\ 0 & & 0 & & & & \end{array}$$

Volume of O_2 used

$15\left(x+\frac{y}{4}\right)=75$

$x+\frac{y}{4}=5$

If further information (i.e., 330 ml) is neglected, option (C_3H_8) only satisfy the above equation.

68. Decomposition of H_2O_2 follows a first order reaction. In fifty minutes the concentration of H_2O_2 decreases from 0.5 to 0.125 M in one such decomposition. When the concentration of H_2O_2 reaches 0.05 M, the rate of formation of O_2 will be:

(1) 1.34×10^{-2} mol min^{-1}

(2) 6.93×10^{-2} mol min^{-1}

(3) 6.93×10^{-4} mol min^{-1}

(4) 2.66 L min^{-1} at STP

Solution: (3)

$2H_2O_2 \rightarrow 2H_2O + O_2$

$[O_2]=\frac{[H_2O_2]}{2}=\frac{[H_2O]}{2}$

For, $t_{\frac{1}{2}}, H_2O_2$ decreases to 0.125M from 0.5M

So, $2 \times t_{\frac{1}{2}} = 50$

$t_{\frac{1}{2}} = 25$

$t_{\frac{1}{2}} = \frac{0.69314}{K}$

$K = \frac{0.69314}{25}$

$[O_2] = \frac{1}{2} \times \frac{0.69314}{25}$

$= 6.93 \times 10^{-4}\ \text{mol min}^{-1}$

69. The pair having the same magnetic moment is:

$[\text{At. No.: Cr} = 24, \text{Mn} = 25, \text{Fe} = 26, \text{Co} = 27]$

(1) $[CoCl_4]^{2-}$ and $[Fe(H_2O)_6]^{2+}$

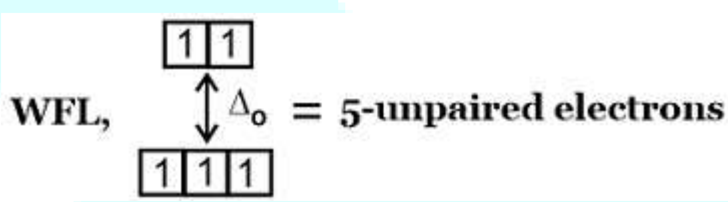
(2) $[Cr(H_2O)_6]^{2+}$ and $[CoCl_4]^{2-}$

(3) $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$

(4) $[Mn\ (H_2O)_6]^{2+}$ and $[Cr(H_2O)_6]^{2+}$

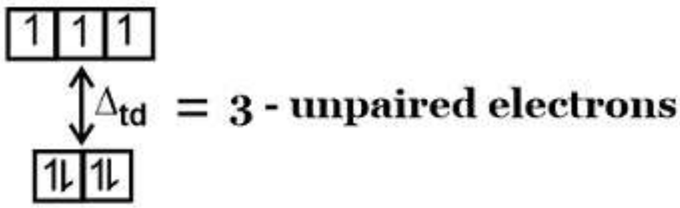
Solution: (3)

In option A: $[Mn(H_2O)_6]^{2+}(3d^5)$ with

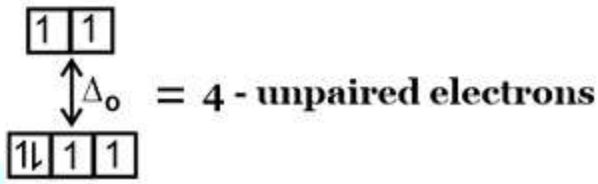


& $[Cr(H_2O)_6]^{2+}, Cr^{2+}(3d^4)$ with W.F.L.,

In option B: $[CoCl_4]^{2-}, Co^{2+}(3d^7)$ with W.F.L.,

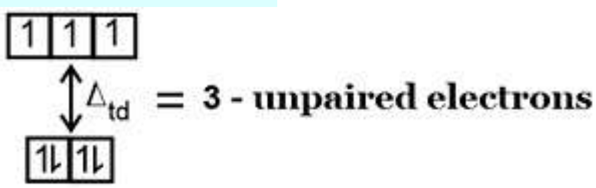


& $[Fe(H_2O)_6]^{2+}, Fe^{2+}(3d^6)$ with W.F.L.,



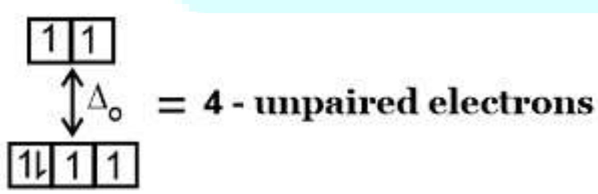
In option C: $[Cr(H_2O)_6]^{2+}, Cr^2 + (3d^4)$ with W.F.L.,

