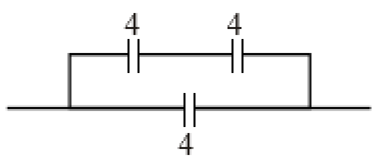
JEE 2016 ONLINE 9th APRIL 2016

Physics

Single correct answer type:

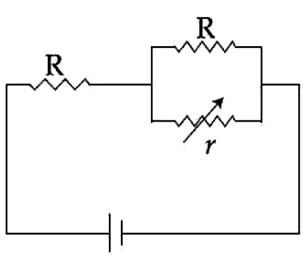
1. Three capacitors each of $4\,\mu F$ are to be connected in such a way that the effective capacitance is $6\,\mu F$. This can be done by connecting them:
   (A) all in series
   (B) all in parallel
   (C) two in parallel and one in series
   (D) two in series and one in parallel

   Solution: (D)



$2+4=6$

2.



In the circuit shown, the resistance r is a variable resistance. If for $r=fR$, the heat generation in r is maximum then the value of f is:

(A) $\frac{1}{2}$

(B) 1

(C) $\frac{1}{4}$

(D) $\frac{3}{4}$

Solution: (A)

Heat energy will be maximum when Resistance will be minimum.

3. The ratio of work done by an ideal monoatomic gas to the heat supplied to it in an isobaric process is

(A) $\frac{2}{5}$

(B) $\frac{3}{2}$

(C) $\frac{3}{5}$

(D) $\frac{2}{5}$

Solution: (A)

$$\eta=\frac{w}{Q}=\frac{R}{C_P}=\frac{R}{\frac{5R}{2}}=\frac{2}{5}$$

4. A rocket is fired vertically from the earth with an acceleration of 2g, where g is the gravitational acceleration. On an inclined plane inside the rocket, making an angle θ with the horizontal, a point object of mass m is kept. The minimum coefficient of friction μ_{min} between the mass and the inclined surface such that the mass does not move is:

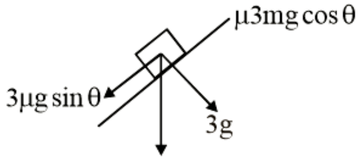
(A) $\tan 2\theta$

(B) $\tan\theta$

(C) $3\tan\theta$

(D) $2\tan\theta$

Solution: (B)



$$3mg\sin\theta = \mu 3mg\cos\theta$$

$$\mu = \tan\theta$$

5. In Young's double slit experiment, the distance between slits and the screen is 1.0 m and monochromatic light of 600 nm is being used. A person standing near the slits is looking at the fringe patter. When the separation between the slits is varied, the interference pattern disappears for a particular distance d_0 between the slits. If the angular resolution of the eye is $\frac{1^o}{60}$, the value of d_0 is close to:

(A) 1 mm
(B) 3 mm
(C) 2 mm
(D) 4 mm

Solution: (B)

$$d : D\theta = 1 \times \frac{\pi}{180} \times \frac{1}{60}$$

$$d_0 = 2 \times 10^{-3} = 2\,mm$$

6. A car of weight W is on an inclined road that rises by 100 m over a distance of 1km and applies a constant frictional force $\frac{W}{20}$ on the car. While moving uphill on the road at a speed of $10\,ms^{-1}$, the car needs power P. If it needs power $\frac{P}{2}$ while moving downhill at speed υ then value of υ is:

(A) $20\,ms^{-1}$

(B) $5\,m\,s^{-1}$

(C) $15\,m\,s^{-1}$

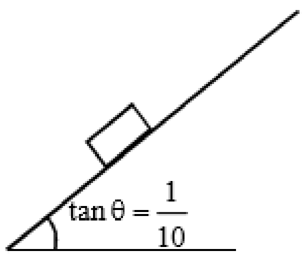
(D) $10\,m\,s^{-1}$

Solution: (C)

$$P=\left(w\sin\theta+\frac{w}{20}\right)10$$

$$P=\left(\frac{w}{10}+\frac{w}{20}\right)10=\frac{3w}{2}$$

$$\frac{P}{2}=\frac{3w}{5}=\left(\frac{w}{10}-\frac{w}{20}\right)w$$



$$\frac{3}{4}=\frac{v}{20}$$

15 m/s = v

7. Two particles are performing simple harmonic motion in a straight line about the same equilibrium point. The amplitude and time period for both particles are same and equal to A and T, respectively. At time $t=0$ one particle has displacement A while the other one has displacement $\frac{-A}{2}$ and they are moving towards each other. If they cross each other at time t, then tis:

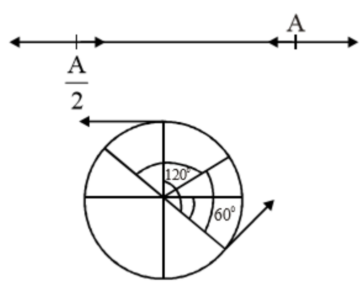
(A) $\frac{5T}{6}$

(B) $\frac{T}{3}$

(C) $\frac{T}{4}$

(D) $\frac{T}{6}$

Solution: (D)

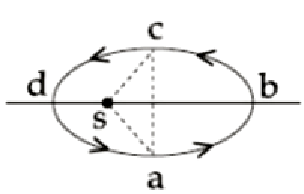


Angle covered to meet ¿ $60^o = \frac{\pi}{3} rad$

$t = \frac{Q}{w}$

¿ $\frac{\pi}{3 \times 2\pi} T = \frac{T}{6}$

8. Figure shows elliptical path abcd of a planet around the sun S such that the area of triangle csa is $\frac{1}{4}$ the area of the ellipse. (see figure) with db as the semi major axis, and ca as the semiminor axis. If t_1 is the time taken for planet to go over path abc and t_2 for path taken over cda then:



(A) $t_1 = 4t_2$

(B) $t_1 = 2t_2$

(C) $t_1 = 3t_2$

(D) $t_1 = t_2$

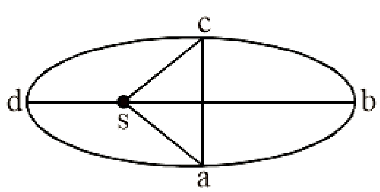
Solution: (C)

Area abca $¿x$

Area $SABCS = x + \frac{1}{2}x$

Area $SADCS = x - \frac{1}{2}x = \frac{1}{2}$

$$\frac{1+\frac{1}{2}}{1-\frac{1}{2}} = \frac{t_1}{t_2}$$



$$\frac{\frac{3}{2}}{\frac{1}{2}} = \frac{t_1}{t_2}$$

$$t_1 = 3t_2$$

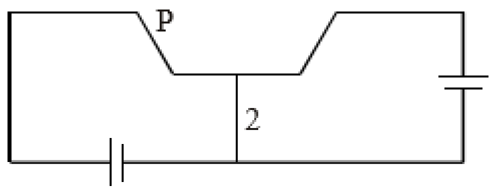
$$\frac{3}{1} = \frac{t_1}{t_2}$$

9. An unknown transistor needs to be identified as a npn pr pnp type. A multimeter, with +ve and –ve terminals, is used to measure resistance

between different terminals of transistor. If terminal is the base of the transistor then which of the following is correct for a pnp transistor?

(A) +ve terminal 2, -ve terminal 3, resistance low
(B) +ve terminal 2, -ve terminal 1, resistance high
(C) +ve terminal 1, -ve terminal 2, resistance high
(D) +ve terminal 3, -ve terminal 2, resistance high

Solution: (C)



10. An experiment is performed to determine the I-V characteristics of a Zener diode, which has a protective resistance of $R=100\,\Omega$, and a maximum power of dissipation rating of 1 W. the minimum voltage range of the DC source in the circuit is:

(A) $0-5V$

(B) $0-24V$

(C) $0-12V$

(D) $0-8V$

Solution: (D)

$$P=\frac{v^2}{R}$$

$$¿\frac{1\times 100}{v}=v\Rightarrow v^2=100$$

11. A uniformly tapering conical wire is made from a material of Young's modulus Y and has a normal, unextended length L. The radii, at the upper and lower ends of this conical wire, have values R and 3R, respectively. The upper end of the wire is fixed to a rigid support and a mass M is suspended from its lower end. The equilibrium extended length, of this wire, would equal:

(A) $L\left(1+\frac{2}{9}\frac{Mg}{\pi Y R^2}\right)$

(B) $L\left(1+\frac{1}{9}\frac{Mg}{\pi Y R^2}\right)$

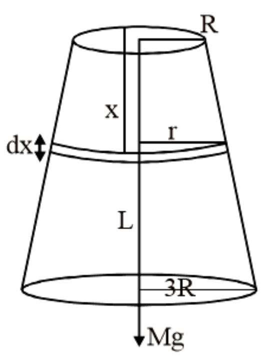
(C) $L\left(1+\frac{1}{3}\frac{Mg}{\pi Y R^2}\right)$

(D) $L\left(1+\frac{2}{3}\frac{Mg}{\pi Y R^2}\right)$

Solution: (C)

$$r=\frac{2R}{L}x+R$$

$$\int dL=\int \frac{Mgdx}{\pi\left[\frac{2R}{L}x\times R\right]^2 y}$$



$$\Delta L=\frac{Mg}{\pi y}\left[\frac{-1}{\left[\frac{2Rx}{L}+R\right]_0^L}\times\frac{L}{2R}\right]$$

$$¿\frac{MgL}{3\pi R^2 y}$$

12. In the following 'I' refers to current and other symbols have their usual meaning. Choose the option that corresponds to the dimensions of electrical conductivity:

(A) $M^{-1}L^{-3}T^{3}I$

(B) $M^{-1}L^{-3}T^{3}I^{2}$

(C) $M^{-1}L^{3}T^{3}I$

(D) $M L^{-3}T^{-3}I^{2}$

Solution: (B)

$$R=\frac{l}{kA}$$

$$K=\frac{L}{RA}$$

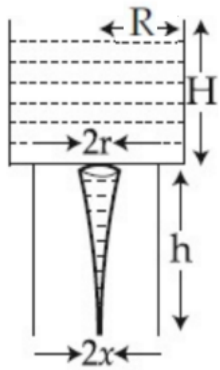
$$\frac{LI}{AV}$$

$$¿\frac{LI}{Aw}It$$

$$¿\frac{L}{L^{2}}\frac{I^{2}T}{M L^{2} T^{2}}$$

$$¿M^{1}L^{-3}T^{-1}I^{2}$$

13.



