Solution

Mathematics

1. $\int \frac{5x^8+7x^6}{(x^2+1+2x^7)^2}\, dx$

$$= \int \frac{5x^8 + 7x^6}{x^{14}\left(\frac{1}{x^5} + \frac{1}{x^7} + 2\right)^2} dx$$

$$= \int \frac{\frac{5}{x^6} + \frac{7}{x^8}}{\left(\frac{1}{x^5} + \frac{1}{x^7} + 2\right)^2} dx$$

Put $\frac{1}{x^5} + \frac{1}{x^7} + 2 = t$

$$\Rightarrow \left(-\frac{5}{x^6} - \frac{7}{x^8}\right) dx = dt$$

$$\therefore -\int \frac{dt}{t^2} = \frac{1}{t} + c$$

$$= \frac{x^7}{x^2 + 1 + 2x^7} + c$$

2. $D = 121 - 24\,\alpha$

for rotational roots D must be perfect square.

$\therefore \alpha = 3, 4, 5$

$\therefore$ number of values of $\alpha = 3$

3. $\left(\frac{1-t^6}{1-t}\right)^3 = (1 - t^6)^3 (1 - t)^{-3}$

$= (1 - t^{18} - 3t^6 + 3t^{12})\,(1 - t)^{-3}$

$\therefore$ Co-efficient of $t^4 = 1 \times {}^{3+4-1}C_4 = {}^6C_4 = 15$

4. Given $\sum_{i=1}^{n}(x_i^2 + 2x_i + 1) = 9n$ ……(i)

$\sum_{i=1}^{n}(x_i^2 - 2x_i + 1) = 5n$ ………(ii)

For $(i) + (ii)$ we get

$\sum_{i=1}^{n}(2x_i^2 + 2) = 14n$

$\Rightarrow \sum_{i=1}^{n} x_i^2 = 6n$

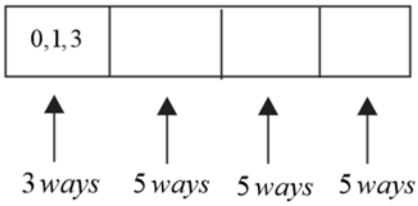
$\sum_{i=1}^{n} x_i = n$

$\therefore$ Variance, $r^2 = E(x^2) - \left(E(x)\right)^2$

$$= \frac{6n}{n} - \left(\frac{n}{n}\right)^2 = 6 - 1 = 5$$

$\therefore$ Standard deviation, $\sigma = \sqrt{5}$

5.



$\therefore$ total number formed less than $7000 = 3 \times 5 \times 5 \times 5 = 375$

$\therefore$ total natural number formed less than $7000 = 375 - 1 = 374$

(number zero has been excluded)

6. Let A is event that $2nd$ drawn ball is red.

$\therefore P(A) = P(R) \times P(A/R) + P(G) \times P\ (A/G)$

$\frac{5}{7} \times \frac{4}{7} + \frac{2}{7} \times \frac{6}{7}$

$= \frac{32}{49}$

7. $t_n = \frac{3n(1^2+2^2+3^2+\cdots+n^2)}{2n+1} = \frac{3n \times n(n+1)(2n+1)}{6\ (2n+1)}$

$= \frac{1}{2}\ (n^3 + n^2)$

$\therefore S_{15} = \sum_{n=1}^{15} tn = \sum_{n=1}^{15} \frac{1}{2} (n^3 + n^2)$

$= \frac{1}{2} \times \left[\left(\frac{15\times16}{2}\right)^2 + \frac{15\times16\times31}{6}\right]$