& $[CoCl_4]^{2-}$, Co^{2+} $(3d^7)$ with W.F.L.,



In option D: $[Cr(H_2O)_6]^{2+}, Cr^{2+}(3d^4)$ with W.F.L.,

& $[Fe(H_2O)_6]^{2+}, Fe^{2+}(3d^6)$ with W.F.L.,



Here both complexes have same unpaired electrons i.e. = 4

70. The species in which the N atom is a state of sp hybridization is:

(1) NO_2

(2) NO_2^+

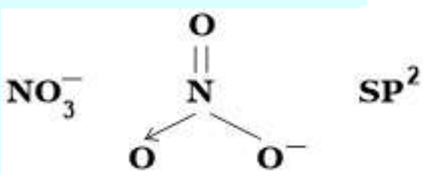
(3) NO_2^-

(4) NO_3^-

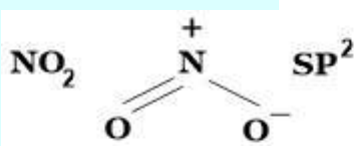
Solution: (2)

N SP

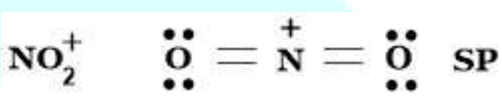
(i)



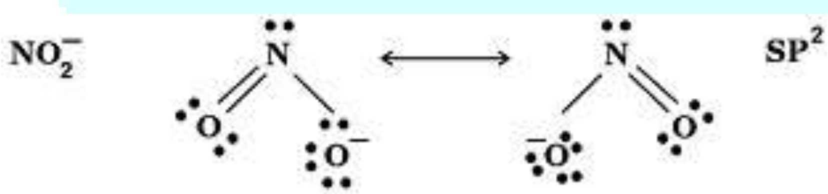
(ii)



(iii)



(iv)



71. Thiol group is present in:

(1) Methionine

(2) Cytosine

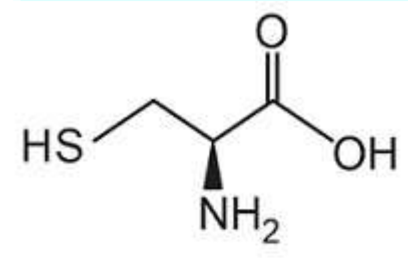
(3) Cystine

(4) Cysteine

Solution: (4)

Thiol group $(-SH)$

Cysteine



72. The pair in which phosphorous atoms have a formal oxidation state of + 3 is:

(1) Pyrophosphorous and pyrophosphoric acids

(2) Orthophosphorous and pyrophosphorous acids

(3) Pyrophosphorous and hypophosphoric acids

(4) Orthophosphorous and hypophospheric acids

Solution: (2)

Acid	Formula	Formal oxidation state of phosphorous
Pyrophosphorous acid	$H_4P_2O_5$	$+3$
Pyrophosphoric acid	$H_4P_2O_7$	$+5$
Orthophosphorous acid	H_3PO_3	$+3$
Hypophosphoric acid	$H_4P_2O_6$	$+4$

Both pyrophosphorous and orthophosphorous acid have $+3$ formal oxidation state

73. The distillation technique most suited for separating glycerol from spent – lye in the soap industry is:

(1) Distillation under reduced pressure

(2) Simple distillation

(3) Fractional distillation

(4) Steam distillation

Solution: (1)

Glycerol (B.P. $290^{o}\mathrm{C}$) is separated from spent – lye in the soap industry by distillation under reduced pressure, as for simple distillation very high temperature is required which might decompose the component.

74. Which one of the following ores is best concentrated by froth floatation method?

(1) Malachite

(2) Magnetite

(3) Siderite

(4) Galena

Solution: (4)

Froth floatation method is used for concentration of sulphide ores.