Consider a water jar of radius R that has water filled up to height H and is kept on a stand of height h (see figure). Through a hole of radius r $(r \ll R)$ at its bottom, the water leaks out and the stream of water coming down towards the ground has a shape like a funnel as shown in the figure. If the radius of the cross-section of water stream when it hits the ground is x. Then:

(A) $x = r\left(\frac{H}{H+h}\right)^{\frac{1}{4}}$

(B) $x = r\left(\frac{H}{H+h}\right)$

(C) $x = r\left(\frac{H}{H+h}\right)^{2}$

(D) $x = r\left(\frac{H}{H+h}\right)^{\frac{1}{2}}$

Solution: (A)

14. Two engines pass each other moving in opposite directions with uniform speed of 30 m/s. one of them is blowing a whistle of frequency 540 Hz. Calculate the frequency heard by driver of second engine before they pass each other. speed of sound is 330 m/sec:

(A) 450 Hz
(B) 540 Hz
(C) 270 Hz
(D) 648 Hz

Solution: (D)

$$\frac{1}{2}\rho v_1^2 + \rho gh = \frac{1}{2}\rho v_2^2$$

$$v_1^2 + 1gh = v_2^2$$

$$2gH + 2gh = v_2^2$$

$$a_1 v_1 = a_2 v_2$$

$$\pi r^2 \sqrt{2gh} = \pi \lambda^2 v_2$$

$$\frac{r^2}{x^2}\sqrt{2gh}=v_2$$

$$2gH+2gh=\frac{r^4}{x^4}2gh$$

$$x=r\left[\frac{H}{H+h}\right]^{\frac{1}{4}}$$

15. An audio signal consists of two distinct sounds: one a human speech signal in the frequency band of 200 Hz to 2700 Hz, while the other is a high frequency music signal in the frequency band of 10200 Hz to 15200 Hz. The ratio of the AM signal band width required to send both the signals together to the AM signal band width required to send just the human speech is:

(A) 2
(B) 5
(C) 6
(D) 3

Solution: (B)

$$f'=\frac{v-v_0}{v-v_s}f$$

$$¿\frac{330+30}{330-30}\times 540$$

$$¿\frac{360}{300}\times 540$$

$$¿648\ Hg$$

16. To know the resistance G of a galvanometer by half deflection method, a battery of emf V_E and resistance R is used to deflect the galvanometer by angle θ. If a shunt of resistance S is needed to get half deflection the G, R and S are related by the equation:

(A) $S(R+G)=RG$

(B) $2S(R+G)=RG$

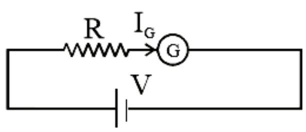
(C) $2G=S$

(D) $2S=G$

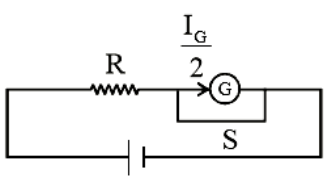
Solution: (A)

$$I_g=\frac{V}{R+G}$$

$$R_c=R+\frac{GS}{G+s}$$



$$I=\frac{V}{R+\frac{GS}{G+S}}$$



$-¿$

$I-I_g^{¿}$

$¿$

$I_g^{'}G=¿$

$$I_g^{'}(G+S)=IS$$

$$\frac{I_g}{2}=\frac{IS}{G+S}$$

$$\frac{V}{2(R+G)}=\frac{V}{R+\frac{GS}{G+S}}\times\frac{S}{G+S}$$

$$\frac{1}{2(R+G)}=\frac{S}{R(G+S)+GS}$$

$$R(G+S)+GS=2S(R+G)$$

$$RG+RS+GS=2S(R+G)$$

$$RG=2S(R+G)-S(R+G)$$

$$RG=S(R+G)$$

17.A hydrogen atom makes a transition from $n=2$ to $n=1$ and emits a photon. This photon strikes a doubly ionized lithium atom $(z=3)$ in excited state and completely removes the orbiting electron. The least quantum number for the excited state of the ion for the process is:

(A) 2
(B) 4
(C) 5
(D) 3

Solution: (B)

$$R_H\left[1-\frac{1}{4}\right]=R_H 9\left[\frac{1}{n}\right]$$

$$\frac{3}{4}=\frac{9}{n^2}$$

$$n=\sqrt{12}$$

¿3.2…..

$\therefore$ The least quatum number must be 4.

18. 200 g water is heated from 40^oC to 60^oC. Ignoring the slight expansion of water, the change in its internal energy is close to (Given specific heat of water ¿ 4184 J/kg/K):

(A) $167.4\,kJ$

(B) $8.4\,kJ$

(C) $4.2\,kJ$

(D) $16.7\,kJ$

Solution: (D)

Assuming Isochoric process

$Q=\Delta U$

$\Delta U=2\times 4184\times 20$

¿ $16.7\,kJ$

19. When photons of wavelength λ_1 are incident on an isolated sphere, the corresponding stopping potential is found to be V. when photons of wavelength λ_2 are used, the corresponding stopping potential was thrice that of the above value. If light of wavelength λ_3 is used then find the stopping potential for this case:

(A) $\frac{hc}{e}\left[\frac{1}{\lambda_3}+\frac{1}{\lambda_2}-\frac{1}{\lambda_1}\right]$

(B) $\frac{hc}{e}\left[\frac{1}{\lambda_3}+\frac{1}{2\lambda_2}-\frac{1}{\lambda_1}\right]$

(C) $\frac{hc}{e}\left[\frac{1}{\lambda_3}-\frac{1}{\lambda_2}-\frac{1}{\lambda_1}\right]$

(D) $\frac{hc}{e}\left[\frac{1}{\lambda_3}+\frac{1}{2\lambda_2}-\frac{3}{2\lambda_1}\right]$

Solution: (D)

$$\frac{hc}{\lambda_1}=\frac{hc}{\lambda_0}+eV \qquad \ldots(i)$$

$$\frac{hc}{\lambda_2}=\frac{hc}{\lambda_0}+eV \qquad \ldots(ii)$$

$$\frac{hc}{\lambda_3}=\frac{hc}{\lambda_0}+3eV' \qquad \ldots.(iii)$$

Equation (i) and (ii)

$$\frac{3}{2\lambda_1}-\frac{2}{2\lambda_2}=\frac{1}{\lambda_0}$$

$$\frac{\lambda c}{\lambda_3}-hc\left[\frac{3}{2\lambda_1}-\frac{1}{2\lambda_2}\right]=eV'$$

$$\frac{hc}{e}\left[\frac{1}{\lambda_3}-\frac{3}{2\lambda_1}+\frac{1}{2\lambda_2}\right]=V'$$

20. A magnetic dipole is acted upon by two magnetic fields which are inclined to each other at an angle of 75^o . One of the fields has a magnitude of 5 mT. the dipole attains stable equilibrium at an angle of 30^o with this field. The magnitude of the other field (in mT) is close to:

(A) 1
(B) 11
(C) 36
(D) 1060

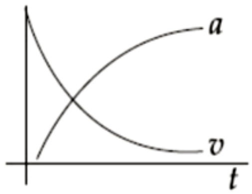
Solution: (B)

$$\frac{x}{\sqrt{2}}=\frac{15}{2}=\frac{15}{\sqrt{2}}\approx 11 \quad .$$

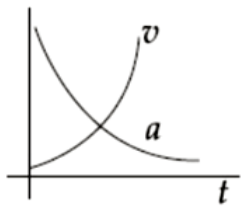
21. Which of the following option correctly describes the variation of the speed υ and acceleration 'a' of a point mass falling vertically in a viscous

medium that applies a force $F = -kv$, where 'k' is a constant, on the body? (Graphs are schematic and not drawn to scale)

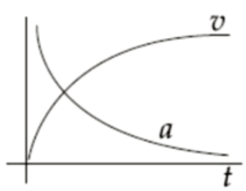
(A)



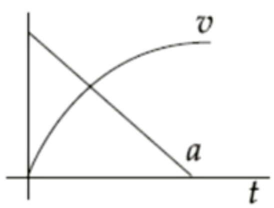
(B)



(C)



(D)



Solution: (D)

Conceptual, (Store's law).

22. The truth table given in fig. represents:

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

(A) OR-Gate
(B) NAND-Gate
(C) AND-Gate

(D) NOR-Gate

Solution: (A)

Conceptual.

23. The potential (in volts) of a charge distribution is given by

$V(z)=30-5z^2$ for $|z|\le 1\,m$

$V(z)=35-10|z|$ for $|Z|\ge 1\,m$.

$V(z)$ does not depend on x and y. If this potential is generated by a constant charge per unit volume ρ_0 (in units of ϵ_0) which is spread over a certain region, then choose the correct statement.

(A) $\rho_0=20\epsilon_0$ in the entire region

(B) $\rho_0=10\epsilon_0$ for $|z|\le 1\,m$ and $\rho_0=0$ else where

(C) $\rho_0=20\epsilon_0$ for $|z|\le 1\,m$ and $\rho_0=0$ else where

(D) $\rho_0=40\epsilon_0$ in the entire region

Solution: (B)

$$\Sigma_1=\frac{-dv}{dr}=10|z|$$

$$\Sigma_2=\frac{-dv}{dr}=10$$ (constant : E)

$\because$ The source is an infinity large non conducting thick plate of thickness 2m.

$$\Rightarrow 10Z\cdot 10A=\frac{\rho\cdot A\propto Z}{\varepsilon_0}$$

$\rho_0=10\,\varepsilon_0$ for $|z|\le 1\,m$

24. Microwave oven acts on the principle of:

(A) giving rotational energy to water molecules
(B) giving translational energy to water molecules
(C) giving vibrational energy to water molecules

(D) transferring electrons from lower to higher energy levels in water molecule

Solution: (D)

Conceptual.

