

## SET A

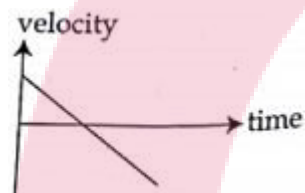
### PART A - PHYSICS

1. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is:

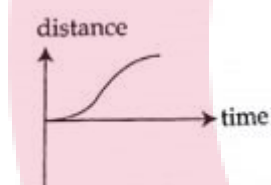
- (A) 2.5%
- (B) 3.5%
- (C) 4.5%
- (D) 6%

2. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.

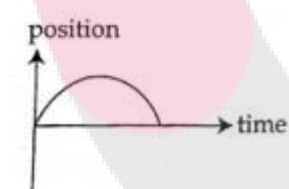
(A)



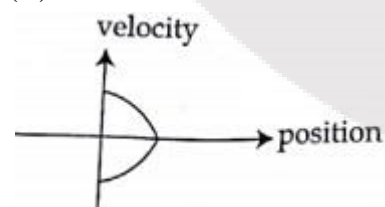
(B)



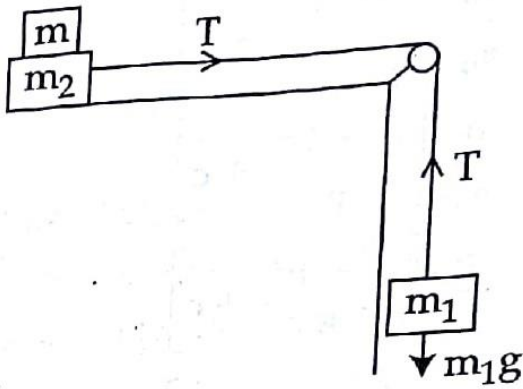
(C)



(D)



3. Two masses  $m_1 = 5 \text{ kg}$  and  $m_2 = 10 \text{ kg}$ , connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight  $m$  that should be put on top of  $m_2$  to stop the motion is:



- (A) 18.3 kg
- (B) 27.3 k
- (C) 43.3 kg
- (D) 10.3 kg

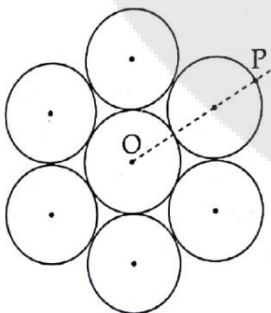
4. A particle is moving in a circular path of radius  $a$  under the action of an attractive potential  $U = -\frac{k}{2r^2}$ . Its total energy is:

- (A)  $-\frac{k}{4a^2}$
- (B)  $\frac{k}{2a^2}$
- (C) Zero
- (D)  $-\frac{3k}{2a^2}$

5. In a collinear collision, a particle with an initial speed  $v_0$  strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is:

- (A)  $\frac{v_0}{4}$
- (B)  $\sqrt{2} v_0$
- (C)  $\frac{v_0}{2}$
- (D)  $\frac{v_0}{\sqrt{2}}$

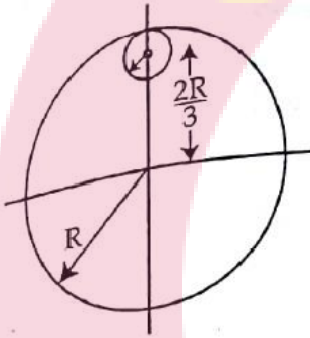
6. Seven identical circular planar disks, each of mass  $M$  and radius  $R$  are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is:



- (A)  $\frac{19}{2} MR^2$

- (B)  $\frac{55}{2}MR^2$
- (C)  $\frac{73}{2}MR^2$
- (D)  $\frac{181}{2}MR^2$

7. From a uniform circular disc of radius  $R$  and mass  $9M$ , a small disc of radius  $\frac{R}{3}$  is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is:



- (A)  $4MR^2$
- (B)  $\frac{40}{9}MR^2$
- (C)  $10MR^2$
- (D)  $\frac{37}{9}MR^2$

8. A particle is moving with a uniform speed in a circular orbit of radius  $R$  in a central force inversely proportional to the  $n^{th}$  power of  $R$ . If the period of rotation of the particle is  $T$ , then:

- (A)  $T \propto R^{3/2}$  for any  $n$
- (B)  $T \propto R^{\frac{n}{2}+1}$
- (C)  $T \propto R^{\frac{n+1}{2}}$
- (D)  $T \propto R^{n/2}$

9. A solid sphere of radius  $r$  made of a soft material of bulk modulus  $K$  is surrounded by a liquid in a cylindrical container. A massless piston of area  $a$  floats on the surface of the liquid, covering entire cross section of

cylindrical container. When a mass  $m$  is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere,  $\left(\frac{dr}{r}\right)$ , is:

- (A)  $\frac{Ka}{mg}$
- (B)  $\frac{Ka}{3mg}$
- (C)  $\frac{mg}{3kA}$
- (D)  $\frac{mg}{Ka}$

10. Two moles of an ideal monoatomic gas occupies a volume  $V$  at  $27^\circ C$ . The gas expands adiabatically to a volume  $2V$ . Calculate (a) the final temperature of the gas and (b) change in its internal energy.