$\downarrow \ \mathrm{Galena} \ \rightarrow \ \mathrm{Pbs}$

75. Which of the following atoms has the highest first ionization energy?

(1) Sc

(2) Rb

(3) Na

(4) K

Solution: (1)

Na is the smallest element in the IA group elements and it has highest IE among K, Rb

Sc has lowest effective nuclear charge

Effective nuclear charge $\times \frac{1}{\text{I.E}}$

Sc has low effective nuclear charge than Na.

So it has I.E. among given elements.

76. In the Hofmann bromamide degradation reaction, the number of moles of NaOH and Br_2 used per mole of amine produced are:

(1) Four moles of NaOH and one mole of Br_2

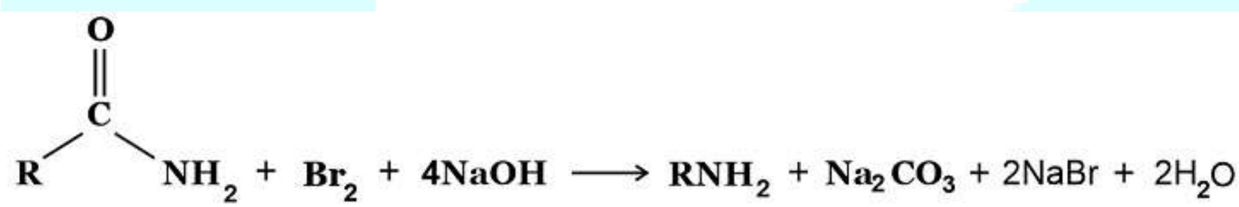
(2) One mole of NaOH and one mole of Br_2

(3) Four moles of NaOH and two moles of Br_2

(4) Two moles of NaOH and two moles of Br_2

Solution: (1)

To find number of moles of NaOH and Br_2 used per mole of amine produced.



77. Which of the following compounds is metallic and ferromagnetic?

(1) MnO_2

(2) TiO_2

(3) CrO_2

(4) VO_2

Solution: (3)

d – block elements are metals.

MnO_2 exhibit strong attraction to magnetic fields and are able to retain their magnetic properties.

So, it exhibits metallic character and it’s ferromagnetic.

78. Which of the following statements about low density polythene is FALSE?

(1) It is used in the manufacture of buckets, dust – bins etc.

(2) Its synthesis requires high pressure

(3) It is a poor conductor of electricity

(4) Its synthesis required dioxygen or a peroxide initiator as a catalyst

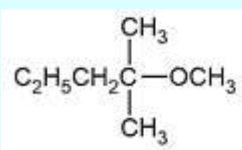
Solution: (1)

Low density polythene: It is obtained by the polymerization of ethene high pressure of 1000-2000 atm. at a temp. of 350 K to 570 K in the pressure of traces of dioxygen or a peroxide initiator (cont).

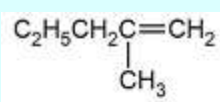
Low density polythene is chemically inert and poor conductor of electricity. It is used for manufacture squeeze bottles. Toys and flexible pipes.

79. 2 – chloro – 2 – methylpentane on reaction with sodium methoxide in methanol yields:

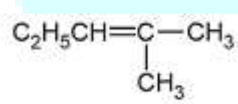
(i)



(ii)



(iii)



