SUMMATIVE ASSESSMENT – II

MATHEMATICS

Time allowed : 3 hours

Maximum Marks : 90
General Instructions:

(i) All questions are compulsory.

(ii) The question paper consists of 31 questions divided into four sections — A, B, C and D.

(iii) Section A contains 4 questions of 1 mark each. Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 11 questions of 4 marks each.

(iv) Use of calculators is not permitted.

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SECTION A

प्रश्न संख्या 1 से 4 तक प्रत्येक प्रश्न 1 अंक का है।

Question numbers 1 to 4 carry 1 mark each.

1. एक समांतर श्रेढ़ी, जिसमें $a_{21} - a_7 = 84$ है, का सार्व अंतर क्या है?

What is the common difference of an A.P. in which $a_{21} - a_7 = 84$?
2. If the angle between two tangents drawn from an external point P to a circle of radius a and centre O, is 60°, then find the length of OP.

3. If a tower 30 m high, casts a shadow $10\sqrt{3}$ m long on the ground, then what is the angle of elevation of the sun?

4. The probability of selecting a rotten apple randomly from a heap of 900 apples is 0.18. What is the number of rotten apples in the heap?

Khanda B

SECTION B

Question numbers 5 to 10 carry 2 marks each.

5. Find the value of p, for which one root of the quadratic equation $px^2 - 14x + 8 = 0$ is 6 times the other.

6. Which term of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, ...$ is the first negative term?
7. Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord.

8. A circle touches all the four sides of a quadrilateral ABCD. Prove that

\[ AB + CD = BC + DA \]

9. A line intersects the y-axis and x-axis at the points P and Q respectively. If (2, -5) is the mid-point of PQ, then find the coordinates of P and Q.

10. If the distances of P(x, y) from A(5, 1) and B(-1, 5) are equal, then prove that

\[ 3x = 2y \]

SECTION C

Question numbers 11 to 20 carry 3 marks each.

11. If \( ad \neq bc \), then prove that the equation

\[ (a^2 + b^2) x^2 + 2 (ac + bd) x + (c^2 + d^2) = 0 \]

has no real roots.

12. The first term of an A.P. is 5, the last term is 45 and the sum of all its terms is 400. Find the number of terms and the common difference of the A.P.
13. On a straight line passing through the foot of a tower, two points C and D are at distances of 4 m and 16 m from the foot respectively. If the angles of elevation from C and D of the top of the tower are complementary, then find the height of the tower.

14. A bag contains 15 white and some black balls. If the probability of drawing a black ball from the bag is thrice that of drawing a white ball, find the number of black balls in the bag.

15. In what ratio does the point \( \left( \frac{24}{11}, y \right) \) divide the line segment joining the points P(2, -2) and Q(3, 7)? Also find the value of y.

16. In the given figure, each of the three circles has a radius of 3 cm. One circle is a unit distance away from the vertex of a right-angled triangle, and the other two circles are also at an equal distance from the vertex. Find the area of the shaded portion.
Three semicircles each of diameter 3 cm, a circle of diameter 4·5 cm and a semicircle of radius 4·5 cm are drawn in the given figure. Find the area of the shaded region.

In the given figure, two concentric circles with centre O have radii 21 cm and 42 cm. If $\angle AOB = 60^\circ$, find the area of the shaded region.

[ Use $\pi = \frac{22}{7}$ ]
18. 5·4 मी. चौड़ा और 1·8 मी. गहरी एक नहर में पानी 25 किमी/घण्टा की गति से बह रहा है । इससे 40 मिनट में कितने क्षेत्रफल की सिंचाई हो सकती है, यदि सिंचाई के लिए 10 सेमी गहरे पानी की आवश्यकता है?

Water in a canal, 5·4 m wide and 1·8 m deep, is flowing with a speed of 25 km/hour. How much area can it irrigate in 40 minutes, if 10 cm of standing water is required for irrigation?

