MATHEMATICS

1. If \( S \) is the set of distinct values of \( 'b' \) for which the following system of linear equations

\[
\begin{align*}
x + y + z &= 1 \\
x + ay + z &= 1 \\
ax + by + z &= 0
\end{align*}
\]

Has no solution, then \( S \) is:

(1) An empty set

(2) An infinite set

(3) A finite set containing two or more elements

(4) A singleton

**Solution:** (4)

\[
D = \begin{vmatrix}
1 & 1 & 1 \\
1 & a & 1 \\
a & b & 1
\end{vmatrix} = 0 \Rightarrow a = 1
\]

and at \( a = 1 \)

\[
D_1 = D_2 = D_3 = 0
\]

But at \( a = 1 \) and \( b = 1 \)

First two equations are and third equation is

\[
\begin{align*}
x + y + z &= 1 \\
x + y + z &= 0
\end{align*}
\]

\( \Rightarrow \) There is no solution.

\( \therefore \) \( b = \{1\} \Rightarrow \) it is a singleton set

2. The following statement

\( (p \rightarrow q) \rightarrow ((\sim p \rightarrow q) \rightarrow q) \) is:
(1) A tautology

(2) Equivalent to \( \sim p \rightarrow q \)

(3) Equivalent to \( p \rightarrow \sim q \)

(4) A fallacy

Solution: (1)

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3. If \( 5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9 \), then the value of \( \cos 4x \) is:

(1) \(-\frac{3}{5}\)

(2) \(\frac{1}{5}\)

(3) \(\frac{2}{5}\)

(4) \(\frac{7}{5}\)

Solution: (4)

\[ 5 \left( \frac{1 - \cos 2x}{1 + \cos 2x} \right) = 2\cos 2x + 9 \]

Let \( \cos 2x = y \)

\[ 5 \left[ -y^2 - 4y + 1 \right] = 4y^2 + 22y + 18 \]

\[ 9y^2 + 42y + 13 = 0 \]
\[ y = -\frac{1}{3} \text{ or } y = -\frac{13}{3} \quad \text{(Not possible)} \]

Now, \[ \cos 4x = 2\cos^2 2x - 1 \]
\[ = 2\left(-\frac{1}{3}\right)^2 - 1 = -\frac{7}{9} \]

4. For three events, \(A, B\) and \(C\), \(P(\text{Exactly one of A or B occurs})\)
\[ = P(\text{Exactly one of B or C occurs}) = \frac{1}{4} \text{ and } P(\text{All the three events occur simultaneously}) = \frac{1}{16} \]

Then the probability that at least one of the events occurs, is:

(1) \( \frac{7}{32} \)
(2) \( \frac{7}{16} \)
(3) \( \frac{7}{64} \)
(4) \( \frac{3}{16} \)

**Solution:** (2)

\[
\begin{figure}
\begin{center}
\includegraphics[width=0.5\textwidth]{chart.png}
\end{center}
\end{figure}
\]

\[ a = \frac{1}{16} \]

\[ c + d + e + f = \frac{1}{4} \quad \text{(ii)} \]
\[ b + d + e + g = \frac{1}{4} \]  
\[ \ldots \text{(ii)} \]

\[ b + f + c + g = \frac{1}{4} \]  
\[ \ldots \text{(iii)} \]

Adding (i), (ii) and (iii)

\[ 2(b + c + d + e + f + g) = \frac{3}{4} \]

\[ b + c + d + e + f + g = \frac{3}{8} \]

Now \[ P(A \cup B \cup C) = a + (b + c + d + e + f + g) \]

\[ = \frac{1}{16} + \frac{3}{8} = \frac{7}{16} \]

5. Let \( \omega \) be a complex number such that \( 2\omega + 1 = z \) where \( z = \sqrt{-3} \). If

\[
\begin{bmatrix}
1 & 1 & 1 \\
1 & -\omega^2 - 1 & \omega^2 \\
1 & \omega^2 & \omega^7
\end{bmatrix} = 3k,
\]

Then \( k \) is equal to:

(1) \( -z \)

(2) \( z \)

(3) \( -1 \)

(4) \( 1 \)

Solution (4)

\[ \omega = -\frac{1}{2} + \frac{\sqrt{3}}{2}i \]  
\[ \text{(cube root of unity)} \]

So \( 1 + \omega + \omega^2 = 0 \)
Now \[
\begin{vmatrix}
1 & 1 & 1 \\
1 & \omega & \omega^2 \\
1 & \omega^2 & \omega
\end{vmatrix} = 3(\omega^2 - \omega)
\]

So \(k = \omega^2 - \omega = \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) - \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\)

\(= -\sqrt{3}i = -z\)

6. Let \(k\) be an integer such that the triangle with vertices \((k, -3k), (5,k)\) and \((-k, 2)\) has area 28 sq. units. Then the orthocenter of this triangle is at the point:

(1) \(\left(2, -\frac{1}{2}\right)\)

(2) \(\left(1, \frac{3}{4}\right)\)

(3) \(\left(1, -\frac{3}{4}\right)\)

(4) \(\left(2, \frac{1}{2}\right)\)

**Solution:** (4)

\[|k(k - 2) + 5(2 + 3k) - k(-4k)| = 56\]

\[5k^2 + 13k - 46 = 0\]

\[k = 2\text{ and } k = -\frac{23}{5}\text{ (Not possible)}\]

Altitude \(AD : x = 2\)
Altitude BE : \( y - 2 = \frac{1}{2}(x - 5) \)

So their point of intersection is \( H \left( 2, \frac{1}{2} \right) \)

7. Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower-bed, is:

(1) 12.5  
(2) 10  
(3) 25  
(4) 30  

**Solution: (3)**

Perimeter \( = 2r + r\theta = 20 \)

![Circular Sector Diagram]

\( \theta = \frac{20 - 2r}{r} \)

Now Area \( A = \frac{r^2\theta}{2} \)

\( A = \frac{r^2}{2} \left( \frac{20 - 2r}{r} \right) = 10r - r^2 \)

\( \frac{dA}{dr} = 10 - 2r = 0 \)
\[
\Rightarrow r = 5 \\
\left( \frac{dA}{dr^2} = 2 \text{ (maxima)} \right) \\
A_{\text{max}} = 10 \times 5 - 25 = 25
\]

8. The area (in sq. units) of the region \(\{(x, y): x \geq 0, x + y \leq 3, x^2 \leq 4y \text{ and } y \leq 1 + \sqrt{x}\}\) is:

(1) \(\frac{59}{12}\)  \\
(2) \(\frac{3}{2}\)  \\
(3) \(\frac{7}{3}\)  \\
(4) \(\frac{5}{2}\)

**Solution: (4)**

**Required Area**

\[
= \int_0^1 (1 + \sqrt{x}) \, dx + \int_1^2 (3 - x) \, dx - \int_0^2 \frac{x^2}{4} \, dx \\
= x + \frac{2}{3}x^\frac{3}{2}\bigg|_0^1 + 3x - \frac{x^2}{2}\bigg|_1^2 - \frac{x^3}{12}\bigg|_0^2 \\
= \left(1 + \frac{2}{3}\right) + \left(4 - \frac{5}{2}\right) - \frac{2}{3} \\
= \frac{5}{2}
\]
9. If the image of the point \( P(1, -2, 3) \) in the plane, \( 2x + 3y - 4z + 22 = 0 \) measured parallel to the line, \( \frac{x}{1} = \frac{y}{4} = \frac{z}{5} \) is \( Q \), then \( PQ \) is equal to:

(1) \( 3\sqrt{5} \)

(2) \( 2\sqrt{42} \)

(3) \( \sqrt{42} \)

(4) \( 6\sqrt{5} \)

**Solution: (2)**

Length of perpendicular from \( P \) to the given plane

\[
PS = \frac{6}{\sqrt{29}}
\]

Angle between plane and line

\[
\sin \theta = \frac{6}{\sqrt{29} \sqrt{42}}
\]

Length \( PQ \)

\[
= 2 \cdot \frac{PS \sin \theta}{6} = 2 \cdot \frac{\frac{6}{\sqrt{29} \sqrt{42}}}{6} = 2\sqrt{42}
\]
10. If for \( x \in \left(0, \frac{1}{\sqrt{3}}\right)\), the derivative of \( \tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^2}\right) \) is \( \sqrt{x} \cdot g(x) \), then \( g(x) \) equals:

(1) \( \frac{9}{1 + 9x^3} \)

(2) \( \frac{3x\sqrt{x}}{1 - 9x^2} \)

(3) \( \frac{3x}{1 - 9x^2} \)

(4) \( \frac{3}{1 + 9x^3} \)