$= 7200 + 620$

$= 7820$

8. $Z_o = w$ or w^2

When $Z_0 = w$

$Z = 3 + 6i \times w^{81} - 3i \times w^{93}$

$= 3 + 6i - 3i = 3 + 3i$

When $Z_0 = w^2$

$Z = 3 + 6i \times (w^2)^{81} - 3i \times (w^2)^{93}$

$= 3 + 6i - 3i = 3 + 3i$

$\therefore \arg(z) = \tan^{-1}\left(\frac{3}{3}\right) = \frac{\pi}{4}$

9. Since $(\vec{a}+\vec{b})$ is perpendicular to $\vec{c}$, hence $(\vec{a}+\vec{b}).\vec{c}=0$

$\Rightarrow \vec{a}.\vec{c}+\vec{b}.\vec{c}=0$

$\Rightarrow 5+1+2+5b_1+b_2+2=0$

$\Rightarrow 5b_1+b_2=-10....(\text{i})$

Given, $\frac{\vec{b}.\vec{a}}{|\vec{a}|}=|\vec{a}|$

$\Rightarrow b_1+b_2+2=4$

$\Rightarrow b_1+b_2=2 ......(\text{ii})$

Solving (i) & (ii), $b_1=-3, b_2-5$

$\therefore |\vec{b}|=\sqrt{9+25+2}=6$

10. $\int_0^{\pi/3} \frac{\tan x}{\sqrt{2k\sec x}}dx$

$=\frac{1}{\sqrt{2k}}\int_0^{\pi/3}\frac{\sin x}{\sqrt{\cos x}}dx$

Put $t=\cos x$

$dt=-\sin x\,dx$, also $x=0, t=1; x=\frac{\pi}{3}, t=\frac{1}{2}$

$\therefore \frac{1}{\sqrt{2k}}\int_1^{1/2}\frac{-dt}{\sqrt{t}}$

$=\frac{1}{\sqrt{2k}}\int_{1/2}^{1} t^{-\frac{1}{2}}dt=\frac{1}{\sqrt{2k}}\times 2\left[t^{\frac{1}{2}}\right]_{1/2}^{1}$

$=\frac{\sqrt{2}}{\sqrt{k}}\left(1-\frac{1}{\sqrt{2}}\right)=\frac{\sqrt{2}-1}{\sqrt{k}}$

Given $\frac{\sqrt{2}-1}{\sqrt{k}}=1-\frac{1}{\sqrt{2}}=\frac{\sqrt{2}-1}{\sqrt{2}}$

$\therefore k=2$

11. $\sin x-\sin 2x+\sin 3x=0$

$\Rightarrow \sin x+\sin 3x-\sin 2x=0$

$\Rightarrow 2\sin 2x.\cos x-\sin 2x=0$

$\Rightarrow \sin 2x=0$ or $\cos x=\frac{1}{2}$

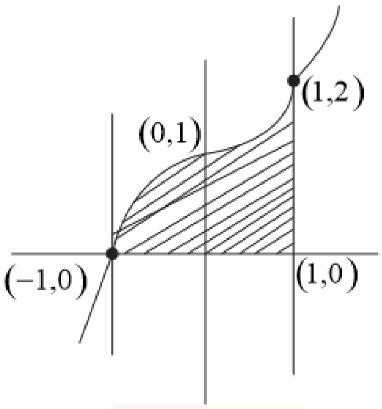
$\Rightarrow 2x=0$ or $x=\frac{\pi}{3}$

$\Rightarrow x=0$ or $x=\frac{\pi}{3}$

$\therefore$ number of solutions $=2$

12. Area enclosed

$= \int_1^0 (-x^2+1)\,dx + \int_0^1 (x^2+1)dx$



$= \left[-\frac{x^3}{3}+x\right]_{-1}^{0} + \left[\frac{x^3}{3}+x\right]_0^1$

$= 2$

13. $\lim\limits_{x\to 0^-} \frac{x\,([x])+|x|\sin[x]}{|x|}$

$\lim\limits_{x\to 0^-} \frac{x(-1-x)\sin(-1)}{-x}$

$\lim\limits_{x\to 0^-} -(1+x)\sin 1$

$-\sin 1$

14. Given relation is $f(xy) = f(x).f(y)\ \forall\ x, y \in R$ .....(i)

On putting $x = y = 0$, we get $f(0) = f^2(0) \Rightarrow f(0) = 0, 1$ but $f(0) \neq 0$

$\Rightarrow f(0) = 1,$

Now if we put $y = 0$ in (i), we get

$f(x) = 1$

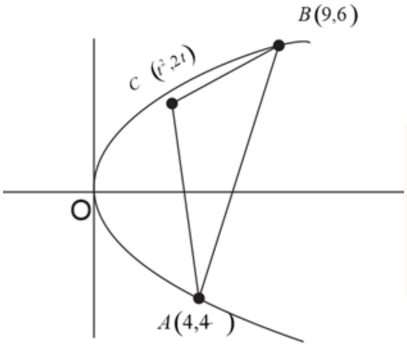
Hence $\frac{dy}{dx} = 1 \Rightarrow y = x + c$

$\Rightarrow y = x + \frac{1}{2}$ (since $y(0) = \frac{1}{2}$)

$\Rightarrow y\left(\frac{1}{4}\right) + y\left(\frac{3}{4}\right) = \left(\frac{1}{4}+\frac{1}{2}\right) + \left(\frac{3}{4}+\frac{1}{2}\right) = 2.$

15. Let the coordinates of C be $(t^2, 2t)$

Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} t^2 & 2t & 1 \\ 9 & 6 & 1 \\ 4 & -4 & 1 \end{vmatrix}$



