Answers

Physics

1. (a) 2. (c) 3. (c) 4. (a) 5. (c) 6. (a) 7. (a) 8. (b) 9. (b) 10. (d)
11. (a) 12. (a) 13. (a) 14. (d) 15. (d) 16. (b) 17. (a) 18. (c) 19. (a) 20. (c)
21. (a) 22. (a) 23. (d) 24. (d) 25. (a) 26. (b) 27. (b) 28. (a) 29. (d) 30. (d)
31. (d) 32. (c) 33. (a) 34. (c) 35. (a) 36. (b) 37. (a) 38. (a) 39. (a) 40. (c)

Chemistry

41. (b) 42. (c) 43. (b) 44. (c) 45. (b) 46. (c) 47. (c) 48. (c) 49. (b) 50. (d)
51. (b) 52. (b) 53. (c) 54. (d) 55. (b) 56. (b) 57. (a) 58. (b) 59. (a) 60. (d)
61. (c) 62. (a) 63. (b) 64. (a) 65. (c) 66. (d) 67. (a) 68. (c) 69. (c) 70. (a)
71. (c) 72. (c) 73. (c) 74. (b) 75. (a) 76. (b) 77. (a) 78. (d) 79. (a) 80. (b)

(a) English Proficiency

81. (b) 82. (d) 83. (a) 84. (d) 85. (d) 86. (a) 87. (b) 88. (c) 89. (c) 90. (b)
91. (c) 92. (b) 93. (b) 94. (c) 95. (c)

(b) Logical Reasoning

96. (c) 97. (a) 98. (d) 99. (c) 100. (d) 101. (b) 102. (d) 103. (a) 104. (d) 105. (b)

Mathematics

106. (a) 107. (c) 108. (c) 109. (b) 110. (c) 111. (b) 112. (d) 113. (b) 114. (b) 115. (b)
116. (b) 117. (b) 118. (d) 119. (b) 120. (d) 121. (b) 122. (b) 123. (c) 124. (a) 125. (a)
126. (b) 127. (a) 128. (c) 129. (d) 130. (b) 131. (d) 132. (b) 133. (a) 134. (d) 135. (c)
136. (b) 137. (d) 138. (c) 139. (c) 140. (c) 141. (c) 142. (d) 143. (a) 144. (c) 145. (c)
146. (d) 147. (a) 148. (b) 149. (b) 150. (b)

Hints & Solutions

Physics

1. The necessary centripetal force required for a planet to move round the sun

= gravitational force exerted on it

$$\frac{mv^2}{R} = \frac{GM_e m}{R^n}$$

or $$v = \left(\frac{GM}{R^{n-1}}\right)^{1/2}$$

as $$T = \frac{2\pi R}{v} = 2\pi R \times \left(\frac{R^{n-1}}{GM}\right)^{1/2}$$

$$T = 2\pi\left[\frac{R^{\frac{(n+1)}{2}}}{(GM_e)^{1/2}}\right]$$

$\therefore$ $$T \propto R^{(n+1)/2}$$

2. $$Y = \frac{FL}{A\Delta L}$$

or $$F = \frac{YA\Delta L}{L} = \frac{YA^2\Delta L}{AL}$$

$$= \frac{YA^2\Delta L}{V} = \frac{YA^2\Delta x}{V}$$

where $AL = V =$ Volume of wire, Young modulus in the same as both the wires are made of same material. It is given that both the wire have same volume and same extension in length

$\therefore$ $$\frac{F'}{F} = \frac{A'^2}{A^2} = \frac{(3A)^2}{A^2} = 9$$

$$F' = 9F$$

3. KE of a satellite

$$E = \frac{1}{2}mv^2$$

or $$mv = \sqrt{2Em}$$

Angular momentum

$$L = mvr = (\sqrt{2Em}) \times r$$

$$= \sqrt{2mEr^2}$$

4. Shunt is given by

$$S = \frac{I_g \times R_g}{I - I_g} = \frac{0.01 \times 100}{10 - 0.01}$$

$$= \frac{0.01 \times 100}{9.99} = 0.100\,\Omega$$

5. Given, $r = 25$ m, $v = 5$ m/s, $m = 500$ kg

$$a_t = 1\text{ m/s}^2,\ a_r = \frac{v^2}{r} = \frac{5 \times 5}{25} = 1\text{ m/s}^2$$

$$a_{net} = \sqrt{a_t^2 + a_r^2} = \sqrt{1^2 + 1^2} = \sqrt{2}\text{ m/s}^2$$

$$F = ma_{net} = 500\sqrt{2}\text{ N}$$

6. From conservation of energy the electron kinetic energy equals the maximum photon energy (we neglect the work function ϕ because it is normally so small compared to eV_0)

$$eV_0 = h\nu_{\max}$$

$$eV_0 = \frac{hc}{\lambda_{\min}}$$

$$V_0 = \frac{hc}{e\lambda_{\min}}$$

$$V_0 = \frac{12400 \times 10^{-10}}{10^{-11}}$$

$$= 124\text{ kV}$$

Hence, accelerating voltage for electrons is X-ray machine should be less than 124 kV.