25. To find the focal length of a convex mirror, a student records the following data:

Object pin	Convex Lens	Convex Mirror	Image Pin
22.2 cm	32.2 cm	45.8 cm	71.2 cm

The focal length of the convex lens is f_1 and that of mirror is f_2. Then taking index correction to be negligibly small, f_1 and f_2 are close to:

(A) $f_1 = 7.8\,cm \quad f_2 = 12.7\,cm$

(B) $f_1 = 12.7\,cm \quad f_2 = 7.8\,cm$

(C) $f_1 = 15.6\,cm \quad f_2 = 25.4\,cm$

(D) $f_1 = 7.8\,cm \quad f_2 = 25.4\,cm$

Solution: (A)

Take $f_2 = 12.07$

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{12.7} = \frac{1}{25.4} + \frac{1}{u}$$

$$\frac{1}{12.7} = \frac{1}{25.4} = \frac{1}{u}$$

$$u = 25.4 = v'$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{f_1}=\frac{1}{25.4+13.6}+\frac{1}{10}$$

$$\frac{1}{f_1}=\frac{1}{39}+\frac{1}{10}$$

$$\frac{1}{f_1}=\frac{390}{49}=7.96$$

The closest answers is (1) as option (3) and (4) are not possible.

26.A $50\,\Omega$ resistance is connected to a battery of 5V. A galvanometer of resistance $100\,\Omega$ is to be used as an ammeter to measure current through the resistance, for this a resistance r_S is connected to the galvanometer. Which of the following connections should be employed if the measured current is within 1% of the current without the ammeter in the circuit?

(A) $r_S=0.5\,\Omega$ in series with galvanometer

(B) $r_S=1\,\Omega$ in series with galvanometer

(C) $r_S=1\,\Omega$ in parallel with galvanometer

(D) $r_S=0.5\,\Omega$ in parallel with the galvanometer

Solution: (D)

$$I=\frac{5}{50}=0.1$$

$$I^{'}=0.099$$

$$R_3=50+\frac{100\,S}{100+S}=\frac{V}{I}$$

$$\frac{100\,S}{100+S}=\frac{5}{0.099}-50$$

$$\frac{100S}{100+S} = 50 \cdot 50 - 50$$

$$\frac{100S}{100+S} = 0.5$$

$$100S = 50 + 0.55$$

$$99.5S = 50$$

$$S = \frac{50}{99.05}$$

$$¿0.5\Omega$$

27. A simple pendulum made of a bob of mass m and a metallic wire of negligible mass has time period 2s at $T = 0^o C$. If the temperature of the wire is increased and the corresponding change in its time period is plotted against its temperature, the resulting graph is a line of slope S. If the coefficient of linear expansion of metal is α then the value of S is:

(A) $\frac{\alpha}{2}$

(B) 2α

(C) α

(D) $\frac{1}{\alpha}$

Solution: (C)

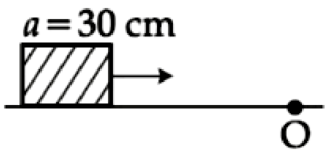
$$\Delta T = \frac{1}{2} T \alpha \Delta \theta$$

$$\frac{\Delta T}{d\theta} = \frac{T}{2}\alpha$$

But $T = 2s$

$$\frac{dT}{d\theta}=\alpha$$

28. A cubical block of side 30 cm is moving with velocity $2\,m\,s^{-1}$ on a smooth horizontal surface. The surface has a bump at a point O as shown in figure. The angular velocity (in rad/s) of the block immediately after in hits the bump, is:



(A) 13.3
(B) 5.0
(C) 9.4
(D) 96.7

Solution: (B)

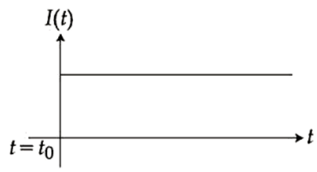
$$mv=I\omega$$

$$\omega=\frac{m.2\frac{R}{2}}{m\left[\frac{R^2}{6}+\left(\frac{R}{\sqrt{2}}\right)^2\right]}$$

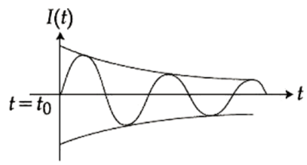
$$\omega=\frac{12}{8R}=\frac{3}{2\times 0.3}=\frac{10}{2}=5$$

29. A series LR circuit is connected do a voltage source with $V(t)=V_0\sin\Omega t$. After very large time, current $I(t)$ behaves as $\left(t_0 \gg \frac{L}{R}\right)$:

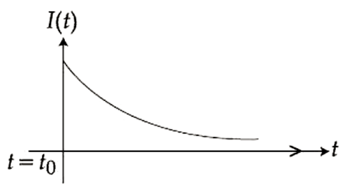
(A)



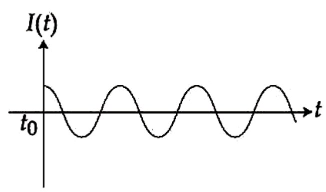
(B)



(C)



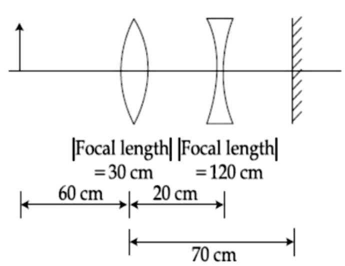
(D)



Solution: (D)

$$I = \frac{v}{\omega L}\left(\omega t - \frac{\pi}{2}\right)$$

30. A convex lens, of focal length 30 cm, a concave lens of focal length 120 cm, and a plane mirror are arranged as shown. For an object kept at a distance of 60 cm from the convex lens, the final image, formed by the combination, is a real image, at a distance of:



(A) 60 cm from the convex lens

(B) 60 cm form the concave lens
(C) 70 cm from the convex lens
(D) 70 cm from the concave lens

Solution: (A)

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

$$\frac{1}{30}=\frac{1}{v}+\frac{1}{60}$$

$$\frac{1}{60}=\frac{1}{v}$$

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

$$\frac{1}{-120}=\frac{1}{v}-\frac{1}{40}$$

Virtual object 10 cm behind plane mirror.

Hence real image 10 cm in from of mirror.

hence 60 cm from convex lens.

Chemistry

Single Correct Answer Type

1. The gas evolved on heating CH_3MgBr in methanol is:

(A) Methane
(B) Ethane
(C) Propane
(D) HBr

Solution: (A)

$$CH_3MgBr + CH_3OH \rightarrow Mg\begin{matrix} \diagup OCH_3 \\ \diagdown Br \end{matrix} + \underset{\text{Methane}}{CH_4 \uparrow}$$

2. The total number of orbitals associated with the principal quantum number 5 is:
(A) 20
(B) 25
(C) 10
(D) 5

Solution: (B) Number of orbitals in a shell ¿ $n^2=(5)^2=25$

3. BOD stands for:
(A) Biochemical oxidation demand
(B) Biological oxygen demand
(C) Biochemical oxygen demand
(D) Bacterial oxidation demand

Solution: (C) BOD stands for Biochemical Oxygen Demand.

4. Identify the incorrect statement regarding heavy water:
(A) It reacts with SO_3 to form deuterated sulphuric acid (D_2SO_4) .
(B) It is used as a coolant in nuclear reactors.
(C) It reacts with CaC_2 to produce C_2D_2 and $Ca(OD)_2$.
(D) It reacts with Al_4C_3 to produce CD_4 and $Al(OD)_3$.

Solution: (B) Heavy water is used in moderator section of nuclear reactors to control the speed of neutrons.

5. Which one of the following complexes will consume more equivalents of aqueous solution of $Ag(NO_3)$?

(A) $Na_2[CrCl_5(H_2O)]$

(B) $Na_3[CrCl_6]$

(C) $[Cr(H_2O)_5Cl]Cl_2$

(D) $[Cr(H_2O)_6]Cl_3$

Solution: (D) More Equivalents of $AgNO_3$ aqueous solution will be consumed by more $\overset{-¿}{Cl^{¿}}$ ions given by the complex

$$-¿(aq)$$
$$-¿(aq) + AgNO_3 \rightarrow AgCl\downarrow + NO_3^{¿}$$
$$Cl^{¿}$$

6. A particular adsorption process has the following characteristics: (i) It arises due to van der Waals forces and (ii) it is reversible. Identify the correct statement that describes the above adsorption process:

(A) Adsorption is monolayer

(B) Adsorption increases with increase in temperature

(C) Enthalpy of adsorption is greater than $100\,kJ\,mol^{-1}$

(D) Energy of activation is low

Solution: (D) In the care of physical Absorption there is almost nil energy of Activation and very low enthalpy change.

7. Bouveault-Blanc reduction reaction involves:

(A) Reduction of an acyl halide with H_2/Pd.

(B) Reduction of an anhydride with $LiAlH_4$

(C) Reduction of an ester with Na/C_2H_5OH .

(D) Reduction of a carbonyl compound with Na/Hg and HCl

Solution: (C) Bouveault-Blanc reduction reaction involves reduction of an ester with Na/C_2H_5OH .

8. At very high pressures, the compressibility factor of one mole of a gas is given by:

(A) $1+\frac{pb}{RT}$

(B) $\frac{pb}{RT}$

(C) $1-\frac{pb}{RT}$

(D) $1-\frac{b}{(VRT)}$

Solution: (A) $\left(P+\frac{a}{V^2}\right)(v-b)=RT$

at very high pressure $P \gg \frac{a}{V^2}$

$P(V-b)=RT$

$PV-Pb=RT$

$\therefore Z=1+\frac{pb}{RT}$ compremibility factor

9. The non-metal that does not exhibit positive oxidation state is:
(A) Chlorine
(B) Iodine
(C) Fluorine
(D) Oxygen

Solution: (C) Fluorine is most electronegative and exhibits O.S. value of –1 only.