$\Delta = 5(6 + t - t^2)$

$\frac{d\Delta}{dt} = 1 - 2t = 0 \Rightarrow t = \frac{1}{2}$

$\Rightarrow c\left(\frac{1}{4}, 1\right)$

16. As we know, if two circles intersect each other, then $|r_1 - r_2| < c_1c_2 < r_1 + r_2$ ....(i)

Now for the first circle $c_1(8, 10)$ and $r_1 = r$

For the second circle $c_2(4, 7)$ and $r_2 = 6$

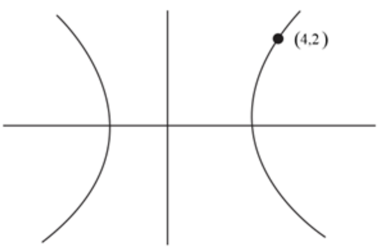
From (i)

$|r - 6| < 5 < r + 6$

$\Rightarrow r \in (1, 11)$

17. From the given information, equation of hyperbola can be taken as

$\frac{x^2}{4} - \frac{y^2}{b^2} = 1, a = 2$



Since it passes through $(4, 2)$,

Hence $b^2 = \frac{4}{3}$

Now $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{1}{3}} = \frac{2}{\sqrt{3}}$

18. Given $x = 3\tan t$ and $y = 3\sec t$

$\Rightarrow \frac{dx}{dt} = 3\sec^2 t$ and $\frac{dy}{dt} = 3\sec t \tan t$

$$\Rightarrow \frac{dy}{dx} = \sin t$$

$$\Rightarrow \frac{d^2y}{dx^2} = \cos t . \frac{dt}{dx} = \cos t . \frac{\cos^2 t}{3} = \frac{\cos^3 t}{3}$$

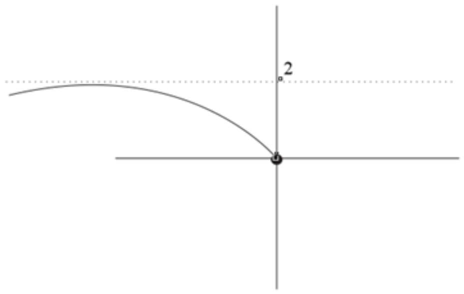
At $t = \frac{\pi}{4}$

$$\frac{d^2y}{dx^2} = \frac{1}{6\sqrt{2}}$$

19. Given function $f(x) = \frac{2x}{x-1}$ and domain of $f(x)$ is $(-\infty, 0)$

Now $f'(x) = \frac{-2}{(x-1)^2} < 0$

$\Rightarrow f$ is always decreasing in the given domain.



Clears given function is one-one.

Since codomain $(-\infty, \infty) \neq$ Range $(0, 2)$, hence f is into.

20. Let the first terms of $A.P.$ be A and common difference be d, then

$a = A + 6d$

$b = A = 10d$

$c = A + 12d$

Given a, b, c and in G. P. $\Rightarrow b^2 = ac$

$\Rightarrow (A + 10d)^2 = (A + 6d)(A + 12d)$

$\Rightarrow A^2 + 100d^2 + 20Ad = A^2 + 18Ad + 72d^2$

$\Rightarrow A = -14d$

Now $\frac{a}{c} = \frac{A+6d}{A+12d} = \frac{-14d+6d}{-14d+12d} = \frac{-8d}{-2d} = 4$

21. Since are vertex of given triangle is o (origin) and other two A & B lies on coordinate axes, hence triangle must be a right angled triangle.

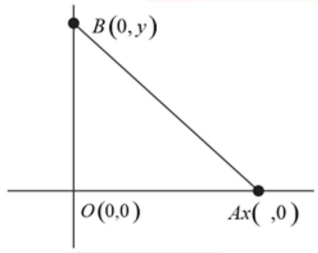
Area $= \frac{1}{2}x.y,\ x, y \in I$

As per given condition

$\frac{1}{2}x.y = 50 \Rightarrow x.y = 100$

Possible ordered pairs of (x, y) when Δ formed in 1^{st} quadrant are 9 i. e. $(1, 100)\ (2, 50), (4, 25)\ (5, 20)\ (10, 10), (20, 5)(25, 4)(50, 2)(100, 1)$.

Δ can be formed in any of the quadrants, hence total number of cases $9 \times 4 = 36$.



22. Given roots of $x^2 - mx + 4 = 0$ are distinct and lies in $(1, 5)$, Following conditions must be true

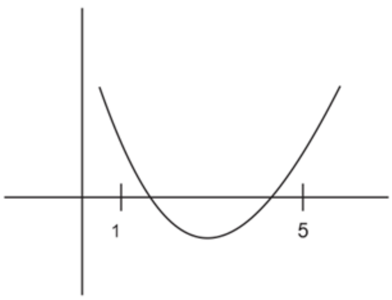
(i) $D > 0 \Rightarrow M \in (-\infty, -4)\ U\ (4, \infty)$