7. Given $l = 60$ cm $= 60 \times 10^{-2}$ m

$$m = 20\text{ kg}, \quad k = 400\text{ N/m}$$

The weight hung from the spring

$$= mg = 20 \times 9.8 = 196\text{ N}$$

Suppose x is the extension produced in spring

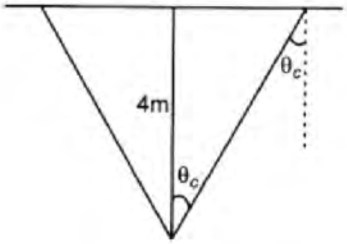
Now, force applied by the spring

= downward force on the spring

$$\therefore \quad kx = mg \Rightarrow x = \frac{mg}{k}$$

$$x = \frac{20 \times 9.8}{4000} = 0.049\text{ m} = 4.9\text{ cm}$$

8. $\sin\theta_c = \frac{1}{\mu} = \frac{3}{5}$



$$\frac{r}{4} = \tan\theta_c = \frac{3}{4}$$

Radius, r = radius = 3 m

Diameter, d = 6 m

9. $y = 2\sin\left(\frac{\pi t}{2} + \phi\right)$

Comparing the equation with the standard equation

$$y = A\sin(\omega t + \phi)$$

So $\quad A = 2\text{ cm}, \omega = \frac{\pi}{2}$

Acceleration of particle is

$$a = \omega^2 x \qquad \text{(numerically)}$$

at $x = +A$, $a = a_{\max}$

$\therefore \quad a_{\max} = \omega^2 A$

$$= \left(\frac{\pi}{2}\right)^2 \times 2$$

$$= 2 \times \frac{\pi^2}{4}$$

$$= \frac{\pi^2}{2}\text{ cm/s}^2$$

10. Apparent frequency heard will be

$$n' = n\left(\frac{v}{v - v_s}\right)$$

v = velocity of sound

v_s = velocity of source of sound

n = frequency

= 3 kHz

$\therefore \quad n' = 3 \times \frac{v}{v - 0.5v}$

$$= 3 \times \frac{v}{0.5v}$$

$$= 6\text{ kHz}$$

11. Bernoulli's theorem is applicable only for tube flow of non-uniform cross-section.

12. From the formula

$$N = N_0\left(\frac{1}{2}\right)^n$$

$$\frac{N}{16} = N_0\left(\frac{1}{2}\right)^n$$

N_0 = original number of atom

$$\left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^n$$

$\Rightarrow \quad n = 4$

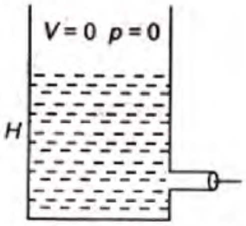
4 half lives

$\therefore \quad 4T_{1/2} = 40$

$$T_{1/2} = \frac{40}{4} = 10\text{ days}$$

13. v = velocity of efflux through an orifice

$$= \sqrt{2gH}$$

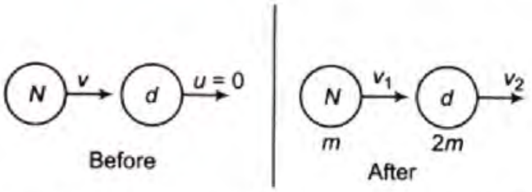


It is independent of the size of orifice.

14. Neutron velocity = v, mass = m

Deuteron contains 1 neutron and 1 proton

= $2m$



In elastic collision, both momentum and KE are conserved

$$p_i = p_f$$

$$mv = m_1v_2 + m_2v_2$$

$$mv = mv_1 + 2mv_2$$

By kinetic energy

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}(2m)v_2^2$$

By solving

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2}v + \frac{2m_2}{(m_1 + m_2)}v$$

$$v_1 = \frac{m_1 - 2m}{3m}$$

$$v_1 = -\frac{v}{3}$$

$$K_i = \frac{1}{2}mv^2$$

$$K_f = \frac{1}{2}mv_1^2$$

$$\frac{K_i - K_f}{K_i} = 1 - \frac{v_1^2}{v^2}$$

$$= 1 - \frac{1}{9}$$

$$= \frac{8}{9} \text{ fractional change in KE}$$

15. Poisson's ratio = 0.5

Since, density is constant therefore change in volume is zero, we have

$$V = A \times l = \text{constant}$$

$$\log V = \log A + \log l$$

or

$$\frac{dA}{A} + \frac{dl}{l} = 0$$

$$\frac{dl}{l} = -\frac{dA}{A}$$

∴ Percentage increase in length = 4%

16. Lenz's law of electromagnetic induction compounds to the law of conservation of energy.

17. Rotational kinetic energy $= \frac{1}{2}I\omega^2$

According to question

$$\frac{1}{2}I\omega^2 = 1500$$

$$\frac{1}{2}I(\alpha t)^2 = 1500$$

$$(1.2) \times (25)^2 \times t^2 = 3000$$

$$1.2 \times 625 \times t^2 = 3000$$

$$t^2 = \frac{3000}{1.2 \times 625} = 4$$

$$t = 2\,\text{s}$$

18. For a solenoid

$$B = \mu_0 ni$$

$$\because \quad n = \frac{N}{2\pi r}$$

$$\therefore \quad B = \frac{\mu_0 NI}{2\pi r}$$

Flux linked with the solenoid

$$\phi = NBA$$

$$\phi = \frac{\mu_0 N^2 IA}{2\pi r}$$

$$= \frac{4\pi \times 10^{-7} \times (1200)^2 \times 12 \times 10^{-4}}{2\pi \times 15 \times 10^{-2}}$$

$$L = 2.3 \times 10^{-3}\ \text{H}$$

$$= 2.3\ \text{mH}$$

19.

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$v = \sqrt{\frac{R}{m}}x$$

$$= \sqrt{\frac{50}{20 \times 10^{-3}}}(10 \times 10^{-2})$$

$$= 50 \times 10^{-1}$$

$$= 5\ \text{m/s}$$

20. Work done, $W = p\Delta V$

$$= 1.013 \times 10^5 \times (1671 - 1) \times 10^{-6}$$

$$= 1.013 \times 10^5 \times 1670 \times 10^{-6}$$

$$= 169.2\ \text{J}$$

21. Volume, $V = l^3 = (1.2 \times 10^{-2}\text{m})^3$

$$= 1.728 \times 10^{-6}\text{m}^3$$

Since length (l) has two significant figure, the volume (V) will also have two significant figure.

