Physics

1. As an electron makes transition from an excited state to the ground state of a hydrogen like atom/ion:
   
   (A) Kinetic energy, potential energy and total energy decrease
   (B) Kinetic energy decreases, potential energy increases but total energy remains same
   (C) Kinetic energy and total energy decreases but potential energy increases
   (D) Its kinetic energy increases but potential energy and total energy decrease

   Answer: (D)

   Solution: \( U = \frac{-e^2}{4\pi\varepsilon_0 r} \)
   
   \( U \) = potential energy
   
   \( k = \frac{e^2}{8\pi\varepsilon_0 r} \)
   
   \( K \) = kinetic energy
   
   \( E = U + k = \frac{-e^2}{8\pi\varepsilon_0 r} \)
   
   \( E \) = Total energy

   As electron de-excites from excited state to ground state \( k \) increases, \( U \) & \( E \) decreases

   Topic: Modern Physics

   Difficulty: Easy (embibe predicted high weightage)

   Ideal time: 30

2. The period of oscillation of a simple pendulum is \( T = 2\pi \sqrt{\frac{L}{g}} \). Measured value of \( L \) is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90s using a wrist watch of 1s resolution. The accuracy in the determination of \( g \) is:
   
   (A) 3%
   (B) 1%
   (C) 5%
   (D) 2%
Answer: (A)

Solution: \( \therefore T = 2\pi \sqrt{\frac{L}{g}} \Rightarrow g = 4\pi^2 \frac{L}{T^2} \)

\( \therefore \) Error in g can be calculated as

\[ \frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta T}{T} \]

\( \therefore \) Total time for n oscillation is \( t = n \) where \( T = \) time for oscillation.

\[ \Rightarrow \frac{\Delta t}{t} = \frac{\Delta T}{T} \]

\[ \Rightarrow \frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta t}{t} \]

Given that \( \Delta L = 10^{-3} \text{ m}, L = 20 \times 10^{-2} \text{ m} \)

\( \Delta t = 1 \text{ s}, t = 90 \text{ s} \)

\[ \text{error} \in g \]

\[ \frac{\Delta g}{g} \times 100 = \left( \frac{\Delta L}{L} + \frac{2\Delta t}{t} \right) \times 100 \]

\[ \frac{\Delta g}{g} \times 100 = \left( \frac{10^{-3}}{20 \times 10^{-2}} + \frac{2 \times 1}{90} \right) \times 100 \]

\[ = \frac{1}{2} + \frac{20}{9} \]

\[ = 0.5 + 2.22 \]

\[ \approx 3 \]

Topic: Unit & Dimensions

Difficulty: Easy (embibe predicted easy to score)

Ideal time: 90

3. A long cylindrical shell carries positive surface charge \( \sigma \) in the upper half and negative surface charge \( -\sigma \) in the lower half. The electric field lines around the cylinder will look like figure given in:

(Figures are schematic and not drawn to scale)
Answer: (D)

Solution:
Consider cross section of cylinders which is circle the half part of circle which has positive charge can be assume that total positive charge is at centre of mass of semicircle. In the same way we can assume that negative charge is at centre of mass of that semicircle.

Now it acts as a dipole now by the properties of dipole and lows of electric field line where two lines should not intersect the graph would be

Topic: Electrostatics

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 90

4. A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz. The frequency of the resultant signal is/are:

(A) 2005 kHz, and 1995 kHz
(B) 2005 kHz, 2000 kHz and 1995 kHz
(C) 2000 kHz and 1995 kHz
(D) 2 MHz only

Answer: (B)

Solution:
5. Consider a spherical shell of radius $R$ at temperature $T$, the black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $p = \frac{1}{3} \left( \frac{U}{V} \right)$. If the shell now undergoes an adiabatic expansion the relation between $T$ and $R$ is:

(A) $T \propto e^{-3R}$
(B) $T \propto \frac{1}{R}$
(C) $T \propto \frac{1}{R^3}$
(D) $T \propto e^{-R}$

Answer: (B)

Solution: $\therefore$ in an adiabatic process.

$$dQ = 0$$

So by first law of thermodynamics
\[ dQ = dU + dW \]
\[ \Rightarrow 0 = dU + dW \]
\[ \Rightarrow dW = -d \]
\[ \therefore dW = PdV \]

\[ \Rightarrow PdV = -dU \quad \text{(i)} \]

Given that \[ \frac{U}{V} \propto T^4 \Rightarrow U = kVT^4 \]

\[ \Rightarrow dU = kd(VT^4) = K(T^4 dV - 4T^3 V dT) \]

Also, \[ P = \frac{1}{3} \frac{U}{V} = \frac{1}{3} \frac{kVT^4}{V} = \frac{KT^4}{3} \]

Putting these values in equation

\[ \Rightarrow \frac{KT^4}{3} dV = -k(T^4 dV + 4T^3 V dT) \]

\[ \Rightarrow \frac{TdV}{3} = -TdV + 4V dT \]

\[ \Rightarrow \frac{4T}{3} dV = 4V dT \]

\[ \Rightarrow \frac{1}{3} dV = -dT \]

\[ \Rightarrow \frac{1}{3} lnV = -lnT \Rightarrow lnV = lnT^{-3} \]

\[ \Rightarrow VT^3 = \text{constant} \]

\[ \Rightarrow \frac{4}{3} \pi R^3 T^3 = \text{constant} \]

\[ \Rightarrow RT = \text{constant} \]

\[ \Rightarrow T \propto \frac{1}{R} \]

**Topic:** Heat & Thermodynamics

**Difficulty:** Difficult (embibe predicted high weightage)

**Ideal time:** 120

6. An inductor \( L = 0.03H \) and a resistor \( R = 0.15k\Omega \) are connected in series to a battery of 15V EMF in a circuit shown below. The key \( K_1 \) has been kept closed for a long time. Then at \( t = 0 \), \( K_1 \) is opened and key \( K_2 \) is closed simultaneously. At \( t = 1ms \), the current in the circuit will be : \( (e^5 \approx 150) \)
Answer: (C)

Solution:

\[ L = 0.03 \text{H} \]

\[ R = 0.15k\Omega = 150\Omega \]

\[ (\sigma^0 = 150 \text{ given}) \]

\[ E = 15\text{V} \]

Case I: \( K_2 \) is closed for long time
for long time, inductor acts as a conducting wire.

\[ i = \frac{V}{R} \]

\[ i = \frac{15}{150} \]

\[ i_0 = 0.1 \text{ A} \]

**Case II: K_1 is open and K_2 is closed**

Current in the circuit

\[ i = I_0 e^{-\frac{t}{\tau}} \]

\[ \tau = \frac{L}{R} \]

After \( t = 1 \text{ ms} = 10^{-3} \text{s} \)

\[ i = I_0 e^{-\frac{10^{-3} \times 150}{3 \times 10^{-2}}} \]
7. A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to $T_M$. If the Young’s modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to:

\[ (g = \text{gravitational acceleration}) \]

(A) $\left( \frac{T_M}{T} \right)^2 - 1 \left( \frac{Mg}{A} \right)$

(B) $\left( 1 - \frac{T_M}{T} \right)^2 \left( \frac{A}{Mg} \right)$

(C) $\left( 1 - \left( \frac{T}{T_M} \right)^2 \right) \left( \frac{A}{Mg} \right)$

(D) $\left( \frac{T_M}{T} \right)^2 - 1 \left( \frac{A}{Mg} \right)$

Answer: (D)

Solution:
Initial length = \( \ell \)

Time period \( T = 2\pi \sqrt{\frac{\ell}{g}} \) \( \ldots \) (i)

After suspending mass \( M \),

Young's modulus \( Y = \frac{\text{Stress}}{\text{Strain}} \)

\[ Y = \frac{F}{\Delta \ell} = \frac{F}{\Delta \ell A} \]

Change in length of wire \( \Delta \ell = \frac{F\ell}{AY} \)

Now Time period \( T_M = 2\pi \sqrt{\frac{\ell + \Delta \ell}{g}} \) \( \ldots \) (ii)

\[ \frac{T}{T_M} = \frac{2\pi \sqrt{\frac{\ell}{g}}}{2\pi \sqrt{\frac{\ell + \Delta \ell}{g}}} \]

\[ \Rightarrow \frac{T^2}{T_M^2} = \frac{\ell + \Delta \ell}{\ell} \]

\[ \frac{T^2}{T_M^2} = \frac{\ell + \frac{F\ell}{AY}}{\ell} \]

[putting \( \Delta \ell \) value]
8. A red LED emits light at 0.1 watt uniformly around it. The amplitude of the electric field of the light at a distance of 1 m from the diode is:

(A) 2.45 V/m
(B) 5.48 V/m
(C) 7.75 V/m
(D) 1.73 V/m

Answer: (A)

Solution:
For a point source of power $P$, then intensity at a point at a separation $x$ from the source is

$$I = \frac{\text{Power}}{\text{Area}} = \frac{P}{\pi x^2}$$

Average intensity of EM wave is given by

$$I = \frac{1}{2} \varepsilon_0 \frac{E^2}{v^2}$$

$$\Rightarrow E_0 = \frac{3P}{4\pi \varepsilon_0 c v^2}$$

$$\Rightarrow \frac{1}{4\pi \varepsilon_0 c} = 9 \times 10^9, P = 0.1 \text{ W}, x = 1 \text{ m}$$

$$c = \text{Speed of light} = 3 \times 10^8 \text{ m/s}$$

$$\Rightarrow E_0 = \sqrt{\frac{2 \times 0.1 \times 9 \times 10^9}{3 \times 10^8 \times 1^2}} \approx 2.45 \text{ V/m}$$

**Topic:** Optics

**Difficulty:** Easy (embibe predicted high weightage)

**Ideal time:** 120

9. Two coaxial solenoids of different radii carry current $I$ in the same direction. Let $\vec{F}_1$ be the magnetic force on the inner solenoid due to the outer one and $\vec{F}_2$ be the magnetic force on the outer solenoid due to the inner one. Then:

(A) $\vec{F}_1$ is radially inwards and $\vec{F}_2$ is radially outwards
(B) $\vec{F}_1$ is radially inwards and $\vec{F}_2 = 0$
(C) $\vec{F}_1$ is radially outwards and $\vec{F}_2 = 0$

**Answer:** (D)

**Solution:**
$S_2$ is solenoid with more radius than $S_1$ field because of $S_1$ on $S_2$ is 0

$\therefore$ force on $S_2$ by $S_1 = 0$

In the uniform field of $S_2$ $S_1$ behaves as a magnetic dipole

$\therefore$ force on $S_1$ by $S_2$ is zero because force on both poles are equal in magnitude and opposite indirection.