(ii) $f(1) > 0 \Rightarrow M < 5$

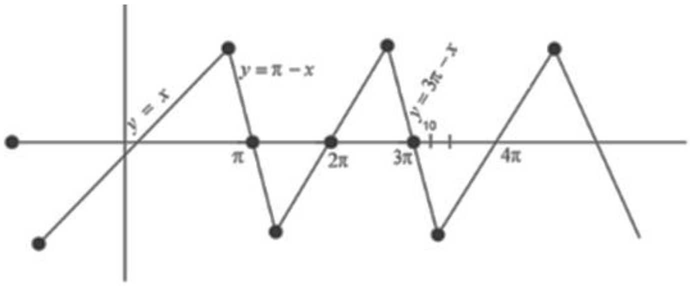
(iii) $f(5) > 0 \Rightarrow M < \frac{29}{5}$

(iv) $1 < -\frac{B}{2A} < 5 \Rightarrow 2 < m < 10$

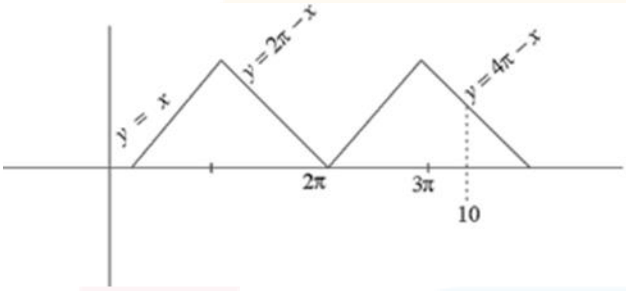
Taking intersection of all conditions, we get $m \in (4, 5)$



23.



From the graph of $y = \sin^{-1}(\sin x)$, it is clear that $\sin^{-1}(\sin 10) = 3\pi - 10$



From the graph of $y = \cos^{-1}(\cos x)$

It is clear that $\cos^{-1}(\cos 10) = 4\pi - 10$

Hence $y - x = (4\pi - 10) - (3\pi - 10) = \pi$

24. Since given matrix A is invertible $\Rightarrow |A| \neq 0$

Now $|A| = \begin{vmatrix} e^t & e^{-t}(\sin t - 2\cos t) & e^{-t}(-2\sin t - \cos t) \\ e^t & -e^{-t}(2\sin t + \cos t) & e^{-t}(\sin t - 2\cos t) \\ e^t & e^{-t}\cos t & e^{-t}\sin t \end{vmatrix}$

$= e^t . e^{-t} . e^{-t} \begin{vmatrix} 1 & \sin t - 2\cos t & -2\sin t - \cos t \\ 1 & 2\sin t + \cos t & \sin t - 2\cos t \\ 1 & \cos t & \sin t \end{vmatrix}$

$= 2e^{-t} \begin{vmatrix} 1 & \sin t - 2\cos t & -2\sin t - \cos t \\ 0 & \sin t & -\cos t \\ 1 & \cos t & \sin t \end{vmatrix}$

$R_1 \rightarrow R_1 - R_2$

$= 2e^{-t} \begin{vmatrix} 1 & -2\cos t & -2\sin t \\ 0 & \sin t & -\cos t \\ 1 & \cos t & \sin t \end{vmatrix}$

$R_1 \rightarrow R_1 + 2R_3$

$= 2e^{-t} \begin{vmatrix} 3 & 0 & 0 \\ 0 & \sin t & -\cos t \\ 1 & \cos t & \sin t \end{vmatrix}$

$|A| = 6\, e^{-t} \neq 0 \;\forall\, t \in R$

Hence given matrix is always invertible.

25. DR's of the first line

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -a & 0 \\ 0 & c & -1 \end{vmatrix} = a\,\hat{\imath} + \hat{\jmath} + c\hat{k}$$

Hence DR's of the first line are $a, 1, c$.

For DR's of the second line

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 0 & -a' \\ 0 & 1 & -c' \end{vmatrix} = a'\hat{\imath} + c'\hat{\jmath} + \hat{k}$$

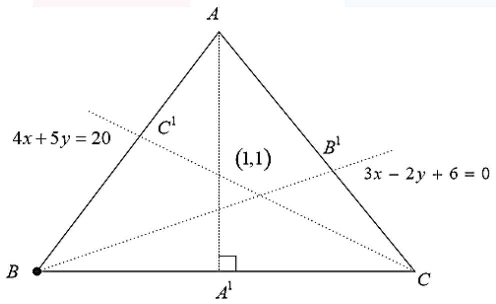
Hence DR's of the 2^{nd} line $= a', c', 1$

Since both the lines are perpendicular,

$a.a' + c' + c = 0$

26. To find equation of BC, first we will find coordinates of B and C.

Equation of BB^1



$$y - 1 = \frac{-2}{3}(x - 1)$$

$\Rightarrow 2x + 3y = 5$ .....(i)

Point of intersection of AB and $BB^1 \Rightarrow B\left(\frac{35}{2}, -10\right)$