**Solution: (1)**

Let \( 3x^{\frac{3}{2}} = y \) then \( y \in \left(0, \frac{3}{\sqrt{3}}\right) \)

\[
\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^2}\right) = \tan^{-1}\left(\frac{2y}{1 - y^{2}}\right)
\]

\[
= 2\tan^{-1}y = 2\tan^{-1}\left(3x^{\frac{3}{2}}\right)
\]

Now \( \frac{d}{dx}2\tan^{-1}\left(3x^{\frac{3}{2}}\right) = \frac{2}{1 + 9x^3} \cdot 3 \cdot \frac{3}{2} \cdot x^{\frac{1}{2}} \cdot \frac{1}{2} \cdot x^{\frac{3}{2}} \)

\[
= \frac{9}{1 + 9x^3} \cdot \sqrt{x}
\]
11. If \((2 + \sin x) \frac{dy}{dx} + (y + 1) \cos x = 0\) and \(y(0) = 1\), then \(y\left(\frac{\pi}{2}\right)\) is equal to:

1. \(\frac{1}{3}\)
2. \(-\frac{2}{3}\)
3. \(-\frac{1}{3}\)
4. \(\frac{4}{3}\)

**Solution:**

\[(2 + \sin x) \frac{dy}{dx} = -(y + 1) \cos x\]

\[\Rightarrow \int \frac{dy}{y + 1} = -\int \frac{\cos x}{\sin x + 2} dx\]

\[\sin x = t\]

\[\cos x dx = dt\]

\[\Rightarrow \log (y + 1) = -\int \frac{dt}{t + 2} = -\log (\sin x + 2) + C\]

\[\Rightarrow \log (y + 1) + \log (\sin x + 2) = C = 0\]

\[\Rightarrow y(0) = 1\]

\[\log (2) + \log (2) = C\]

\[C = \log 4\]

\[y\left(\frac{\pi}{2}\right)\]

\[\log (y + 1) + \log (3) - \log (4) = 0\]

\[\Rightarrow \log (y + 1) = \log \left(\frac{4}{3}\right)\]
\[ y + 1 = \frac{4}{3} \]
\[ y = \frac{4}{3} - 1 = \frac{1}{3} \]

12. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that AP = 2AB. If \( \angle BPC = \beta \), then \( \tan \beta \) is equal to:

(1) \( \frac{6}{7} \)

(2) \( \frac{1}{4} \)

(3) \( \frac{2}{5} \)

(4) \( \frac{4}{5} \)

**Solution:** (3)

\[ AB = 2(AC) = 2(BC) \]

\[ AP = 2AB \Rightarrow AP = 4(AC) = 4(BC) \]

\[ \tan \alpha = \frac{AC}{AP} = \frac{AC}{4(AC)} = \frac{1}{4} \]

\[ \tan (\alpha + \beta) = \frac{AB}{AP} = \frac{2AB}{2(AB)} = \frac{1}{2} \]
Let $\tan \beta = x$

\[
\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta} = \frac{1}{2}
\]

\[
\Rightarrow \quad \frac{1}{4 + x - \frac{x}{2}} = \frac{1}{2}
\]

\[
\Rightarrow \quad 1 + 4x - 4 + x = 0
\]

\[
\Rightarrow \quad 9x - 2 = 0
\]

\[
\Rightarrow \quad x = \frac{2}{9}
\]

13. If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then $\text{adj} \left( 3A^2 + 12A \right)$ is equal to:

(1) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$

(2) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$

(3) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$

(4) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$

Solution: (2)

$A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$
\[ A^2 = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 4 + 12 & -6 - 3 \\ -8 - 4 & 12 + 1 \end{bmatrix} = \begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix} \]

\[ 3A^2 = \begin{bmatrix} 48 & -27 \\ -36 & 39 \end{bmatrix} \]

\[ 12A = \begin{bmatrix} 24 & -36 \\ -48 & 12 \end{bmatrix} \]

\[ (3A^2 + 12A) = \begin{bmatrix} 72 & -63 \\ 84 & 51 \end{bmatrix} \]

\[ \text{Adj}(3A^2 + 12A) = \begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix} = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix} \]

14. For any three positive real numbers \( a, b \) and \( c \), \( 9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c) \). Then:

(1) \( b, c \) and \( a \) are in G.P.

(2) \( b, c \) and \( a \) are in A.P.

(3) \( a, b \) and \( c \) are in A.P.

(4) \( a, b \) and \( c \) are in G.P.

**Solution:** (2)

If means

\[ 15a = 3b = 5c = k \] (let say)

\[ a = \frac{k}{15} \]

\[ b = \frac{k}{3} \] and
\[ c = \frac{k}{5} \]
\[ a + b = \frac{2k}{5} = 2(c) \]

So, b, c, a are in A.P.

15. The distance of the point \((1, 3, -7)\) from the plane passing through the point \((1, -1, -1)\), having normal perpendicular to both the lines \[ \frac{x-1}{1} = \frac{y+2}{2} = \frac{z-4}{3} \quad \text{and} \quad \frac{x-2}{2} = \frac{y+1}{1} = \frac{z+7}{-1}, \] is:

- (1) \( \frac{20}{\sqrt{4}} \)
- (2) \( \frac{10}{\sqrt{83}} \)
- (3) \( \frac{5}{\sqrt{83}} \)
- (4) \( \frac{10}{\sqrt{4}} \)

**Solution:** [2]

\[ \ell - 2m + 3n = 0 \]
\[ 2\ell - m - n = 0 \]
\[ \frac{x - 1}{1} = \frac{y + 2}{-2} = \frac{y - 4}{3} \]
\[ \frac{x - 2}{2} = \frac{y + 1}{-1} = \frac{3 + 7}{-1} = -1 \]
\[ \frac{\ell}{2 + 3} = \frac{m}{6 + 1} = \frac{n}{-1 + 4} = \lambda \]
\[ \ell = 5\lambda, \ m = 7\lambda, \ n = 3\lambda \]
\[ 5x + 7y + 3y + d = 0 \]
\[ 5 - 7 - 3 + d = 0 \]
\[ d = 5 \]
\[ 5x + 7y + 3y + 5 = 0 \]

\[ PQ = \frac{|5 + 21 - 21 + 5|}{\sqrt{(5)^2 + (7)^2 + (3)^2}} = \frac{10}{\sqrt{25 + 49 + 9}} = \frac{10}{\sqrt{83}} \]

16. Let \( I_n = \int \tan^n x \, dx, \) (\( n > 1 \)). If \( I_4 + I_6 = \tan^5 x + bx^5 + C, \) where \( C \) is a constant of integration, then the ordered pair \((a, b)\) is equal to:

(1) \( \left( -\frac{1}{5}, 1 \right) \)

(2) \( \left( \frac{1}{5}, 0 \right) \)

(3) \( \left( \frac{1}{5}, -1 \right) \)

(4) \( \left( -\frac{1}{5}, 0 \right) \)

**Solution:** (2)

\[ \int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \, dx \]
\[ = \int \tan^4 x \, dx + \int \frac{\tan^5 x}{5} \, dx - \int \tan^4 x \, dx \]

\[ l_4 + l_6 \Rightarrow a\tan^5 x + bx^5 = \int \frac{\tan^5 x}{5} \]

\[ a = \frac{1}{5}, b = 0 \]

\[ \begin{pmatrix} \frac{1}{5} \\ 0 \end{pmatrix} \]

17. The eccentricity of an ellipse whose center is at the origin is \( \frac{1}{2} \). If one of its directives is \( x = -4 \), then the equation of the normal to it at \( \left(1, \frac{3}{2}\right) \) is:

(1) \( 2y - x = 2 \)

(2) \( 4x - 2y = 1 \)

(3) \( 4x + 2y = 7 \)

(4) \( x + 2y = 4 \)

**Solution:** (2)

\[ e = \frac{1}{2} = \frac{c}{a} \]
\[ \frac{a}{e} = 4 \]
\[ a = 4 \times \frac{1}{2} = 2 \]
\[ c = 1 \]
\[ b = \sqrt{a^2 - c^2} = \sqrt{4 - 1} = \sqrt{3} \]

\[ \frac{x^2}{4} + \frac{y^2}{3} = 1 \]

\[ \frac{2x}{4} + \frac{2y}{3} \frac{dy}{dx} = 0 \]

\[ \frac{dy}{dx} = \frac{-x}{4} \times \frac{3}{y} = \frac{-3x}{4y} \]

\[ \frac{dx}{dy} = \frac{4y}{3x} \]

\[ \therefore \text{ Equation of normal is } \left( y - \frac{3}{2} \right) = \frac{4 \times 3}{2 \times 3 \times 1} (x - 1) \]

\[ y - \frac{3}{2} = 2x - 2 \Rightarrow 2y - 3 = 4x - 4 \]

\[ 4x - 2y = 1 \]