Topic: Magnetism

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 120

10. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increase as $V^q$, where $V$ is the volume of the gas. The value of $q$ is:

$$y = \frac{C_p}{C_v}$$

(A) $\frac{3y-5}{6}$

(B) $\frac{y+1}{2}$

(C) $\frac{y-1}{2}$

(D) $\frac{3y+5}{6}$

Answer: (B)
Average time of collision

\[ t = \frac{\text{mean free path}}{\text{average speed}} \]

\[ t \propto \frac{1}{v} \]

\[ \lambda \propto \frac{1}{\text{no. of molecules per unit volume}} \]

\[ \lambda \propto \frac{1}{(\frac{N}{V})} \]

\[ \Rightarrow \lambda \propto V \]

And \( \nu \propto \sqrt{T} \):

\[ \Rightarrow \nu \propto \sqrt{PV} \]

\[ \Rightarrow P \propto V^{-y} \]

for adiabatic process where \( y = \text{adiabatic coefficient} \):

\[ \Rightarrow \nu \propto \sqrt{V^{1-y}} \]

\[ \Rightarrow \nu \propto V^{\frac{1-y}{2}} \]

So average time

\[ t_{avg} \propto \frac{\nu}{V^{\frac{1-y}{2}}} \]

\[ t_{avg} \propto V^{1-(1-y)} \]

\[ t_{avg} \propto V^{1-y} \]

\[ \therefore q = \frac{1+y}{2} \]

Topic: Heat & Thermodynamics

Difficulty: Difficult (embibe predicted high weightage)

Ideal time: 120
11. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to $Q_0$ and then connected to the L and R as shown below:

If a student plots graphs of the square of maximum charge ($Q_{Max}^2$) on the capacitor with time (t) for two different values $L_1$ and $L_2$ ($L_1 > L_2$) of L then which of the following represents this graph correctly? (plots are schematic and not drawn to scale)

(A) 
(B) 
(C) 
(D) 

Answer: (D) 

Solution:
Comparing to damped pendulum

We write

\[ m \frac{dv}{dt} = -kv - bv; \textit{by is resistive force} \]

\[ \Rightarrow \text{Amplitude } A = A_0 e^{-\frac{bt}{2m}} \]

Comparing results, we can write

\[ q = + L \frac{di}{dt} + iR \]

as charge decreasing

\[ \frac{q}{C} = L \frac{d^2 q}{dt^2} - \frac{dq}{dt} \cdot R. \]

\[ \Rightarrow A = q; R = b, m = L \]

\[ q = q_0 e^{-\frac{R_1}{2L}} \]

\[ q^2 = q_0^2 e^{-\frac{R_1}{L}} \]

Exponentially decreasing function more ‘L’ losses will \( \left( \frac{R}{L} \right) \) and more will be \( e^{-\left( \frac{R_1}{L} \right)} \):

\[ L_1 \text{ graph has higher values than } L_2 \]

**Topic:** Magnetism

**Difficulty:** Difficult (embibe predicted high weightage)

**Ideal time:** 150
From a solid sphere of mass M and radius R, a spherical portion of radius $\frac{R}{2}$ is removed, as shown in the figure. Taking gravitational potential $V = 0$ and $r = \infty$, the potential at the center of the cavity thus formed is:

\[ (G = \text{gravitational constant}) \]

(A) $-\frac{G}{R}$

(B) $-\frac{2G}{3R}$

(C) $-\frac{2G}{R}$

(D) $-\frac{GM}{2R}$

Answer: (A)

Solution:

Potential due to whole sphere if cavity is not there at distance $\frac{R}{2}$ from centre

\[ = -\frac{GM}{R^2} \left( \frac{3}{2}R^2 - \frac{1}{2} \cdot \frac{R^2}{3} \right) \]

\[ = -\frac{GM}{R^2} \left( \frac{13}{6} R^2 - \frac{R^2}{3} \right) \]

\[ = -\frac{GM}{R^2} \cdot \frac{11R^2}{6} \]
13. A train is moving on a straight track with speed \(20\text{ms}^{-1}\). It is blowing its whistle at the frequency of \(1000\text{ Hz}\). The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound \(320\text{ms}^{-1}\)) close to:

(A) 12%
(B) 18%
(C) 24%
(D) 6%

Answer: (A)

Solution:
Before $f_0 = 1000 \ Hz$ 

$$f' = \left( \frac{\nu}{\nu - v_2} \right) \times f_0$$

$$= \left( \frac{320}{320 - 20} \right) \times f_0$$

$$= \left( \frac{320}{300} \right) \times f_0$$

$$= \frac{16}{15} f_0$$

$$f'' = \left( \frac{\nu}{\nu + v_2} \right) \times f_0$$

$$f'' = \left( \frac{320}{320 + 20} \right) f_0$$

$$= \left( \frac{320}{340} \right) f_0$$

$$= \left( \frac{16}{17} \right) f_0$$

Change in frequency

$$= \left( \frac{16}{15} \times \frac{16}{17} \right) f_0$$

$\therefore$ Percentage change in frequency

$$= \left( \frac{16}{15} \times \frac{16}{17} \right) f_0 \times 100 \approx 12.5\% \text{ nearly}$$
Given in the figure are two blocks A and B of weight 20N and 100N, respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is:

(A) 80 N  
(B) 120 N  
(C) 150 N  
(D) 100 N

Answer: (B)

Solution:

For complete state equilibrium of the system. The state friction on the block B by wall will balance the total weight 120 N of the system.

Topic: Laws of Motion

Difficulty: Moderate (embibe predicted Low Weightage)

Ideal time: 60

15. a solid uniform cone its vertex is \( z_0 \). If the radius of its base is \( R \) and its height is \( h \) then \( z_0 \) is equal to:

(A) \( \frac{3h}{4} \)
Consider an elementary disc of radius $r$ and thickness $dy$.

If total mass of cone $= M$ and density $= \rho$

Then mass of elementary disc is $dm = \rho \, dv = \rho \times \pi r^2 dy$ .... (1)

In similar $\triangle$'s AOE and AO'C

$\frac{y}{h} = \frac{r}{R} \Rightarrow r = \frac{y}{h} R$ .... (2)
16. A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is placed in different orientations as shown in the figure below:

If there is a uniform magnetic field of 0.3 T in the positive z direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium?

(A) (a) and (c), respectively
Answer: (B)

Solution: For a magnetic dipole placed in a uniform magnetic field the torque is given by
\[ \vec{\tau} = \vec{M} \times \vec{B} \]
and potential energy \( U \) is given as
\[ U = -\vec{M} \cdot \vec{B} = -MB\cos\theta \]

When \( \vec{M} \) is in the same direction as \( \vec{B} \) then \( \vec{\tau} = 0 \) and \( U \) is min = - MB as \( \theta = 0^\circ \)
\[ \Rightarrow \text{Stable equilibrium is (b). and when } \vec{M} \text{ then } \vec{\tau} = 0 \text{ and } U \text{ is max = + MB} \]
As \( \theta = 180^\circ \)

Unstable equilibrium in (d)

Topic: Magnetism

Difficulty: Easy (embibe predicted high weightage)

Ideal time: 30

17.

In the circuit shown, the current in the 1Ω resistor is:

(A) 0A
(B) 0.13 A, from Q to P
(C) 0.13 A, from P to Q
(D) 1.3 A, from P to Q

Answer: (B)

Solution:
The distribution of current according to Kirchhoff's first law is as shown in the circuit. By Kirchhoff's second law (voltage rule)

In loop APQBA using sign curve line

\[ 6 - 3I - I_1 = 0 \]

\[ \Rightarrow 3I + I_1 = 6 \quad \ldots(i) \]

In loop CD & PQ

\[ \Rightarrow -3(I - I_1) + 9 - 2(I - I_1) + 1 \times I_1 = 0 \]

\[ \Rightarrow 9 - 5(I - I_1) + I_1 = 0 \]

\[ \Rightarrow 0 + 6I_1 - 5I = 0 \]

\[ \Rightarrow 5I - 6I_1 = 0 \quad \ldots(ii) \]

(Multiplying \((i)\) by 5) - (Multiplying \((ii)\) by 3)

\[ \Rightarrow 15I + 5I_1 = 30 \]

\[ 15I - 18I_1 = 27 \]

\[
\begin{align*}
23I_1 &= 3 \\
I_1 &= \frac{3}{23} \text{ A}
\end{align*}
\]

+ve sign of \(I_1\) shows that current 0.13 A flows from Q to P.

Topic: Electrostatics

Difficulty: Easy (embibe predicted high weightage)

Ideal time: 90
18. A uniformly charged solid sphere of radius \( R \) has potential \( V_0 \) (measured with respect to \( \infty \)) on its surface. For this sphere the equipotential surfaces with potential \( \frac{3V_0}{2} \), \( \frac{5V_0}{4} \) and \( \frac{3V_0}{4} \) have radius \( R_1 \), \( R_2 \), \( R_3 \) and \( R_4 \) respectively. Then

(A) \( R_1 \neq 0 \) and \( (R_2 - R_1) > (R_4 - R_3) \)
(B) \( R_1 = 0 \) and \( R_2 < (R_4 - R_3) \)
(C) \( 2R < R_4 \)
(D) \( R_1 = 0 \) and \( R_2 > (R_4 - R_3) \)

Answer: (B)

Solution:

\[
\text{Potential for uniformly charged solid sphere}
\]

\[
v = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r} \quad \text{outside i.e. } r > R
\]

\[
v = \frac{1}{4\pi \varepsilon_0} \frac{Q}{R} \quad \text{on the surface}
\]

\[
v = \frac{1}{4\pi \varepsilon_0} \frac{Q}{R} \left[ \frac{3}{2} - \frac{1}{2} \left( \frac{r}{R} \right)^2 \right] \quad \text{inside i.e. } r < R
\]

Clearly potential is decreasing with \( r \).

\[
\Rightarrow \quad \frac{3V_0}{2}, \quad \frac{5V_0}{4} \quad \text{are inside potentials } [v > V_0]
\]
\( \frac{3v_0}{4}, \quad \frac{v_0}{4} \) are outside potentials \( \Rightarrow v < v_0 \)

To get \( R_1 \):
\[
\frac{3v_0}{2} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R} \left[ \frac{3}{2} - \frac{1}{2} \frac{R_1^2}{R^2} \right]
\]
\[
v_0 = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R}
\]
\[
\frac{3}{2} = \frac{3}{2} - \frac{1}{2} \frac{R_1^2}{R^2} \Rightarrow R_1 = 0
\]