Similarly point $c\left(-13, -\frac{33}{2}\right)$

Equation of BC $y + 10 = \frac{13}{61}\left(x - \frac{35}{2}\right)$

27. Given functional in equality is

$|f(x) - f(y)| \leq 2\,|x - y|^{3/2}$

Put $x = y + h,$ we get

$|f(y + h)| - f(y)| \leq 2.|h|^{3/2}$

$$\Rightarrow \left|\frac{f(y + h) - f(y)}{h}\right| \leq 2\,|h|^{1/2}$$

$$\Rightarrow \lim_{h\to 0}\left|\frac{f(y+h)-f(y)}{h}\right| \le \lim_{h\to 0} 2\,|h|^{1/2}$$

$\Rightarrow |f'(y)| \le 0$

$\Rightarrow |f'(y)| = 0$

$\Rightarrow f'(y) = 0$

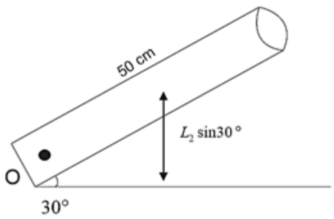
$\Rightarrow f(y) = c$

$\Rightarrow f(y) = 1$ (since $f(0) = 1$)

Now $\int_0^1 f^2(x)dx = \int_0^1 1\ dx = 1$

Physics

1. From conservation of mechanical energy



$$mg\frac{l}{2}\sin 30 = \frac{1}{2}I\omega^2$$

$$mg\frac{l}{2}\sin 30 = \frac{1}{2}\frac{ml^2}{3}\omega^2$$

$$\omega = \sqrt{\frac{3g}{2l}} = \sqrt{\frac{3\times 10}{2\times 50\times 10^{-2}}} = \sqrt{30}\,rad/s$$

2. 75 sec, $\frac{4}{10}$, Application (i) Velocity

$x = A\cos\omega t$

$y = A\sin\omega t$

$z = A\,\omega t$

$v_x = -A\omega\sin\omega t$

$v_y = A\omega\cos\omega t$

$v_2 = A\omega$

$$v = \sqrt{v_x^2 + v_y^2 + v_2^2}$$

$$= \sqrt{(-A\omega\sin\omega t)^2 + (A\omega\cos\omega t)^2 + (A\omega)^2}$$

$$= A\omega\sqrt{2}$$

3. 60 sec, $\frac{3}{10}$, Application (i) Transformer

(ii) Efficiency of a transformer

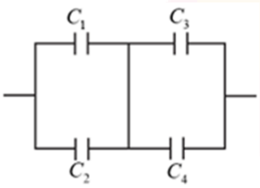
Efficiency $y = 90\%$

$\Rightarrow 0.9 \times \text{power in} = \text{Power out}$

$0.9 \times 2300 \times 5 = 230 \times I_{\text{out}}$

$I_{\text{out}} = 45\ A$

4. 75 sec, $\frac{4}{10}$, Application, (i) Capacitance with dielectric



$$C_1 = \frac{\epsilon_0 \frac{A}{2} K_1}{\frac{d}{2}} = \frac{\epsilon_0\ AK_1}{d}$$

$$C_2 = \frac{\epsilon_0\ AK_2}{d}$$

$$C_3 = \frac{\epsilon_0\ AK_3}{d}$$

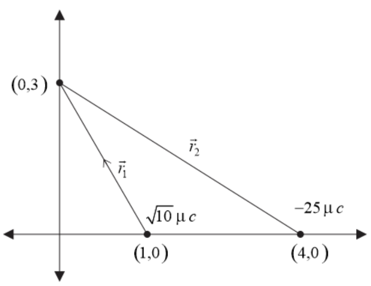
$$C_4 = \frac{\epsilon_0\ AK_4}{d}$$

$$C_{\text{eq}} = \frac{(C_1 + C_2)(C_3 + C_4)}{C_1 + C_2 + C_3 + C_4}$$

$$\frac{\epsilon_0\ A\ K_{\text{eq}}}{d} = \frac{\epsilon_0\ A}{d} \frac{(K_1 + K_2)(K_3 + K_4)}{K_1 + K_2 + K_3 + K_4}$$

$$K_{\text{eq}} = \frac{(K_1 + K_2)(K_3 + K_4)}{K_1 + K_2 + K_3 + K_4}$$

5. Electric field due to $\sqrt{10}\ \mu c$



$$\vec{E}_1 = \frac{1}{4\pi \in_0} \frac{\sqrt{10} \times 10^{-6}}{r_1^3} \vec{r}_1$$

$$= \frac{1}{4\pi \in_0} \frac{\sqrt{10} \times 10^{-6}}{\left(\sqrt{10}\right)^3} (-\hat{\imath} + 3\hat{\jmath})$$