18. A hyperbola passes through the point \( P(\sqrt{2}, \sqrt{3}) \) and has foci at \( (\pm 2, 0) \). Then the tangent to this hyperbola at \( P \) also passes through the point:
(1) \( (3\sqrt{2}, 2\sqrt{3}) \)

(2) \( (2\sqrt{3}, 3\sqrt{2}) \)

(3) \( (\sqrt{3}, \sqrt{2}) \)

(4) \( (-\sqrt{2}, -\sqrt{3}) \)

**Solution:**

\[ ac = \pm 2 \]

\[ c = 2 \]

\[ b^2 = c^2 - a^2 = (4 - a^2) \]

\[ \frac{x^2}{a^2} - \frac{y^2}{4 - a^2} = 1 \]

\[ \frac{2}{a^2} - \frac{3}{4 - a^2} = 1 \]

\[ a^2 = t \]

\[ 2(4 - t) - 3t - t(4 - t) = 0 \]

\[ 8 - 2t - 3t - 4t + t^2 = 0 \]

\[ t^2 - 8t - t + 8 = 0 \]

\[ t = 8, 1 \]

\[ a^2 = 8, 1 \]

\[ a = 2\sqrt{2}, 1 \]

\[ \frac{x^2}{1} - \frac{y^2}{3} = 1 \]

The equation of tangent at \( P \) is \( \sqrt{2}x - \frac{y}{\sqrt{3}} = 1 \)
19. The function \( f : \mathbb{R} \to \left[ -\frac{1}{2}, \frac{1}{2} \right] \) defined as \( f(x) = \frac{x}{1 + x^2} \) is:

(1) Invertible
(2) Injective but not surjective
(3) Surjective but not injective
(4) Neither injective nor surjective

**Solution:** (3)

\[
f(x) = \left( \frac{x}{1 + x^2} \right)
\]

\[
y = \frac{x}{1 + x^2}
\]

\[
y + yx^2 - x = 0
\]

\[
x^2y - x + y = 0
\]

\[
x = \frac{1 \pm \sqrt{1 - 4y^2}}{2y}
\]

\[
1 - 4y^2 \geq 0
\]

\[
4y^2 \leq 1
\]

\[
y \in \left( -\frac{1}{2}, \frac{1}{2} \right]
\]

Co-Domain = Range, surjective in nature

\[
\frac{x_1}{1 + x_1^2} - \frac{x_2}{1 + x_2^2}
\]

\[
x_1 + x_2x_2^2 - x_2 - x_2x_1 = 0
\]

\[
(x_1 - x_2) + x_1x_2(x_2 - x_1) = 0
\]
\((x_2 - x_1)(x_1 x_2 - 1) = 0\)

\(x_2 = x_1\) or \(x_1 = \frac{1}{x_2}\)

Not injective.

20. \(\lim_{{x \to \frac{\pi}{2}}} \frac{\cot x - \cos x}{(\pi - 2x)^3}\) equals:

(1) \(\frac{1}{24}\)

(2) \(\frac{1}{16}\)

(3) \(\frac{1}{8}\)

(4) \(\frac{1}{4}\)

**Solution:** (2)

\[\lim_{{x \to \frac{\pi}{2}}} \frac{\cos x - \cos x}{(\pi - 2x)^3}\]

\[= \lim_{{h \to 0}} \frac{\cot \left(\frac{\pi}{2} - h\right) - \cos \left(\frac{\pi}{2} - h\right)}{(\pi - 2 \times \left(\frac{\pi}{2} - h\right))^3}\]

\[= \lim_{{h \to 0}} \frac{\tan h - \sin h}{\left(\pi - 2 \times \frac{\pi}{2} + 2h\right)^3}\]

\[= \lim_{{h \to 0}} \frac{\tanh h - \sin h}{8h^3}\]
\[
\frac{1}{8} \lim_{h \to 0} \frac{\sin h [\sec h - 1]}{h^{2}} = 1 \cdot \lim_{h \to 0} \frac{\sin h [1 - \cos h]}{\cos h \cdot h^{2}} = \frac{1}{8} \lim_{h \to 0} \frac{2 \sin^{2} \left(\frac{h}{2}\right)}{h^{2}} \cdot 4 = \frac{1}{16}
\]

21. Let \( \vec{a} = 2\hat{i} + \hat{j} - 2\hat{k} \) and \( \vec{b} = \hat{i} + \hat{j} \).

Let \( \vec{c} \) be a vector such that \( |\vec{c} - \vec{a}| = 3 \), \( |(\vec{a} \times \vec{b}) \times \vec{c}| = 3 \) and the angle between \( \vec{c} \) and \( \vec{a} \times \vec{b} \) be 30°. Then \( \vec{a} \cdot \vec{c} \) is equal to:

(1) \( \frac{25}{8} \)

(2) 2

(3) 5

(4) \( \frac{1}{8} \)

**Solution:** (2)

\( \vec{a} = 2\hat{i} + \hat{j} - 2\hat{k} \)

\( |\vec{c} - \vec{a}| = 3 \)

\( |(\vec{a} \times \vec{b}) \times \vec{c}| = 3 \)

\( |\vec{c} - \vec{a}| = 3 \)

\( |\vec{a} \times \vec{b}| \|\vec{c}\| \cos 30° = 3 \)

\( |\vec{a} \times \vec{b}| \|\vec{c}\| = 6 \)

\( c^2 + a^2 - 2\vec{a} \cdot \vec{c} = 9 \)
\[ |c| = \begin{vmatrix} 6 \\ i \\ j \\ k \end{vmatrix} = \begin{vmatrix} 6 \\ 2 \\ 1 \\ 1 \\ 1 \end{vmatrix} = \frac{6}{|i(2) - 9(2) + 9k(1)|} \]

\[ |c| = \frac{6}{3} = 2 \]

\[ \overrightarrow{a} \cdot \overrightarrow{c} = 2 \]

22. The normal to the curve \( y(x - 2) \ (x - 3) = x + 6 \) at the point where the curve intersects the y-axis passes through the point:

(1) \( \left( -\frac{1}{2}, \frac{1}{2} \right) \)

(2) \( \left( -\frac{1}{2}, \frac{1}{2} \right) \)

(3) \( \left( -\frac{1}{2}, -\frac{1}{2} \right) \)

(4) \( \left( \frac{1}{2}, \frac{1}{2} \right) \)

Solution: (2)

For \( y \) axis

Put \( x = 0 \)

\[ +6y = 6, \quad xy = +1 \]

On differentiating

\[ \frac{dy}{dx} \ (x - 2) \ (x - 3) + y(2x - 5) = 1 \]

Put \( x = 0, \ y = +1 \)
\[ \frac{dy}{dx}(x + 6) + 1(-5) = 1 \]

\[ \frac{dy}{dx} = 1 \]

Slope of normal = -1

Equation \[ y - 1 = -1(x - 0) \]

\[ x + y - 1 = 0 \]

Point \[ \left( \frac{1}{4}, 1 \right) \] satisfy the normal.

23. If two different numbers are taken from the set \((0, 1, 2, 3, \ldots, 10)\); then the probability that their sum as well as absolute difference are both multiple of 4, is:

(1) \[ \frac{6}{55} \]

(2) \[ \frac{12}{55} \]

(3) \[ \frac{14}{45} \]

(4) \[ \frac{7}{55} \]

**Solution: (1)**

\((0, 4), (0, 8)\)

\((2, 6), (2, 10)\)

\((4, 8), (6, 10)\)

\[ h(e) = 6 \]

\[ h(s) = \frac{11}{2} = \frac{11 \times 10}{2} = 55 \]
24. A man $X$ has 7 friends, 4 of them are ladies and 3 are men. His wife $Y$ also has 7 friends, 3 of them are ladies and 4 are men. Assume $X$ and $Y$ have no common friends. Then the total number of ways in which $X$ and $Y$ together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of $X$ and $Y$ are in this party is:

(1) 485

(2) 468

(3) 469

(4) 484

Solution: (1)

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Total = \(9 \times 16 + 1 + 9 \times 36 + 16\)

= \(144 + 1 + 324 + 16\)

= 485

25. The value of

\[\left( \binom{21}{1}c_1 - \binom{10}{1}c_1 \right) + \left( \binom{21}{2}c_2 - \binom{10}{2}c_2 \right) + \left( \binom{21}{3}c_3 - \binom{10}{3}c_3 \right) + \left( \binom{21}{4}c_4 - \binom{10}{4}c_4 \right) + \ldots + \left( \binom{21}{10}c_{10} - \binom{10}{10}c_{10} \right)\] is:
(1) $2^{21} - 2^{11}$
(2) $2^{21} - 2^{10}$
(3) $2^{20} - 2^9$
(4) $2^{20} - 2^{10}$