Therefore, the correct answer is

$$V = 1.7 \times 10^{-6}\text{m}^3$$

22. $t = \sqrt{\dfrac{2h}{g}}$ and $t' = \sqrt{\dfrac{2(2h)}{g}}$

$$\therefore \quad \frac{t}{t'} = \frac{1}{\sqrt{2}}$$

23. Given, $P = a + bt^2$

$$\frac{dP}{dt} = 2bt$$

$$\because \quad F = \frac{dP}{dt}$$

$$\therefore \quad F = 2bt$$

or $\quad F \propto t$

24. A ball bearing striking another ball bearing is not an example of perfectly inelastic collision.

25. By Kepler's third law, $T^2 \propto R^3$

$$\therefore \left(\frac{T_2}{365 \text{ days}}\right)^2 = \left(\frac{2r}{r}\right)^3$$

$$T_2 = 365 \times 2\sqrt{2} = 1032 \text{ days}$$

26. Weight, $w = mg = m\dfrac{4}{3}\pi GR\rho$

$$\therefore \quad \frac{w'}{w} = \frac{R'\rho'}{R\rho} = \left(\frac{1}{2}\right) \times 4 = 2$$

$$w' = 2w$$

27. Force on electron

$$|F| = qE = eE = mg$$

$$E = \frac{mg}{e}$$

28. Work done, $W = \dfrac{q^2}{2C}$

$$= \frac{(8 \times 10^{-18})^2}{2 \times 100 \times 10^{-6}}$$

$$= 32 \times 10^{-32} \text{ J}$$

29. In a metallic conductor of non-uniform cross section, only the current remains constant along the entire length of the conductor.

30. Here, $I_g = 25 \times 4 \times 10^{-4} \text{A} = 10^{-2}$ A

To convert the galvanometer into a voltmeter we must join a series resistance of

$$R = \frac{V}{I_g} - G$$

$$= \frac{25}{10^{-2}} - 50 = 2500 - 50$$

$$= 2450\,\Omega$$

31. Cyclotron of frequency, $\nu = \dfrac{Bq}{2\pi m}$

$$= \frac{1 \times 1.6 \times 10^{-19}}{2\pi \times 9.1 \times 10^{-31}}$$

$$= 2.8 \times 10^{10} \text{ Hz}$$

$$= 28 \text{ GHz}$$

32. Torque, $\tau = \mathbf{M} \times \mathbf{B}$

33. Energy is stored in an inductor in the form of magnetic potential energy.

34. In a photoelectric cell, optical energy is being transformed into electrical energy because light photons are being absorbed and photoelectric current is being produced.

35. Ionisation energy $E = eV$

$$= 1.6 \times 10^{-19} \times 24.6 \text{ J}$$

$$= 24.6 \text{ eV}$$

36. Fast neutrons can easily be slowed down by passing them through water. Slowing down process is due to collision between neutron and hydrogen nucleus present in water.

37. As linear magnification $m = 4$,

hence, a real magnification $m_a = m^2$

$$= (4)^2 = 16$$

$\therefore$Surface area of film image on the screen

$$= 16 \times 100 = 1600 \text{ cm}^2$$

38. Under identical pressure and temperature condition, speed of sound in moist air is more than that in dry air, *i.e.*, $v_m > v_d$

39. Heat flow through vacuum is possible in radiation mode due to which temperature of hot body falls.

40. Given, $T_2 = -13°\text{C} = 260$ K and $\beta = 5$

$$\beta = \frac{T_2}{T_1 - T_2}$$

$$5 = \frac{260}{T_1 - 260}$$

$$T_1 = 312 \text{ K}$$

or $\quad T_1 = 312 \text{ K} = 39°\text{C}$

Chemistry

41. $40\text{g NaOH} = \frac{40}{40} = 1 \text{ mol NaOH}$

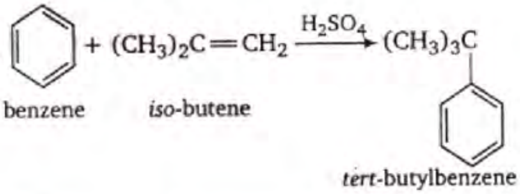
$60\text{g } CH_3COOH = \frac{60}{60} = 1 \text{ mol } CH_3COOH$

Since acetic acid is a weak acid, some of the heat is utilised to ionise it. So, enthalpy of neutralisation of 1 mol of NaOH by 1 mol CH_3COOH is less than 57.1 kJ.

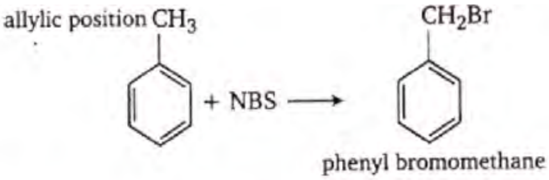
Enthalpy of neutralisation of a strong acid by a strong base is always 57.1 kJ.

42. Ionic size varies inversely with nuclear charge. Higher the nuclear charge, smaller the radii. Thus, P^{5+} because of the high nuclear charge, has the smallest size.

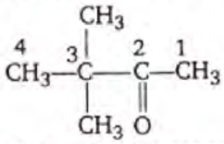
43.



44. NBS (N-bromosuccinimide) causes bromination at allylic position.



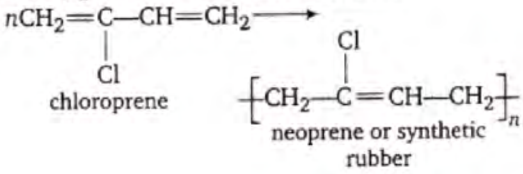
45. The structure of pinacolone is



$$\overset{4}{C}H_3-\overset{3}{C}(CH_3)_2-\overset{2}{C}(=O)-\overset{1}{C}H_3$$

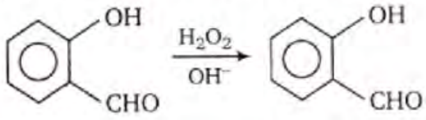
3,3-dimethyl-2-butanone

46. Neoprene is synthetic rubber. It is a polymer of chloroprene and is resistant to the action of oils, gasoline and other solvents.