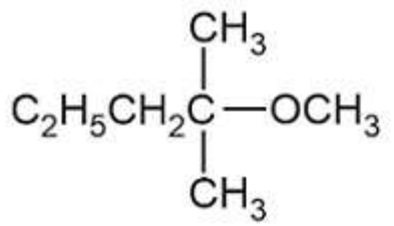
(1) i and ii

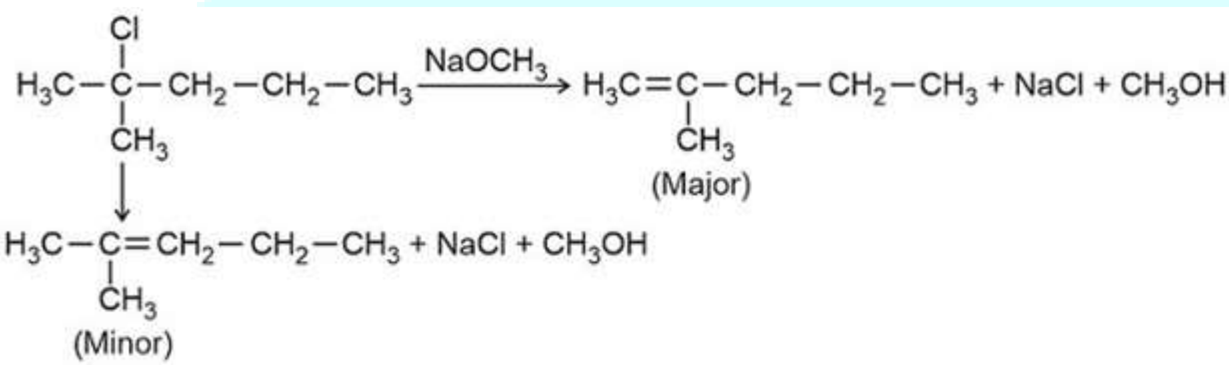
(2) All of these

(3) i and iii

(4) iii only

Solution: (2)





80. A stream of electrons from a heat filament was passed between two charge plates kept at a potential difference V esu. If e and m are charge and mass of an electron, respectively, then the value of $\frac{h}{\lambda}$ (where λ is wavelength associated with electron wave) is given by:

(1) $\sqrt{2\,\mathrm{meV}}$

(2) $\mathrm{me\,V}$

(3) $2\,\mathrm{me\,V}$

(4) $\sqrt{\mathrm{meV}}$

Solution: (1)

Given stream of electron from heated filament was passed between two charge plates at potential difference V

e, m are charge and mass of electron

$V = \frac{E}{e}$

$eV = \frac{1}{2} \times m \times V^2$

$\lambda = \frac{h}{mv} \quad V = \frac{h}{m\lambda}$

$\mathrm{eV} = \frac{1}{2} \times \mathrm{m} \times \left[\frac{\mathrm{h}}{\mathrm{m}\lambda}\right]^2$

$\frac{\mathrm{h}}{\lambda} = \sqrt{2\,\mathrm{meV}}$

81. 18 g glucose $(C_6H_{12}O_6)$ is added to 178.2 g water. The vapor pressure of water (in torr) for this aqueous solution is:

(1) 759.0

(2) 7.6

(3) 76.0

(4) 752.4

Solution: (4)

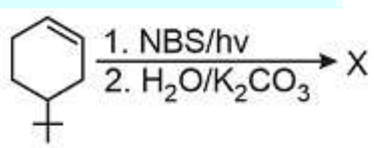
$\frac{\Delta P}{P^o}$ = mol. Fraction of glucose

$\frac{760-P_{Soln}}{760} = \frac{\frac{W_1}{Mwt_1}}{\frac{W_1}{M.wt_1}+\frac{W_2}{M.wt_2}} = \frac{\frac{18}{180}}{\frac{18}{180}+\frac{178.2}{18}} = \frac{0.1}{0.1+9.9} = \frac{1}{100}$

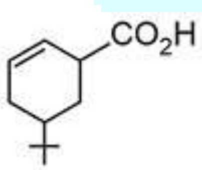
$760 - P_{Soln} = \frac{760}{100}$

$P_{Sol} = 752.4$

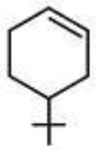
82. The product of the reaction give below is:



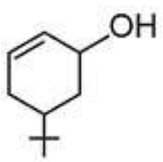
(1)



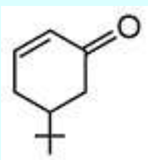
(2)



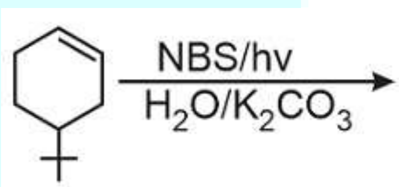
(3)

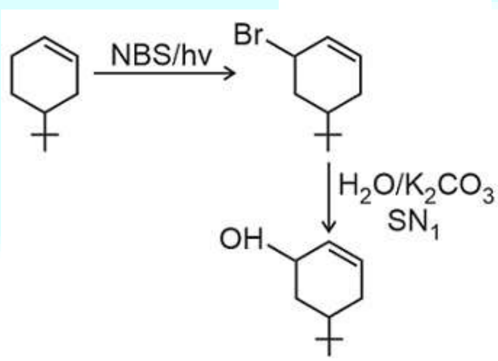


(4)