10. 5 L of an alkane requires 25 L of oxygen for its complete combustion. If all volumes are measured at constant temperature and pressure, the alkane is:
(A) Isobutane
(B) Ethane
(C) Butane
(D) Propane

Solution: (D) Combustion of Hydrocarbon:

$$C_xH_y + \left(X + \frac{y}{4}\right)O_2 \rightarrow XCO_2 + \frac{y}{2}H_2O$$

$$1 : \left(X + \frac{y}{4}\right)$$

$\therefore x + \frac{y}{4} = 5$ which is satisfied by C_3H_8 (Propane)

11. The correct order of the solubility of alkaline-earth metal sulphates in water is:
(A) Mg > Ca > Sr > Ba
(B) Mg > Sr > Ca > Ba
(C) Mg < Ca < Sr < Ba
(D) Mg < Sr < Ca < Ba

Solution: (A) Hydration power energy $\propto \frac{\text{¿¿}\,Charge}{\text{¿}}$

$\because$ Solubility order of IIA group element sulphates in water:

$$MgSO_4 > CaSO_4 > SrSO_4 > BaSO_4$$

12. The group of molecules having identical shape is:
(A) PCl_5, IF_5, XeO_2F_2
(B) BF_3, PCl_3, XeO_3
(C) SF_4, XeF_4, CCl_4

$+¿$

(D) $Cl\,F_3, XeO\,F_2, Xe\,F_3^{¿}$

Solution: (D) T-shaped Hybridization

$$Cl\,F_3 \rightarrow p=3+\frac{1}{2}[7-3]=5(s\,p^3 d)$$

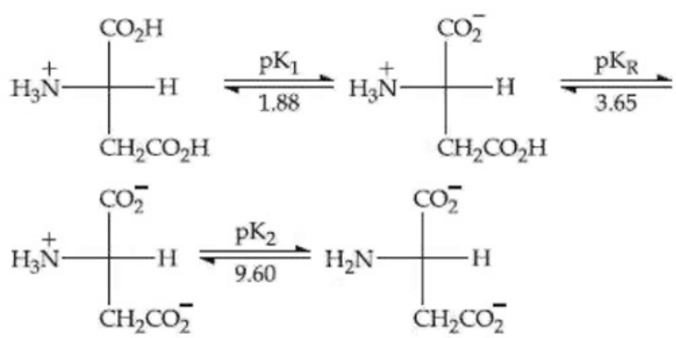
$$XeO\,F_2 \rightarrow p=3+\frac{1}{2}[8-4]=5(s\,p^3 d)$$

$$+¿ \rightarrow p=3+\frac{1}{2}[8-3-1]=5(s\,p^3 d)$$

$Xe\,F_3^{¿}$

All molecules have 3 Bond pairs and 2 core pairs.

13. Consider the following sequence for aspartic acid:



The pI (Isoeelectric point) of aspartic acid is:

(A) 3.65
(B) 2.77
(C) 5.74
(D) 1.88

Solution: (B) $PH=\frac{p\,K_1+p\,K_2}{2}$

$$¿\frac{1.88+3.65}{2}$$

$¿\frac{5.53}{2}$

$¿2.765 \approx 2.77$

At Isoelectric point of Aspartic Acid both carboxylic as well as amino from co-exist.

14. What will occur if a block of copper metal is dropped into a beaker containing a solution of 1 M $ZnSO_4$?

(A) The copper metal will dissolve with evolution of oxygen gas.
(B) The copper metal will dissolve with evolution of hydrogen gas.
(C) No reaction will occur.
(D) The copper metal will dissolve and zinc metal will be deposited.

Solution: (C) No reaction will occur. As the Displacement reaction is
$Zn_{(s)}+Cu^{+2}_{(aq)} \rightarrow Cu(s)+Zu^{+2}_{(aq)}$ which is feasible

15. Which intermolecular force is most responsible in allowing xenon gas to liquefy?

(A) Instantaneous dipole - induced dipole
(B) Ion-dipole
(C) Ionic
(D) Dipole-dipole

Solution: (A) London dispersian Force (LDF) are also kumun as londen forces, or instantaneous dipole-induced dipole forces, or loosely Vander-waals forces.

16. Assertion: Rayon is a semisynthetic polymer whose properties are better than natural cotton.
Reason: Mechanical and aesthetic properties of cellulose can be improved by acetylation.

(A) Both assertion and reason are correct, but the reason is not the correct explanation for the assertion.
(B) Both assertion and reason are correct, and the reason is the correct explanation for the assertion.
(C) Assertion in incorrect statement, but the reason is correct.
(D) Both assertion and reason are incorrect.

Solution: (A) The strength of cellulose is improved by acetylation and then for making packing material. Rayon which semi synthetic polymer has superior properties than natural cotton.

17. Match the items in Column I with its main use listed in Column II:

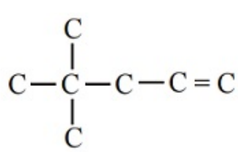
	Column I		Column II
A.	Silica gel	i.	Transistor
B.	Silicon	ii.	Ion-exchanger
C.	Silicone	iii.	Drying agent
D.	Silicate	iv.	Sealant

(A) A – iii, B – i, C – iv, D – ii
(B) A – iv, B – i, C – ii, D – iii
(C) A – ii, B – i, C – iv, D – iii
(D) A – ii, B – iv, C – i, D – iii

Solution: (A) Silica gel is used as drying agent. Silicon is used in Transistors. Silicon is a sealant while silicates including Zeotites are ion-exchanges.

18. The hydrocarbon with seven carbon atoms containing a neopentyl and a vinyl group is:
(A) 2, 2-dimethyl-4-pentene
(B) 4, 4-dimethylpentene
(C) Isopropyl-2-butene
(D) 2, 2-dimethyl-3-pentene

Solution: (B)

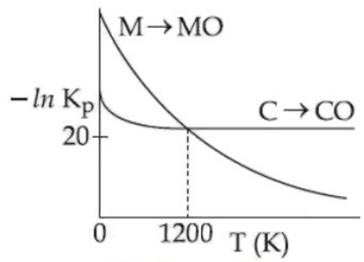


Hydrocarbon of 7 carbons having neopentyl a vinyl group.

19. The plot shows the variation of $-\ln K_P$ Versus temperature for the two reactions.

$M(s)+\frac{1}{2}O_2(g) \rightarrow MO(s)$ and

$C(s)+\frac{1}{2}O_2(g) \rightarrow CO(s)$



Identify the correct statement:

(A) At T < 1200 K, oxidation of carbon is unfavourable
(B) Oxidation of carbon is favourable at all temperatures
(C) At T < 1200 K, the reaction $MO(s)+C(s) \rightarrow M(s)+CO(g)$ is spontaneous
(D) At T > 1200 K, carbon will reduce MO(s) to M(s)

Solution: (D) $\Delta G^o = -RT \ln Kp$

$$\therefore \ln K_p = \frac{\Delta G^o}{RT}$$

$\Delta G^o \propto$ Temperature. At high temperature ΔG^o value is very high so non-feasible or non-spontaneous process takes place.

20. An organic compound contains C, H and S. The minimum molecular weight of the compound containing 8 Sulphur is:

(A) $600\, g\, mol^{-1}$

(B) $200\, g\, mol^{-1}$

(C) $400\, g\, mol^{-1}$

(D) $300\, g\, mol^{-1}$

Solution: (C) of Sulphur $¿\frac{wt\ of\ sulphur}{wt\ of\ compound}\times 100=8$

$$\therefore \frac{32}{wt.\ of\ compound}=\frac{8}{100}$$

$$\therefore wt.\ of\ compound=400\ g\ mol^{-1}$$

21. The most appropriate method of making egg-albumin sol is:
(A) Break an egg carefully and transfer the transparent part of the content to 100 mL of 5 w/V saline solution and stir well
(B) Keep the egg in boiling water for 10 minutes. After removing the shell, transfer the yellow part of the content to 100 mL of 5 w/V saline solution and homogenize with a mechanical shaker
(C) Keep the egg in boiling water for 10 minutes. After removing the shell, transfer the white part of the content to 100 mL of 5 w/V saline solution and homogenize with a mechanical shaker
(D) Break an egg carefully and transfer only the yellow part of the content to 100 mL of 5 w/V saline solution and stir well

Solution: (C) The process of egg – Albumin sol is involving boiling and then dissolving is saline water for homogeneity.