47. HOCl and Cl_2 are formed over Antarctica. These are converted back into reactive Cl atoms which start the chain reaction with O_3 causing its depletion.

48. The reduction of —CHO group to —OH by H_2O_2/OH^- is called Dakin reaction.



Thus, the above reaction is Dakin reaction.

49. Conjugated base of a stronger acid is weak. The corresponding acids of the given conjugated bases are as (in order of acidity)

$$C_2H_5OH < CH_3COOH < HF < HNO_3$$

$\therefore$ The order of basicity is

$$C_2H_5O^- > CH_3COO^- > F^- > NO_3^-$$

Thus, NO_3^- is the weakest base among the given.

50. $\Delta T_b = ik_b \cdot m = 2 \times 0.52 \times 0.1 = 0.104°C$

$T_b = 100 + 0.104°C = 100.104°C$

51. $\underset{1\text{ mol}}{P_2O_5} + 6H_2O \longrightarrow \underset{4\text{ mol}}{4H_3PO_4}$

$\therefore$ 1 L solution contains 4 mol H_3PO_4.

$\therefore$ Molarity of $H_3PO_4 = 4$ M

Normality = Molarity × Basicity

$= 4 \times 3 \text{ N} = 12 \text{ N}$

52. $$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\Rightarrow \frac{10^{-6} \times 1000}{298} = \frac{760 \times V_2}{273}$$

$V_2 = 1.2 \times 10^{-6}$cc (at STP)

$$\text{No. of molecules} = \frac{6.02 \times 10^{23}}{22400} \times 1.2 \times 10^{-6}$$

$$= 3.2 \times 10^{13}$$

53. $\lambda = h / mv$

For same velocity $\lambda \propto 1/m$

Electron has the least mass, so its wavelength is maximum.

54. Radioactivity does not depend upon the state of combination so it remains unaffected.

55. In $SO_4^{2-} \Rightarrow bp = 4; lp = 0 \therefore$ hybridisation sp^3

In $SF_4 \Rightarrow bp = 4; lp = 1 \therefore$ hybridisation sp^3d

In $SF_2 \Rightarrow bp = 2; lp = 2 \therefore$ hybridisation sp^3

In $S_8 \Rightarrow bp = 2; lp = 2 \therefore$ hybridisation sp^3

Thus, only SF_4 does not have sp^3 hybridisation.

56. $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

$$K = \frac{[NH_3]^2}{[N_2][H_2]^3} = \frac{\left[\frac{\text{mol}}{\text{L}}\right]^2}{\left[\frac{\text{mol}}{\text{L}}\right]\left[\frac{\text{mol}}{\text{L}}\right]^3}$$

$$= \left[\frac{\text{mol}}{\text{L}}\right]^{-2} = \text{L}^2\text{mol}^{-2}$$

57. $pH = 3$

$$[H^+] = 1 \times 10^{-3} \text{ mol/L}$$

$$H_2SO_4 \longrightarrow 2H^+ + SO_4^{2-}$$

$$[H_2SO_4] = \frac{1 \times 10^{-3}}{2} = \frac{1}{2000} \text{ M}$$

$$N = 2\text{ M} \qquad (\text{for } H_2SO_4)$$

$$\text{Normality} = \frac{2}{2000} = \frac{1}{1000} \text{ N}$$

58. $NOClO_4$ is actually $NO^+ClO_4^-$.

Let the oxidation state of N in NO^+ is x.

$NO^+ \qquad x + (-2) = +1$

$x = +1 + 2 = +3$

Let the oxidation state of Cl in ClO_4^- is y.

$ClO_4^- \qquad y + (-2) \times 4 = -1$

$y - 8 = -1\,; y = +7$

59. Pyrolusite is MnO_2. Thus, it is an oxide ore.

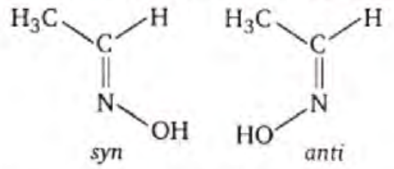
60. $K_4[Fe(CN)_6] + 6H_2SO_4 + 6H_2O \longrightarrow$

$2K_2SO_4 + FeSO_4 + 3(NH_4)_2SO_4 + 6CO$

61. When sodium carbonate is treated with CO_2 and H_2O, it gets converted into sodium bicarbonate.

$$Na_2CO_3 + CO_2 + H_2O \longrightarrow \underset{\text{sodium bicarbonate or sodium hydrogen carbonate}}{2NaHCO_3}$$

62. Among the given only $CH_3CH{=}NOH$ (oxime) satisfy the conditions essential for exhibiting geometrical isomerism. So, it will exhibit *syn-anti* geometrical isomerism.


syn *anti*

63. Octane number is defined as the percentage of *iso*-octane (by volume) in a mixture of *iso*-octane and *n*-heptane which has the same anti-knocking properties as the fuel under consideration.

Thus, the octane number of the given fuel is 70 as it contains 70% *iso*-octane.