- (A) (a)  $189 K$  (b)  $2.7 kJ$
- (B) (a)  $195 K$  (b)  $-2.7 kJ$
- (C) (a)  $189 K$  (b)  $-2.7 kJ$
- (D) (a)  $195 K$  (b)  $2.7 kJ$

11. The mass of a hydrogen molecule is  $3.32 \times 10^{-27} kg$ . If  $10^{23}$  hydrogen molecules strike, per second, a fixed wall of area  $2 cm^2$  at an angle of  $45^\circ$  to the normal, and rebound elastically with a speed of  $10^3 m/s$ , then the pressure on the wall is nearly:

- (A)  $2.35 \times 10^3 N/m^2$
- (B)  $4.70 \times 10^3 N/m^2$
- (C)  $2.35 \times 10^2 N/m^2$
- (D)  $4.70 \times 10^2 N/m^2$

12. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of  $10^{12}/sec$ . What is the force constant of the bonds connecting one atom with the other? (Mole wt. of silver = 108 and Avagadro number =  $6.02 \times 10^{23} gm\ mole^{-1}$ )

- (A)  $6.4 N/m$
- (B)  $7.1 N/m$
- (C)  $2.2 N/m$
- (D)  $5.5 N/m$

13. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is  $2.7 \times 10^3 kg/m^3$  and its Young's modulus is  $9.27 \times 10^{10} Pa$ . What will be the fundamental frequency of the longitudinal vibrations?

- (A) 5 kHz
- (B) 2.5 kHz
- (C) 10 kHz
- (D) 7.5 kHz

14. Three concentric metal shells A, B and C of respective radii  $a$ ,  $b$  and  $c$  ( $a < b < c$ ) have surface charge densities  $+\sigma$ ,  $-\sigma$  and  $+\sigma$  respectively. The potential of shell B is:

- (A)  $\frac{\sigma}{\epsilon_0} \left[ \frac{a^2 - b^2}{a} + c \right]$
- (B)  $\frac{\sigma}{\epsilon_0} \left[ \frac{a^2 - b^2}{b} + c \right]$

- (C)  $\frac{\sigma}{\epsilon_0} \left[ \frac{b^2 - c^2}{b} + a \right]$   
 (D)  $\frac{\sigma}{\epsilon_0} \left[ \frac{b^2 - c^2}{c} + a \right]$

15. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20V. If a dielectric material of dielectric constant  $K = \frac{5}{3}$  is inserted between the plates, the magnitude of the induced charge will be:

- (A) 1.2 nC  
 (B) 0.3 nC  
 (C) 2.4 nC  
 (D) 0.9 nC

16. In an a.c circuit, the instantaneous e.m.f. and current are given by

$$e = 100 \sin 30t$$

$$r = 20 \sin \left( 30t - \frac{\pi}{4} \right)$$

In one cycle of a.c., the average power consumed by the circuit and the wattless current are, respectively:

- (A) 50, 10  
 (B)  $\frac{1000}{\sqrt{2}}$ , 10  
 (C)  $\frac{50}{\sqrt{2}}$ , 10  
 (D) 50, 0

17. Two batteries with e.m.f. 12 V and 13 V are connected in parallel across a load resistor of  $10\Omega$ . The internal resistances of two batteries are  $1\Omega$  and  $2\Omega$  respectively. The voltage across the load lies between:

- (A) 11.6 V and 11.7 V  
 (B) 11.5 V and 11.6 V  
 (C) 11.4 and 11.5 V  
 (D) 11.7 V and 11.8 V

18. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii  $r_e, r_p, r_\alpha$  respectively in a uniform magnetic field B. The relation between  $r_e, r_p, r_\alpha$  is:

- (A)  $r_e > r_p = r_\alpha$                       (B)  $r_e < r_p = r_\alpha$   
 (C)  $r_e < r_p < r_\alpha$                       (D)  $r_e < r_\alpha < r_p$

19. The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is  $B_1$ . When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is  $B_2$ . The ration  $\frac{B_1}{B_2}$  is:

- (A) 2  
 (B)  $\sqrt{3}$   
 (C)  $\sqrt{2}$   
 (D)  $\frac{1}{\sqrt{2}}$

20. For an RLC circuit driven with voltage of amplitude  $v_m$  and frequency  $\omega_0 = \frac{1}{\sqrt{LC}}$  the current exhibits resonance. The quality factor, Q is given by

- (A)  $\frac{\omega_0 L}{R}$
- (B)  $\frac{\omega_0 R}{L}$
- (C)  $\frac{R}{(\omega_0 C)}$
- (D)  $\frac{CR}{\omega_0}$

21. An EM wave from air enters a medium. The electric fields are  $\vec{E}_1 = E_{01} \hat{x} \cos \left[ 2\pi v \left( \frac{z}{c} - t \right) \right]$  in air and  $\vec{E}_2 = E_{02} \hat{x} \cos [k(2z - ct)]$  in medium, where the wave number k and frequency v refer to their values in air. The medium is non-magnetic. If  $\epsilon_{r_1}$  and  $\epsilon_{r_2}$  refer to relative permittivities of air and medium respectively, which of the following, option is correct?