To get \( R_2 \):
\[
\frac{3}{4} v_0 = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R} \left[ \frac{3}{2} - \frac{1}{2} \frac{R_2^2}{R^2} \right]
\]
\[
\frac{5}{4} = \frac{3}{2} - \frac{1}{2} \frac{R_2^2}{R^2}
\]
\[
\frac{1}{2} \frac{R_2^2}{R^2} = \frac{1}{4}
\]
\[
R_2 = \frac{R}{\sqrt{2}}
\]

To get \( R_3 \):
\[
\frac{3v_0}{4} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R_3}
\]
\[
\frac{3}{4} \frac{Q}{4\pi\varepsilon_0} \frac{1}{R} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R_3}
\]
\[
\frac{3}{4R} = \frac{1}{R_3}
\]
\[
R_3 = \frac{4}{3} R
\]
19. In the given circuit, charge $Q_2$ on the $2\mu F$ capacitor changes as $C$ is varied from $1\mu F$ to $3\mu F$. $Q_2$ as a function of ‘$C$’ is given properly by: (figure are drawn schematically and are not to scale)
Answer: (A)

Solution:

\[ \therefore 1 \land 2 \mu F \text{ are in parallel.} \]

\[ \therefore \text{Equivalent capacitance of the series combination is } C_{eq} = \frac{3C}{C+3} \]

So total charge supplied by battery is \( Q = C_{eq} = \frac{3CE}{C+3} \)

\[ \therefore \text{Potential difference across parallel combination of } 1 \mu F \text{ ad } 2 \mu F \text{ is } \]

\[ \Delta V = \frac{Q}{3} = \frac{CE}{C+3} \]

So charge on \( 2 \mu F \) capacitor is
\[ Q_2 = C_2 \Delta V = \frac{2CE}{C+3} \]
\[ \Rightarrow \frac{Q_2}{2E} = \frac{C}{C+3} \Rightarrow \frac{Q_2}{2E} = \frac{C+3-3}{C+3} \]
\[ \Rightarrow \frac{Q_2}{2E} = 1 - \frac{3}{C+3} \Rightarrow \left( \frac{Q_2}{2E} - 1 \right) = \frac{-3}{C+3} \]
\[ \Rightarrow (Q_2 - 2E)(C + 3) = -6E \]
Which is of the form \((y - \alpha)(x + \beta) < 0\)
So the graph is a hyperbola. With downward curve line, i.e
\[
\begin{array}{c|c}
\text{-}Q_2 & \\
\hline 
C & \\
\end{array}
\]
Topic: Electrostatics
Difficulty: Moderate (embibe predicted high weightage)
Ideal time: 120

20. A particle of mass m moving in the \(x\) direction with speed \(2v\) is hit by another particle of mass \(2m\) moving in the \(y\) direction with speed \(v\). If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to:

(A) 50%
(B) 56%
(C) 62%
(D) 44%

Answer: (B)
Solution:
The initial momentum of system is \(\vec{P_i} = m(2V)\hat{k} + (2m)v\hat{j}\)

According to question as
On perfectly inelastic collision the particles stick to each other so.

\[ \overrightarrow{P}_f = 3m\overrightarrow{V}_f \]

By conservation of linear momentum principle

\[ \overrightarrow{P}_f = \overrightarrow{P}_i \Rightarrow 3m\overrightarrow{V}_f = m2Vt + 2mVf \]

\[ \Rightarrow \overrightarrow{V}_f = \frac{2V}{3} (t + f) \Rightarrow V_f = \frac{2\sqrt{2}}{3}V \]

\[ \therefore \text{loss in KE of system } K_{\text{initial}} - K_{\text{final}} \]

\[ \frac{1}{2} m(2V)^2 + \frac{1}{2} (2m)V^2 - \frac{1}{2} (3m)\left(\frac{2\sqrt{2}V}{3}\right)^2 \]

\[ 2mV^2 + mV^2 - \frac{4}{3} mV^2 = 3mV^2 - \frac{4mV^2}{3} \]

\[ \frac{5}{3} mV^2 \]

\[ \% \text{ change in KE } 100 \times \frac{\Delta K}{K_i} = \frac{\frac{5}{3} mV^2}{3mV^2} = \frac{5}{9} \times 100 \]

\[ \frac{500}{9} = 56 \]

Topic: Conservation of Momentum

Difficulty: Moderate (embibe predicted easy to score (Must Do))

Ideal time: 90

21. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is \( \mu \), a ray, incident at an angle \( \theta \), on the face AB would get transmitted through the face AC of the prism provided:
(A) $\theta < \sin^{-1} \left[ \mu \sin \left( A - \sin^{-1} \left( \frac{1}{\mu} \right) \right) \right]$

(B) $\theta > \cos^{-1} \left[ \mu \sin \left( A + \sin^{-1} \left( \frac{1}{\mu} \right) \right) \right]$

(C) $\theta < \cos^{-1} \left[ \mu \sin \left( A + \sin^{-1} \left( \frac{1}{\mu} \right) \right) \right]$

(D) $\theta > \sin^{-1} \left[ \mu \sin \left( A - \sin^{-1} \left( \frac{1}{\mu} \right) \right) \right]$

Answer: (D)

Solution:

For emergence $r_2 < \text{critical angle}$

$\Rightarrow r_2 < \sin^{-1} \left( \frac{1}{\mu} \right)$

$A = r_1 + r_2$

$\Rightarrow A - r_1 = r_2$

$\Rightarrow A - r_1 < \sin^{-1} \left( \frac{1}{\mu} \right)$
22. From a solid sphere of mass $M$ and radius $R$ a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is:

(A) $\frac{MR^2}{3\sqrt{2} \pi}$

(B) $\frac{MR^2}{3\sqrt{3} \pi}$

(C) $\frac{4MR^2}{3\sqrt{3} \pi}$

(D) $\frac{MR^2}{3\sqrt{2} \pi}$

Answer: (B)

Solution: Let $a$ be length of cube for cube with maximum possible volume diagonal length = $2R$

$\Rightarrow \sqrt{3}a = 2R \Rightarrow a = \frac{2R}{\sqrt{3}}$
As densities of sphere and cube are equal. Let $M'$ be mass of cube

$$\frac{M}{\frac{4\pi R^3}{3}} = \frac{M'}{a^3}$$

$$M' = \frac{3Ma^3}{4\pi R^3}$$

Moment of inertial of cube about an axis passing through centre

$$\frac{M'(2a^2)}{12}$$

$$\frac{3Ma^3}{4\pi R^3} \times \frac{2a^2}{12}$$

$$\frac{Ma^5}{8\pi R^3}$$

$$a = \frac{2}{\sqrt{3}}R$$

$$\frac{M \times 32R^5}{8\pi \times 9\sqrt{3}R^3}$$

$$\frac{4MR^2}{9\sqrt{3}\pi}$$

**Topic:** Rotational Mechanics

**Difficulty:** Moderate (embibe predicted Low Weightage)

**Ideal time:** 300

23. Match list-I (Fundamental Experiment) with List-II (its conclusion) and select the correct option from the choices given below the list:

<table>
<thead>
<tr>
<th>List-I</th>
<th>List-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(i)</td>
</tr>
<tr>
<td>B</td>
<td>(ii)</td>
</tr>
<tr>
<td>C</td>
<td>(iii)</td>
</tr>
<tr>
<td>D</td>
<td>(iv)</td>
</tr>
</tbody>
</table>

(A) $A - (ii); B - (iv); C - (iii)$

(B) $A - (ii); B - (i); C - (iii)$

(C) $A - (iv); B - (iii); C - (ii)$
Answer: (B)

Solution: Frank-Hertz experiment demonstrated the existence of excited states in mercury atoms helping to confirm the quantum theory which predicted that electrons occupied only discrete quantized energy states.

Phot-electric experiment = Demonstrate that photon is the field particle of light which can transfer momentum and energy due to collision.

Davisson-Germer experiment = this experiment shows the wave nature of electron.

Topic: Modern Physics

Difficulty: Easy (embibe predicted high weightage)

Ideal time: 30

24. When 5V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is \(2.5 \times 10^{-4} \text{ m/s}\). If the electron density in the wire is \(8 \times 10^{28} \text{ m}^{-3}\), the resistivity of the material is close to:

(A) \(1.6 \times 10^{-7} \Omega \text{m}\)

(B) \(1.6 \times 10^{-6} \Omega \text{m}\)

(C) \(1.6 \times 10^{-5} \Omega \text{m}\)

(D) \(1.6 \times 10^{-8} \Omega \text{m}\)

Answer: (C)

Solution:
Potential difference = 5V

\[ \text{length} = 0.1 \text{m} = \ell \]

Electron speed = drift velocity \( v_d = 2.5 \times 10^{-4} \text{m/s} \)

electron density \( (n) = 8 \times 10^{28} \text{m}^{-3} \)

charge on each electron(e) = \( 1.6 \times 10^{-19} \text{C} \)

We know \( i = nAe v_d \quad \cdots \text{(i)} \)

And \( v = iR \quad \cdots \text{(ii)} \)

\[
\begin{align*}
\text{Resistance } R \text{ is also equal } & \frac{\rho \ell}{A} \\
R &= \frac{\rho \ell}{A} \\
\rho &= \frac{Al}{\ell} \quad [\rho = \text{Resistivity}] \\
&= \frac{A}{\ell} \times \frac{v}{1} \quad [\text{from (i)}] \\
&= \frac{A v}{\ell nA e v_d} \quad & [\text{from (ii)}] \\
&= \frac{v}{\ell nA e v_d} \\
&= \frac{5}{0.1 \times 8 \times 10^{28} \times 1.6 \times 10^{-19} \times 2.5 \times 10^{-4}} \\
&= 0.16 \times 10^{-4} \frac{\text{m}}{\text{V}} = 1.6 \times 10^{-5} \frac{\text{m}}{\text{V}}
\end{align*}
\]

Topic: Electrostatics

Difficulty: Easy (embibe predicted high weightage)

Ideal time: 120
25. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly? (Graphs are schematic and not drawn to scale)

(A)

(B)

(C)

(D)

Answer: (A)

Solution:
For simple pendulum performing simple harmonic motion, displacement

\[ y = A \sin \omega t \]

Velocity

\[ \frac{dy}{dt} = V = \omega A \cos \omega t \]

\[ = A \omega \sqrt{1 - \sin^2 \omega t} \]

\[ = A \omega \sqrt{1 - \frac{y^2}{A^2}} \]

\[ = \omega \sqrt{A^2 - y^2} \]

Kinetic energy

\[ \frac{1}{2} mv^2 \]

\[ = \frac{1}{2} \times m \times \omega^2 (A^2 - y^2) \]

at \[ y = A \] (extreme positions)

Kinetic energy

\[ \frac{1}{2} \omega^2 m (A^2 - A^2) = 0 \]

Similarly potential energy

\[ \frac{1}{2} m\omega^2 y^2 \]

On plotting graphs of potential energy & Kinetic energy

---

**Topic:** Simple Harmonic Motion

**Difficulty:** Easy (embibe predicted high weightage)

**Ideal time:** 60

26. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first?