**Solution:**

$$\left( \binom{21}{1} + \binom{21}{2} + \binom{21}{3} + \ldots + \binom{21}{10} \right) - \left( \binom{10}{1} + \binom{10}{2} + \binom{10}{3} + \ldots + \binom{10}{10} \right)$$

$$= \frac{1}{21} \left( 2^{21} - 2^{10} \right) - \left( 2^{10} - 1 \right)$$

$$= \frac{1}{2} \left( 2^{21} - 2 \right) - \left( 2^{10} - 1 \right)$$

$$= \frac{1}{2} \times 2^{10} \times 2^{11} - 1 - 2^{10} + 1 = 2^{11} - 2^{10}$$

26. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is:

(1) $\frac{12}{5}$
(2) 6
(3) 4
(4) $\frac{6}{25}$

**Solution:** (1)

For Bernoulli's trials variance = n.p.q

n = 10
\[
p = \frac{3}{5}, \quad q = \frac{2}{5}
\]

\[
\text{Variance} = 10 \times \frac{3}{5} \times \frac{2}{5} = \frac{12}{5}
\]

27. Let \(a, b, c \in R\). If \(f(x) = ax^2 + bx + c\) is such that \(a + b + c = 3\) and \(f(x + y) = f(x) + f(y) + xy, \forall x, y \in R\), then \(\sum_{n=1}^{10} f(n)\) is equal to:

(1) 330  
(2) 165  
(3) 190  
(4) 255

**Solution:**

\[
f(x + y) = f(x) + f(y) + xy, \quad \forall x, y \in \mathbb{R}
\]

Put \(y = 1\)

\[
f(x + 1) = f(x) + f(1) + x
\]

\[
f(x + 1) = f(x) + 3 + x
\]

\[
f(2) = f(1 + 1) = f(1) + 3 + 1 = 3 + 3 + 1 = 7
\]

\[
f(3) = f(2 + 1) = f(2) + 3 + 2 = 7 + 3 + 2 = 12
\]

\[
f(4) = f(3 + 1) = f(3) + 3 + 3 = 12 + 3 + 3 = 18
\]

\[
\sum_{i=1}^{10} f(i) = 3 + 7 + 12 + 18 + \ldots + T_r
\]

\[
\sum_{i=1}^{10} f(i) = 3 + 7 + 12 + \ldots + T_r
\]

\[
0 = 3 + 4 + 5 + 6 + \ldots + T_r
\]
\[ T_r^2 = 3 + 4 + 5 + 6 \ldots 10 \]
\[ = \frac{r}{2} \left[ 6 + r - 1 \right] = \frac{sr}{2} + \frac{r^2}{2} \]
\[ \sum_{r=1}^{10} f(x)^2 = \sum_{r=1}^{10} r + \frac{1}{r} \sum_{r=1}^{10} r^2 \]
\[ = \frac{5}{2} \times \frac{10(11)}{2} + \frac{1}{2} \times \frac{10(11)}{6} \]
\[ = \frac{275}{2} + \frac{385}{2} = \frac{660}{2} = 330 \]

28. The radius of a circle, having minimum area, which touches the curve \( y = 4 - x^2 \) and the lines, \( y = |x| \) is:

(1) \( 2(\sqrt{2} + 1) \)

(2) \( 2(\sqrt{2} - 1) \)

(3) \( 4(\sqrt{2} - 1) \)

(4) \( 4(\sqrt{2} + 1) \)

**Solution:** (3)

\[ k = \frac{1}{\sqrt{2}} \]

\[ r = |4 - k|, \quad r = 4 - k = \frac{k}{\sqrt{2}} \]

\[ 4 = \frac{k + \sqrt{2}k}{\sqrt{2}}, \quad k = \frac{4\sqrt{2}}{\sqrt{2} + 1} \]

\[ r = \frac{4\sqrt{2}}{(\sqrt{2} + 1)\sqrt{2}} \]
29. If, for a positive integer \( n \), the quadratic equation,

\[ x(x + 1) + (x + 1)(x + 2) + \ldots + (x + n - 1)(x + n) = 10n \]

has two consecutive integral solutions, then \( n \) is equal to:

(1) 12
(2) 9
(3) 10
(4) 11

**Solution:** (4)

One simplifying we get the quadratic equations as

\[ nx^2 + n^2x + \frac{n(n^2 - 1)}{3} = 10n \]

\[ x^2 + nx + \frac{n^2 - 31}{3} = 0 \]

Difference of roots \( \frac{\sqrt{D}}{a} = 1 \)

\[ n^2 - \frac{4}{3}(n^2 - 31) = 1 \]
\( n^2 = 121 \)

\( n = 11 \)

30. The integral \( \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1 + \cos x} \) is equal to:

(1) \(-2\)

(2) 2

(3) 4

(4) \(-1\)

**Solution:**

\[
\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1 + \cos x} = \frac{1}{2} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sec^2 \frac{x}{2} dx
\]

\[
= \left[ \tan \frac{x}{2} \right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}
\]

\[
= \tan \frac{3\pi}{8} - \tan \frac{\pi}{8} = (\sqrt{2} + 1) - (\sqrt{2} - 1) = 2
\]
31. A radioactive nucleus A with a half life $T$, decays into a nucleus B. At $t = 0$, there is no nucleus B. At sometime $t$, the ratio of the number of B to that of A is 0.3. Then, $t$ is given by:

(1) $t = \frac{T}{\log(1.3)}$

(2) $t = \frac{T \log 2}{2\log 1.3}$

(3) $t = T \frac{\log 1.3}{\log 2}$

(4) $t = T \log (1.3)$

**Solution: (3)**

$N = N_0 e^{-\lambda t}$

$\lambda = \frac{\log 2}{T}$

Given that $N_B = 1.3N_A$
32. The following observations were taken for determining surface tension \( T \) of water by capillary method:

- diameter of capillary, \( D = 1.25 \times 10^{-2} \text{m} \)
- rise of water, \( h = 1.45 \times 10^{-2} \text{m} \).

Using \( g = 9.80 \text{ m/s}^2 \) and the simplified relation \( T = \frac{rhg}{2} \times 10^3 \text{N/m} \), the possible error in surface tension is closest to:

(1) 10%
(2) 0.15%
(3) 1.5%
(4) 2.4%

**Solution:** [3]

Percentage error in surface tension \( = \frac{\Delta T}{T} = \frac{\Delta r}{r} + \frac{\Delta h}{h} + \frac{\Delta g}{g} \)

\( \Delta r = 0.01 \)
\( \Delta h = 0.01 \)
\( \Delta g = 0.01 \)

After calculation we get \( \frac{\Delta T}{T} = 1.6\% \approx 1.5\% \)
33. An electron beam is accelerated by a potential difference $V$ to hit a metallic target to produce X-rays. It produces continuous as well as characteristic X-rays. If $\lambda_{\text{min}}$ is the smallest possible wavelength of X-ray in the spectrum, the variation of $\log \lambda_{\text{min}}$ with $V$ is correctly represented in:

(1) \[
\begin{align*}
\log \lambda_{\text{min}} & \quad \rightarrow \quad \log V \\
\end{align*}
\]

(2) \[
\begin{align*}
\log \lambda_{\text{min}} & \quad \rightarrow \quad \log V \\
\end{align*}
\]

(3) \[
\begin{align*}
\log \lambda_{\text{min}} & \quad \rightarrow \quad \log V \\
\end{align*}
\]

(4) \[
\begin{align*}
\log \lambda_{\text{min}} & \quad \rightarrow \quad \log V \\
\end{align*}
\]
Solution: (2)

\[ V = \frac{hc}{\lambda} \]

Taken log both side

\[ \log V = \log hc - \log \lambda \]

\[ y = -ax + c \] straight line of decreasing slope

34. The moment of inertia of a uniform cylinder of length \( l \) and radius \( R \) about its perpendicular bisector is \( I \). What is the ratio \( I/R \) such that the moment of inertia is minimum?