- (A)  $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = 4$
- (B)  $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = 2$
- (C)  $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{4}$
- (D)  $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{2}$

22. Unpolarized light of intensity I passes through an ideal polarizer A. Another identical polarizer B is placed behind A. The intensity of light beyond B is found to be  $\frac{1}{2}$ . Now another identical polarizer C is placed between A and B. The intensity beyond B is now found to be  $\frac{1}{8}$ . The angle between polarizer A and C is

- (A)  $0^\circ$
- (B)  $30^\circ$
- (C)  $45^\circ$
- (D)  $60^\circ$

23. The angular width of the central maximum in a single slit diffraction pattern is  $60^\circ$ . The width of the slit is  $1\mu\text{m}$ . The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it,

Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance?

(i.e. distance between the centres of each slit.)

- (A)  $25 \mu m$
- (B)  $50 \mu m$
- (C)  $75 \mu m$
- (D)  $100 \mu m$

24. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let  $\lambda_n, \lambda_g$  be the de Broglie wavelength of the electron in the  $n^{th}$  state and the ground state respectively. Let  $\Lambda_n$  be the wavelength of the emitted photon in the transition from the  $n^{th}$  state to the ground state. For large n, (A, B are constants)

- (A)  $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$
- (B)  $\Lambda_n \approx A + B\lambda_n$
- (C)  $\Lambda_n^2 \approx A + B\lambda_n^2$
- (D)  $\Lambda_n^2 \approx \lambda$

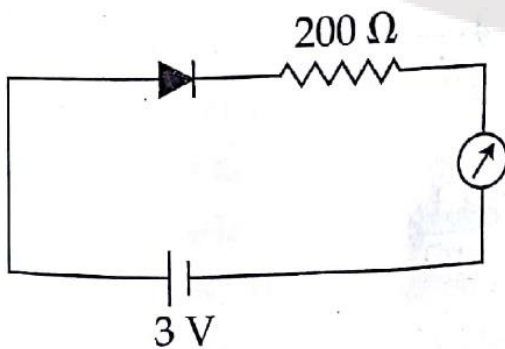
25. If the series limits frequency of the Lyman series is  $\nu_L'$  then the series limits frequency of the P fund series is:

- (A)  $25\nu_L$
- (B)  $16\nu_L$
- (C)  $\frac{\nu_L}{16}$
- (D)  $\frac{\nu_L}{25}$

26. It is found that if a neutron suffers an elastic collision with deuterium at rest, fractional loss of its energy is  $P_d$  ; while for its similar collision with carbon nucleus at rest, fractional loss of energy is  $P_c$ . The values of  $P_d$  and  $P_c$  are respectively.

- (A) (.89, .28)
- (B) (.28, .89)
- (C) (0 0)
- (D) (0, 1)

27. The reading of the ammeter for a silicon diode in the given circuit is:



- (A) 0
- (B)  $15 mA$
- (C)  $11.5 mA$

(D) 13.5 mA

28. A telephonic communication service is working at carrier frequency of 10 GHz. Only 10% of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz?

- (A)  $2 \times 10^3$
- (B)  $2 \times 10^4$
- (C)  $2 \times 10^5$
- (D)  $2 \times 10^6$

29. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance



of  $5\ \Omega$ , balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell.

- (A)  $1\ \Omega$
- (B)  $1.5\ \Omega$
- (C)  $2\ \Omega$
- (D)  $2.5\ \Omega$

30. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combination is  $1\ k\Omega$ . How much was the resistance on the left slot before interchanging the resistances?

- (A)  $990\ \Omega$
- (B)  $505\ \Omega$
- (C)  $550\ \Omega$
- (D)  $910\ \Omega$

## PART B - CHEMISTRY

31. The ratio of mass percent of C and H of an organic compound ( $C_XH_YO_Z$ ) is 6 : 1. If one molecule of the above compound ( $C_XH_YO_Z$ ) contains half as much oxygen as required to burn one molecule of compound  $C_XH_Y$  completely to  $CO_2$  and  $H_2O$ . The empirical formula of compound  $C_XH_YO_Z$  is:

- (A)  $C_3H_6O_3$
- (B)  $C_2H_4O$
- (C)  $C_3H_4O_2$
- (D)  $C_2H_4O_3$

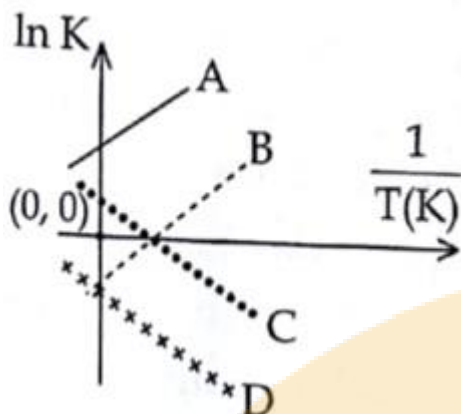
32. Which type of 'defect' has the presence of cations in the interstitial sites?