(Assume stones do not rebound after hitting the ground and neglect air resistance, take \( g = 10 \text{ms}^{-2} \))

(the figure are schematic and not drawn to scale)

(A)
Solution: \[ S_1 = 10t - \frac{1}{2}gt^2 \]

When \( S_1 = -240 \)

\[ \Rightarrow -240 = 10t - \frac{1}{2}gt^2 \]

\[ \Rightarrow t = 8s \]

So at \( t = 8 \) seconds first stone will reach ground

\[ S_2 = 20t - \frac{1}{2}gt^2 \]

Till \( t = 8 \) seconds

\[ S_2 - S_1 = 30t \]

But after 8 second \( S_1 \) is constant \(-240\)

Relative to stone \( t_1 > 8 \) seconds, displacements of stone 2 \( S_2 + 240 \)
\[ \Rightarrow s_2 + 240 = 20t - \frac{1}{2}gt^2 \]

And at \( t = 12 \) s seconds stone will reach ground.

The corresponding graph of relative position of second stone w.r.t. first is

[Graph]

Topic: Kinematics

Difficulty: Moderate (Embibe predicted high weightage)

Ideal time: 240

27. A solid body of constant heat capacity \( 1 \text{ J}/^\circ\text{C} \) is being heated by keeping it in contact with reservoirs in two ways:

(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.
(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

In both the cases body is brought from initial temperature \( 100^\circ\text{C} \) to final temperature \( 200^\circ\text{C} \). Entropy change of the body in the two cases respectively is:

(A) \( \ln 2, \ln 2 \)
(B) \( 2\ln 2, 2\ln 2 \)
(C) \( 2\ln 2, 8\ln 2 \)
(D) \( \ln 2, 4\ln 2 \)

Answer: (A)

Solution:
Change in entropy $ds = \frac{dq}{T}$

$\Delta Q = \text{heat supplied} = C \Delta T$

$dQ = c dT$

$ds = \frac{c dT}{T}$

Integrating both sides

$\int_{S_i}^{S_f} ds = \int_{T_1}^{T_2} C \frac{dT}{T}$

$S_f - S_i = \Delta S = C \ln \frac{T_2}{T_1}$

$= C \ln \frac{200}{100}$

$\Delta S = C \ln 2$

$C = 1 \text{J/}^\circ\text{C}$

$\Rightarrow \Delta S = \ln 2$

Entropy change is same for both cases as $C$ is constant, and temperature change (i.e. from 100 to 200) in same.

Topic: Heat & Thermodynamics

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 90

28. Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is:

(A) 30 $\mu$m
(B) 100 $\mu$m
(C) 300 $\mu$m
(D) 1 $\mu$m

Answer: (A)
Solution:

By Fraunhofer diffraction through a circular aperture $\theta = \frac{1.22 \lambda}{D}$

\[ D = \text{diameter of pupil} = 2 \times 0.25 = 0.5 \text{ cm} \]

$\lambda = 500 \text{ nm}$

First dark ring is formed by the light diffracted from the hole at an angle $\theta$ with the axis.

Viewing distance $D = 25 \text{ cm}$

$\therefore$ minimum separation between $2$ objects $= D\theta$

\[ = \frac{25 \times 10^{-2} \times 1.22 \times 500 \times 10^{-6}}{5 \times 10^{-1}} \]

\[ = 30 \times 10^{-6} \text{ m} \]

\[ = 30 \mu \text{m} \]

Topic: Optics

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 120
Two long current carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle \( \theta \) with the vertical. If wires have mass \( \lambda \) per unit length then the value of I is:

\( g = \) gravitational acceleration

(A) \( 2\sin \theta \sqrt{\frac{\pi \lambda g L}{\mu_0 \cos \theta}} \)

(B) \( 2 \sqrt{\frac{\pi g L}{\mu_0} \tan \theta} \)

(C) \( \sqrt{\frac{\pi \lambda g L}{\mu_0} \tan \theta} \)

(D) \( 2 \sqrt{\frac{\pi g L}{\mu_0} \tan \theta} \)

Answer: (A)

Solution

Two wires will repel each other due to current in the same direction and due to magnetic force.

\[ B = \frac{\mu_0 I}{2\pi L \sin \theta} \]

\[ \therefore \text{ magnetic force per unit length is} \]

\[ F = \frac{dF}{dl} = \frac{\mu_0 I^2}{2\pi (2L \sin \theta)} = \frac{\mu_0 I^2}{4\pi L \sin \theta} \]

and mass per unit length of each wire \( = \frac{dm}{dl} = \lambda \)

So, magnetic force on total length \( \ell' \) of the mix is

\[ F_m = \frac{\mu_0 I^2 \ell'}{4\pi L \sin \theta} \]

and weight \( = \lambda \ell' g \)

By equilibrium of mix,
On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygens’ principle leads us to conclude that as it travels, the light beam:

(A) Goes horizontally without any deflection
(B) Bends downwards
(C) Bends upwards
(D) Becomes narrower

Answer: (C)

Solution: Consider air layers with increasing refractive index.
At critical angle it will bend upwards at interface. This process continues at each layer, and light ray bends upwards continuously.

Topic: Optics

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 60
Chemistry

1. Which of the following is the energy of a possible excited state of hydrogen?
   (A) $-6.8\, eV$
   (B) $-3.4\, eV$
   (C) $+6.8\, eV$
   (D) $+13.6\, eV$

   **Solution:** (B) $E_n = \frac{-13.6}{n^2} \, eV$
   
   Where $n = 2 \Rightarrow E_2 = -3.40\, eV$

2. In the following sequence of reactions:
   
   $Toluene \xrightarrow{KMnO_4} ASOCl_2 B H_2/Pd, BaSO_4 C$

   The Product C is:
   (A) $C_6 H_5 CH_3$
   (B) $C_6 H_5 CH_2 OH$
   (C) $C_6 H_5 CHO$
   (D) $C_6 H_5 COOH$

   **Solution:** (C)

3. Which compound would give 5-keto-2-methyl hexanal upon ozonolysis:
   (A)
4. The ionic radii (Å) of $3^-, O$ and $F^-$ are respectively:

(A) 1.36, 1.71 and 1.40
(B) 1.71, 1.40 and 1.36
(C) 1.71, 1.36 and 1.40
(D) 1.36, 1.40 and 1.71
Solution: (B) As $\frac{Z}{r}$ ↑ ionic radius decreases for isoelectronic species.

$$\begin{align*}
3 - \frac{Z}{r} &= \frac{7}{10} \\
N \\
2 - \frac{Z}{r} &= \frac{8}{10} \\
O \\
- \frac{Z}{r} &= \frac{9}{10} \\
F \\
2\rightarrow F \\
3\rightarrow O \\
N
\end{align*}$$

5. The color of $KMnO_4$ is due to:

(A) $d - d$ transition
(B) $L \rightarrow M$ charge transfer transition
(C) $\sigma - \sigma$ transition
(D) $M \rightarrow L$ charge transfer transition

Solution: (B) Charge transfer from $O$ to empty d-orbitals of metal ion ($Mn^{4+}$)

6. Assertion: Nitrogen and Oxygen are the main components in the atmosphere but these do not react to form oxides of nitrogen.
Reason: The reaction between nitrogen and oxygen requires high temperature.

(A) Both Assertion and Reason are correct, but the Reason is not the correct explanation for the Assertion.
(B) The Assertion is incorrect but the Reason is correct.
(C) Both the Assertion and Reason are incorrect.
(D) Both Assertion and Reason are correct and the Reason is the correct explanation for the Assertion.

Solution: (D) $N_2(g)$ & $O_2(g)$ react under electric arc at $2000^\circ C$ to form $NO(g)$. Both assertion and reason are correct and reason is correct explanation.

7. Which of the following compounds is not an antacid?
(A) Cimetidine
(B) Phenezine
(C) Rantidine
Aluminium hydroxide

Solution: (B) Ranitidine, Cimetidine and metal hydroxides i.e. Aluminum hydroxide can be used as antacid but not phenelzine. Phenelzine is not an antacid. It is an antidepressant. Antacids are a type of medication that can control the acid levels in stomach. Working of antacids: Antacids counteract (neutralize) the acid in stomach that’s used to aid digestion. This helps reduce the symptoms of heartburn and relieves pain.

8. In the context of the Hall-Heroult process for the extraction of Al, which of the following statements is false?

(A) $Al_2O_3$ is mixed with $CaO$ which lowers the melting point of the mixture and brings conductivity.

(B) $3 \overset{Al}{\rightarrow} \overset{3+}{\rightarrow} \overset{Al}{\rightarrow}$ is reduced at the cathode to form Al.

(C) $Na_2AlF_6$ serves as the electrolyte.

(D) $CO$ and $CO_2$ are produced in this process.

Solution: (C) Hall-Heroult process for extraction of Al. $Al_2O_3$ is electrolyte $Na_2AlF_6$ reduces the fusion temperature and provides good conductivity.

9. Match the catalysts to the correct process:

<table>
<thead>
<tr>
<th>Catalyst</th>
<th>Process</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>TiCl$_3$ i. Wacker process</td>
</tr>
<tr>
<td>B</td>
<td>PdCl$_2$ ii. Ziegler – Natta polymerization</td>
</tr>
<tr>
<td>C</td>
<td>CuCl$_2$ iii. Contact process</td>
</tr>
<tr>
<td>D</td>
<td>V$_2$O$_5$ iv. Deacon’s process</td>
</tr>
</tbody>
</table>

(A) $A \rightarrow ii, B \rightarrow i, C \rightarrow iv, D \rightarrow iii$

(B) $A \rightarrow ii, B \rightarrow iii, C \rightarrow iv, D \rightarrow i$

(C) $A \rightarrow iii, B \rightarrow i, C \rightarrow ii, D \rightarrow iv$

(D) $A \rightarrow iii, B \rightarrow ii, C \rightarrow iv, D \rightarrow i$

Solution: (A) The Wacker process originally referred to the oxidation of ethylene to acetaldehyde by oxygen in water in the presence of tetrachloropalladate (II) as the catalyst.