(1) \( \frac{3}{4} \sqrt{c} \)

(2) \( \frac{R}{\sqrt{c}} \)

(3) \( \frac{\sqrt{c}}{2} \)

(4) 1

Solution: (2)

\[ m = \rho (\pi R^2 l) \]

\[ dm = \rho \pi (2R \, dR \, R l + R^2 \, dl) \]

\[ O = \rho \pi (2R \, dR + R^2 \, dl) \]

\[ 2R \, dl = -R^2 \, dl \]
\[ l = \frac{MR^2}{4} + \frac{Ml^2}{12} \]

\[ l = \frac{M}{12}(3R^2 + l^2) \]

\[ dl = \frac{M}{12}(6T(dR) + 2l
dl) = 0 \]

\[ 6R \left( \frac{-R^2 dl}{2Rl} \right) + 2l \; dl = 0 \]

\[ \left( \frac{-3R^2}{l} + 2l \right) dl = 0 \]

\[ -3R^2 + 2l^2 = 0 \]

\[ 2l^2 = 3R^2 \]

\[ \frac{l}{R} = \frac{\sqrt{3}}{\sqrt{2}} \]

35. A slender uniform rod of mass \( M \) and length \( l \) is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle \( \theta \) with the vertical is:
(1) \( \frac{2g}{3} \cos \theta \)

(2) \( \frac{3g}{2l} \sin \theta \)

(3) \( \frac{2g}{3} \sin \theta \)

(4) \( \frac{3g}{2l} \cos \theta \)

Solution: (2)

Moment of about O is \( l \alpha = mg \frac{g}{2} \sin \theta \)

\[
\alpha = \frac{mgl}{2ml^2} \sin \theta = \frac{3}{3} \sin \theta
\]

36. \( C_p \) and \( C_v \) are specific heats at constant pressure and constant volume respectively. It is observed that

\( C_p - C_v = \alpha \) for hydrogen gas

\( C_p - C_v = b \) for nitrogen gas

The correct relation between \( a \) and \( b \) is:

(1) \( a = 2b \)
(2) \( a = \frac{1}{14} b \)

(3) \( a = b \)

(4) \( a = 14b \)

**Solution: (4)**

\[ C_p - C_v = \frac{R}{M} \]

For hydrogen, \( C_p - C_v = \frac{R}{2} = a \)

For nitrogen, \( C_p - C_v = \frac{R}{28} = b \)

\[ \frac{a}{b} = 14 \]

\[ a = 14b \]

37. A copper ball of mass 100 gm is at a temperature \( T \). It is dropped in a copper calorimeter of mass 100 gm, filled with 170 gm of water at room temperature. Subsequently, the temperature of the system is found to be 75\(^\circ\)C. \( T \) is given by:

(Given: room temperature = 30\(^\circ\)C, specific heat of copper = 0.1 cal/gm\(^\circ\)C)

(1) 825\(^\circ\)C

(2) 800\(^\circ\)C

(3) 885\(^\circ\)C

(4) 1250\(^\circ\)C

**Solution: (3)**

\[ m_{cb} = 100 \text{ gm} \quad (T^\circ \text{C}) \]

\[ m_c = 100 \text{ gm} \quad (30^\circ \text{C}) \]

\[ m_w = 170 \text{ gm} \quad (30^\circ \text{C}) \]

Net heat loss = Net heat gain
100 \text{gm} \times 0.1 \times (T - 75) = 100 \times 0.1 \times (75 - 30) + 170 \times 1 \times (75 - 30)

\Rightarrow T - 75 = 45 + 17 \times 45

= T = 885^\circ \text{C}

38. In amplitude modulation, sinusoidal carrier frequency used is denoted by \( \omega_c \) and the signal frequency is denoted by \( \omega_m \). The bandwidth \( (\Delta \omega_m) \) of the signal is such that \( \Delta \omega_m \ll \omega_c \). Which of the following frequencies is not contained in the modulated wave?

(1) \( \omega_c - \omega_m \)

(2) \( \omega_m \)

(3) \( \omega_c \)

(4) \( \omega_m + \omega_c \)

Solution: (2)

\( \omega_m \) will not be there in the frequencies.

39. The temperature of an open room of volume 30 \( m^3 \) increases from \( 17^\circ \text{C} \) to \( 27^\circ \text{C} \) due to the sunshine. The atmospheric pressure in the room remains \( 1 \times 10^5 \) Pa. If \( n_i \) and \( n_f \) are the number of molecules in the room before and after heating, then \( n_f - n_i \) will be:

(1) \(- 2.5 \times 10^{25}\)

(2) \(- 1.61 \times 10^{23}\)

(3) \(1.38 \times 10^{23}\)

(4) \(2.5 \times 10^{25}\)

Solution: (1)

\[
\frac{P V}{R} \left[ \frac{1}{T_f} - \frac{1}{T_i} \right]
\]


\[ = -2.5 \times 10^{-25} \]

40. In a Young's double slit experiment, slits are separated by 0.5 mm, and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide is:

(1) 15.6 mm
(2) 1.56 mm
(3) 7.8 mm
(4) 9.75 mm

**Solution:**

\[
y_1 = \frac{n_1 \lambda_1 D}{d} \quad \text{for bright fringes}
\]

\[
y_2 = \frac{n_2 \lambda_2 D}{d} \quad \text{for bright fringes}
\]

To coincide

\[ n_1 \lambda_1 = n_2 \lambda_2 \]

\[ n_1 \times 650 = n_2 \times 520 \]

\[ \therefore \frac{n_1}{n_2} = \frac{4}{5} \]

For minimum value of \( n_1 \) and \( n_2 \)

\[ n_1 = 4 \quad n_2 = 5 \]

\[ (y_1)_{min} = 4 \times 650 \times nm \times \frac{150 m}{0.5 \times 10^{-3} m} \]

\[ = 7800 \times 10^{-6} m \]
41. A particle A of mass m and initial velocity \( v \) collides with a particle B of mass \( \frac{m}{2} \) which is at rest. The collision is head-on, and elastic. The ratio of the de-Broglie wavelengths \( \frac{\lambda_A}{\lambda_B} \) after the collision is:

1. \( \frac{\lambda_A}{\lambda_B} = \frac{1}{2} \)
2. \( \frac{\lambda_A}{\lambda_B} = \frac{1}{3} \)
3. \( \frac{\lambda_A}{\lambda_B} = 2 \)
4. \( \frac{\lambda_A}{\lambda_B} = \frac{2}{3} \)

**Solution:** (3)

Before collision:

\[ m \quad \text{A} \quad \rightarrow \quad v \quad \text{B} \quad \rightarrow \quad 0 \]

After collision:

\[ \text{A} \quad \rightarrow \quad v_A \quad \text{B} \quad \rightarrow \quad v_B \quad \text{After collision} \]

\[ m \cdot v = m \cdot v_A + \frac{m}{2} \cdot v_B \]

\[ V = v_A + \frac{v_B}{2} \quad \text{......(i)} \]

\[ V = v_B - v_A \quad \text{......(ii)} \]

Solving (i) and (ii)

\[ 2V = \frac{3v_B}{2} \]

\[ \therefore \quad v_B = \frac{4V}{3} \]
\[ V_A = V_B - V = \frac{V}{3} \]

\[ P_A = \frac{mV}{3} \quad P_B = \frac{m \cdot 4V}{2 \cdot 3} \]

\[ \frac{\lambda_A}{\lambda_B} = \frac{h \times P_B}{P_A \times h} = \frac{2mV}{\frac{3}{mV}} = 2 \]

42. A magnetic needle of magnetic moment \(6.7 \times 10^{-2} \text{ Am}^2\) and moment of inertia \(7.5 \times 10^{-6} \text{ kg m}^2\) is performing simple harmonic oscillations in a magnetic field of 0.01T. Time taken for 10 complete oscillations is:

(1) 8.76s
(2) 6.65s
(3) 8.89s
(4) 6.98s

**Solution: (2)**

\[ T = 2\pi \sqrt{\frac{I}{MB}} \]

\[ = 2\pi \sqrt{\frac{7.5 \times 10^{-6}}{6.7 \times 10^{-2} \times 0.01}} \text{ sec} \]

\[ = 0.6647 \text{ sec} \]

\[ t = 10T = 6.647 \text{ sec} = 6.65 \text{ sec} \]
43. An electric dipole has fixed dipole moment \( \vec{p} \), which makes angle \( \theta \) with respect to x-axis. When subjected to an electric field \( \vec{E}_1 = E\hat{i} \), it experiences a torque \( \vec{T}_1 = \tau\hat{k} \). When subjected to another electric field \( \vec{E}_2 = \sqrt{3} E\hat{j} \), it experiences a torque \( \vec{T}_2 = -\vec{T}_1 \). The angle \( \theta \) is:

(1) \( 90^\circ \)
(2) \( 30^\circ \)
(3) \( 45^\circ \)
(4) \( 60^\circ \)