22. The amount of arsenic pentasulphide that can be obtained when 35.5 g arsenic acid is treated with excess H_2S in the presence of conc. HCl (assuming 100% conversion) is:
(A) 0.25 mol
(B) 0.50 mol
(C) 0.333 mol
(D) 0.125 mol

Solution: (D) $2H_3AsO_4+5H_2S \xrightarrow{Conc.\ HCl} As_2S_5+8H_2O$

2 moles of Arsenic Acid $\rightarrow$ 1 mole of Arsenic Pentasulphide

1 moles of Arsenic Acid $\rightarrow$ 1/2 mole of Arsenic Pentasulphide

$$\frac{Molar\ mass\ of\ H_3AsO_4 = 141}{Molar\ mass\ of\ As_2S_5 = 308}$$

$\therefore$ Number of moles of $H_3AsO_4 = \frac{35.5}{141} = 0.25$

$\therefore$ Number of moles of $As_2S_5 = \frac{0.25}{2} = 0.125\,mol$

23. The reaction of ozone with oxygen atoms in the presence of chlorine atoms can occur by a two step process shown below:

$O_3(g) + Cl^{\bullet}(g) \rightarrow O_2(g) + ClO^{\bullet}(g)$ —— (i)

$k_i = 5.2 \times 10^9\, L\,mol^{-1}s^{-1}$

$ClO^{\bullet}(g) + O^{\bullet}(g) \rightarrow O_2(g) + Cl^{\bullet}(g)$ —— (ii)

$k_{ii} = 2.6 \times 10^{10}\, L\,mol^{-1}s^{-1}$

The closest rate constant for the overall reaction $O_3(g) + O^{\bullet}(g) \rightarrow 2\,O_2(g)$ is:

(A) $1.4 \times 10^{20}\, L\,mol^{-1}s^{-1}$

(B) $3.1 \times 10^{10}\, L\,mol^{-1}s^{-1}$

(C) $5.2 \times 10^{9}\, L\,mol^{-1}s^{-1}$

(D) $2.6 \times 10^{10}\, L\,mol^{-1}s^{-1}$

Solution: (A) Overall Rate constant ¿ $K_i \times K_{ii}$

¿ $5.2 \times 10^9 \times 2.6 \times 10^{10}$

$\approx 1.4 \times 10^{20}\, mol^{-1}L\,s^{-1}$

$$O_{3(g)} + Cl^{\bullet}_{(g)} \longrightarrow O_{2(g)} + ClO^{\bullet}_{(g)} \quad K_i$$

$$O_{3(g)} + Cl^{\bullet}_{(g)} \longrightarrow O_{2(g)} + ClO^{\bullet}_{(g)} \quad K_i$$

$$ClO^{\bullet}_{(g)} + O^{\bullet}_{(g)} \longrightarrow O_{2(g)} + Cl^{\bullet}_{(g)} \quad K_{ii}$$

$$+ \text{Add}$$

$$O_{3_{(g)}} + O^{\bullet}_{(g)} \longrightarrow 2O_{2(g)} \quad K_{Rate} = K_i \times K_{ii}$$

24. The artificial sweetener that has the highest sweetness value in comparison to cane sugar is:
(A) Sucralose
(B) Aspartane
(C) Saccharin
(D) Alitame

Solution: (D) Alitame is an artificial sweetness which is have mone 1000 times sweetness value in comparison to Cane sugar.

25. Which one of the following species is stable in aqueous solution?
(A) Cr^{2+}
(B) MnO_4^{2-}
(C) MnO_4^{3-}
(D) Cu^{+}

Solution: (B) In MnO_4^{2-} manganese is in +6 oxidation state which is having highest stability.

Multiple Correct Answer Type

26. The test to distinguish primary, secondary and tertiary amines is:
(A) Sandmeyer's reaction
(B) Carbylamine reaction
(C) Mustard oil test
(D) $C_6H_5SO_2Cl$

Solution: (C,D) Mustard oil test and Benzene sulphonyl chloride both are used to distinguish primary, secondary and tertiary amines.

Single Correct Answer Type

27. The solubility of N_2 in water at 300 K and 500 torr partial pressure $0.01\,g\,L^{-1}$. The solubility (in $h\,L^{-1}$) at 750 torr partial pressure is:
(A) 0.0075
(B) 0.005
(C) 0.02
(D) 0.015

Solution: (D) Partial Pressure = Mole fraction $\times$ solubility

$$\frac{p_1}{p_2}=\frac{s_1}{s_2}\Rightarrow\frac{500}{0.01}=\frac{750}{x}$$

$$\therefore x=0.015\,g/L$$

28. For the reaction, $A(g)\rightarrow B(g)\rightarrow C(g)+D(g)$, ΔH^o and ΔS^o are, respectively, $-29.8\,kJ\,mol^{-1}$ and $-0.100\,kJ\,K^{-1}mol^{-1}$ at 298 K. The equilibrium constant for the reaction at 298 k is:
(A) 1.0×10^{-10}
(B) 10
(C) 1
(D) 1.0×10^{10}

Solution: (C) $\Delta G^o = \Delta H^o - T\Delta S^o$

$\Delta G^o = -29.8\, KJ/mol + 0.1 \times 298\, KJ\, mol^{-1}$

$\Delta G^o = -2.303\, RT \log K$

If log K = 0

$\therefore K = 1$

29. A reaction at 1 bar is non-spontaneous at low temperature but becomes spontaneous at high temperature. Identify the correct statement about the reaction among the following:

(A) ΔH is negative while ΔS is positive

(B) Both ΔH and ΔS are negative

(C) ΔH is positive while ΔS is negative

(D) Both ΔH and ΔS are positive

Solution: (D) If $\Delta H > 0$ and $\Delta S > 0$ then the process is feasible at light temperature and non-feasible at low temperature.

30. Identify the correct trend given below:
(Atomic No. = Ti : 22, Cr : 24 and Mo : 42)

(A) $2+\text{¿}$ / $2+\text{¿}>[Mo(H_2O)_6]^{\text{¿}}$ / $\Delta_0 of [Cr(H_2O)_6]^{\text{¿}}$ and $2+\text{¿}$ / $3+\text{¿}>[Ti(H_2O)_6]^{\text{¿}}$ / $\Delta_0 of [Ti(H_2O)_6]^{\text{¿}}$

(B) $2+\text{¿}$ / $2+\text{¿}>[Mo(H_2O)_6]^{\text{¿}}$ / $\Delta_0 of [Cr(H_2O)_6]^{\text{¿}}$ and $2+\text{¿}$ / $3+\text{¿}<[Ti(H_2O)_6]^{\text{¿}}$ / $\Delta_0 of [Ti(H_2O)_6]^{\text{¿}}$

(C) $2+\text{¿}$ / $2+\text{¿}<[Mo(H_2O)_6]^{\text{¿}}$ / $\Delta_0 of [Cr(H_2O)_6]^{\text{¿}}$ and $2+\text{¿}$ / $3+\text{¿}>[Ti(H_2O)_6]^{\text{¿}}$ / $\Delta_0 of [Ti(H_2O)_6]^{\text{¿}}$

(D) $\begin{matrix} 2+¿ \\ 2+¿<\left[Mo\left(H_2O\right)_6\right]^{¿} \\ \Delta_0 of\left[Cr\left(H_2O\right)_6\right]^{¿} \end{matrix}$ and $\begin{matrix} 2+¿ \\ 3+¿<\left[Ti\left(H_2O\right)_6\right]^{¿} \\ \Delta_0 of\left[Ti\left(H_2O\right)_6\right]^{¿} \end{matrix}$

Solution: (A) Stability of a complex depends inversely on the size of the cation, Nucleophility of the ligand CFSE values, chelating ligands.

$$\begin{matrix} Cr^{+2} \\ Mo^{+2} \\ Ti^{+3} \\ Ti^{+2} \\ ¿(¿)<¿(¿)\vee¿(¿)<¿(¿) \end{matrix}$$

Mathematics

Single Correct answer Type:

1. The shortest distance between the lines $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $\frac{y-4}{8}=\frac{z-5}{4}$ lies in the interval:

(A) $(3,4]$ (B) $(2,3]$ (C) $[1,2)$ (D) $[0,1)$

Solution: (B)

Shortest distance

$$\frac{\begin{vmatrix} x_2 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\begin{vmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}$$

$$¿\frac{\begin{vmatrix} -2 & 4 & 5 \\ 2 & 2 & 1 \\ -1 & 3 & 4 \end{vmatrix}}{\begin{vmatrix} i & j & k \\ 2 & 2 & 1 \\ -1 & 8 & 4 \end{vmatrix}}=\frac{254}{20.1}=2.7$$

2. If m and M are the minimum and the maximum values of $4+\frac{1}{2}\sin^2 2x-2\cos^4 x, x\in R,$ then $M-m$ is equal to:

(A) $\frac{9}{4}$ (B) $\frac{15}{4}$ (C) $\frac{7}{4}$ (D) $\frac{1}{4}$

Solution: (A)

$$4+\frac{1}{2}\sin^2 2x-2\cos^4 x$$

$$4+2\left(1-\cos^2 x\right)\cos^2 x-2\cos^4 x$$

$$-4\left\{\cos^4 x-\frac{\cos^2 x}{2}-1+\frac{1}{16}-\frac{1}{16}\right\}$$

$$-4\left\{\left(\cos^2 x-\frac{1}{4}\right)^2-\frac{17}{16}\right\}$$

$$0\le\cos^2 x\le 1$$

$$\frac{-1}{4}\le\cos^2 x-\frac{1}{4}\le\frac{3}{4}$$

$$0\le\left(\cos^2 x-\frac{1}{4}\right)^2\le\frac{9}{16}$$

$$\frac{-17}{16}\le\left(\cos^2 x-\frac{1}{4}\right)^2-\frac{17}{16}\le\frac{9}{16}-\frac{17}{16}$$

$$\frac{17}{14} \geq -4\left\{\left(\cos^2 x - \frac{1}{4}\right)^2 - \frac{17}{16}\right\} \geq \frac{1}{2}$$

$$M = \frac{17}{4}$$

$$M = \frac{1}{2}$$

$$M - M = \frac{17}{4} - \frac{2}{4} = \frac{15}{4}$$

3. If the equations $x^2 + bx - 1 = 0$ and $x^2 + x + b = 0$ have a common root different from $-1,$ then $|b|$ is equal to:

(A) 2 (B) 3 (C) $\sqrt{3}$ (D) $\sqrt{2}$

Solution: (C)

$x^2 + bx - 1 = 0$ common root

$x^2 + x + b = 0$

— — —

$$x = \frac{b+1}{b-1}$$

Put $x = \dfrac{b+1}{b-1}$ in equation ......(ii)

$$\left(\frac{b+1}{b-1}\right)^2 + \left(\frac{b+1}{b-1}\right) + b = 0$$

$$(b+1)^2 (b+1)(b-1) + b(b-1)^2 = 0$$

$b^2+1+2b+b^2-1+b(b62-2b+1)=0$

$2b^2+2b+b^3-2b^2+b=0$

$b^3+3b=0$

$b(b^2+3)=0$

$b^2=-3$

$b=\pm\sqrt{3}i$

$|b|=\sqrt{3}$

4. A circle passes through $(-2,4)$ and touches the y – axis at (0, 2). Which one of the following equations can represent a diameter of this circle ?