64. When SO_2 is passed in the sodium carbonate solution, CO_2 gas is evolved and sodium carbonate is converted into $NaHSO_3$ (sodium bisulphite).

$$Na_2CO_3 + 2SO_2 + H_2O \rightarrow 2NaHSO_3 + CO_2$$

65. $t_{1/2} \propto \frac{1}{(a)^{n-1}}$

Given, $\qquad t_{1/2} \propto \frac{1}{a}$

On comparing, $\qquad a^{n-1} = a^1$

$n - 1 = 1$

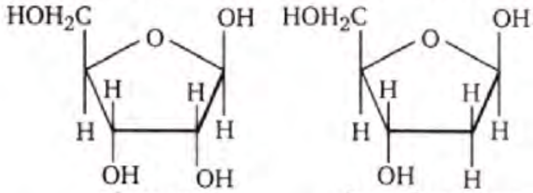
$n = 1 + 1 = 2$

66. In $HC{\equiv}C{-}CH_2OH$ two active hydrogen atoms are present, hence it will react with two moles of CH_3MgBr (Grignard reagent).

$$HC{\equiv}C{-}CH_2OH + 2CH_3MgBr \longrightarrow$$
$$2CH_4 + BrMgC{\equiv}CCH_2OMgBr$$

67. Haemoglobin molecule contains four polypeptide chains.

68. The structure of sugar of DNA (*i.e.*, deoxyribose) and that of RNA (*i.e.*, ribose) is as


ribose sugar deoxyribose sugar

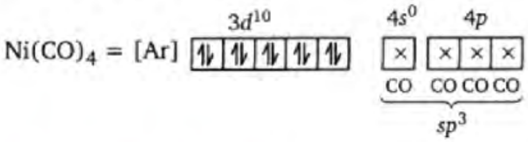
Thus, it is clear that these have furanose structure.

69. Tetrazine is an artificial edible colour.

70. In nickel carbonyl, $Ni(CO)_4$, Ni is present as Ni.

$Ni = [Ar]\ 3d^8 4s^2$

CO being strong field ligand, shifts electrons from 4s to 3d orbital.



Therefore, number of unpaired electrons in nickel carbonyl is 0.

71. For a first order reaction,

$$k = \frac{2.303}{t} \log \frac{a}{a-x}$$

$$t = \frac{2.303 \times t_{1/2}}{0.693} \log \frac{100}{(100-90)}$$

$$\left[\because k = \frac{t_{1/2}}{0.693}\right]$$

$$= \frac{2.303 \times t_{1/2}}{0.693} \log 10$$

$$= 3.3\, t_{1/2}$$

$= 3.3$ times that of half-life

72. Given, $pK_a = 4$

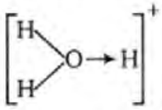
$$\therefore \quad K_a = 1 \times 10^{-4}$$

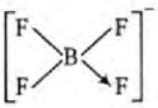
$$[H^+] = \sqrt{K_a \cdot C} = \sqrt{1 \times 10^{-4} \times 0.01}$$

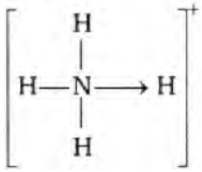
$$= \sqrt{10^{-6}} = 10^{-3} M$$

$$pH = -\log[H^+] = -\log 10^{-3} = 3$$

73. Among the given, only HF_2^- has H-bonding $[F—H..F]^-$. Rest all the molecules have coordinate bonds.







74. Anti-bonding molecular orbital is raised more in energy than the energy by which bonding molecular orbital is lowered.

75.

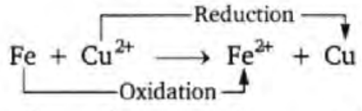
$$E^\circ = \frac{0.059}{n} \log K_c$$

$$0.46 = \frac{0.059}{2} \log K_c$$

$$\log K_c = \frac{0.46 \times 2}{0.059} = 15.6$$

$$K_c = \text{antilog } 15.6$$

76.



$Fe + Cu^{2+} \longrightarrow Fe^{2+} + Cu$

$$E^\circ_{cell} = E^\circ_{Fe/Fe^{2+}} + E^\circ_{Cu^{2+}/Cu} = 0.44 + 0.32$$

$$= 0.76\ V$$

77. $\underset{sp^3}{\overset{III}{CH_3}}—\underset{sp^3}{\overset{II}{CH_2}}—\underset{sp}{C}\equiv\underset{sp}{\overset{I}{CH}}$

Electronegativity ∝ *s*-character.

In *sp* hybrid orbitals, *s* character = 50%

and in sp^3 hybrid orbitals, *s* character = 33.3%

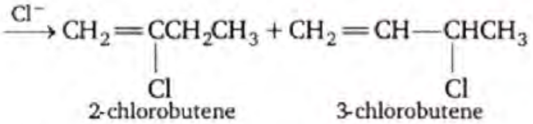
Thus, I is the most electronegative.

78. Dehydrohalogenation of alkyl halide gives alkenes but not alkane.

$$R—CH_2—CH_2X \xrightarrow[-HX]{\text{Alco. KOH}} RCH=CH_2$$

79. $CH_2=C=CH—CH_3 + H^+ \longrightarrow$

$$[CH_2=\overset{+}{C}—CH_2CH_3 + CH_2=CH—\overset{+}{C}HCH_3]$$



$$\xrightarrow{Cl^-} \underset{\text{2-chlorobutene}}{CH_2=\underset{Cl}{\underset{|}{C}}CH_2CH_3} + \underset{\text{3-chlorobutene}}{CH_2=CH—\underset{Cl}{\underset{|}{C}}HCH_3}$$

80. When formaldehyde reacts with ammonia, a well known urinary antiseptic urotropine (also called hexamethylene tetramine) is obtained.

$$6HCHO + 4NH_3 \rightarrow (CH_2)_6N_4 + 4H_2O$$

structure of urotropine

(b) Logical Reasoning

96. The pattern of the series is

$$\begin{tikzcd} & & & \boxed{26} & \\ 5 & 10 & 17 & 24 & 37 \\ 2^2+1 \arrow[u] & 3^2+1 \arrow[u] & 4^2+1 \arrow[u] & 5^2+1 \arrow[u] & 6^2+1 \arrow[u] \end{tikzcd}$$

Hence, 24 is the wrong number.