**Solution: (4)**

\[
\vec{p} = P\cos \theta \hat{i} + P\sin \theta \hat{j}
\]

\[
\vec{T}_1 = (P\cos \theta \hat{i} + P\sin \theta \hat{j}) \times (E\hat{i})
\]

\[= -PE\sin \theta \hat{k}\]

\[
\vec{T}_2 = (P\cos \theta \hat{i} + P\sin \theta \hat{j}) \times (\sqrt{3}E\hat{j})
\]

\[= \sqrt{3}PE\cos \theta \hat{k}\]

\[
\vec{T}_2 = -\vec{T}_1
\]

\[
\therefore \sin \theta = \sqrt{3}\cos \theta
\]

\[
\therefore \tan \theta = \sqrt{3}
\]

\[
\therefore \theta = 60^\circ
\]

44. In a coil of resistance 100\( \Omega \), a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is:
45. A time dependent force $F = 6t$ acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 sec. will be:

(1) 18 J
(2) 4.5 J
(3) 22 J
(4) 9 J

Solution: (2)
\[ m = 1 \text{ kg} \]

\[ F = 6t \]

\[ a = \frac{F}{m} = 6t \]

\[ \frac{dv}{dt} = 6t \]

\[ \therefore \int_0^v dv = \int_0^t 6tdt \]

\[ v_0 = 6 \times \frac{2}{2} = 3 \text{ m/sec} \]

\[ \Delta w = k_f - k_i = \frac{1}{2} \times 1 \times 9 - 0 = 4.5f \]

46. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths \( r = \frac{\lambda_1}{\lambda_2} \) is given by:

(1) \( r = \frac{1}{3} \)

(2) \( r = \frac{4}{3} \)

(3) \( r = \frac{2}{3} \)
(4) \( r = \frac{3}{4} \)

**Solution:**

\[
\lambda_1 = \frac{hc}{\Delta E_1} = \frac{hc}{E_0}
\]
\[
\lambda_2 = \frac{hc}{\Delta E_2} = \frac{hc \times 3}{E}
\]

\[
\frac{\lambda_1}{\lambda_2} = \frac{1}{3}
\]

47.

In the above circuit the current in each resistance is:

(1) 0 A
(2) 1 A
(3) 0.25 A
(4) 0.5 A

**Solution:**

(1)
p.d across each resistor is zero. Hence correct is zero in each resistor.

48. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time?

(1)

(2)

(3)
Velocity becomes negative after some time and the negative slope is constant.

49. A capacitance of $2 \mu F$ is required in an electrical circuit across a potential difference of 1.0 kV. A large number of $1 \mu F$ capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is:

(1) 32
50. In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be:

\[ CE \left( \frac{r_1}{r_1 + r} \right) \]

(1) \( CE \left( \frac{r_1}{r_1 + r} \right) \)

(2) \( CE \)

(3) \( CE \left( \frac{r_1}{r_2 + r} \right) \)
(4) \( CE \frac{r_2}{r + r_2} \)

**Solution: (4)**

In steady state current through capacitor will be zero, so current through \( r_1 \) will be zero,

\[
I = \frac{E}{r + r_2}
\]

\[
V_2 = I \times r_2 = \frac{E r_2}{r + r_2}
\]

\[
\therefore q = CV_2 = \frac{CE r_2}{r + r_2}
\]

51. In a common emitter amplifier circuit using an n-p-n transistor, the phase difference between the input and the output voltages will be:

(1) 180°
(2) 45°
(3) 90°
(4) 135°

**Solution: (1)**

Voltage gain of the amplifier is

\[
A_v = \frac{v_o}{v_i} = \frac{\Delta V_{CF}}{r \Delta I_B}
\]

\[
= \frac{-B_{ac} R_l}{K}
\]
-Ve sign indicates phase difference is $180^\circ$.

52. Which of the following statements is false?

(1) Kirchhoff’s second law represents energy conservation

(2) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude

(3) In a balanced Wheatstone bridge if the cell and the galvanometer are exchanged, the null point is disturbed

(4) A rheostat can be used as a potential divider

Solution: (3)

Null point does not change.
53. A particle is executing simple harmonic motion with a time period T. At time $t = 0$, it is at its position of equilibrium. The kinetic energy – time graph of the particle will look like:

(1)

(2)

(3)

(4)
Solution: (1)

T of KE = \( \frac{T}{2} \) of oscillation and at \( t = 0 \) KE is maximum.

The period of KE graph is half of the period of displacement graph.

Comment:- The points on the graph of x-axis should be differentiable.

In all option mentioned, the graph has cusps at the points on x-axes.

54. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer?

(speed of light = \( 3 \times 10^8 \text{ m s}^{-1} \))

(1) 15.3 GHz
(2) 10.1 GHz
(3) 12.1 GHz
(4) 17.3 GHz

Solution: (4)

\[
\frac{c}{f} = \frac{c}{\lambda} \sqrt{\frac{1 - \beta}{1 + \beta}}
\]

\[
\beta = \frac{1}{2}
\]
\[
\beta = \frac{\nu}{c}
\]

\[
\nu = \frac{1}{2}
\]
\[
\frac{1}{f'} = \frac{1}{f} \cdot \frac{1 - \frac{1}{2}}{1 + \frac{1}{2}} = \frac{1}{\sqrt{3}}
\]

\[f' = \sqrt{3}f\]

\[f' = 1.73(10)\]

\[f' = 17.3 \text{ GHz}\]

55. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of:

(1) \(\frac{1}{81}\)

(2) 9

(3) \(\frac{1}{9}\)

(4) 81

**Solution:** [2]

Stress = \(\frac{F}{A} = \frac{Mg}{A}\)

\[= \frac{\rho \text{(Volume)} g}{A}\]

\[= \frac{\rho (9^3 x^3) g}{9^2 x^2}\]

= 9 times

56. When a current of 5mA is passed through a galvanometer having a coil of resistance 15Ω, it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into a voltmeter of range 0 – 10 V is:

(1) \(4.005 \times 10^3 \Omega\)
(2) $1.985 \times 10^3 \Omega$

(3) $2.045 \times 10^3 \Omega$

(4) $2.535 \times 10^3 \Omega$

Solution: (2)

$10 = 5 \times 10^{-3} (x + 15)$

$x = 1.985 \times 10^3 \Omega$

57. The variation of acceleration due to gravity $g$ with distance $d$ from the center of the Earth is best represented by ($R =$ Earth’s radius):

(1)

(2)

(3)

(4)
58. An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by:

(1) \( \frac{3PK}{\alpha} \)

(2) \( \frac{P}{3aK} \)

(3) \( \frac{P}{aK} \)

(4) \( \frac{3a}{PK} \)

Solution: (2)
\[ \Delta V = V \gamma \Delta T, \quad \frac{\Delta V}{V} = \gamma \Delta T \]

\[ \frac{\Delta V}{V} = 3a \Delta T \]

\[ K = \frac{P}{\frac{\Delta V}{V} \frac{P}{3a \Delta T}} \]

\[ \Delta T = \frac{P}{3ak} \]

59. A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is:

(1) Real and at a distance of 6 cm from the convergent lens
(2) Real and at a distance of 40 cm from convergent lens
(3) Virtual and at a distance of 40 cm from convergent lens
(4) Real and at a distance of 40 cm from the divergent lens

**Solution:** (2)

Diverging lens forms the image at its focus. The image formed by diverging lens in the object to converging lens.

\[ \text{Distance of the object from converging lens is 40 cm i.e., object is at double focal point.} \]
A body of mass \(m = 10^{-2} \text{ kg}\) is moving in a medium and experiences a frictional force \(F = -kv^2\). Its initial speed is \(v_0 = 10 \text{ m/s}\). If, after 10s, its energy is \(\frac{1}{8}mv_0^2\), the value of \(k\) will be:

1. \(10^{-1} \text{ kg m}^{-1}\text{s}^{-1}\)
2. \(10^{-3} \text{ kg m}^{-1}\)
3. \(10^{-3} \text{ kg s}^{-1}\)
4. \(10^{-4} \text{ kg m}^{-1}\)

**Solution: (4)**

After 10 sec, \(\frac{1}{2}mv^2 = \frac{1}{8}mv_0^2\)

\[v = \frac{v_0}{2} = 5 \text{ m/sec}\]

\[a = -\frac{kv^2}{m} \Rightarrow \frac{dv}{dt} = -\frac{kv^2}{m}\]

\[= \frac{5}{10} \int_0^{10} \frac{dV}{v^2} = -\frac{k}{m} \int_0^{10} dt\]

\(10^{-4} \text{ kg m}^{-1}\)
CHEMISTRY

61. 1 gram of a carbonate \( \left( M_2CO_3 \right) \) on treatment with excess HCl produces 0.01186 moles of \( CO_2 \). The molar mass of \( M_2CO_3 \) in g mol\(^{-1} \) is:

(1) 84.3
(2) 118.6
(3) 11.86
(4) 1186

Solution: (1)

\[ M_2CO_3 + 2HCl \rightarrow 2MCl + H_2O + CO_2 \]