Similarly electric field due to $-25\,\mu c$

$$\vec{E}_2 = \frac{1}{4\pi \in_0} \frac{25 \times 10^{-6}}{(5)^3} (4\hat{\imath} - 3\hat{\jmath})$$

net electric field

$$\vec{E} = 9 \times 10^9 \times 10^{-6} \left(\frac{-\hat{\imath}}{10} + \frac{3\hat{\jmath}}{10} + \frac{4}{5}\hat{\imath} - \frac{3}{5}\hat{\jmath}\right)$$

$$= 9 \times 10^3 \left(\frac{7\hat{\imath}}{10} - \frac{3\hat{\jmath}}{10}\right)$$

$$= (63\hat{\imath} - 27\hat{\jmath}) \times 10^2 \frac{N}{C}$$

6. $E_1 + \left(\frac{-GMm}{R}\right) = \frac{-GMm}{R+h}$

$E_2 = \frac{1}{2} mV^2$ and $\frac{mv^2}{R+h} = \frac{GMm}{(R+h)^2}$

$\Rightarrow E_2 = \frac{1}{2}\left(\frac{GMm}{R+h}\right)$

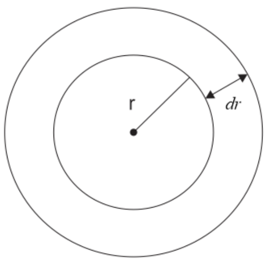
$E_1 = E_2 \Rightarrow \frac{GMm}{R} - \frac{GMm}{R+h} = \frac{GMm}{2(R+h)}$

$\frac{h}{R(h+R)} = \frac{1}{2(R+h)}$

$h = \frac{R}{2} = 3.2 \times 10^3\ km$

7. $B = B_O[\sin(3.14 \times 10^7 Ct) + \sin(6.28 \times 10^7 Ct)]$

8. $\rho(r) = \frac{A}{r^2}\, e^{-\frac{2r}{a}}$



Charge enclosed between r and $r + dr$ is

$dq = \rho(r)\, 4\pi r^2 dr$

To get total charge 'Q',

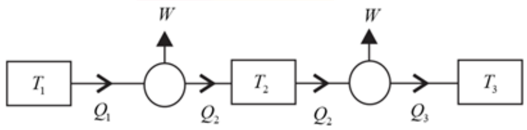
$$Q = \int dq = \int_0^R \frac{A}{r^2} e^{-2r/a}\ 4\pi r^2 dr$$

$$= -4\pi A \frac{a}{2}\left[e^{-2r/a}\right]^R$$

$$Q = -4\pi\ A \frac{a}{2}\left[e^{-2R/a} - 1\right]$$

$$R = -\ln\left[1 - \frac{Q2}{4\pi Aa}\right]\frac{a}{2}$$

$$= \frac{a}{2}\ln\left[\frac{2\pi QA}{2\pi Aa - Q}\right]$$

9. 

$$W = Q_1 - Q_2 = Q_2 - Q_3$$

$$\frac{Q_1}{Q_2} + \frac{Q_3}{Q_2} = 2$$

$$\frac{T_1}{T_2} + \frac{T_3}{T_2} = 2$$

$$T_2 = \frac{T_1 + T_3}{2} = 500K$$

10. $S = 3t^2 + 5$

Velocity $V = 6t + O$

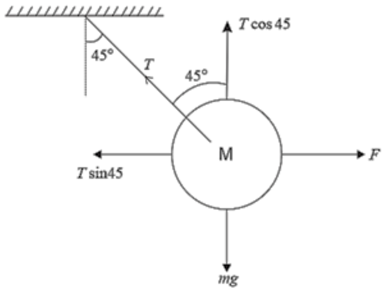
From work energy theorem

Work done $= \Delta KE$

$$= \frac{1}{2} \times 2 \times (6 \times 5)^2 - \frac{1}{2} \times 2 \times (6 \times 0)^2$$

$$= 900\, J$$

11.



$T\cos 45 = mg$

$T\sin 45 = F$

$Tan\ 45 = \dfrac{F}{mg}$

$F = mg = 100\ N$

12. Current through R_4 (as well as R_3)

is $i_1 = \frac{V}{R} = \frac{5}{500} = 10\ mA$

Potential drop across 400 Ω is

$V_{400} = 18 - 10 \times 10^{-3} \times 600$

$= 12\ V$

Current through battery

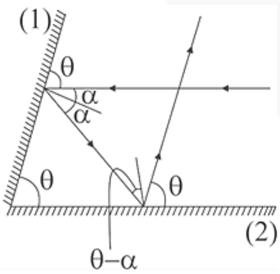
$I = \dfrac{12}{400} = 30\ mA$

$\therefore$ Current through R_2 $i_2 = 30 - 10$

$= 20\ mA$

$\therefore R_2 = \dfrac{6}{20 \times 10^{-3}} = 300\ \Omega$

13.