In contact process, Platinum used to be the catalyst for this reaction, however as it is susceptible to reacting with arsenic impurities in the sulphur feedstock, vanadium (V) oxide ($V_2O_5$) is now preferred.

In Deacon’s process, The reaction takes place at about 400 to 450°C in the presence of a variety of catalysts, including copper chloride($CuCl_2$).
In Ziegler-Natta catalyst, Homogenous catalysts usually based on complexes of Ti, Zr or Hf used. They are usually used in combination with different organ aluminium co-catalyst.

10. In the reaction,

\[
\text{Product E is: (A)}
\]

\[
\text{(B)}
\]

\[
\text{(C)}
\]

\[
\text{(D)}
\]
11. Which polymer is used in the manufacture of paints and lacquers?
(A) Glyptal
(B) Polypropene
(C) Poly vinyl chloride
(D) Bakelite

Solution: (A) Glyptal is polymer of glycerol and phthalic anhydride.

12. The number of geometric isomers that can exist for square planar $\{\text{Pt(}Cl\text{)(py)}(\text{NH}_3)(\text{NH}_2\text{OH})\}$ is (py = pyridine):
(A) 3
(B) 4
(C) 6

(D) 2

Solution: (A) Complexes with general formula \([Mabcd]\) square planar complex can have three isomers.

13. Higher order (3) reactions are rare due to:
   (A) Increase in entropy and activation energy as more molecules are involved
   (B) Shifting of equilibrium towards reactants due to elastic collisions.
   (C) Loss of active species on collision
   (D) Low probability of simultaneous collision of all the reacting species

Solution: (D) Probability of an event involving more than three molecules in a collision are remote.

14. Which among the following is the most reactive?
   (A) \(Br_2\)
   (B) \(I_2\)
   (C) \(ICl\)
   (D) \(Cl_2\)

Solution: (C) \(I – Cl\) bond strength is weaker than \(I_2, Br_2\) and \(Cl_2\) (Homonuclear covalent).

15. Two Faraday of electricity is passed through a solution of \(CuSO_4\). The mass of copper deposited at the cathode is: (Atomic mass of Cu = 63.5 amu)

   (A) 63.5g
   (B) 2g
   (C) 127g
   (D) 0g

Solution: (A) \(2F \equiv 2Eqs of Cu\)

\[
2 \times \frac{63.5}{2} = 63.5g
\]

16. 3 g of activated charcoal was added to 50 mL of acetic acid solution (0.06N) in a flask. After an hour it was filtered and the strength of the filtrate was found to be 0.042 N. The amount of acetic acid adsorbed (per gram of charcoal) is:

   (A) 36 mg
(B) 42 mg
(C) 54 mg
(D) 18 mg

Solution: (D) Meqs of \( CH_3COOH \) (initial) \( 50 \times 0.06 = 3 \text{Meqs} \)

Meqs \( CH_3COOH \) (final) \( 50 \times 0.042 = 2.1 \text{Meqs} \)

\[ CH_3COOH_{\text{adsorbed}} = 3 - 2.1 = 0.9 \text{Meqs} \]

\[ 9 \times 10^{-1} \times 60 \text{ g/Eq} \times 10^{-3} \text{ g} \]

\[ 540 \times 10^{-4} = 0.054 \text{ g} \]

\[ 54 \text{ mg} \]

\[ \text{Per gram} = \frac{54}{3} = 18 \text{ mg} / \text{of Charcoal} \]

17. The synthesis of alkyl fluorides is best accomplished by:

(A) Sandmeyer’s reaction
(B) Finkelstein reaction
(C) Swarts reaction
(D) Free radical fluorination

Solution: (C) Alkyl fluorides is best accomplished by swarts reaction i.e. heating an alkyl chloride/bromide in the presence of metallic fluoride such as \( \text{AgF, } Hg_2F_2, CoF_2, SbF_3 \). 

\[ CH_3Br + AgF \rightarrow CH_3 - F + AgBr \]

The reaction of chlorinated hydrocarbons with metallic fluorides to form chlorofluoro hydrocarbons, such as \( CCl_2F_2 \) is known as swarts reaction.

18. The molecular formula of a commercial resin used for exchanging ions in water softening is \( C_8H_7SO_3Na \) (Mol. wt. 206). What would be the maximum uptake of \( \text{Ca}^{2+} \) ions by the resin when expressed in mole per gram resin?

(A) \( \frac{1}{206} \)
(B) \( \frac{2}{309} \)
(C) \( \frac{1}{412} \)
(D) \( \frac{1}{103} \)
19. Which of the vitamins given below is water soluble?
(A) Vitamin D
(B) Vitamin E
(C) Vitamin K
(D) Vitamin C

Solution: (D) B complex vitamins and vitamin C are water soluble vitamins that are not stored in the body and must be replaced each day.

20. The intermolecular interaction that is dependent on the inverse cube of distance between the molecules is:
(A) Ion-dipole interaction
(B) London force
(C) Hydrogen bond
(D) Ion-ion interaction

Solution: (C) $\text{Ion} - \text{ion interaction} \propto \frac{1}{r^2}$

Ion-dipole interaction $\propto \frac{1}{r^4}$

London forces $\propto \frac{1}{r^6}$

And Hydrogen bond $\propto \frac{1}{r^3}$

21. The following reaction is performed at 298 K.

$$2NO(g) + O_2(g) \rightleftharpoons 2NO_2(g)$$

The standard free energy of formation of $NO(g)$ is 86.6 kJ/mol at 298 K. What is the standard free energy of formation of $NO_2(g)$ at 298 K? ($K_p = 1.6 \times 10^{12}$)
22. Which of the following compounds is not colored yellow?

(A) \( K_3[Co(NO_2)_6] \)
(B) \( (NH_4)_3[As(Mo_3O_{10})_4] \)
(C) \( BaCr_4 \)
(D) \( Fe(CN)_6 \)

Solution: (D) Cyanides not yellow.

\[ BaCrO_4 - \]
\[ K_3[Co(NO_2)_6] - \]
\[ (NH_4)_3[As(Mo_3O_{10})_4] - \]

23. In Carius method of estimation of halogens, 250 mg of an organic compound gave 141 mg of AgBr. The percentage of bromine in the compound is: (Atomic mass: Ag = 108, Br = 80)

(A) 36
(B) 48
(C) 60
(D) 24

Solution: (D) \( R = Br_{Carus\ method}AgBr \)
250 mg organic compound is RBr

141 mgAgBr \rightarrow 141 \times \frac{80}{188} mgBr

Br in organic compound

141 \times \frac{80}{188} \times \frac{1}{250} \times 100 = 24

24. Sodium metal crystallizes in a body centred cubic lattice with a unit cell edge of 4.29Å. The radius of sodium atom is approximately:

(A) 3.22Å
(B) 5.72Å
(C) 0.93Å
(D) 1.86Å

Solution: (D)

For B.C.C

4r = \sqrt{3}a

r = \frac{\sqrt{3}}{4} a = \frac{1.732}{4} \times 4.29

1.86Å

25. Which of the following compounds will exhibit geometrical isomerism?

(A) 3 – Phenyl – 1- butene
(B) 2 – Phenyl – 1- butene
(C) 1,1 – Diphenyl – 1- propane
(D) 1 – Phenyl – 2 – butene

Solution: (D) 1 - Phenyl - 2 - butene:

![Geometrical Isomerism Diagram]

26. The vapour pressure of acetone at 20°C is 185 torr. When 1.2 g of a non-volatile substance was dissolved in 100 g of acetone at 20°C, its vapour pressure was 183 torr. The molar mass (g mol\(^{-1}\)) of the substance is:
27. From the following statements regarding \( H_2O_2 \), choose the incorrect statement:

- (A) It decomposes on exposure to light
- (B) It has to be stored in plastic or wax lined glass bottles in dark.
- (C) It has to be kept away from dust.
- (D) It can act only as an oxidizing agent.

Solution: (D) It can act both as oxidizing agent and reducing agent.

28. Which one of the following alkaline earth metal sulphates has its hydration enthalpy greater than its lattice enthalpy?

- (A) \( BeS_4 \)
- (B) \( BaSO_4 \)
- (C) \( SrSO_4 \)
- (D) \( CaSO_4 \)

Solution: (A) \( \Delta H_{\text{Hydration}} > \Delta H_{\text{Lattice}} \)

Salt is soluble. \( BeSO_4 \) is soluble due to high hydration energy of small \( Be^{2+} \) ion. \( K_{sp} \) for \( BeSO_4 \) is very high.
29. The standard Gibbs energy change at 300 K for the reaction \(2A \rightleftharpoons B + C\) is 2494.2 J. At a
given time, the composition of the reaction mixture is \([A] = \frac{1}{2}, [B] = 2\) and \([C] = \frac{1}{2}\). The
reaction proceeds in the: \(R = 8.314 J/\)mol, \(e = 2.718\)

(A) Reverse direction because \(Q > K_c\)
(B) Forward direction because \(Q < K_c\)
(C) Reverse direction because \(Q < K_c\)
(D) Forward direction because \(Q > K_c\)

Solution: (A) \(\Delta G^o = -RT \ln K_c\)

\[
2494.2 = -8.314 \times 300 \ln K_c
\]

\[
2494.2 = -8.314 \times 300 \times 2.303 \log K_c
\]

\[
\frac{-249.2}{2.303 \times 300 \times 8.314} = -0.44 = \log K_c
\]

\[
\log K_c = -0.44 = 1.56
\]

\(K_c = 0.36\)

\[
Q_c = \frac{2 \times \frac{1}{2}}{(\frac{1}{2})} = 4
\]

\(Q_c > K_c\) reverse direction.