From reaction,

Moles of \( M_2CO_3 \) = Moles of \( CO_2 \)

\[ \frac{1}{M.wt.} = 0.01186 \]

Molecular Mass (M. wt.) = \( \frac{1}{0.01186} = 84.3 \) g/mol

62. Given:
\[ C_{(\text{graphite})} + O_2(g) \rightarrow CO_2(g); \Delta_f H^0 = -393.5 \text{ kJ mol}^{-1} \]

\[ H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(l); \Delta_f H^0 = -285.8 \text{ kJ mol}^{-1} \]

\[ CO_2(g) + 2H_2O(l) \rightarrow CH_4(g) + 2O_2(g); \Delta_f H^0 = +890.3 \text{ kJ mol}^{-1} \]

Based on the above thermochemical equations, the value of \( \Delta_f H^0 \) at 298 K for the reaction

\[ C_{(\text{graphite})} + 2H_2(g) \rightarrow CH_4(g) \] will be:

1. + 144.0 kJ mol\(^{-1}\)
2. − 74.8 kJ mol\(^{-1}\)
3. − 144.1 kJ mol\(^{-1}\)
4. + 74.8 kJ mol\(^{-1}\)

**Solution:** (2)

For reaction

\[ CO_2(g) + 2H_2O(l) \rightarrow CH_4(g) + 2O_2(g) \]

\[ \Delta_f H^0 = \sum(\Delta_f H^0)_{\text{products}} - \sum(\Delta_f H^0)_{\text{reactants}} \]

\[ = \left[ \Delta_f H^0(CH_4) + 2 \times 0 \right] - \left[ \Delta_f H^0(CO_2) + 2 \Delta_f H^0(H_2O(l)) \right] \]

\[ + 890.3 = \left[ \Delta_f H^0(CH_4) \right] - \left[ -393.5 + 2 \times (-285.8) \right] \]

\[ \Delta_f H^0(CH_4) = -74.8 \text{ kJ mol}^{-1} \]

63. The freezing point of benzene decreases by 0.45°C when 0.2 g of acetic acid is added to 20 g of benzene. If acetic acid associates to form a dimer in benzene, percentage association of acetic acid in benzene will be:
\( (K_f \text{ for benzene} = 5.12 \text{ } K \text{ } kg \text{ } mol^{-1}) \)

(1) 80.4%
(2) 74.6%
(3) 94.6%
(4) 64.6%

Solution: (3)

\[ 2\text{CH}_2\text{COOH} \rightleftharpoons (\text{CH}_2\text{COOH})_2 \]

\[ 1 - \alpha \quad \frac{a}{2i} = 1 - \frac{\alpha}{2} \]

\[ \Delta T_f = i K_f \times m \]

\[ 0.45 = i \times 5.12 \times \frac{0.2 \times 1000}{60 \times 20} \]

\[ i = 0.527 \quad 1 - \frac{\alpha}{2} \Rightarrow \alpha = 0.946 \text{ or } 94.6\% \]

64. The most abundant elements by mass in the body of a healthy human adult are:

Oxygen (61.4%); Carbon (22.9%), Hydrogen (10.0%); and Nitrogen (2.6%). The weight which a 75kg person would gain if all \(^1\text{H}\) atoms are replaced by \(^2\text{H}\) atoms is:

(1) 37.5 kg
(2) 7.5 kg
(3) 10 kg
(4) 15 kg

Solution: (2)

Weight of Hydrogen in body of 75 kg person = \( 75 \times \frac{10}{100} = 7.5 kg \)
On replacing $^1H$ by $^2H$, the new weight of $H = 7.5 \times 2 = 15 kg$

Weight increase = 7.5 kg

65. $\Delta U$ is equal to:
   (1) Isobaric work
   (2) Adiabatic work
   (3) Isothermal work
   (4) Isochoric work

   Solution: (2)

   $\Delta U = q + w$

   For an adiabatic process,

   $q = 0$, $\Delta U = W$

66. The formation of which of the following polymers involves hydrolysis reaction?
   (1) Bakelite
   (2) Nylon 6,6
   (3) Terylene
   (4) Nylon 6

   Solution: (4)

   Nylon – 6 formed from caprolactam first step of which involve ring opening by hydrolysis (base catalyzed).

67. Given
$E_{\text{Cu}^2+/\text{Cu}^-} = 1.36\, V$, $E_{\text{Cr}^{3+}/\text{Cr}^{2+}} = -0.74\, V$ 

$E_{\text{Cr}^{2+}/\text{Cr}^{3+}} = 1.33\, V$, $E_{\text{Mn}^{2+}/\text{Mn}^{3+}} = 1.51\, V$.

Among the following, the strongest reducing agent is:

(1) $\text{Mn}^{2+}$
(2) $\text{Cr}^{3+}$
(3) $\text{Cl}^-$
(4) $\text{Cr}$

**Solution:** (4)

Lower the reduction potential, stronger the reducing agent.

68. The Tyndall effect is observed only when following conditions are satisfied:

(i) The diameter of the dispersed particles is much smaller than the wavelength of the light used
(ii) The diameter of the dispersed particle is not much smaller than the wavelength of the light used
(iii) The refractive indices of the dispersed phase and dispersion medium are almost similar in magnitude
(iv) The refractive indices of the dispersed phase and dispersion medium differ greatly in magnitude

(1) (ii) and (iv)
(2) (i) and (iii)
(3) (ii) and (iii)
(4) (i) and (iv)

**Solution:** (1)

(ii) and (iv)

The Tyndall effect is observed only when:
(ii) The diameter of the dispersed particle is not much smaller than the wavelength of the light used

(iv) The refractive indices of the dispersed phase and dispersion medium differ greatly in magnitude

69. In the following reactions, ZnO is respectively acting as a/an:

(i) \( ZnO + Na_2O \rightarrow Na_2ZnO_2 \)

(ii) \( ZnO + CO_2 \rightarrow ZnCO_3 \)

(1) base and base

(2) acid and acid

(3) acid and base

(4) base and acid

**Solution: (3)**

ZnO is amphoteric, with \( Na_2O \) it is acting as acid and with \( CO_2 \) as base.

70. Which of the following compounds will behave as reducing sugar in an aqueous KOH solution?

(1)

![Diagram](image1)

(2)

![Diagram](image2)
71. The major product obtained in the following reactions is:

\[ \text{C}_6\text{H}_5\text{CH} = \text{CHC}_6\text{H}_5 \]
(2) \( (+) \text{C}_6\text{H}_5\text{CH}(\text{O}^+\text{Bu})\text{CH}_2\text{C}_6\text{H}_5 \)

(3) \( (-) \text{C}_6\text{H}_5\text{CH}(\text{O}^+\text{Bu})\text{CH}_2\text{C}_6\text{H}_5 \)

(4) \( \text{C}_6\text{H}_5\text{CH}(\text{O}^+\text{Bu})\text{CH}_2\text{C}_6\text{H}_5 \)

**Solution:** (1)

Bulky base favors elimination reaction.

72. Which of the following species is not paramagnetic?

(1) \( \text{CO} \)

(2) \( \text{O}_2 \)

(3) \( \text{B}_2 \)

(4) \( \text{NO} \)

**Solution:** (4)

\( \text{CO} \) has no unpaired electron while other have (Apply M.O.T)

73. On treatment of 100mL of 0.1 M solution of \( \text{CoCl}_3\cdot6\text{H}_2\text{O} \) with excess \( \text{AgNO}_3; 1.2 \times 10^{-22} \) ions are precipitated. The complex is:

(1) \( [\text{Co}(\text{H}_2\text{O})_3\text{Cl}]_3\text{H}_2\text{O} \)

(2) \( [\text{Co}(\text{H}_2\text{O})_6\text{Cl}]_3\text{H}_2\text{O} \)

(3) \( [\text{Co}(\text{H}_2\text{O})_5\text{Cl}]\text{Cl}_2\cdot\text{H}_2\text{O} \)

(4) \( [\text{Co}(\text{H}_2\text{O})_4\text{Cl}]_2\text{Cl}\cdot2\text{H}_2\text{O} \)

**Solution:** (3)
No. of ionisable $Cl^-$ = Moles of AgCl

\[
\frac{\text{Moles of AgCl}}{\text{Moles of compounds}} = \frac{1.2 \times 10^{22}}{6.02 \times 10^{23} \times 0.1 \times 100 \times 10^{-3}}
\]

= 2

So, complex is $[Co(H_2O)_3Cl][Cl_2H_2O]$

74. $pK_a$ of a weak acid (HA) and $pK_b$ of a weak base (BOH) are 3.2 and 3.4, respectively. The pH of their salt (AB) solution is:

(1) 6.9
(2) 7.0
(3) 1.0
(4) 7.2

Solution: (1)

For salt of weak acid and weak base

\[
pH = \frac{1}{2}(pK_w + pK_a - pK_b)
\]

= $\frac{1}{2}(14 + 3.2 - 3.4)$

= 6.9

75. The increasing order of the reactivity of the following halides for the $S_n1$ reaction is:
(1) (II) < (I) < (III)
(2) (I) < (III) < (II)
(3) (II) < (III) < (I)
(4) (III) < (II) < (I)

**Solution:** (3)

Rate of $S_N 1$ reaction is directly dependent on stability of carbocation intermediate

$I \rightarrow 2^\circ$ carbocation

$II \rightarrow 1^\circ$ carbocation

$III \rightarrow$ Benzylic carbocation further stabilized by +M effect of $OCH_3$ group.