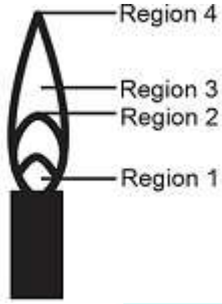


Solution: (3)





83. The hottest region of Bunsen flame shown in the figure below is:



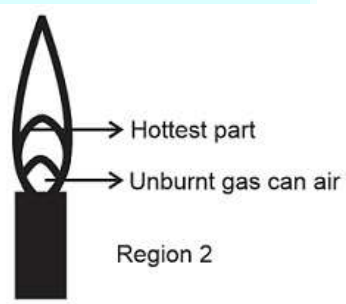
(1) Region 4

(2) Region 1

(3) Region 2

(4) Region 3

Solution: (3)



84. The reaction of zinc with dilute and concentrated nitric acid, respectively, produces:

(1) NO_2 and N_2O

(2) N_2O and NO_2

(3) NO_2 and NO

(4) NO and N_2O

Solution: (2)

Zn on reaction with HNO_3

$4Zn + 10HNO_3 \rightarrow 4Zn(NO_3)_2 + N_2O + 5H_2O$

$Zn + 4HNO_{3(con)} \rightarrow Zn(NO_3)_2 + 2NO_2 + 2H_2O$

85. Which of the following is an anionic detergent?

(1) Glyceryl oleate

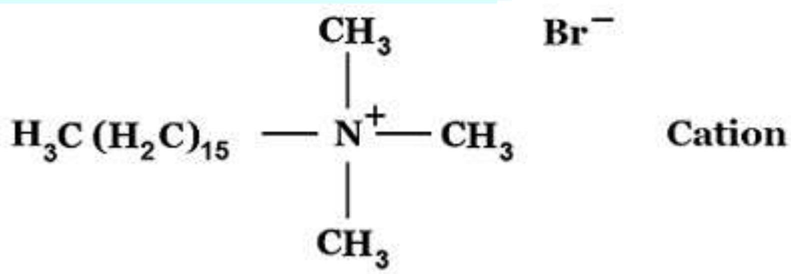
(2) Sodium stearate

(3) Sodium lauryl sulphate

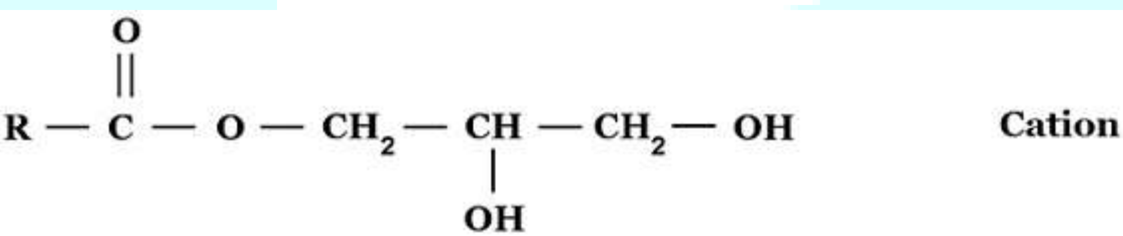
(4) Cetyltrimethyl ammonium bromide

Solution: (3)