97. The pattern of the series is

$$\begin{tikzcd} E \arrow[r, bend right, "+1"'] & F \arrow[r, bend right, "+2"'] & H \arrow[r, bend right, "+3"'] & K \arrow[r, bend right, "+4"'] & O \arrow[r, bend right, "+5"'] & T \arrow[r, bend right, "+6"'] & Z \end{tikzcd}$$

98. As,

$$\begin{array}{l} M \xrightarrow{+1} N \\ O \xrightarrow{-1} N \\ N \xrightarrow{+1} O \\ K \xrightarrow{-1} J \\ E \xrightarrow{+1} F \\ Y \xrightarrow{-1} X \end{array} \quad \text{Similarly,} \quad \begin{array}{l} T \xrightarrow{+1} U \\ A \xrightarrow{-1} Z \\ R \xrightarrow{+1} S \\ G \xrightarrow{-1} F \\ E \xrightarrow{+1} F \\ T \xrightarrow{-1} S \end{array}$$

99. $L > S > (P, Q) > N > M$

Hence, L is the tallest among six friends.

100. Number of boys towards the left of Manick

$$= (40 - 14) = 26$$

So, Manick is 27th from the left end.

101. The sum of the two numbers in the upper part is seven times the number in the lower part.

$$(25 + 17) \div 7 = 6$$

$$(38 + 18) \div 7 = 8$$

So, missing term $= (89 + 16) \div 7$

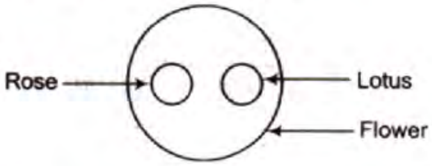
$$= 105 \div 7$$

$$= 15$$

102. Clearly, the given letters, when arranged in the order of '6, 3, 5, 2, 4, 1' form the word 'BRANCH'.

103. Rose and Lotus are different type of flowers.

Hence, the diagram will be



104. Answer figure (d) will appear when a piece of paper is folded and cut.

105. Answer figure (b) will complete the question figure.

Mathematics

106. Given, $x^2 + y^2 = a^2$

On differentiating w.r.t. x, we get

$$2x + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{x}{y}$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(x', y')} = -\frac{x'}{y'}$$

$\therefore$ Equation of normal is

$$y - y' = \frac{y'}{x'}(x - x')$$

$$\Rightarrow \quad x' y - y' x' = xy' - y' x'$$

$$\Rightarrow \quad x' y - xy' = 0$$

107. Given equations are

$$3x + 4y + 5 = 0$$

and

$$12x - 5y - 7 = 0$$

$$\therefore \quad a_1a_2 + b_1b_2 = 3 \times 12 + 4 \times (-5)$$

$$= 16 > 0$$

$\therefore$ For acute angle bisector

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = -\frac{(a_2x + b_2y + c_2)}{\sqrt{a_2^2 + b_2^2}}$$

$$\therefore \quad \frac{3x + 4y + 5}{\sqrt{9 + 16}} = -\frac{(12x - 5y - 7)}{\sqrt{12^2 + (-5)^2}}$$

$$\Rightarrow \quad \frac{3x + 4y + 5}{5} = -\frac{(12x - 5y - 7)}{13}$$

$$\Rightarrow 39x + 52y + 65 = -60x + 25y + 35$$

$$\Rightarrow \quad 99x + 27y + 30 = 0$$

108. Given, $z = \cos\theta + i \sin\theta$

$$\therefore z^n + \frac{1}{z^n} = (\cos\theta + i \sin\theta)^n + (\cos\theta + i\sin\theta)^{-n}$$

$$= \cos n\theta + i \sin n\theta + \cos n\theta - i \sin n\theta$$

$$= 2\cos n\theta$$

109. Since, α and β are the roots of $x^2 - 2x + 4 = 0$

$\therefore \quad \alpha + \beta = 2$ and $\alpha\beta = 4$

Now, $\quad (\alpha - \beta) = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$$= \sqrt{4-16} = 2\sqrt{3}\,i$$

On solving, we get

$$2\alpha = 2 + 2\sqrt{3}\,i$$

$$\Rightarrow \quad \alpha = 2\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$

$$= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

and $\quad \beta = \dfrac{2 - 2\sqrt{3}\,i}{2} = 2\left(\cos\dfrac{\pi}{3} - i\sin\dfrac{\pi}{3}\right)$

$$\therefore \quad \alpha^n + \beta^n = \left[2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right]^n + \left[2\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)\right]^n$$

$$= 2^n\left[2\cos\frac{n\pi}{3}\right] = 2^{n+1}\cos\frac{n\pi}{3}$$

110. Now, $A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$\therefore A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence, option (c) is the correct answer.

111. Applying $R_1 \to R_1 + R_2 + R_3$ and taking common from R_1, we get

$$\Delta = (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$\Delta = (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 0 & -(a+ \end{vmatrix}$$

$$= (a+b+c)^3$$

112. Given,

$$(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots + C_n x^n$$

Put $x = -1$, we get

$$C_0 - C_1 + C_2 - \ldots + (-1)^n \cdot C_n = 0$$

113. Let a and b be two numbers, then

$$\text{AM} = \frac{a+b}{2}$$

$$\Rightarrow \quad 27 = \frac{a+b}{2}$$

$$\Rightarrow \quad a + b = 54$$

and $\quad \text{HM} = \dfrac{2ab}{a+b}$

$$\Rightarrow \quad 12 = \frac{2ab}{54}$$

$$\Rightarrow \quad ab = 324$$

$$\therefore \quad \text{GM} = \sqrt{ab} = \sqrt{324} = 18$$

114. Given, $\quad P(A \cup B) = \dfrac{5}{6}, \quad P(A \cap B) = \dfrac{1}{3},$

$P(B) = \dfrac{1}{2},$

$\because \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\therefore \quad P(A) = \frac{5}{6} - \frac{1}{2} + \frac{1}{3}$$

$$= \frac{5-3+2}{6} = \frac{4}{6} = \frac{2}{3}$$

115. $\therefore$ Required probability $= \dfrac{{}^3C_1}{{}^8C_1} = \dfrac{3}{8}$