(A) $2x-3y+10=0$ (B) $3x+4y-3=0$

(C) $4x+5y-6=0$ (D) $5x+2y+4=0$

Solution: (A)

As $y-axis$ is tangent to the circle therefore the centre of the circle would be (h, 2) and distance of centre from (0, 2) and (-2, 4) should be equal.

$\because \quad \sqrt{x^2+0^2}=\sqrt{(x+2)^2}\times 2^2$

$x^2=x^2+4x+4+4$

$x=-2$

Hence centre = (-2, 2)

$\therefore \quad 2x-3y+10=0$ is diameter.

5. If a variable line drawn through the intersection of the lines $\frac{x}{3}+\frac{y}{4}=1$ and $\frac{x}{4}+\frac{y}{3}=1$, meets the coordinate axes at A and B, $(A \neq B)$, then the locus of the midpoint of AB is:

(A) $7xy=6(x+y)$ (B) $4(X+y)^2-28(x+y)+48=0$

(C) $6xy=7(x+y)$ (D) $14(x+y)^2-97(x+y)+168=0$

Solution: (A)

$L_1 : 4x+3y-12=0$

$L_2 : 3x+4y-12=0$

$L_1+\lambda L_2=0$

$(4x+3y-12)+\lambda(3x+4y-12)=0$

$x(4+3\lambda-12)+y(3+4\lambda)-12(1+\lambda)=0$

Point $A\left(\frac{12(1+\lambda)}{4+3\lambda},0\right)$

Point $B\left(0,\frac{12(1+\lambda)}{3+4\lambda}\right)$

Mid point $\Rightarrow h=\frac{6(1+\lambda)}{4+3\lambda}$ ....(i)

$k=\frac{6(1+\lambda)}{3+4\lambda}$ ....(ii)

Eliminate λ from (i) and (ii) then

$6(h+k)=$ ¿ hk

$6(x+y)=$ ¿ xy

6. If $2\int_0^1 \tan^{-1}x\,dx = \int_0^1 \cot^{-1}\left(1-x+x^2\right)dx,$ then $\int_0^1 \tan^{-1}\left(1-x+x^2\right)dx$ is equal to:

(A) $\frac{\pi}{2}+\log 2$ (B) $\log 2$ (C) $\frac{\pi}{2}-\log 4$ (D) $\log 4$

Solution: (B)

$$2\int_0^1 \tan^{-1}x\,dx = \int_0^1 \left(\frac{\pi}{2}-\tan^{-1}\left(1-x+x^2\right)\right)dx$$

$$2\int_0^1 \tan^{-1}x dx = \int_0^1 \frac{\pi}{2}dx - \int_0^1 \tan^{-1}\left(1-x+x^2\right)dx$$

$$\int_0^1 \tan^{-1}\left(1-x+x^2\right)dx = \frac{\pi}{2} - 2\int_0^1 \tan^{-1}x dx$$

Let

$$I_1 = \int_0^1 \tan^{-1}x dx$$

$$¿\left[\left(\tan^{-1}x\right)x\right]_0^1 - \int_0^1 \frac{1}{1+x^1}xdx$$

$$¿\frac{\pi}{4} - \int_0^1 \frac{x}{1+x^2}dx$$

$$¿\frac{\pi}{2} - \frac{1}{2}\ln 2$$

By equation ......(i)

$$\frac{\pi}{2} - 2\left[\frac{\pi}{2} - \frac{1}{2}\ln\right] = \ln 2$$

7. The number of $x \in [0, 2\pi]$ for which $\left|\sqrt{2\sin^4 x + 18\cos^2 x} - \sqrt{2\cos^4 x + 18\sin^2 x}\right| = 1$ is:

(A) 2 (B) 6 (C) 4 (D) 8

Solution: (D)

$$\left|\sqrt{2\sin^4 x + 18\cos^2 x} - \sqrt{2\cos^4 x + 18\sin^2 x}\right| = 1$$

$$\sqrt{2\sin^4 x + 18\cos^2 x} - \sqrt{2\cos^4 x + 18\sin^2 x} = \pm 1$$

$$\sqrt{2\sin^4 x + 18\cos^2 x} = \pm 1 + \sqrt{2\cos^4 x + 18\sin^2 x}$$

By squaring both the sides we will get B solutions

8. The distance of the point (1, -2, 4) from the plane passing through the point (1, 2, 2) and perpendicular to the planes $x - y + 2z = 3$ and $2x - 2y + z + 12 = 0,$ is :

(A) 2 (B) $\sqrt{2}$ (C) $2\sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$

Solution: (C)

Let equation of plane be

$a(x-1) + b(y-2) + c(z-2) = 0$ ....(i)

i) is perpendicular to given planes then

$$a - b + 2c = 0$$

$$2a - 2b + c = 0$$

Solving above equations $c = 0$ and $a = b$ equation of plane i) can be

$$x + y - 3 = 0$$

Distance from (1, -2, 4) will be

$$D = \frac{|1 - 2 - 3|}{\sqrt{1+1}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

9. If the tangent at a point on the ellipse $\frac{x^2}{27}+\frac{y^2}{3}=1$ meets the coordinate axes at A and B, and O is the origin, then the minimum area (in sq. units) of the triangle OAB is :

(A) $3\sqrt{3}$ (B) $\frac{9}{2}$ (C) 9 (D) $9\sqrt{3}$

Solution: (C)

Equation of tangent to ellipse

$$\frac{x}{\sqrt{27}}\cos\theta+\frac{y}{\sqrt{3}}\sin\theta=1$$

Area bounded by line and co – ordinate axis

$$\Delta=\frac{1}{2}.\frac{\sqrt{27}}{\cos\theta}.\frac{\sqrt{3}}{\sin\theta}=\frac{9}{\sin 2\theta}$$

Δ will be minimum when $\sin 2\theta=1$

$$\Delta_{min}=9$$

10. If $f(x)$ is a differentiable function in the interval $(0,\infty)$ such that $f(1)=1$ and $\lim_{t\to x}\frac{t^2 f(x)-x^2 f(t)}{t-x}=1,$ for each $x>0,$ then $f\left(\frac{3}{2}\right)$ is equal to:

(A) $\frac{23}{18}$ (B) $\frac{13}{6}$ (C) $\frac{25}{9}$ (D) $\frac{31}{18}$

Solution: (C)

Let $L=\lim_{t\to x}\frac{t^2 f(x)-x^2 f(t)}{t-x}=1$

Applying L.H. rule

$$L=\lim_{t\to x}\frac{2tf(x)-x^2f(x)}{1}=1$$

Or $2xf(x)-x^2f'(x)=1$

Solving above differential equation we get

$$f(x)=\frac{2}{3}x^2+\frac{1}{3x}$$

Put $x=\frac{3}{2}$

$$f\left(\frac{3}{2}\right)=\frac{2}{3}\times\left(\frac{3}{2}\right)^2+\frac{1}{3}\times 3\times 2$$

$$¿\frac{3}{2}+\frac{2}{9}=\frac{27+4}{18}=\frac{31}{18}$$

11. If the mean deviation of the numbers $1,1+d,\ldots,1+100d$ from their mean is 255, then a value of d is :

(A) 10.1 (B) 5.05 (C) 20.2 (D) 10

Solution: (A)

$$\acute{x}=\frac{1}{101}\left[1+(1+d)+1(1+2d)\right]\ldots.(1+100d)¿$$

$$¿\frac{1}{101}\times\frac{101}{2}\left[1+(1+100d)\right]=1+50d$$

Mean deviation from mean

$$¿\frac{1}{101}\left[11-(1+50d)1+1(1+d)-(1+50d)\right|\ldots.\left|[1+100d]-(1+50d)\right]$$

$$¿\frac{21dt}{101}(1+2+3\ldots.+50)$$

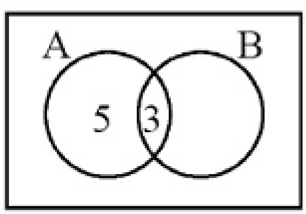
$$¿\frac{21\,dl}{101}\times\frac{50\times 51}{2}=\frac{2556}{101}dl$$

$$¿\frac{2550}{101}|d|=225\Rightarrow |d|=10.1$$

12. If A and B are any two events such that $P(A)=\frac{2}{5}$ and $P(A\cap B)=\frac{3}{20}$, then the conditional probability, $P\left(A\middle|\left(A'\cup B'\right)\right)$, where A' denotes the complement of A, is equal to:

(A) $\frac{11}{20}$ (B) $\frac{5}{17}$ (C) $\frac{8}{17}$ (D) $\frac{1}{4}$

Solution: (B)



$$P(A)=\frac{2}{5}=\frac{8}{20};P(A\cap B)=\frac{3}{20}$$

$$P(A\acute{\cap} B)=1-\frac{3}{20}$$

$$\Rightarrow \quad P(\acute{A}\cup\acute{B})=\frac{17}{20}$$

$$A\cap\left(A'\cup B'\right)$$

$$¿A-(A\cap B)$$

$$\quad P(A-(A\cap B))=\frac{5}{20}$$

$$P\left(\frac{A}{\left(A^{'}\cap B^{'}\right)}\right)=\frac{P\left(A-\left(A\cap B\right)\right)}{P\left(\acute{A}\cup\acute{B}\right)}=\frac{5}{17}$$