- (A) Schottky defect
- (B) Vacancy defect
- (C) Frenkel defect
- (D) Metal deficiency defect

33. According to molecular orbital theory, which of the following will not be a viable molecule?

- (A)  $He_2^{2+}$
- (B)  $He_2^+$
- (C)  $H_2^-$
- (D)  $H_2^{2-}$

34. Which of the following lines correctly show the temperature dependence of equilibrium constant, K, for an exothermic reaction?



- (A) A and B  
 (B) B and C  
 (C) C and D  
 (D) A and D

35. The combustion of benzene ( $l$ ) gives  $CO_2(g)$  and  $H_2O(l)$ . Given that heat of combustion of benzene at constant volume is  $-3263.9 \text{ kJ mol}^{-1}$  at  $25^\circ C$ ; heat of combustion (in  $\text{kJ mol}^{-1}$ ) of benzene at constant pressure will be:

( $R = 8.314 \text{ JK}^{-1} \text{ mol}^{-1}$ )

- (A) 4152.6                      (B) -452.46                      (C) 3260                      (D) -3267.6

36. For 1 molal aqueous solution of the following compounds, which one will show the highest freezing point?

- (A)  $[Co(H_2O)_6]Cl_3$   
 (B)  $[Co(H_2O)_5Cl]Cl_2 \cdot H_2O$   
 (C)  $[Co(H_2O)_4Cl_2]Cl \cdot 2H_2O$   
 (D)  $[Co(H_2O)_3Cl_3] \cdot 3H_2O$

37. An aqueous solution contains  $0.10 \text{ M } H_2S$  and  $0.20 \text{ M } HCl$ . If the equilibrium constants for the formation of  $HS^-$  from  $H_2S$  is  $1.0 \times 10^{-7}$  and that of  $S^{2-}$  from  $HS^-$  ions is  $1.2 \times 10^{-13}$  then the concentration of  $S^{2-}$  ions in aqueous solution is:

- (A)  $5 \times 10^{-8}$   
 (B)  $3 \times 10^{-20}$   
 (C)  $6 \times 10^{-21}$   
 (D)  $5 \times 10^{-19}$

38. An aqueous solution contains an unknown concentration of  $Ba^{2+}$ . When  $50 \text{ mL}$  of a  $1 \text{ M}$  solution of  $Na_2SO_4$  is added,  $BaSO_4$  just begins to precipitate. The final volume is  $500 \text{ mL}$ . The solubility product of  $BaSO_4$  is  $1 \times 10^{-10}$ . What is the original concentration of  $Ba^{2+}$ ?

- (A)  $5 \times 10^{-9} \text{ M}$   
 (B)  $2 \times 10^{-9} \text{ M}$   
 (C)  $1.1 \times 10^{-9} \text{ M}$

(D)  $1.0 \times 10^{-10} M$

39. At  $518^\circ C$ , the rate of decomposition of a sample of gaseous acetaldehyde, initially at a pressure of 363 Torr, was  $1.00 \text{ Torr s}^{-1}$  when 5% had reacted and  $0.50 \text{ Torr s}^{-1}$  when 33% had reacted. The order of the reaction is:

- (A) 2
- (B) 3
- (C) 1
- (D) 0

40. How long (approximate) should water be electrolyzed by passing through 100 amperes current so that the oxygen released can completely burn 27.66 g of diborane? (Atomic weight of B = 10.8 u)

- (A) 6.4 hours
- (B) 0.8 hours
- (C) 3.2 hours
- (D) 1.6 hours

41. The recommended concentration of fluoride ion in drinking water is up to 1 ppm as fluoride ion is required to make teeth enamel harder by converting  $[3Ca_3(PO_4)_2 \cdot Ca(OH)_2]$  to:

- (A)  $[CaF_2]$
- (B)  $[3(CaF_2) \cdot Ca(OH)_2]$
- (C)  $[3Ca_3(PO_4)_2 \cdot CaF_2]$
- (D)  $[3\{Ca(OH)_2\} \cdot CaF_2]$

42. Which of the following compounds contain(s) no covalent bond(s)?

$KCl, PH_3, O_2, B_2H_6, H_2SO_4$

- (A)  $KCl, B_2H_6, PH_3$
- (B)  $KCl, H_2SO_4$
- (C)  $KCl$
- (D)  $KCl, B_2H_6$

43. Which of the following are Lewis acids?

- (A)  $PH_3$  and  $BCl_3$
- (B)  $AlCl_3$  and  $SiCl_4$
- (C)  $PH_3$  and  $SiCl_4$
- (D)  $BCl_3$  and  $AlCl_3$

44. Total number of lone pair of electrons in  $I_3^-$  ions is:

- (A) 3
- (B) 6
- (C) 9
- (D) 12

45. Which of the following salts is the most basic in aqueous solution?

- (A)  $Al(CN)_3$
- (B)  $CH_3COOK$
- (C)  $FeCl_3$
- (D)  $Pb(CH_3COO)_2$

Solution: (B)

$CH_3COOK$

46. Hydrogen peroxide oxidises  $[Fe(CN)_6]^{4-}$  to  $[Fe(CN)_6]^{3-}$  in acidic medium but reduces  $[Fe(CN)_6]^{3-}$  to  $[Fe(CN)_6]^{4-}$  in alkaline medium. The other products formed are, respectively:

- (A)  $(H_2O + O_2)$  and  $H_2O$   
 (B)  $(H_2O + O_2)$  and  $(H_2O + OH^-)$   
 (C)  $H_2O$  and  $(H_2O + O_2)$   
 (D)  $H_2O$  and  $(H_2O + OH^-)$

47. The oxidation states of  $Cr$  in  $[Cr(H_2O)_6]Cl_3$ ,  $[Cr(C_6H_6)_2]$ , and  $K_2[Cr(CN)_2(O)_2(NH_3)]$  respectively are:

- (A) +3, +4 and +6  
 (B) +3, +2 and +4  
 (C) +3, 0 and +6  
 (D) +3, 0 and +4

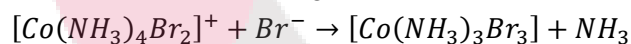
48. The compound that does not produce nitrogen gas by thermal decomposition is:

- (A)  $Ba(N_3)_2$                       (B)  $(NH_4)_2Cr_2O_7$   
 (C)  $NH_4NO_2$                       (D)  $(NH_4)_2SO_4$

49. When metal 'M' is treated with NaOH, a white gelatinous precipitate 'X' is obtained, which is soluble in excess of NaOH. Compound 'X' when heated strongly gives an oxide which is used in chromatography as an adsorbent. The metal 'M' is:

- (A) Zn                      (B) Ca                      (C) Al                      (D) Fe

50. Consider the following reaction and statement:



- (i) Two isomers are produced if the reactant complex ion is a cis – isomer.  
 (ii) Two isomers are produced if the reactant complex ion is a trans – isomer.  
 (iii) Only one isomer is produced if the reactant complex ion is a trans – isomer.  
 (iv) Only one is produced if the reactant complex ion is a cis – isomer.

The correct statements are:

- (A) (i) and (ii)  
 (B) (i) and (iii)  
 (C) (iii) and (iv)  
 (D) (ii) and (iv)

51. Glucose on prolonged heating with HI gives:

- (A) n – Hexane  
 (B) 1 – Hexene  
 (C) Hexanoic acid  
 (D) 6 – iodohexanal

52. The trans-alkenes are formed by the reduction of alkynes with:

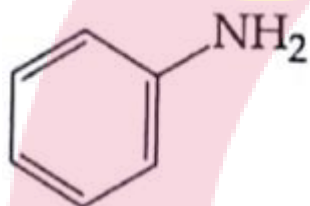
- (A)  $H_2 - Pd/C, BaSO_4$   
 (B)  $NaBH_4$  (C)  $Na/liq. NH_3$                       (D)  $Sn - HCl$

53. Which of the following compounds will be suitable for Kjeldahl's method for nitrogen estimation?

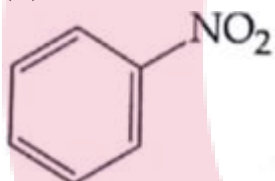
(A)



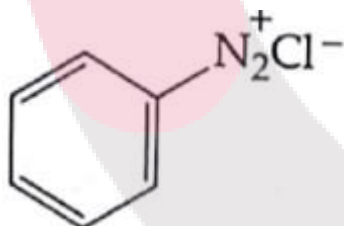
(B)



(C)

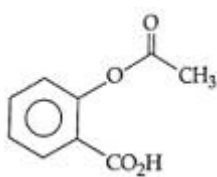


(D)

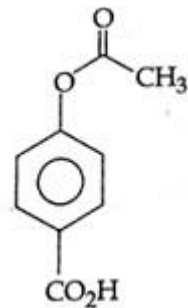


54. Phenol on treatment with  $CO_2$  in the presence of NaOH followed by acidification produces compound X as the major product. X on treatment with  $(CH_3CO)_2$  in the presence of catalytic amount of  $H_2SO_4$  produces:

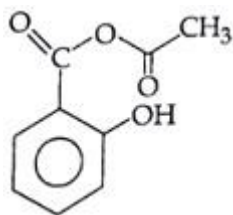
(A)



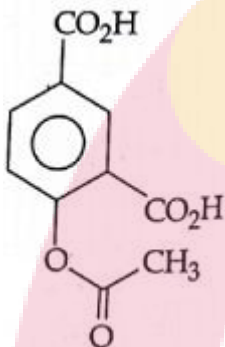
(B)



(C)



(D)

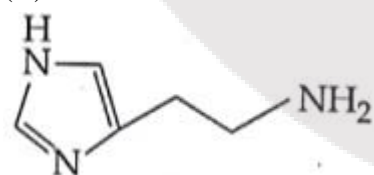


55. An alkali is titrated against an acid with methyl orange as indicator, which of the following is a correct combination?