116. Since, scalar triple product is zero, then **a**, **b** and **c** are coplanar.

117. Since, given vectors are perpendicular.

$\therefore (2\mathbf{i} + \mathbf{j} - \mathbf{k}) \cdot (\mathbf{i} - 4\mathbf{j} + \lambda\mathbf{k}) = 0$

$$\Rightarrow \quad 2-4-\lambda=0$$

$$\Rightarrow \quad \lambda=-2$$

118. $\int_1^2 f(x)dx = \int_1^2 \frac{d}{dx}(\phi(x))dx$

$$=[\phi(x)]_1^2 = \phi(2)-\phi(1)$$

119. $\int_0^2 |1-x|\,dx = \int_0^1 (1-x)\,dx + \int_1^2 (x-1)\,dx$

$$=\left[x-\frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2}-x\right]_1^2$$

$$=1-\frac{1}{2}+\left[2-2-\left(\frac{1}{2}-1\right)\right]$$

$$=\frac{1}{2}+\frac{1}{2}=1$$

120. Let $I=\int \frac{\sin 2x}{\sin^4 x+\cos^4 x}dx$

$$=\int \frac{\sin 2x}{(\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x}dx$$

$$=\int \frac{\sin 2x}{1-\frac{1}{2}(\sin 2x)^2}dx$$

$$=\int \frac{\sin 2x}{1-\frac{1}{2}(1-\cos^2 2x)}dx$$

$$=\int \frac{\sin 2x}{\frac{1}{2}(1+\cos^2 2x)}dx$$

Put $\cos 2x=t \Rightarrow -2\sin 2x\,dx=dt$

$$\therefore \quad I=-\int\frac{dt}{1+t^2}=-\tan^{-1}t+C$$

$$=-\tan^{-1}(\cos 2x)+C$$

121. Let $y=\sin x+\cos x$

$$=\sqrt{2}\left(\sin\left(\frac{\pi}{4}+x\right)\right)$$

Here, y will be maximum when

$$x=\frac{\pi}{4}$$

122. Let $y=x^x$

$$\Rightarrow \quad \log y = x\log x$$

On differentiating w.r.t. x, we get

$$\frac{1}{y}\frac{dy}{dx}=\frac{x}{x}+\log x$$

$$\Rightarrow \quad \frac{dy}{dx}=y(1+\log x)$$

$$\Rightarrow \quad =x^x(\log e+\log x)$$

$$=x^x(\log ex)$$

123. $\lim_{x\to 0}\frac{\sin x}{x}=\lim_{x\to 0}\frac{\cos x}{1}$

$$=\cos 0=1$$

124. Since, $x^2=16 \Rightarrow x=\pm 4$

and $2x=6 \Rightarrow x=3$

Hence, no value of x is satisfied.

$$\therefore \quad A=\phi$$

125. Required number of ways

$$=4^5=2^{10}=1024$$

126. Since, $(\sqrt{5}+1)^5-(\sqrt{5}-1)^5$

$$=2\{{}^5C_1(\sqrt{5})^4+{}^5C_3(\sqrt{5})^2+{}^5C_5\cdot 1\}$$

$$=2\{5\times 25+50+1\}$$

$$=2(176)=352$$

127. $\therefore 7\log\left(\frac{16}{15}\right)+5\log\left(\frac{25}{24}\right)+3\log\left(\frac{81}{80}\right)$

$$=\log\left[\left(\frac{16}{15}\right)^7\cdot\left(\frac{25}{24}\right)^5\cdot\left(\frac{81}{80}\right)^3\right]$$

$$=\log 2$$

128. Here, $T_n=\frac{n(n+1)}{n!}=\frac{n-1+2}{(n-1)!}$

$$=\frac{1}{(n-2)!}+\frac{2}{(n-1)!}$$

$$\therefore \quad S=\sum_{n=1}^{\infty}T_n=\sum_{n=1}^{\infty}\frac{1}{(n-2)!}+2\sum_{n=1}^{\infty}\frac{1}{(n-1)!}$$

$$=e+2e=3e$$

129. $\log_4 2-\log_8 2+\log_{16} 2-\dots$

$$=\left(\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\dots\right)-1+1$$

$$=-\left(1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\dots\right)+1$$

$$=1-\log(1+1)=1-\log_e 2$$

130. Now, $x^2-6x+7=(x-3)^2-2$

It is obvious that minimum value is -2 and maximum value is ∞.

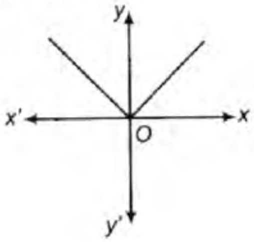
131. $$\lim_{x\to 0}\frac{\cos(\sin x)-1}{x^2}=\lim_{x\to 0}\frac{-2\sin^2\left(\frac{\sin x}{2}\right)}{x^2}$$

$$=-2\lim_{x\to 0}\frac{\sin^2\left(\frac{\sin x}{2}\right)\cdot\left(\frac{\sin x}{2}\right)^2}{\left(\frac{\sin x}{2}\right)^2\times x^2}$$

$$=-2(1)^2\frac{1}{4}=-\frac{1}{2}$$

132. $\lim_{x\to 0} f(x)=f(0)=\lim_{x\to 0}(1+x)^{1/x}=e$

133. Given, $f(x)=|x|$



It is clear from the graph, $f(x)$ is continuous but non-differentiable at $x=0$.