13. The value of $\sum_{r=1}^{15} r^2\left(\frac{{}^{15}C_r}{{}^{15}C_{r-1}}\right)$ is equal to:

(A) 1240 (B) 560 (C) 1085 (D) 680

Solution: (D)

$$\sum_{r=1}^{15} r^2\left(\frac{{}^{15}C_r}{{}^{15}C_{r-1}}\right)$$

$$\frac{{}^{15}C_r}{{}^{15}C_{r-1}}=\frac{\dfrac{\llcorner 15}{\llcorner 15-r}}{\dfrac{\llcorner 15}{\llcorner r-1\llcorner 16-r}}=\frac{\dfrac{1}{\llcorner 15-r\,r\llcorner r-1}}{\dfrac{1}{r\llcorner r-1 r\llcorner 15-r.\left(16-r\right)}}$$

$$¿\frac{16-r}{r}$$

$$\sum_{r=1}^{15} r^2\left(\frac{16-r}{r}\right)$$

$$\sum_{r=1}^{15} r\left(16-r\right)$$

$$16\sum_{r=1}^{15} r\left(16-r\right)$$

$$16\sum_{r=1}^{15} r-\sum_{r=1}^{15} r^2$$

$$\frac{16\times 15\times 16}{2}-\frac{15\times 31\times 16}{6}$$

$$8\times 15\times 16-5\times 8\times 31$$

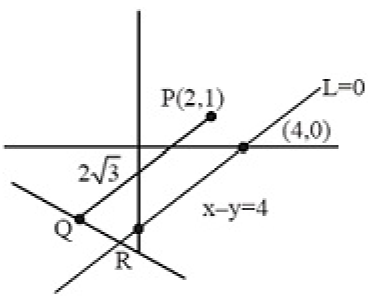
$1920-1240=680$

14. The point (2, 1) is translated parallel to the line $L: x-y=4$ by $2\sqrt{3}$ units. If the new point Q lies in the third quadrant, then the equation of the line passing through Q and perpendicular to L is :

(A) $x+y=2-\sqrt{6}$ (B) $2x+2y=1-\sqrt{6}$

(C) $x+y=3-3\sqrt{6}$ (D) $x+y=3-2\sqrt{6}$

Solution: (D)



$x-y=4$

To find equation of R

Slope of $L=0$ is 1

$\Rightarrow$ Slope of QR = -1

Let QR is $y=mx+c$

$y=-x+c$

$x+y-c=0$

Distance of QR from (2, 1) is $2\sqrt{3}$

$$2\sqrt{3}=\frac{|2+1-c|}{\sqrt{2}}$$

$2\sqrt{6}=|3-c|$

$c-3=\pm 2\sqrt{6} c=3\pm 2\sqrt{6}$

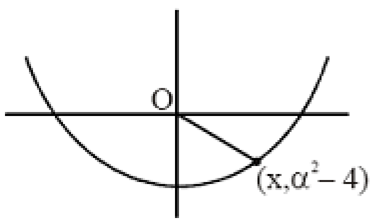
Line can be $x+y=3\pm 2\sqrt{6}$

$x+y=3-2\sqrt{6}$

15. The minimum distance of a point on the curve $y=x^2-4$ from the origin is :

(A) $\frac{\sqrt{15}}{2}$ (B) $\sqrt{\frac{19}{2}}$ (C) $\sqrt{\frac{15}{2}}$ (D) $\frac{\sqrt{19}}{2}$

Solution: (A)



$D=\sqrt{\alpha^2+(\alpha^2-4)^2}$

$D^2=\alpha^2+\alpha^4+16-8\alpha^2$

¿ $\alpha^4-7\alpha^2+16$

$\frac{dD^2}{d\alpha}=4\alpha^3-14\alpha=0$

$2\alpha(2\alpha^2-7)=0$

$\alpha^2=\frac{7}{2}$

$D^2=\frac{49}{4}-\frac{49}{2}+16$

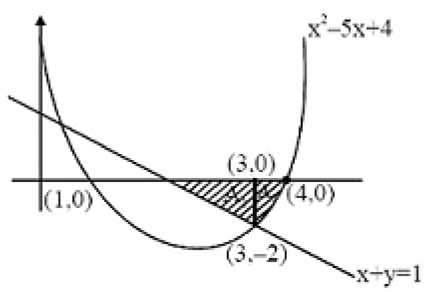
¿$-\frac{49}{2}+16=\frac{15}{4}$

$D=\frac{\sqrt{15}}{2}$

16. The area (in sq. units) of the region described by $A=\{x, y\} \mid y\geq x^2-5x+4, x+y\geq 1, y\leq 0\}$ is:

(A) $\frac{19}{6}$ (B) $\frac{17}{6}$ (C) $\frac{7}{2}$ (D) $\frac{13}{6}$

Solution: (A)



Required area ¿A_1+A_2

¿$\frac{1}{2}\times 2\times 2\times \left|\int_3^4 (x^2-5x+4)\,dx\right|$

¿$2+\frac{7}{6}=\frac{19}{6}\,sq.units$

17. In a triangle ABC, right angled the vertex A, if the position vectors of A, B and C are respectively $3\hat{i}+\hat{j}-\hat{k}, -\hat{i}+3\hat{j}+p\hat{k}$ and $5\hat{i}+9\hat{j}-4\hat{k}$, then the point (p, q) lies on a line:

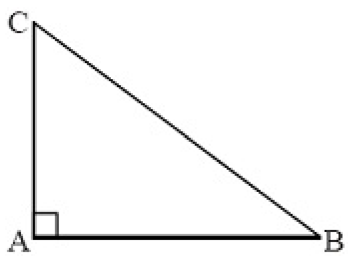
(A) Making ab obtuse angle with the positive direction of $x-axis$

(B) Parallel to $x-axis$

(C) Parallel to $y-axis$

(D) Making an acute angle with the positive direction of $x-axis$

Solution: (D)



$\acute{A}B=-4\hat{i}+2\hat{j}+(p+1)\hat{k}$

$\acute{A}C=2\hat{i}+(a_n-1)\hat{j}-3\hat{k}$

$\acute{A}B \perp \acute{A}C$

$\Rightarrow \quad \acute{A}B.\acute{A}C=0$

$-8+2(a_n-1)-3(p+1)=0$

$3p-2q+13=0$

(p, q) lies on $3x-2y+13=0$

Slope ¿ $\frac{3}{2}$

$\therefore$ Acute angle with $x-axis$

18. If $\int \frac{dx}{\cos^3 x\sqrt{2\sin 2x}}=(\tan x)^A+C(\tan x)^B+k,$ where k is a constant of integration, then $A+B+C$ equals :

(A) $\frac{16}{5}$ (B) $\frac{27}{10}$ (C) $\frac{7}{10}$ (D) $\frac{21}{5}$

Solution: (A)

$$\frac{\int dx}{\cos^3 x\sqrt{4\sin x\cos x}}=\int\frac{dx}{2\cos^4 x\sqrt{\tan x}}$$

Let $\tan x=t^2 \Rightarrow sec^2 x=1+t^4$

$$sec^2 xdz=2t\,dt$$

$$¿\int\frac{sec^4 xdx}{2\sqrt{\tan x}}=\int\frac{sec^2 x\left(sec^2 x\,dx\right)}{2\sqrt{\tan x}}$$

$$¿\int\frac{\left(1+t^4\right)2t\,dt}{2t}=\int\left(1+t^4\right)dt$$

$$¿t+\frac{t^5}{5}+k$$

$$¿\sqrt{\tan x}+\frac{1}{5}\tan^{\frac{5}{2}}x+k\left[t=\sqrt{\tan x}\right]$$

$$A=\frac{1}{2},B=\frac{5}{2},C=\frac{1}{5}$$

$$A+B+C=\frac{16}{5}$$

19. Consider the following two statements:

P : If 7 is an odd number, then 7 is divisible by 2.

Q : If 7 is a prime number, then 7 is an odd number.

If V_1 is the truth value of the contrapositive of P and V_2 is the truth value of contrapositive of Q then the ordered pair $\left(V_1,V_2\right)$ equals:

(A) (F, F) (B) (F, T) (C) (T, F) (D) (T, T)

Solution: (B)

Contra positive of P :

T is not divisible by 2 $\Rightarrow$ T is not odd number

$T \Rightarrow F : F\ (V_1)$

Contra positive of Q :

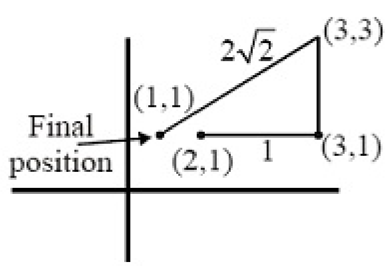
T is not odd number $\Rightarrow$ T is not a prime number

$F \Rightarrow F : T\ (V_2)$

20. The point represented by $2+i$ in the Argand plane moves 1 unit eastwards, then 2 units northwards and finally from there $2\sqrt{2}$ units in the south – west wards direction. Then its new position in the Argand plane is at the point represented by:

(A) $1+i$ (B) $2+2i$ (C) $-2-2i$ (D) $-1-i$

Solution: (A)



21. For $x \in R, x \neq 0, x \neq 1,$ let $f_0(x) = \frac{1}{1-x}$ and $f_{n+1}(x) = f_0(f_n(x)), n = 0, 1, 2, \ldots..$ Then the value of $f_{100}(3) + f_1\left(\frac{2}{3}\right) + f_2\left(\frac{3}{2}\right)$ is equal to :

(A) $\frac{8}{3}$ (B) $\frac{4}{3}$ (C) $\frac{5}{3}$ (D) $\frac{1}{3}$

Solution: (C)

$$f_1(x)=f_{0+1}(x)=f_0(f_0(x))=\frac{1}{1-\frac{1}{1-x}}=\frac{x-1}{x}$$

$$f_2(x)=f_{1+1}(x)=f_0(f_1(x))=\frac{1}{1-\frac{x-1}{x}}=x$$

$$f_3(x)=f_{2+1}(x)=f_0(f_2(x))=f_0(x)=\frac{1}{1-x}$$

$$f_4(x)=f_{3+1}(x)=f_0(f_0(x))=\frac{x-1}{x}$$

$$\therefore \quad f_0=f_3=f_6=\ldots=\frac{1}{1-x}$$

$$f_1=f_4-f_5=f_9=\ldots=\frac{x-1}{x}$$

$$f_2=f_5=f_8=\ldots=x$$

$$f_{100}(3)=\frac{3-1}{3}=\frac{2}{3}f_1\left(\frac{2}{3}\right)=\frac{\frac{2}{3}-1}{\frac{2}{3}}=\frac{-1}{2}$$

$$f_2\left(\frac{3}{2}\right)=\frac{3}{2}$$

$$\therefore \quad f_{100}(3)+f_1\left(\frac{2}{3}\right)+f_2\left(\frac{3}{2}\right)=\frac{5}{3}$$