$90 - \propto + \theta + x$

$x = 90 + \propto - \theta$

$90 + \propto - \theta = \theta$

and $90 {-} \propto = \theta$

$180 - \theta = 2\theta$

$\theta = 60^o$

14. $\frac{1}{2}a_1 t_0^2 = \frac{1}{2}a_2(t_0 + t)^2$

$v_1 = a_1 t_0 \quad v_2 = a_2(t_0 + t)$

$$v = v_1 - v_2 = (a_1 - a_2)t_0 - a_2 t$$

$$\sqrt{\frac{a_1}{a_2}}\ t_0 = t_0 + t$$

$$t_0 = \frac{t}{\sqrt{\frac{a_1}{a_2}} - 1}$$

$$v = (a_1 - a_2)\frac{t}{\sqrt{\frac{a_1}{a_2}} - 1} - a_2 t$$

$$= t\left(\frac{a_1\sqrt{a_2}}{\sqrt{a_1} - \sqrt{a_2}} - \frac{a_2\sqrt{a_2}}{\sqrt{a_1} - \sqrt{a_2}} - a_2\right)$$

$$= t\left(\frac{a_1\sqrt{a_2} - a_2\sqrt{a_1}}{\sqrt{a_1} - \sqrt{a_2}}\right)$$

$$= t\left(\sqrt{a_1}\,\sqrt{a_2}\right)$$

15. $KE = \frac{1}{2}kx^2 \quad PE = \frac{1}{2}K(A^2 - x^2)$

$$\frac{1}{2}Kx^2 = \frac{1}{2}K(A^2 - x^2)$$

$$x^2 = \frac{A^2}{2}$$

$$x = \frac{A}{\sqrt{2}}$$

16. $T = 2\pi\sqrt{\frac{I}{C}}$

$$f = \frac{1}{2\pi}\sqrt{\frac{C}{I}}$$

$$f_{\text{initial}} = \frac{1}{2\pi}\sqrt{\frac{C_3}{MC^2}}$$

$$f_{\text{final}} = \frac{1}{2\pi}\sqrt{\frac{C}{\left(\frac{MC^2}{3} + 2m\frac{L^2}{4}\right)}}$$

$$0.2\ \frac{1}{2\pi}\sqrt{\frac{3C}{ML^2}} = \frac{1}{2\pi}\sqrt{\frac{C}{\frac{MC^2}{3} + \frac{2MC^2}{4}}}$$

$$= 0.04\frac{3}{M} = \frac{12}{4M + 6M}$$

$$4M = 0.16M + 0.24\,M$$

$$\frac{m}{M} = \frac{3.84}{0.24} = 16$$

17. $V_{rms} = \sqrt{\frac{3RT}{M}}$

T must be increased 4 times to get V_{rms} doubled.

$$\Delta Q = nC_v\Delta T$$

$$= \frac{15}{28} \times \frac{5}{2} \times 8.314 \times (1200 - 300)$$

$$= 10021.3J = 10\ kJ$$

19. Frequency of flute $f = 2 \times \frac{V}{2l}$

$$= 660\ Hz$$

From Doppler effect, observed frequency $f = f_0\left(\frac{V + V_0}{V}\right)$

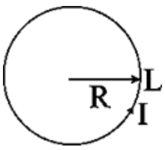
$$= 660\left(\frac{330+10\times\frac{5}{18}}{330}\right)$$

$$= 665\ Hz$$

20. For circular loop, $r = \frac{L}{2\pi}$

Magnetic field at centre $B_2 = \frac{\mu_0 i}{2r}$

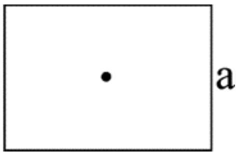
$$B_C = \frac{\mu_0 i\ 2\pi}{2L}$$



For square, magnetic field

$$B_3 = \frac{4\mu_0 i}{4\pi\left(\frac{2}{8}\right)}\sqrt{2}$$

$$\frac{B_2}{B_3} = \frac{2\pi\ \pi\left(\frac{2}{8}\right)}{2L\sqrt{2}}$$



$$= \frac{\pi^2}{8\sqrt{2}}$$

21.