30. Which one has the highest boiling point?

(A) Ne
(B) Kr
(C) Xe
(D) He

Solution: (C) Due to higher Vander Waal’s forces. Xe has the highest boiling point.
Mathematics

1. The sum of coefficients of integral powers of \( x \) in the binomial expansion of \( \left(1 - 2\sqrt{x}\right)^{50} \) is:

(A) \( \frac{1}{2} (3^{50}) \)

(B) \( \left( \frac{3}{|50 - 1|} \right)^{\frac{1}{2}} \)

(C) \( \frac{1}{2} (2^{50} + 1) \)

(D) \( \frac{1}{2} (3^{50} + 1) \)

Answer: (D)
Solution:
Integral powers \( \Rightarrow \) odd terms

\[
\sum_{terms} = \frac{(1 - 2\sqrt{x})^{50} + (1 + 2\sqrt{x})^{50}}{2} \cdot \frac{1 + 3^{50}}{2}
\]

2. Let \( f(x) \) be a polynomial of degree four having extreme values at \( x = 1 \) and \( x = 2 \). If \( \lim_{x \to 0} \left[ 1 + \frac{f(x)}{x^2} \right] = 3 \), then \( f(2) \) is equal to:

(A) \( -4 \)

(B) \( 0 \)

(C) \( 4 \)

(D) \( -8 \)
Answer: (B)

Solution:
Let \( f(x) = ax^4 + bx^3 + cx^2 + dx + e \)

\[
\lim_{x \to 0} \left[ 1 + \frac{f(x)}{x^2} \right] = 3
\]

\[
\lim_{x \to 0} \left[ 1 + ax^2 + bx + c + \frac{d}{x} + \frac{e}{x^2} \right] = 3
\]

This limit exists when \( d = e = 0 \)

So, \( \lim_{x \to 0} \left[ 1 + ax^2 + bx + \right] = 3 \)

\[ \Rightarrow c + 1 = 3 \]

\[ c = 2 \]

It is given, \( x = 1 \ \& \ x = 2 \) are solutions of \( f'(x) = 0 \)

\[ f'(x) = 4ax^3 + 3bx^2 + 2cx \]

\[ x(4ax^2 + 3bx + 2c) = 0 \]

1,2 are roots of quadratic equation

\[ \Rightarrow \sum of \ roots = \frac{-3b}{4a} = 1 + 2 = 3 \]

\[ \Rightarrow b = -4a \]

Product of roots = \( \frac{2c}{4a} = 1.2 = 2 \)

\[ \Rightarrow a = \frac{c}{4} \]

\[ a = \frac{1}{2}, b = -2 \]

\[ \therefore f(x) = \frac{1}{2}x^4 - 2x^3 + 2x^2 \]

\[ f(2) = 8 - 16 + 8 \]

0
3. The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data, is:

(A) 16.0
(B) 15.8
(C) 14.0
(D) 16.8

Answer: (C)
Solution:
Given, \[ \frac{\sum_{i=1}^{16} x_i + 16}{16} = 16 \]
\[ \Rightarrow \sum_{i=1}^{15} x_i + 16 = 256 \]
\[ \Rightarrow \sum_{i=1}^{15} x_i = 240 \]

Required mean = \[ \frac{\sum_{i=1}^{15} x_i + 3+4+5}{18} = \frac{240+3+4+5}{18} \]
\[ \frac{252}{18} = 14 \]

4. The sum of first 9 terms of the series \[ \frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \cdots \] is:

(A) 96
(B) 142
(C) 192
(D) 71

Answer: (A)
Solution:
\[ T_n = \frac{1^3+2^3+\cdots+n^3}{1+3+\cdots+(2n-1)} = \frac{n^2(n+1)^2}{4} = \frac{(n+1)^2}{4} \]
Sum of 9 terms = ∑_{n=1}^{9} \frac{(n+1)^2}{4}
\frac{1}{4} \times [2^2 + 3^2 + \cdots + 10^2]
\frac{1}{4} \left[(1^2 + 2^2 + \cdots + 10^2) - 1\right]
\frac{1}{4} \left[\frac{10 \times 11 \times 21}{6} - 1\right]
\frac{1}{4} [384] = 96

5. Let O be the vertex and Q be any point on the parabola, \(x^2 = 8y\). If the point P divides the line segment OQ internally in the ratio 1:3, then the locus of P is:

(A) \(y^2 = x\)
(B) \(y^2 = 2x\)
(C) \(x^2 = 2y\)
(D) \(x^2 = y\)

Answer: (C)
Solution:

General point on \(x^2 = 8y\) is \(Q(4t, 2t^2)\)

Let P(h, k) divide OQ in ratio 1:3

\[
(h, k) = \left(\frac{1(4t) + 3(0)}{1+3}, \frac{1(2t^2) + 3(0)}{1+3}\right)
\]

\[
(h, k) \left( t, \frac{2t^2}{4}\right)
\]

\[
h = t \text{ and } k = \frac{t^2}{2}
\]
6. Let \( \alpha \) and \( \beta \) be the roots of equation \( x^2 - 6x - 2 = 0 \). If \( a_n = \alpha^n - \beta^n \), for \( n \geq 1 \), then the value of \( \frac{a_{10} - 2a_8}{2a_9} \) is equal to:

(A) -6  
(B) 3  
(C) -3  
(D) 6

Answer: (B)

Solution:

Given, \( \alpha, \beta \) are roots of \( x^2 - 6x - 2 = 0 \)

\[ \Rightarrow \alpha^2 - 6\alpha - 2 = 0 \quad \text{and} \quad \beta^2 - 6\beta - 2 = 0 \]

\[ \Rightarrow \alpha^2 - 6 = 6\alpha \quad \text{and} \quad \beta^2 - 2 = 6\beta \quad \ldots (1) \]

\[ \frac{a_{10} - 2a_8}{2a_9} = \frac{(\alpha^{10} - \beta^{10}) - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)} \]

\[ = \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{2(\alpha^9 - \beta^9)} \]

\[ = \frac{\alpha^8(6\alpha) - \beta^8(6\beta)}{2(\alpha^9 - \beta^9)} \]

\[ = \frac{6\alpha^9 - 6\beta^9}{2(\alpha^9 - \beta^9)} = 3 \quad \ldots (1) \]

7. If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls is:

(A) \( 55 \left( \frac{2}{3} \right)^{10} \)

(B) \( 220 \left( \frac{1}{3} \right)^{12} \)

(C) \( 22 \left( \frac{1}{3} \right)^{11} \)
(D) \( \frac{55}{3} \left( \frac{2}{3} \right)^{11} \)

Answer : (C)
Solution: (A)

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Total number of possibilities = 19.
Number of favorable case = 5
Required probability $\frac{5}{19}$

INCORRECT SOLUTION

Required Probability $^{12}C_9 \cdot \left(\frac{1}{3}\right)^3 \cdot \left(\frac{2}{3}\right)^9$

\[55 \cdot \left(\frac{2}{3}\right)^{11}\]

The mistake is “we can’t use $^nC_r$ for identical objects”.

8. A complex number $z$ is said to be unimodular if $|z| = 1$. Suppose $z_1 \land z_2$ are complex numbers such that $\frac{z_1 - z_2}{2 - z_1 \bar{z}_2}$ is unimodular and $z_2$ is not unimodular. Then the point $z_1$ lies on a:

(A) Straight line parallel to y-axis.
(B) Circle of radius 2
(C) Circle of radius $\sqrt{2}$
(D) Straight line parallel to x-axis.

Answer: (B)

Solution:

Given $\frac{z_1 - z_2}{2 - z_1 \bar{z}_2}$ is unimodular

$\Rightarrow \left|\frac{z_1 - z_2}{2 - z_1 \bar{z}_2}\right| = 1$

$|z_1 - z_2| = |2 - z_1 \bar{z}_2|$

Squaring on both sides.

$|z_1 - z_2|^2 = |2 - z_1 \bar{z}_2|^2$

$(z_1 - z_2)(\bar{z}_1 - 2 \bar{z}_2) = (2 - z_1 \bar{z}_2)(2 - z_1 \bar{z}_2)$
9. The integral \( \int \frac{dx}{x^2(x^4+1)^{\frac{3}{2}}} \) equals:

(A) \((x^4 + 1)^\frac{1}{2} + c\)

(B) \(- (x^4 + 1)^\frac{1}{2} + c\)

(C) \(- \left(\frac{x^4+1}{x^4}\right)^\frac{1}{2} + c\)

(D) \(\left(\frac{x^4+1}{x^4}\right)^\frac{1}{2} + c\)

Answer: (A)

Solution:

\[
I = \int \frac{dx}{x^2(x^4+1)^{\frac{3}{2}}}
\]

\[
\int \frac{dx}{x^2x^3\left(1+\frac{1}{x^4}\right)^{\frac{3}{2}}}
\]

\[\text{Put} \ 1 + \frac{1}{x^4} = t\]

\[\Rightarrow -\frac{4}{x^5}dx = dt\]
\[ \therefore I = \int \frac{-dt}{t^4} \]

\[ -\frac{1}{4} \times \int t^{-\frac{3}{4}} dt \]

\[ -\frac{1}{4} \times \left( \frac{t^{\frac{1}{4}}}{\frac{1}{4}} \right) + c \]

\[ - \left( 1 + \frac{1}{x^7} \right)^{\frac{1}{2}} + c \]

10. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0, 41) and (41, 0), is:

(A) 861

(B) 820

(C) 780

(D) 901

Answer: (C)
Solution:

\[ P(x_1, y_1) \text{ lies inside the triangle } \Rightarrow x_1, y_1 \in N \]

\[ x_1 + y_1 < 41 \]

\[ \therefore 2 \leq x_1 + y_1 \leq 40 \]

Number of points inside = Number of solutions of the equation
Number of non-negative integral solutions of \( x_1 + x_2 + \ldots + x_n = r \) is \( {}^nC_{r+1} \).

\[
\text{Number of solutions} = \begin{cases}
2^{n-2} \binom{n-2}{1} + n-1 \binom{n-2}{2} = \frac{n-1}{2} & \text{for } 2 \leq n \leq 40 \\
1 + 2 + \ldots + 39 & \text{for } n = 41
\end{cases}
\]

We have

\[
2 \leq n \leq 40,
\]

\[
\therefore \text{Number of solutions} = 1 + 2 + \ldots + 39 = \frac{39 \times 40}{2} = 780.
\]

11. The distance of the point \((1, 0, 2)\) from the point of intersection of the line \(\frac{x-2}{1} = \frac{y+1}{12} = \frac{z}{3}\) and the plane \(x - y + z = 16\), is

(A) 8
(B) \(3\sqrt{21}\)
(C) 13
(D) \(2\sqrt{14}\)

Answer: (C)

Solution:

\[
\frac{x-2}{3} = \frac{y+1}{4} = \frac{z}{12} = t \text{ (say)}
\]

\[
\Rightarrow \text{General point is } (2 + 3t, -1 + 4t, 2 + 12t)
\]

It lies on the plane \(x - y + z = 16\)

\[
\Rightarrow 2 + 3t + 1 - 4t + 2 + 12t = 16
\]

\[
\Rightarrow 11t = 11
\]

\[
\Rightarrow t = 1
\]

\(\therefore\) The point of intersection will be \((5, -3, 14)\).
\[(2 + 3(1), -1 + 4(1), 2 + 12(1))\]
\[(5, 3, 14)\]

Distance from \((1, 0, 2)\) = \[\sqrt{(5 - 1)^2 + (3 - 0)^2 + (14 - 2)^2}\]
\[\sqrt{4^2 + 3^2 + 12^2}\]
\[13\]