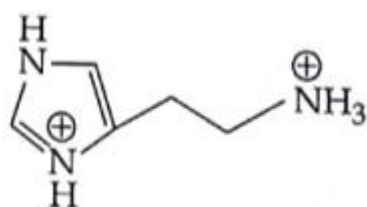
- |     |        |        |                       |
|-----|--------|--------|-----------------------|
| (A) | Base   | Acid   | End point             |
|     | Weak   | Strong | Colourless to pink    |
| (B) | Base   | Acid   | End point             |
|     | Strong | strong | Pinkish red to yellow |
| (C) | Base   | Acid   | End point             |
|     | Weak   | Strong | Yellow to pinkish red |
| (D) | Base   | Acid   | End point             |
|     | Strong | Strong | Pink to colourless    |

56. The predominant form of histamine present in human blood is ( $pK_a$ , Histidine = 6.0.)

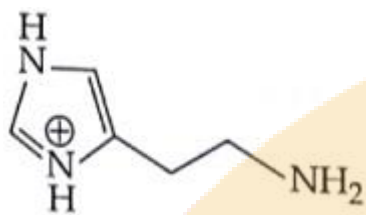
(A)



(B)



(C)

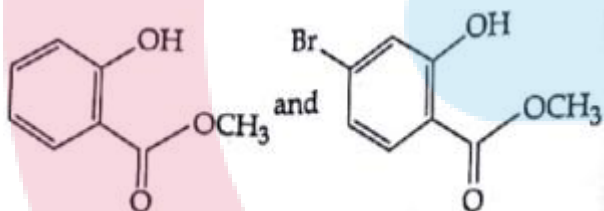


(D)

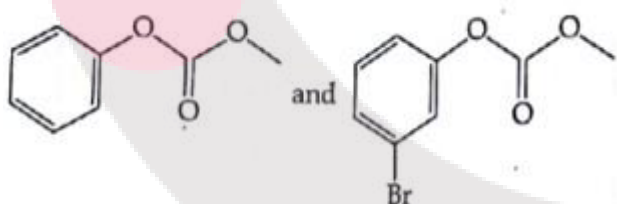


57. Phenol reacts with methyl chloroformate in the presence of  $NaOH$  to form product A. A reacts with  $Br_2$  to form product B. A and B are respectively:

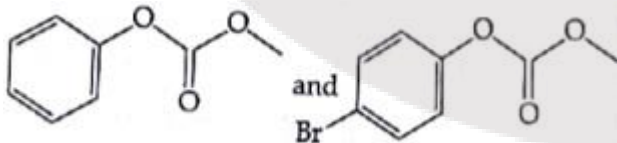
(A)



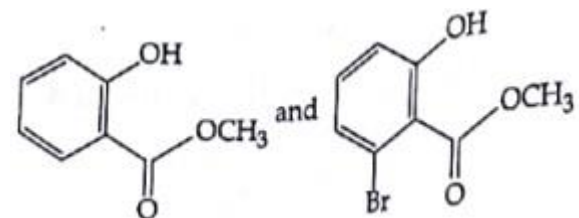
(B)



(C)



(D)

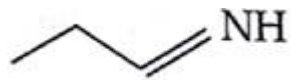


58. The increasing order of basicity of the following compounds is:

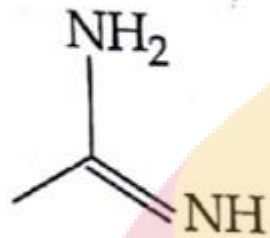
(i)



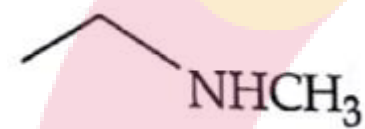
(ii)



(iii)



(iv)



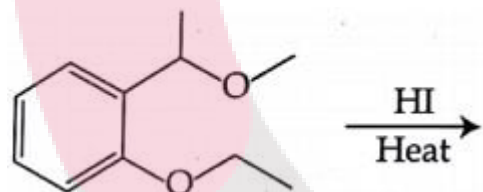
(A) (i) < (ii) < (iii) < (iv)

(B) (ii) < (i) < (iii) < (iv)

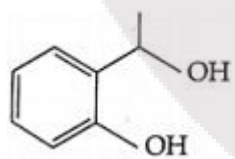
(C) (ii) < (i) < (iv) < (i)

(D) (iv) < (ii) < (i) < (iii)

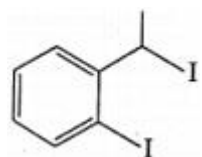
59. The major product formed in the following reaction is:



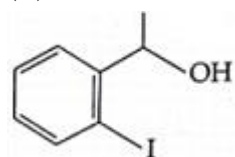
(A)



(B)

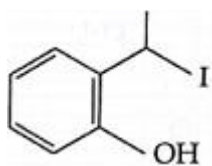


(C)

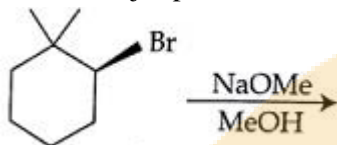




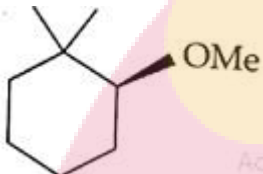
(D)



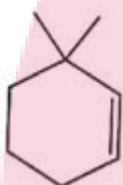
60. The major product of the following reaction is:



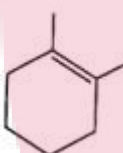
(A)



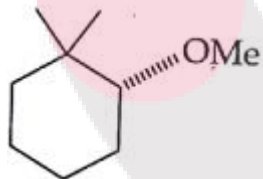
(B)