134. Let any point on the curve be (h, k).

Then, $h^2=2k$

$\therefore$ Distance $D=\sqrt{h^2+(k-5)^2}$

$\Rightarrow$ $D=\sqrt{2k+(k-5)^2}$

$\Rightarrow$ $\frac{dD}{dk}=\frac{1}{2\sqrt{2k+(k-5)^2}}\times\{2+2(k-5)\}=0$ (say)

$\therefore$ $k=4$

Then, point will be $(\pm 2\sqrt{2}, 4)$.

135. Let $y=x^{\frac{1}{x}}\Rightarrow \log y=\frac{1}{x}\log x$

$\Rightarrow$ $\frac{1}{y}\frac{dy}{dx}=\frac{1}{x^2}-\frac{\log_e x}{x^2}$

$\Rightarrow$ $\frac{dy}{dx}=x^{1/x}\left(\frac{1-\log_e x}{x^2}\right)$

For $1<x<\infty, x^{1/x}>0$

and $\frac{1-\log_e x}{x^2}>0$ in $(1, e)$

and $\frac{1-\log_e x}{x^2}<0$ in (e, ∞)

Hence, $f(x)$ is increasing in $(1, e)$ and decreasing in (e, ∞).

136. $\therefore$ Required area $=\int_0^\pi \sin x\,dx$

$=[-\cos x]_0^\pi$

$=[+1+1]$

$=2$ sq units

137. Given, $\sqrt{\frac{dy}{dx}}-4\frac{dy}{dx}-7x=0$

On squaring, we get

$$\frac{dy}{dx}=16\left(\frac{dy}{dx}\right)^2+49x^2+56x\frac{dy}{dx}$$

Here, order is 1 and degree is 2.

138. $\therefore$ Required ratio $=-\frac{(-1+1-4)}{(5+7-4)}=\frac{1}{2}$

139. Here, $a+b=1-1=0$

Hence, pair of lines are perpendicular.

140. Length of tangent $=\sqrt{S_1}$

$=\sqrt{5^2+1^2+30-4-3}$

$=\sqrt{49}=7$

141. Given equation can be rewritten as

$$(x-1)^2+(y-3)^2=\left(\frac{5x-12y+17}{13}\right)^2$$

$\Rightarrow$ $SP=PM$

Here, focus is $(1, 3)$, directrix

$5x-12y+17=0$

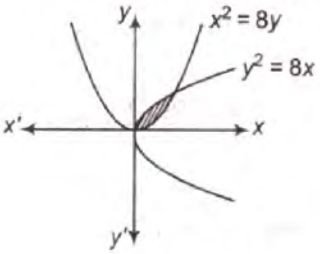
$\therefore$ The distance of the focus from the directrix

$$=\left|\frac{5-36+17}{\sqrt{25+144}}\right|$$

$$=\frac{14}{13}=2a$$

$\therefore$ Latusrectum $=2\times\frac{14}{13}=\frac{28}{13}$

142. It is clear that angle between the curves



= angle between the x-axis and y-axis is $\frac{\pi}{2}$.

143. Given centre (0, 0), focus (0, 3), $b = 5$

$\Rightarrow be = 3 \Rightarrow e = \frac{3}{5}$

$$\therefore \quad a = b\sqrt{1-e^2} = 5\sqrt{1-\frac{9}{25}} = 4$$

Hence, required equation is

$$\frac{x^2}{16} + \frac{y^2}{25} = 1$$

144. The equation of director circle to the hyperbola is $x^2 + y^2 = a^2 - b^2$

$$\therefore \quad \text{Radius} = \sqrt{a^2 - b^2}$$

145. Let d be the length of line, then projection on x-axis $= dl = 3$, projection of y-axis $= dm = 4$ and projection on z-axis $= dn = 5$

Now, $\quad d^2(l^2 + m^2 + n^2) = 50$

$\Rightarrow \quad d^2 = 50 \Rightarrow d = 5\sqrt{2}$

146. $\therefore \tan(\theta + \phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta \cdot \tan\phi}$

$$= \left(\frac{1}{2} + \frac{1}{3}\right) \Big/ \left(1 - \frac{1}{2} \cdot \frac{1}{3}\right) = 1$$

$\Rightarrow \quad \theta + \phi = \frac{\pi}{4}$

147. $\because \sin\theta = \frac{1}{2} = \sin\frac{\pi}{6}$

$\Rightarrow \quad \theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}$

and $\quad \tan\theta = \frac{1}{\sqrt{3}} = \tan\left(\frac{\pi}{6}\right)$

$\Rightarrow \quad \theta = \frac{\pi}{6}, \pi + \frac{\pi}{6}$

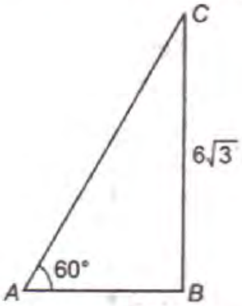
$\therefore$ Common value of θ is $\frac{\pi}{6}$.

$\therefore$ General value of θ is

$$2n\pi + \frac{\pi}{6}, \forall n \in I$$

148. $\therefore \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \sin^{-1}\left(\sin\left(-\frac{\pi}{3}\right)\right) = -\frac{\pi}{3}$

149. $\therefore$ Length of ladder, $AC = \frac{6\sqrt{3}}{\sin 60^\circ} = 12$ m



150. Since. $A + B + C = \pi$

But $\quad 2B = A + C$

$\therefore \quad 3B = \pi \Rightarrow B = \pi/3$

$$\frac{a+c}{b} = \frac{\sin A + \sin C}{\sin B}$$

$$= \frac{2\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{(A-C)}{2}\right)}{\sin\frac{\pi}{3}}$$

$$= \frac{2\sin\frac{\pi}{3}\cos\left(\frac{A-C}{2}\right)}{\sin\frac{\pi}{3}}$$

$$= 2\cos\left(\frac{A-C}{2}\right)$$