12. The equation of the plane containing the line \(2x - 5y + z = 3; x + y + 4z = 5\), and parallel to the plane, \(x + 3y + 6z = 1\), is:

(A) \(x + 3y + 6z = -7\)

(B) \(x + 3y + 6z = 7\)

(C) \(2x + 6y + 12z = -13\)

(D) \(2x + 6y + 12z = 13\)

Answer: (B)

Solution:

Any plane containing the line of intersection of \(2x - 5y + z = 3, x + y + 4z = 5\) will be of the form
\[(2x - 5y + z - 3) + \lambda(x + y + 4z - 5) = 0\]
\[(2 + \lambda)x - (5 - \lambda)y + (1 + 4\lambda)z - (3 + 5\lambda) = 0\]
\[(2 + \lambda)x - (5 - \lambda)y + (1 + 4\lambda)z - (3 + 5\lambda) = 0\]

It is parallel to plane \(x + 3y + 6z = 1\)

\[\frac{2 + \lambda}{1} = \frac{\lambda - 5}{3} = \frac{1 + 4\lambda}{6} = \frac{-(3 + 5\lambda)}{11}\]

\[2 + \lambda = \frac{\lambda - 5}{3} \Rightarrow 6 + 3\lambda = \lambda - 5\]
\[2\lambda = -11\]
\[ \lambda = -\frac{11}{2} \]

\[ \therefore \text{ Required plane is} \]

\[(2x - 5y + z - 3)\left(-\frac{11}{2}\right)(x + y + 4z - 5) = 0\]

\[2(2x - 5y + z - 3) - 11(x + y + 4z - 5) = 0\]

\[4x - 10y + 2z - 6 - 11x - 11y - 44z + 55 = 0\]

\[-7x - 21y - 42z + 49 = 0\]

\[\Rightarrow x + 3y + 6z - 7 = 0\]

13. The area (in sq. units) of the region described by \([x, y]: y^2 \leq 2x \land y \geq 4x - 1]\) is

(A) \(\frac{5}{64}\)

(B) \(\frac{15}{64}\)

(C) \(\frac{9}{32}\)

(D) \(\frac{7}{32}\)

Answer: (C)

Solution

Let us find the points intersections of \(y^2 = 2x \text{ and } y = 4x - 1\)

\[(4x - 1)^2 = 2x\]

\[16x^2 - 10x + 1 = 0\]

\[(8x - 1)(2x - 1) = 0\]

\[x = \frac{1}{2}, \frac{1}{8}\]

\[x = \frac{1}{2} \Rightarrow y = 4\left(\frac{1}{2}\right) - 1 = 1\]

\[x = \frac{1}{8} \Rightarrow y = 4\left(\frac{1}{8}\right) - 1 = \frac{1}{2}\]
14. If \( m \) is the A.M. of two distinct real numbers \( l \land n(l, n > 1) \), \( G_1, G_2 \land G_3 \) are three geometric means between \( l \land n \), then \( G_1^3 + 2G_2^3 + G_3^3 \) equals.

(A) \( 4lm^2n \)

(B) \( 4lm^2 \)

(C) \( 4l^2m^2n^2 \)
(D) \(4l^2mn\)

Answer: (A)
Solution:
m is the A.M. of \(l, n\)

\[ m = \frac{l + n}{2} \quad ... (1) \]

\(G_1, G_2, G_3\) are G.M. of between \(l\) and \(n\)

\[ \Rightarrow l, G_1, G_2, G_3, n \text{ are in G.P.} \]

\(n = l(r)^4 \quad \Rightarrow r^4 = \frac{n}{l} \quad ... (2)\)

\[ G_1 = lr, \quad G_2 = lr^2, \quad G_3 = lr^3 \]

\[ G_1^4 + 2G_2^4 + G_3^4 = l^4r^4 + 2 \times l^4r^8 + l^4 \times r^{12} \]

\[ l^4r^4[1 + 2r^4 + r^8] \]

\[ l^4 \times \frac{n}{l} [1 + r^4]^2 = l^3n \times \left[ 1 + \frac{n}{l} \right]^2 \]

\[ l^3 \times n \times \frac{(l+n)^2}{l^2} \]

\[ ln \times (2m)^2 \quad \therefore \]

\[ 4lm^2n \]

15. Locus of the image of the point (2, 3) in the line \((2x - 3y + 4) + k(x - 2y + 3) = 0, k \in \mathbb{R}\), is a :

(A) Straight line parallel to y-axis.

(B) Circle of radius \(\sqrt{2}\)

(C) Circle of radius \(\sqrt{3}\)
D) Straight line parallel to x-axis

Answer: (B)
Solution:
Given, family of lines \( L_1 + L_2 = 0 \)
Let us take the lines to be
\[
L_2 + \lambda(L_1 - L_2) = 0
\]
\[
(x - 2y + 3) + \lambda(x - y + 1) = 0
\]
\[
(1 + \lambda)x - (2 + \lambda)y + (3 + \lambda) = 0
\]
Let, Image of (2, 3) be (h, k)

From (1):
\[
(h - 2)^2 + (k - 3)^2 = \frac{4}{(\lambda + 1)^2 + (\lambda + 2)^2}
\]  
(3)

From (1) and (3):
\[
\frac{h - 2}{\lambda + 1} = \frac{2}{(\lambda + 1)^2 + (\lambda + 2)^2}
\]
\[
5 - h - k = \frac{1}{2}[(h - 2)^2 + (k - 3)^2]
\]
\[
10 - 2h - 2k = h^2 + k^2 - 4h - 6k + 13
\]
x^2 + y^2 - 2x - 4y + 3 = 0

Radius \( \sqrt{1 + 4 - 3} = \sqrt{2} \).

16. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse \( \frac{x^2}{9} + \frac{y^2}{5} = 1 \), is:

(A) 18

(B) \( \frac{27}{2} \)

(C) 27

(D) \( \frac{27}{4} \)
Answer: (C)
Solution:

\[
\frac{x^2}{9} + \frac{y^2}{5} = 1
\]
\[
e = \sqrt{1 - \frac{5}{9}} = \frac{2}{3}
\]
\[
a^2 = 9, b^2 = 5
\]

Focii = (±ae, 0) = (±2, 0)
Ends of latus recta \((±ae, ±\frac{b^2}{a})\)

\[
\left(±2, ±\frac{5}{3}\right)
\]

Tangent at ‘L’ is \(T = 0\)

\[
\frac{2 \cdot x}{9} + \frac{5}{3} \cdot \frac{y}{5} = 1
\]

It cut coordinates axes at \(P\left(\frac{9}{2}, 0\right)\) and \(Q(0,3)\)

Area of quadrilateral PQRS = 4(Area of triangle OPQ)

\[
4 \left(\frac{1}{2} \cdot \frac{9}{2} \cdot 3\right) = 27 \text{square units.}
\]

17. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is:

(A) 192
(B) 120
(C) 72
(D) 216
Answer: (B)
Solution:
4 digit and 5 digit numbers are possible

4 digit numbers:
\[
\frac{6}{7/8} \quad \frac{3}{4} \quad \frac{f}{32} \quad \frac{f}{f} = \frac{343}{2}
\]
Total number possible = \(3 \cdot 4 \cdot 3 \cdot 2 = 72\)

5 digit numbers:
\[
\frac{5}{4} \quad \frac{3}{2} \quad \frac{1}{1}
\]
Total number possible = \(5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120\)
\[
\therefore \text{Total number} = 72 + 120 = 192.
\]

18. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set \(A \times B\), each having at least three elements is:

(A) 256

(B) 275

(C) 510

(D) 219

Answer: (D)
Solution:
\[
n(A) = 4 \\
n(B) = 2
\]
Number of elements in \(A \times B = 2 \cdot 4 = 8\)
Number of subsets having at least 3 elements
\[
2^8 - \binom{8}{0} - \binom{8}{1} - \binom{8}{2} - \frac{2^8}{219}
\]

19. Let \(\tan^{-1}y = \tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)\), where \(|x| < \frac{1}{\sqrt{3}}\). Then a value of y is:

(A) \(\frac{3x-x^3}{1-3x^2}\)
(B) $\frac{3x-x^2}{1+3x^2}$

(C) $\frac{3x+x^3}{1+3x^2}$

(D) $\frac{3x-x^3}{1-3x^2}$

Answer: (D)
Solution:
\[
tan^{-1}y \tan^{-1}x + tan^{-1}\left(\frac{2x}{1-x^2}\right), |x| < \frac{1}{\sqrt{3}}
\]

\[
tan^{-1}x + 2tan^{-1}x
\]

\[
3tan^{-1}x
\]

\[
tan^{-1}y \ tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)
\]

\[
\Rightarrow y = \frac{3x-x^3}{1-3x^2}
\]

20. The integral $\int_{2}^{4} \frac{\log x^2}{\log x^2 + \log(6-x)^2} dx$ is equal to:

(A) 4

(B) 1

(C) 6

(D) 2

Answer: (B)
Solution:
\[
I = \int_{2}^{4} \frac{\log x^2}{\log x^2 + \log(6-x)^2} dx
\]

\[
I = \int_{2}^{4} \frac{\log(6-x)^2}{\log(6-x)^2 + \log x^2} dx
\]

\[
[\because \int_{a}^{b} f(x)dx = \int_{a}^{b} f(a + b - x)dx]
\]
21. The negation of \( s \lor (r \land s) \) is equivalent to:

(A) \( s \land (r \land \sim s) \)

(B) \( s \lor (r \lor \sim s) \)

(C) \( s \land r \)

(D) \( s \land \sim r \)

Answer: (C)

Solution:

\[
\begin{align*}
((s) \lor (~r \land s)) &= s \land [~(~r) \land s] \\
&s \land (r \lor (~s)) \\
&s \land r \lor (s \land (~s)) \\
&(s \land r) \lor F
\end{align*}
\]

\( s \land r \) \((F \rightarrow \text{Fallacy})\)

22. If the angles of elevation of the top of a tower from three collinear points A, B and C, on a line leading to the foot of the tower, are 30°, 45° and 60° respectively, then the ratio, \( AB:BC \) is:

(A) \( \sqrt{3}: \sqrt{2} \)

(B) 1: \( \sqrt{3} \)

(C) 2: 3

(D) \( \sqrt{3}: 1 \)

\[
2I = \int_{2}^{4} \log x^2 + \log(6 - x)^2 \, dx
\]

\[
2I = \int_{2}^{4} \log(6 - x)^2 + \log x^2 \, dx
\]

\[
2I = \int_{2}^{4} 1 \, dx
\]

\[
2I = [x]_{2}^{4}
\]

\[
2I = 4 - 2
\]