19. एक शंकु के छिटक की तिर्यक ऊँचाई 4 सेमी है तथा इसके वृत्तीय सिरों के परिमाप 18 सेमी और 6 सेमी हैं । इस छिटक का बनक पृष्ठीय क्षेत्रफल ज्ञात कीजिए ।

The slant height of a frustum of a cone is 4 cm and the perimeters of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

20. एक टोस लोहे के घनाभ की विमाएँ 4·4 मी. × 2·6 मी. × 1·0 मी. हैं । इसे पिघलाकर 30 सेमी आंतरिक त्रिज्या और 5 सेमी मोटाई का एक खोखला बेलनाकार पाइप बनाया गया है । पाइप की लंबाई ज्ञात कीजिए ।

The dimensions of a solid iron cuboid are 4·4 m × 2·6 m × 1·0 m. It is melted and recast into a hollow cylindrical pipe of 30 cm inner radius and thickness 5 cm. Find the length of the pipe.

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SECTION D

प्रश्न संख्या 21 से 31 तक प्रत्येक प्रश्न के 4 अंक हैं ।

Question numbers 21 to 31 carry 4 marks each.

21. x के लिए हल कीजिए :

\[
\frac{1}{x+1} + \frac{3}{5x+1} = \frac{5}{x+4}, \quad x \neq -1, \quad -\frac{1}{5}, \quad -4
\]

Solve for x :\n
\[
\frac{1}{x+1} + \frac{3}{5x+1} = \frac{5}{x+4}, \quad x \neq -1, \quad -\frac{1}{5}, \quad -4
\]
22. Two taps running together can fill a tank in $3\frac{1}{13}$ hours. If one tap takes 3 hours more than the other to fill the tank, then how much time will each tap take to fill the tank?

23. If the ratio of the sum of the first n terms of two A.Ps is $(7n + 1) : (4n + 27)$, then find the ratio of their 9$^{th}$ terms.

24. Prove that the lengths of two tangents drawn from an external point to a circle are equal.

25. In the given figure, $XY$ and $X'Y'$, $O$ being a circle, two tangents at $A$ and $B$, the other two tangents at $X'$ and $Y'$ respectively. Prove that $\angle AOB = 90^\circ$.
In the given figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C, is intersecting XY at A and X'Y' at B. Prove that \( \angle AOB = 90^\circ \).

26. एक त्रिभुज ABC की रचना कीजिए जिसमें भुजा BC = 7 सेमी, \( \angle B = 45^\circ \), \( \angle A = 105^\circ \) हो। तब एक अन्य त्रिभुज की रचना कीजिए जिसकी भुजाएँ \( \Delta ABC \) की संगत भुजाओं की \( \frac{3}{4} \) गुनी हों।

Construct a triangle ABC with side BC = 7 cm, \( \angle B = 45^\circ \), \( \angle A = 105^\circ \). Then construct another triangle whose sides are \( \frac{3}{4} \) times the corresponding sides of the \( \Delta ABC \).

27. एक हवाई जहाज़ भूतल से ऊपर 300 मी. की ऊंचाई पर उड़ रहा है। इस ऊंचाई पर उड़ते हुए हवाई जहाज़ से एक नदी के दोनों किनारों पर फर्स्ट विस्तृत दिशाओं में स्थित दो बिंदुओं के अवनमन कोण क्रमशः 45° तथा 60° हैं। नदी की चौड़ाई ज्ञात कीजिए।

\[ \sqrt{3} = 1.732 \] प्रयोग कीजिए।

An aeroplane is flying at a height of 300 m above the ground. Flying at this height, the angles of depression from the aeroplane of two points on both banks of a river in opposite directions are 45° and 60° respectively. Find the width of the river. [Use \( \sqrt{3} = 1.732 \)]

28. यदि बिंदु \( A(k + 1, 2k) \), \( B(3k, 2k + 3) \) तथा \( C(5k - 1, 5k) \) संरेख हों, तो k का मान ज्ञात कीजिए।

If the points \( A(k + 1, 2k) \), \( B(3k, 2k + 3) \) and \( C(5k - 1, 5k) \) are collinear, then find the value of k.
29. Two different dice are thrown together. Find the probability that the numbers obtained have
(i) even sum, and
(ii) even product.

30. In the given figure, ABCD is a rectangle of dimensions 21 cm $\times$ 14 cm. A semicircle is drawn with BC as diameter. Find the area and the perimeter of the shaded region in the figure.
In a rain-water harvesting system, the rain-water from a roof of 22 m × 20 m drains into a cylindrical tank having diameter of base 2 m and height 3·5 m. If the tank is full, find the rainfall in cm. Write your views on water conservation.