(C)



(D)



## PART C - MATHEMATICS

61. Two sets A and B are as under:

$$A = \{(a, b) \in R \times R : |a - 5| < 1 \text{ and } |b - 5| < 1\};$$

$$B = \{(a, b) \in R \times R : 4(a - 6)^2 + 9(b - 5)^2 \leq 36\}. \text{ Then:}$$

(A)  $B \subset A$

(B)  $A \subset B$

(C)  $A \cap B = \phi$  (an empty set)

(D) Neither  $A \subset B$  nor  $B \subset A$

62. Let  $S = \{x \in R : x \geq 0 \text{ and } 2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0\}$ . Then S:

- (A) Is an empty set
- (B) Contains exactly one element
- (C) Contains exactly two elements
- (D) Contains exactly four elements

63. If  $\alpha, \beta \in C$  are the distinct roots, of the equation  $x^2 - x + 1 = 0$ , then  $\alpha^{101} + \beta^{107}$  is equal to:

- (A) -1
- (B) 0
- (C) 1
- (D) 2

64. If  $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$ , then the ordered pair (A, B) is equal to:

- (A) (-4, -5)
- (B) (-4, 3)
- (C) (-4, 5)
- (D) (4, 5)

65. If the system of linear equations

$$x + ky + 3z = 0$$

$$3x + ky - 2z = 0$$

$$2x + 4y - 3z = 0$$

has a non-zero solution  $(x, y, z)$ , then  $xz/y^2$  is equal to:

- (A) -10
- (B) 10
- (C) -30
- (D) 30

66. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is:

- (A) At least 1000
- (B) Less than 500
- (C) At least 500 but less than 750
- (D) At least 750 but less than 1000

67. The sum of the co-efficient of all odd degree terms in the expansion of  $(x + \sqrt{x^3 - 1})^5 + (x - \sqrt{x^3 - 1})^5$ ,  $(x > 1)$  is:

- (A) -1
- (B) 0
- (C) 1
- (D) 2

68. Let  $a_1, a_2, a_3, \dots, a_{49}$  be in A.P. such that  $\sum_{k=0}^{12} a_{4k+1} = 416$  and  $a_9 + a_{43} = 66$ . If  $a_1^2 + a_2^2 + \dots + a_{17}^2 = 140m$ , then m is equal to:

- (A) 66
- (B) 68
- (C) 34
- (D) 33

69. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series

$$1^2 + 2 \cdot 2^2 + 3^2 + 2 \cdot 4^2 + 5^2 + 2 \cdot 6^2 + \dots$$

If  $B - 2A = 100\lambda$ , then  $\lambda$  is equal to:

- (A) 232
- (B) 248
- (C) 464
- (D) 496

70. For each  $t \in \mathbb{R}$ , let  $[t]$  be the greatest integer less than or equal to  $t$ . Then  $\lim_{x \rightarrow 0^+} x \left( \left[ \frac{1}{x} \right] + \left[ \frac{2}{x} \right] + \dots + \left[ \frac{15}{x} \right] \right)$
- (A) Is equal to 0  
 (B) Is equal to 15  
 (C) Is equal to 120  
 (D) Does not exist (in  $\mathbb{R}$ )
71. Let  $S = \{t \in \mathbb{R} : f(x) = |x - \pi| \cdot (e^{|x|} - 1) \sin|x| \text{ is not differentiable at } t\}$ . Then the set  $S$  is equal to:
- (A)  $\phi$  (an empty set) (B)  $\{0\}$   
 (C)  $\{\pi\}$  (D)  $\{0, \pi\}$
72. If the curves  $y^2 - 6x, 9x^2 + by^2 = 16$  intersect each other at right angles, then the value of  $b$  is:
- (A) 6 (B)  $\frac{7}{2}$  (C) 4 (D)  $\frac{9}{2}$
73. Let  $f(x) = x^2 + \frac{1}{x^2}$  and  $g(x) = x - \frac{1}{x}, x \in \mathbb{R} - \{-1, 0, 1\}$ . If  $h(x) = \frac{f(x)}{g(x)}$ , then the local minimum value of  $h(x)$  is:
- (where  $C$  is a constant of integration)
- (A) 3 (B) -3 (C)  $-2\sqrt{2}$  (D)  $2\sqrt{2}$
74. The integral  $\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2} dx$  is equal to:
- (A)  $\frac{1}{3(1+\tan^3 x)} + C$  (B)  $\frac{-1}{3(1+\tan^3 x)} + C$   
 (C)  $\frac{1}{1+\cot^3 x} + C$  (D)  $\frac{-1}{1+\cot^3 x} + C$
75. The values of  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx$  is:
- (A)  $\frac{\pi}{8}$  (B)  $\frac{\pi}{2}$  (C)  $4\pi$  (D)  $\frac{\pi}{4}$
76. Let  $g(x) = \cos x^2, f(x) = \sqrt{x}$ , and  $\alpha, \beta (\alpha < \beta)$  be the roots of the quadratic equation  $18x^2 - 9\pi x + \pi^2 = 0$ . Then the area (in sq. units) bounded by the curve  $y = (g \circ f)(x)$  and the lines  $x = \alpha, x = \beta$  and  $y = 0$ , is:
- (A)  $\frac{1}{2}(\sqrt{3} - 1)$  (B)  $\frac{1}{2}(\sqrt{3} + 1)$   
 (C)  $\frac{1}{2}(\sqrt{3} - \sqrt{2})$  (D)  $\frac{1}{2}(\sqrt{2} - 1)$
77. Let  $y = y(x)$  be the solution of the differential equation  $\sin x \frac{dy}{dx} + y \cos x = 4x, x \in (0, \pi)$ . If  $y\left(\frac{\pi}{2}\right) = 0$ , then  $y\left(\frac{\pi}{6}\right)$  is equal to:
- (A)  $\frac{4}{9\sqrt{3}}\pi^2$  (B)  $\frac{-8}{9\sqrt{3}}\pi^2$  (C)  $-\frac{8}{9}\pi^2$  (D)  $-\frac{4}{9}\pi^2$