22. If the function $f(x)=\begin{cases} -x, & x<1 \\ a+\cos^{-1}(x+b), & 1\le x\le 2 \end{cases}$ is differentiable at $x=1$, then $\frac{a}{b}$ is equal to :

(A) $\frac{\pi+2}{2}$ (B) $\frac{\pi-2}{2}$ (C) $\frac{-\pi-2}{2}$ (D) $-1-\cos^{-1}(2)$

Solution: (A)

$$f(x)=\begin{cases} -x & x<1 \\ a+\cos^{-1}(x+b) & 1\le x\le 2 \end{cases}$$

$f(x)$ is continuous

$$\Rightarrow \quad \begin{matrix} x\to 1^{+¿} a+\cos^{-1}(x+b)=f(x) \\ x\to 1^{-¿} f(x)=\lim_{¿} ¿ \\ \lim_{¿} ¿ \end{matrix}$$

$\Rightarrow \quad -1=a+\cos^{-1}(1+3)$

$\cos^{-1}(1+b)=-1-a$ ....(i)

$f(x)$ is differentiate

$\Rightarrow$ LHD = RHD

$\Rightarrow \quad -1=\frac{-1}{\sqrt{1-(1+b)^2}}$

$\Rightarrow \quad 1-(1+b)^2=1$

$\Rightarrow \quad b=-1$ ......(ii)

From (i) $\Rightarrow \quad \cos^{-1}(0)=-1-a$

$\therefore \quad -1-a=\frac{\pi}{2}$

$$a = -1 - \frac{\pi}{2}$$

$$a = \frac{-\pi - 2}{2} \qquad \text{......(iii)}$$

$$\therefore \qquad \frac{a}{b} = \frac{\pi + 2}{2}$$

23. If $\lim\limits_{x \to \infty} \left(1 + \frac{a}{x} - \frac{4}{x^2}\right)^{2x} = e^3$, then 'a' is equal to:

(A) 2 (B) $\frac{3}{2}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$

Solution: (B)

$$\lim \left(1 + \frac{a}{2} - \frac{4}{x^2}\right)^{2x} \qquad (1\infty\, form)$$

$$¿ e \left[\lim_{x \to \infty} \left(1 + \frac{a}{2} - \frac{4}{x^2} - 1\right) 2x\right]$$

$$¿ e \lim_{x \to \infty} \left(2a - \frac{8}{x}\right) = e^{2a}$$

$$\therefore \qquad 2a = 3 \Rightarrow a = \frac{3}{2}$$

24. Let $x.y.z$ be positive real numbers such that $x + y + z = 12$ and $x^3 y^4 z^5 = (0.1)(600)^3$. Then $x^3 + y^3 + z^3$ is equal to :

(A) 342 (B) 216 (C) 258 (D) 270

Solution: (B)

$x+y+z=12$

$AM \geq GM$

$$\frac{3\left(\frac{x}{3}\right)+4\left(\frac{y}{4}\right)+5\left(\frac{z}{5}\right)}{12} \geq = \sqrt[12]{\left(\frac{x}{3}\right)^3\left(\frac{y}{4}\right)^4\left(\frac{z}{5}\right)^5}$$

$$\frac{x^3 y^4 z^5}{3^3 4^4 5^5} \leq 1$$

$x^3 y^4 z^5 \leq 3^3 . 4^4 . 5^5$

$x^3 y^4 z^5 \leq (0.1)(6000)^3$

But, given $x^3 y^4 z^5 = (0.1)(600)^3$

$\therefore$ Al the numbers are equal.

$\therefore$ $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}(¿k)$

$x=3k ; y=4k ; z=5k$

$x+y+z=12$

$3k+4k+5y=12$

$k=1$

$\therefore$ $x=3 ; y=4' z=5$

$\therefore$ $x^3+y^3+z^3=216$

25. The number of distinct real roots of the equation, $\begin{vmatrix} \cos x & \sin x & \sin x \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$ in the interval $\left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$ is :

(A) 1 (B) 4 (C) 2 (D) 3

Solution: (C)

$$\begin{vmatrix} \cos x & \sin x & \sin x \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$

$R_2 \to R_2 - R_3$

$$\begin{vmatrix} \cos x - \sin x & \sin x - \cos x & 0 \\ 0 & \cos x - \sin x & \sin x - \cos x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$$

$C_2 \to C_2 + C_3$

$$\begin{vmatrix} \cos x - \sin x & \sin x - \cos x & 0 \\ 0 & 0 & \sin x - \cos x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$$

Expanding using first row

$2 \sin x (\sin x - \cos x)^2 = 0$

$\sin x = 0 \vee \sin x = \cos x$

$x = 0$ or $x = \frac{\pi}{4}$

26. Let a and b respectively be the semi-transverse and semi – conjugate axes of a hyperbola whose eccentricity satisfies the equation $9e^2-18e+5=0$. If $S(5,0)$ is a focus and $5x=9$ is the corresponding directrix of this hyperbola, then a^2-b^2 is equal to :

(A) -7 (B) -5 (C) 5 (D) 7

Solution: (A)

$s(5_{10}) \Rightarrow a_e=5(focus)$ ......(i)

$x=\frac{a}{5} \Rightarrow \frac{a}{e}=\frac{9}{5}$ (directrix) ......(ii)

(i) And (ii) $\Rightarrow a^2=9$

(i) $\Rightarrow e=\frac{5}{3}$

$b^2=a^2(e^2-1) \Rightarrow b^2=16$

$a^2-b^2=9-16=-7$

27. If $P=\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, A=\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q=PAP^T$, then $P^TQ^{2015}P$ is :

(A) $\begin{bmatrix} 0 & 2015 \\ 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 2015 & 0 \\ 1 & 2015 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 2015 \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 2015 & 1 \\ 0 & 2015 \end{bmatrix}$

Solution: (C)

$$P=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ \frac{-1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix} P^T=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{-1}{2}\\ \frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$$

$$P\,P^T=P^T\,P=I$$

$\theta^{2005}=\left(PA\,P^T\right)\left(PA\,P^T\right)$ ......(2005 terms)

$¿\,P\,A^{2005}\,P^T$

$$P^T\,\theta^{2005}\,P=A^{2005}$$

$$A^2=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$$

$$A^3=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}$$

 $A^{2005}=\begin{bmatrix}1 & 2005\\ 0 & 1\end{bmatrix}$

28. For $x\in R, x\neq -1,$ if $(1+x)^{2016}+x(1+x)^{2015}+x^2(1+x)^{2014}+\ldots+x^{2016}=\sum_{i=0}^{2016} a_i x^i$, then a_{17} is equal to:

(A) $\frac{2017!}{17!\,2000!}$ (B) $\frac{2016!}{17!\,1999!}$ (C) $\frac{2016!}{16!}$ (D) $\frac{2017!}{2000!}$

Solution: (A)

$$S=(1+x)^{2016}+x(1+x)^{2015}+x^2(1+x)^{2014}+\ldots\ldots+x^{2015}(1+x)+x^{2016}$$

$$\frac{\left(\frac{x}{1+x}\right)S = x(1+x)^{2015} + x^2(1+x)^{2014} + \ldots + x^{2016} + \frac{x^{2017}}{1+x}}{\frac{S}{1+x} = (1+x)^{2016} - \frac{x^{2017}}{1+x}}$$

$\therefore \quad S = (1+x)^{2017} - x^{2017}$

$a_{17} = coeffecient\ of\ x^{17} = {}^{2017}C_{17}\, x^7 = \frac{2017!}{17!\,2000!}$

29. If the four letter words (need not be meaningful) are to be formed using the letters from the word "MEDITERRANEAN" such that the first letter is R and the fourth letter is E, then the total number of all such words is:

(A) 110 (B) 59 (C) $\frac{11!}{(2!)^3}$ (D) 56

Solution: (B)

M, EEE, D.I, T, RR, AA, NN

$\underline{R}\,\underline{\square}\,\underline{\square}\,\underline{E}$

Two empty places can filled with identical letters [EE, AA, NN] $\Rightarrow$ 3 ways

Two empty places, can be filled with distinct letters [M, E, D, I,T, R, A, N] $\Rightarrow {}^8P_2$

$\therefore$ Number of words $3 + {}^8P_2 = 59$

30. If the tangent at a point P, with parameter t, on the curve $x = 4t^2 + 3,\ y = 8t^3 - 1,\ t \in R,$ meets the curve again at a point Q, then the coordinates of Q are :

(A) $(16t^2+3, -64t^3-1)$ (B) $(4t^2+3, -8t^3-1)$

(C) $\left(t^2+3, t^3-1\right)$ (D) $\left(t^2+3, -t^3-1\right)$

Solution: (D)

$P\left(4t^2+3, 8t^3-1\right)$

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{dy}{dx}=3t$$ (slope of tangent at P)

Let $Q=\left(4\lambda^2+3, 8\lambda^3-1\right)$

Slope of $PQ=3t$

$$\frac{8t^3-8\lambda^3}{4t^2-4\lambda^2}=3t$$

$\Rightarrow \quad t^2+t\lambda-2\lambda^2=0$

$\left(t-\lambda\right)\left(t+2\lambda\right)=0$

$t=\lambda$ (or) $\lambda=\frac{-t}{2}$

$\therefore \quad Q\left[t^2+3, -t^3-1\right]$