\( I = 1. \)
Answer: (D)
Solution:

\[ \tan 30^\circ = \frac{h}{x + y + z}, \tan 45^\circ = \frac{h}{y + z}, \tan 60^\circ = \frac{h}{z} \]

\[ \Rightarrow x + y + z = \sqrt{3}h \]

\[ y + z = h \]

\[ z = \frac{h}{\sqrt{3}} \]

\[ y = h \left( 1 - \frac{1}{\sqrt{3}} \right) \]

\[ x = (\sqrt{3} - 1)h \]

\[ \frac{x}{y} = \frac{h(\sqrt{3} - 1)}{\frac{\sqrt{3}}{1}} \]

\[ \frac{x}{y} = \frac{\sqrt{3}}{1} \]

23. \( \lim_{x \to 0} \frac{1 - \cos x}{x \tan 4x} \) is equal to:

(A) 3
(B) 2
(C) \( \frac{1}{2} \)
(D) 4

Answer: (B)
Solutions:
24. Let \( \mathbf{a}, \mathbf{b}, \mathbf{c} \) be three non-zero vectors such that no two of them are collinear and
\[
(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = \frac{1}{3} ||\mathbf{b}|| ||\mathbf{c}|| |\mathbf{a}|
\]
If \( \theta \) is the angle between vectors \( \mathbf{b} \wedge \mathbf{c} \), then a value of \( \sin \theta \) is:

(A) \( -\frac{\sqrt{2}}{3} \)

(B) \( \frac{2}{3} \)

(C) \( -\frac{2\sqrt{3}}{3} \)

(D) \( \frac{2\sqrt{2}}{3} \)

Answer: (D)

Solution:

\[
(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = \frac{1}{3} ||\mathbf{b}|| ||\mathbf{c}|| |\mathbf{a}|
\]

\[
(\mathbf{a} \cdot \mathbf{c}) \mathbf{b} - (\mathbf{b} \cdot \mathbf{c}) \mathbf{a} = \frac{1}{3} ||\mathbf{b}|| ||\mathbf{c}|| |\mathbf{a}|
\]

\[
\Rightarrow \mathbf{a} \cdot \mathbf{c} = 0 \wedge \mathbf{b} \cdot \mathbf{c} = -\frac{1}{3} ||\mathbf{b}|| ||\mathbf{c}||
\]

\[
|\mathbf{b}| |\mathbf{c}| \cos \theta = -\frac{1}{3} ||\mathbf{b}|| ||\mathbf{c}||
\]
25. If \( A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \) is a matrix satisfying the equation \( AA^T = 9I \), where \( I \) is a \( 3 \times 3 \) identity matrix, then the ordered pairs \( (a, b) \) is equal to:

(A) \((-2, 1)\)

(B) \((2, 1)\)

(C) \((-2, -1)\)

(D) \((2, -1)\)

Answer: (C)

Solution:

\[ AA^T = 9I \]

\[ \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} \]

\[
\begin{align*}
1(1) + 2(2) + 2(2)1(2) + 2(1) + 2(-2) &= 2(2) + 2(b) \\
2(1) + 1(2) - 2(2)2(2) + 1(1) - 2(-2)1(a) + 2(a) + a(a) + 1(2) - 2(b) \\
a(1) + 2(2) + b(2)a(2) + 2(1) + b(-2) &= 2(2) + b(b)
\end{align*}
\]

\[
\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}
\]

\[
\begin{bmatrix} 9 & 0 & a + 2b + 4 \\ 0 & 9 & 2a - 2b + 2 \\ a + 2b + 4 & 2a - 2b + 2 & a^2 + b^2 + 4 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}
\]

Comparing the corresponding elements
\[
\begin{align*}
2a - 2b &= +2 = 0 \\
2a + 2b &= +4 = 0 \\
\hline
3a + 6 &= 0 \\
\Rightarrow \ a &= -2 \Rightarrow -2 + 2b + 4 = 0 \\
b &= -1
\end{align*}
\]

The third equation is useful to verify whether this multiplication is possible.

26. If the function, 
\[ g(x) = \begin{cases} 
    k\sqrt{x+1}, & 0 \leq x \leq 3 \\
    mx + 2, & 3 < x \leq 5
\end{cases} \]
is differentiable, then the value of \( k + m \) is:

(A) \( \frac{16}{5} \)

(B) \( \frac{10}{3} \)

(C) 4

(D) 2

Answer: (D)

Solution:

\[ g(x) \text{ is differentiable at } x = 3 \]
\[ \Rightarrow g(x) \text{ is continuous at } x = 3 \]
\[ \therefore \text{ LHL = RHL} \]
\[ x \to 3^+ g(x) = g(3) \]
\[ x \to 3^- g(x) = \lim \]
\[ \lim_{x \to 3^-} k\sqrt{x+1} = m(3) + 2 \]
\[ 2k = 3m + 2 \]
\[ k = \frac{3}{2} m + 1 \quad \text{(1)} \]

\[ g(x) \text{ is differentiable} \]
\[ \Rightarrow LHD = RHD \]
\[ g'(x) = \begin{cases} 
    k \sqrt{x+1}, & 0 < x < 3 \\
    m, & 3 < x < 5
\end{cases} \]
\[ \lim_{x \to 3^+} g'(x) = \lim_{x \to 3^-} g'(x) \]
\[ \lim_{x \to 3^-} \frac{k}{2\sqrt{3} + 1} = m \]
\[ k = 4m \]

From (1), \( 4m = \frac{3}{2} m + 1 \)
\[ \frac{5m}{2} = 1 \]
\[ \Rightarrow m = \frac{2}{5}, k = \frac{8}{5} \]
\[ k + m = \frac{2}{5} + \frac{8}{5} = 2. \]

27. The set of all values of \( \lambda \) for which the system of linear equations:
\[
\begin{align*}
2x_1 - 2x_2 + x_3 &= \lambda x_1 \\
2x_1 - 3x_2 + 2x_3 &= \lambda x_2 \\
-x_1 + 2x_2 &= \lambda x_3
\end{align*}
\]
has a non-trivial solution,

(A) Is a singleton
(B) Contains two elements
(C) Contains more than two elements
(D) Is an empty set

Answer: (B)

Solution:
\[
(2 - \lambda)x_1 = 2x_2 + x_3 = 0 \\
2x_1 - (\lambda + 3)x_2 + 2x_3 = 0 \\
-x_1 + 2x_2 - \lambda x_3 = 0
\]
The systems of linear equations will have a non-trivial solution
\[
\Rightarrow \begin{vmatrix} 2 - \lambda & -2 & 1 \\ 2 & -\lambda - 3 & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0
\]
\[ \Rightarrow (2 - \lambda)[\lambda^2 + 3\lambda - 4] + 2[-2\lambda + 2] + 14 - \lambda - 3 = 0 \]
\[ 2\lambda^2 + 6\lambda - 8 - 3\lambda^2 - 4\lambda + 4\lambda + 4 + 1 - \lambda = 0 \]
\[ -\lambda^3 - \lambda^2 + 5\lambda - 3 = 0 \]
\[ \lambda^3 + \lambda^2 - 5\lambda + 3 = 0 \]
\[ (\lambda - 1)(\lambda^2 + 2\lambda - 3) = 0 \]
\[ (\lambda - 1)(\lambda + 3)(\lambda - 1) = 0 \]
\[ \Rightarrow \lambda = 3, -1, -1 \]

28. The normal to the curve, \( x^2 + 2xy - 3y^2 = 0 \), at \((1, 1)\):

(A) Meets the curve again in the second quadrant
(B) Meets the curve again in the third quadrant
(C) Meets the curve again in the fourth quadrant
(D) Does not meet the curve again

Answer: (C)
Solution:

\[ x^2 + 2xy - 3y^2 = 0 \]

\[(x + 3y)(x - y) = 0 \]
Pair of straight lines passing through origin.

\[ x + 3y = 0 \quad \text{or} \quad x - y = 0 \]
Normal exists at \((1, 1)\), which is on \(x - y = 0\)
⇒ Slope of normal at \((1,1) = -1\)
∴ Equation of normal will be

\[ y - 1 = -1(x - 1) \]
\[ y - 1 = -x + 1 \]
\[ x + y = 2 \]

Now, find the point of intersection with \(x + 3y = 0\)

\[ x + y = 2 \]
\[ x + 3y = 0 \]

\[ -2y = 2 \Rightarrow y = -1, x = 3 \]

\((3, 1)\) lies in fourth quadrant.

29. The number of common tangents to the circles \(x^2 + y^2 - 4x - 6y - 12 = 0\) and \(x^2 + y^2 + 6x + 18y + 26 = 0\), is:

(A) 2
(B) 3
(C) 4
(D) 1

Answer: (B)
Solution:

\[ x^2 + y^2 - 4x - 6y - 12 = 0 \]

\[ C_1(2,3), r_1 = \sqrt{2^2 + 3^2 + 12} = \sqrt{25} = 5 \]

\[ x^2 + y^2 + 6x + 18y + 26 = 0 \]

\[ C_2(-3,-9), r_2 = \sqrt{3^2 + 9^2 - 26} \]

\[ \sqrt{90 - 26} = 8 \]

\[ C_1C_2 = \sqrt{5^2 + 12^2} = 13 \]

\[ C_1C_2 = r_1 + r_2 \]
⇒ Externally touching circles
⇒ 3 common tangents.

30. Let \( y(x) \) be the solution of the differential equation \( (x \log x) \frac{dy}{dx} + y = 2x \log x, (x \geq 1) \).
Then \( y(e) \) is equal to:

(A) 0
(B) 2
(C) 2e
(D) e

Answer: (A)

Solution:
\[
\frac{dy}{dx} + \left( \frac{1}{x \log x} \right) y = 2
\]

Integrating factor = \( e^{\int \frac{1}{x \log x} dx} \)

The solution will be
\[
y \cdot e^{\int \frac{1}{x \log x} dx} = \int Q \cdot e^{\int \frac{1}{x \log x} dx} + c
\]
\[
y \cdot \log x = 2 \cdot \log x + c
\]
\[
x - x
\]
\[
x \log + c
\]
\[
x = 2
\]
\[
y \cdot \log
\]

Let \( P(1, y_1) \) be any point on the curve
\[
y_1(0) = 2(0 - 1) + c
\]
\[
c = 2
\]
\[
\begin{align*}
\text{e} & \quad \text{e} - e \\
e \log + 2 & \quad x = e, y
\end{align*}
\]

When

\[
\log = 2
\]

\[
x = e, y
\]

\[
y = 2
\]