(i) Cetyltrimethyl ammonium bromide

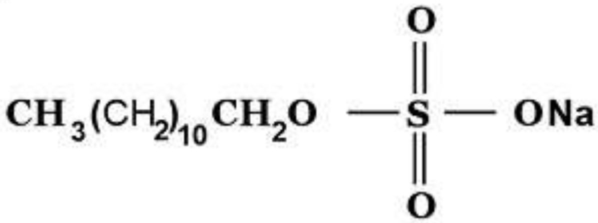


(ii) Glyceryl oleate



(iii) Sodium stearate

(iv) Anionic surfactant



86. The reaction of propene with HOCl $(Cl_2 + H_2O)$ proceeds through the intermediate:

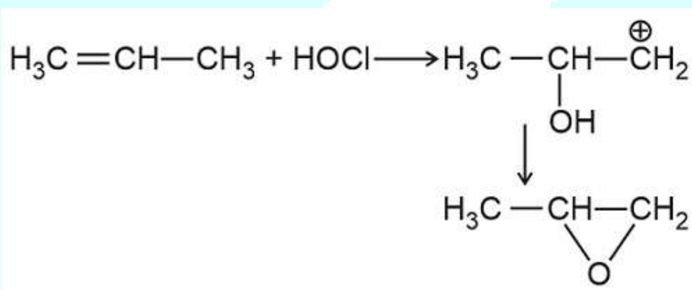
(1) $CH_3 - CHCl - CH_2^+$

(2) $CH_3 - CH^+ - CH_2 - OH$

(3) $CH_3 - CH^+ - CH_2 - Cl$

(4) $CH_3 - CH(OH) - CH_2^+$

Solution: (3)



87. For a linear plot of log $\left(\frac{x}{m}\right)$ versus log p in a Freundlich adsorption isotherm, which of the following statements is correct? (k and n are constants)

(1) $\log\left(\frac{1}{n}\right)$ appears as the intercept

(2) Both k and $\frac{1}{n}$ appear in the slope term

(3) $\frac{1}{n}$ appears as the intercept

(4) Only $\frac{1}{n}$ appears as the slope

Solution: (4)

According to Freundlich isotherm

$\frac{x}{m} = k.p^{\frac{1}{n}}$

$\log\frac{x}{m} = \log k + \frac{1}{n}\log P$

So intercept is log k and slope is $\frac{1}{n}$

88. The main oxides formed on combustion of Li, Na and K in excess of air are, respectively:

(1) Li_2O, Na_2O_2 and KO_2

(2) Li_2O, Na_2O and KO_2

(3) LiO_2, Na_2O_2 and K_2O

(4) Li_2O_2, Na_2O_2 and KO_2

Solution: (1)

In 1A group

Li on $r \times n$ with excess air

$4Li + O_2 \rightarrow 2Li_2O$

Na on $r \times n$ with excess air

$2Na + O_2 \rightarrow Na_2O_2$

K on $r \times n$ with excess air

$K + O_2 \rightarrow KO_2$

89. The equilibrium constant at 298 K for a reaction $A + B \rightleftharpoons C + D$ is 100, If the initial concentration of all the four species were 1 M each, then equilibrium concentration of D (in mol L^{-1}) will be:

(1) 1.182

(2) 0.182

(3) 0.818

(4) 1.818

Solution: (4)

K at 298 K for the reaction

$A + B \rightleftharpoons C + D$ is 100

Given initial concentration of all four species is 1M

At t = 0,

$\therefore \ \underset{1}{A} + \underset{1}{B} \rightleftharpoons \underset{1}{C} + \underset{1}{D}$

At equilibrium

$$\underset{1-x}{A} + \underset{1-x}{B} \rightleftharpoons \underset{1+x}{C} + \underset{1+x}{D}$$

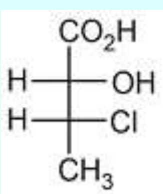
$$K = \frac{(1+x)\,(1+x)}{(1-x)\,(1-x)} = 100$$

$$\frac{(1-x)}{1-x} = 10$$

$$x = \frac{9}{11} = 0.818$$

$$[D] = 1 + x = 1 + 0.818 = 1.818$$

90. The absolute configuration of



is:

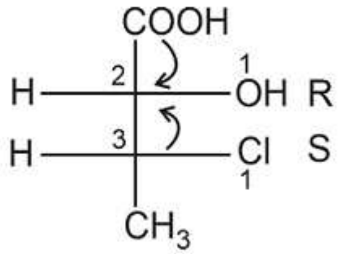
(1) $(2R, 3R)$

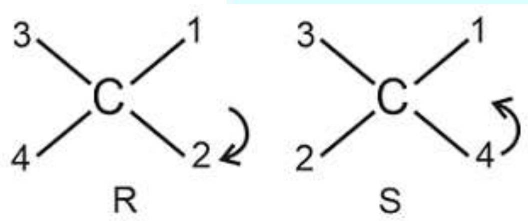
(2) $(2R, 3S)$

(3) $(2S, 3R)$

(4) $(2S, 3S)$

Solution: (3)





∴ Configuration is 2S, 3R.