78. A straight line through a fixed point  $(2, 3)$  intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is:

- (A)  $3x + 2y = 6$                       (B)  $2x + 3y = xy$   
 (C)  $3x + 2y = xy$                       (D)  $3x + 2y = 6xy$

79. Let the orthocentre and centroid of a triangle be  $A(-3, 5)$  and  $B(3, 3)$  respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is:

- (A)  $\sqrt{10}$                                   (B)  $2\sqrt{10}$                       (C)  $3\sqrt{\frac{5}{2}}$                       (D)  $\frac{3\sqrt{5}}{2}$

80. If the tangent at  $(1, 7)$  to the curve  $x^2 = y - 6$  touches the circle  $x^2 + y^2 + 16x + 12y + c = 0$  then the value of c is:

- (A) 195                      (B) 185                      (C) 85                      (D) 95

81. Tangent and normal are drawn at  $P(16, 16)$  on the parabola  $y^2 = 16x$ , which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and  $\angle CPB = \theta$ , then a value of  $\tan \theta$  is:

- (A)  $\frac{1}{2}$                       (B) 2                      (C) 3                      (D)  $\frac{4}{3}$

82. Tangents are drawn to the hyperbola  $4x^2 - y^2 = 36$  at the points P and Q. If these tangents intersect at the point  $T(0, 3)$  then the area (in sq. units) of  $\Delta PTQ$  is:

- (A)  $45\sqrt{5}$                       (B)  $54\sqrt{3}$                       (C)  $60\sqrt{3}$                       (D)  $36\sqrt{5}$

83. If  $L_1$  is the line of intersection of the planes  $2x - 2y + 3z - 2 = 0, x - y + z + 1 = 0$  and  $L_2$  is the line of intersection of the planes  $x + 2y - z - 3 = 0, 3x - y + 2z - 1 = 0$ , then the distance of the origin from the plane, containing the lines  $L_1$  and  $L_2$  is:

- (A)  $\frac{1}{4\sqrt{2}}$                       (B)  $\frac{1}{3\sqrt{2}}$                       (C)  $\frac{1}{2\sqrt{2}}$                       (D)  $\frac{1}{\sqrt{2}}$

84. The length of the projection of the line segment joining the points  $(5, -1, 4)$  and  $(4, -1, 3)$  on the plane,  $x + y + z = 7$  is:

- (A)  $\frac{2}{\sqrt{3}}$                       (B)  $\frac{2}{3}$                       (C)  $\frac{1}{3}$                       (D)  $\sqrt{\frac{2}{3}}$

85. Let  $\vec{u}$  be a vector coplanar with the vectors  $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$  and  $\vec{b} = \hat{j} + \hat{k}$ . If  $\vec{u}$  is perpendicular to  $\vec{a}$  and  $\vec{u} \cdot \vec{b} = 24$ , then  $|\vec{u}|^2$  is equal to:

- (A) 336                      (B) 315                      (C) 256                      (D) 84

86. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is:

- (A)  $\frac{3}{10}$       (B)  $\frac{2}{5}$       (C)  $\frac{1}{5}$       (D)  $\frac{3}{4}$

88. If sum of all the solutions of the equation  $8 \cos x \cdot \left( \cos \left( \frac{\pi}{6} + x \right) \cdot \cos \left( \frac{\pi}{6} - x \right) - \frac{1}{2} \right) = 1$  in  $[0, \pi]$  is  $k\pi$ , then  $k$  is equal to:

- (A)  $\frac{2}{3}$       (B)  $\frac{13}{9}$       (C)  $\frac{8}{9}$       (D)  $\frac{20}{9}$

89. PQR is a triangular park with  $PQ = PR = 200\text{m}$ . A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively  $45^\circ$ ,  $30^\circ$  and  $30^\circ$ , then the height of the tower (in m) is:

- (A) 100      (B) 50      (C)  $100\sqrt{3}$       (D)  $50\sqrt{2}$

90. The Boolean expression  $\sim(p \vee q) \vee (\sim p \wedge q)$  is equivalent to:

- (A)  $\sim p$       (B)  $p$       (C)  $q$       (D)  $\sim q$