76. Both lithium and magnesium display several similar properties due to the diagonal relationship; however, the one which is incorrect, is:

(1) Both form soluble bicarbonates

(2) Both form nitrides

(3) Nitrates of both Li and Mg yield $NO_2$ and $O_2$ on heating

(4) Both form basic carbonates

**Solution:** (4)

Both form basic carbonates
77. The correct sequence of reagents for the following conversion will be:

(1) $\text{CH}_3\text{MgBr}, H^+ / \text{CH}_3\text{OH}, [\text{Ag} (\text{NH}_3)_2]^+ \text{OH}^-$

(2) $\text{CH}_3\text{MgBr}, [\text{Ag} (\text{NH}_3)_2]^+ \text{OH}^- H^+ / \text{CH}_3\text{OH}$

(3) $[\text{Ag} (\text{NH}_3)_2]^+ \text{OH}^- , \text{CH}_3\text{MgBr}, H^+ / \text{CH}_3\text{OH}$

(4) $[\text{Ag} (\text{NH}_3)_2]^+ \text{OH}^- , H^+ / \text{CH}_3\text{OH}, \text{CH}_3\text{MgBr}$

Solution: (4)
78. The products obtained when chlorine gas reacts with cold and dilute aqueous NaOH are:

(1) $ClO_2^-$ and $ClO_3^-$
(2) $Cl^-$ and $ClO^-$
(3) $Cl^-$ and $ClO_2^-$
(4) $ClO^-$ and $ClO_3^-$

**Solution:** (2)

$Cl_2$ undergoes disproportionation

$Cl_2 + 2 NaOH \rightarrow NaCl + NaOCl + H_2O$

79. Which of the following compounds will form significant amount of meta product during mono-nitration reaction?

(1)
Solution: (2)

Strong acidic medium changes orientation of $NH_2$ group by converting it to $NH_3^+$ group.
80. Methyl-pent-2-ene on reaction with HBr in presence of peroxide forms an addition product. The number of possible stereoisomers for the product is:

(1) Zero  (2) Two  (3) Four  (4) Six

Solution: [3]

\[
\begin{align*}
\text{CH}_3\text{CH} = \text{C} &\quad \text{CH}_2\text{CH}_3 \\
\text{CH}_3 &\quad \text{HBr} \\
\text{Peroxide} &\quad \text{Br}
\end{align*}
\]

2 chiral centres

No. of stereoisomers \( = 2^2 = 4 \)

81. Two reactions \( R_1 \) and \( R_2 \) have identical pre-exponential factors. Activation energy of \( R_1 \) exceeds that of \( R_2 \) by 10 kJ mol\(^{-1}\). If \( k_1 \) and \( k_2 \) are rate constants for reactions \( R_1 \) and \( R_2 \) respectively at 300 K, then \( \ln \left( \frac{k_2}{k_1} \right) \) is equal to:

\( R = 8.314 \text{ J mol}^{-1}\text{K}^{-1} \)

(1) 12  (2) 6  (3) 4  (4) 8

Solution: [3]

From Arrhenius equation

\[
k = Ae^{-\frac{E_a}{RT}}
\]

\[
k_1 = A_1e^{-\frac{E_{a1}}{RT}}
\]

\[
k_2 = A_2e^{-\frac{E_{a2}}{RT}}
\]
\[
\frac{k_2}{k_1} = e^{\frac{(E_{a_1} - E_{a_2})RT}{RT}}
\]

\[
\ln \frac{k_2}{k_1} = \frac{E_{a_1} - E_{a_2}}{RT} = \frac{10 \times 1000}{8.314 \times 300} = 4
\]

82. Which of the following molecules is least resonance stabilized?

(1)

(2)

(3)

(4)

Solution: (3)

Entire ring does not involve resonance.

83. The group having isoelectronic species is:
(1) $O^-, F^-, Na, Mg^{++}$  
(2) $O^{2-}, F^-, Na, Mg^{2+}$

(3) $O^-, F^-, Na^+, Mg^{2+}$  
(4) $O^{2-}, F^-, Na^+, Mg^{2+}$

Solution: (4)

All have 10 electrons

84. The radius of the second Bohr orbit for hydrogen atom is:

(Planck's Constant, $h = 6.6262 \times 10^{-34}$ Js; mass of electron = $9.1091 \times 10^{-31}$ kg; charge of electron, $e = 1.6021 \times 10^{-19}$ C; permittivity of vacuum, $\varepsilon_0 = 8.854185 \times 10^{-12}$ kg$^{-1}$ m$^{-3}$ A$^{-2}$)

(1) 4.76Å  
(2) 0.529Å  
(3) 2.12Å  
(4) 1.65Å

Solution: (3)

$$r_n = \frac{n^2 \lambda}{2}$$

$$= 0.529 \times \frac{(2)^2}{1} \text{Å}$$

$$= 2.12\text{Å}$$

85. The major product obtained in the following reaction is:

[Diagram of a chemical reaction]

(1)
DIBAL-H (Diisobutyl Aluminium Hydride) reduces carboxylic acids and derivatives to aldehydes.

86. Which of the following reactions is an example of a redox reaction?

(1) \( XeF_2 + PF_5 \rightarrow [XeF]^+ PF_6^- \)

(2) \( XeF_6 + H_2O \rightarrow XeOF_4 + 2HF \)

(3) \( XeF_6 + 2H_2O \rightarrow XeO_2F_2 + 4HF \)
(4) \( XeF_4 + O_2F_2 \rightarrow XeF_6 + O_2 \)

Solution: (4)

\( XeF_4 \) is oxidized while \( O_2F_2 \) is reduced

87. A metal crystallizes in a face centered cubic structure. If the edge length of its unit cell is \( 'a' \), the closest approach between two atoms in metallic crystal will be:

(1) \( 2\sqrt{2} a \)

(2) \( \sqrt{2} a \)

(3) \( \frac{a}{\sqrt{2}} \)

(4) \( 2a \)

Solution: (3)

\( \sqrt{2} a = 4r \)

Closest distance, \( 2r = \frac{\sqrt{2} a}{2} = \frac{a}{\sqrt{2}} \)

88. Sodium salt of an organic acid \( 'X' \) produces effervescence with conc. \( H_2SO_4 \) \( 'X' \) reats with the acidified aqueous \( CaCl_2 \) solution to give a white precipitate which decolourises acidic solution of \( KMnO_4 \) \( 'X' \) is:

(1) \( HCOONa \)

(2) \( CH_3COONa \)

(3) \( Na_2C_2O_4 \)

(4) \( C_6H_5COONa \)

Solution: (3)

\( Na_2C_2O_4 + H_2SO_4 \rightarrow Na_2SO_4 + CO + CO_2 + H_2O \)
\[ \text{Na}_2\text{C}_2\text{O}_4 + \text{CaCl}_2 \rightarrow \text{CaC}_2\text{O}_4 \downarrow + 2\text{NaCl} \]

\(\text{white ppt} \downarrow \text{KMnO}_4\)

\(\text{decolourised}\)

89. A water sample has ppm level concentration of following anions

\(F^- = 10; SO_4^{2-} = 100; NO_3^- = 50\)

The anion/anions that make/makes the water sample unsuitable for drinking is/are:

(1) Both \(SO_4^{2-}\) and \(NO_3^-\)

(2) Only \(F^-\)

(3) Only \(SO_4^{2-}\)

(4) Only \(NO_3^-\)

**Solution:** (2)

\(F^- = 1 \text{ ppm}\)

\(SO_4^{2-} < 500 \text{ ppm}\)

\(NO_3^- = 50 \text{ ppm}\)

90. Which of the following, upon treatment with tert-BuONa followed by addition of bromine water, fails to decolourize the colour of bromine?

(1) 

![Chemical structure](image)
Compounds given in options (1), (2) and (3) first undergo dehydrohalogenation to give alkene which decolourise $Br_2$ water while compound (4) has no $\beta$ - $H$ to undergo elimination.