Color	Value
Green	5
Orange	3
Yellow	4

$R = 53 \times 10^4 \pm 58$

22. For particle to move in circle under magnetic field

$qVB = \frac{mv^2}{R} \quad \ldots(\text{i})$

For particle to move in straight line

In both magnetic field and electric field

$qVB = 2E$

$\Rightarrow V = \frac{E}{B} \quad \ldots(\text{ii})$

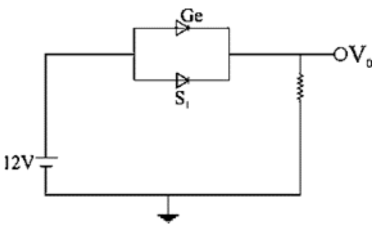
From (i) to (ii)

$$qB = \frac{mv}{R}$$

$$= \frac{m.E}{BR}$$

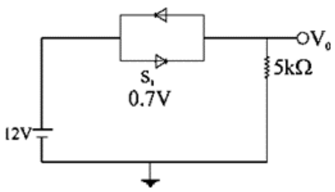
$$\Rightarrow m = \frac{qE^2R}{E} = \frac{1.6 \times 10^{-13} \times 0.5^2 \times 10^{-2}}{0.15} kg = \frac{4}{3} \times 10^{-20} kg$$

23.



$V_0 = 12\,V - V_{G_e}$

$= (12 - 0.3)V$



$= 11.7\ V$

$V_0 = 12V - 0.7V$

$= 11.3V$

$\therefore \Delta V_0 = 11.7 - 11.3 = 0.4V$

25. Let $[T] = [G]^{\alpha}[L]^{\beta}[C]^{\gamma}$

$= [M^{-1}L^3T^{-2}]^9 \cdot [ML^2T^{-1}]^{\beta}[LT^{-1}]^{\gamma}$

$\Rightarrow\ -\alpha + \beta = 0$

$3 \propto +2\beta + \gamma = 0$

$-2\alpha - \beta - \gamma = 1$

On solving we get

$\alpha = \frac{1}{2}, \beta = \frac{1}{2}, \gamma = \frac{5}{2}$

Chemistry

1. Among the alkali metals, only lithium is able to form a stable nitride.

$6Li + N_2 \rightarrow 2Li_3N$

Lithium nitride

2. According to molecular orbital theory

Bond order $= \frac{(e^- \text{ in bonding molecular orbital}) - (e^- \text{ in Anti bonding Molecular orbital})}{2}$

Molecular orbital configuration.

$(A)\ O_2 \Longrightarrow \sigma 1s^2 < \overset{*}{\sigma} 1s^2 < \sigma 2s^2 < \overset{*}{\sigma} 2s^2 < \sigma 2p_x^2 < \pi 2p_y^2 = \pi 2p_z^2 < \overset{*}{\sigma} 2p_y^1 = \overset{*}{\pi} 2p_y^1$

$\text{B. O.} = \frac{6-2}{2} = 2$

$O_2^+ \Longrightarrow \sigma 1s^2 < \overset{*}{\sigma} 1s^2 < \sigma 2s^2 < \overset{*}{\sigma} 2s^2 < \sigma 2p_x^2 < \pi 2p_y^2 = \pi 2p_z^2 < \overset{*}{\pi} 2p_y^1$

$\text{B. O.} = \frac{6-3}{2} = 2.5$

Both are paramagnetic because having unpaired e^-

bond order increase $O_2 \rightarrow O_2^+$

$(B) NO = \sigma 1s^2 < \overset{*}{\sigma} 1s^2 < \sigma 2s^2 < \overset{*}{\sigma} 2s^2 < \sigma 2p_x^2 < \pi 2p_y^2 = \pi 2p_z^2 < \overset{*}{\pi} 2p_y^1 = \overset{*}{\pi} 2p_z^0$

$\text{B.O.} = \frac{6-1}{2} = 2.5$

1 Unpaired e^- so paramagnetic

$NO^+ \Rightarrow \sigma 1s^2 < \overset{*}{\sigma} 1s^2 < \sigma 2s^2 < \overset{*}{\sigma} 2s^2 < \sigma 2px^2 < \pi 2p_y^2 = \pi 2p_z^2$

$\text{B.O.} = \frac{6-0}{2} = 3$

doesn't have unpaired e^- so diamagnetic

$NO \rightarrow NO^+$ {Bond order increases and paramagnetic to diamagnetic charcter change}

$(C)\ O_2 \rightarrow \sigma 1s^2 < \overset{*}{\sigma} 1s^2 < \sigma 2s^2 < \overset{*}{\sigma} 2s^2 < \sigma 2p_x^2 < \pi 2p_y^2 = \pi 2p_z^2 < \overset{*}{\pi} 2p_y^1 = \overset{*}{\pi} 2p_y^1$

$\text{B.O.} = \frac{6-2}{2} = 2$

Having 2 – unpaired e^- so paramagnetic

$O_2^- \rightarrow \sigma 1s^2 < \overset{*}{\sigma} 1s^2 < \sigma 2s^2 < \overset{*}{\sigma} 2s^2 < \sigma 2s^2 < \pi 2p_y^2 = \pi 2p_z^2 < \overset{*}{\pi} 2p_y^2 = \overset{*}{\pi} 2p_y^1$

$\text{B.O.} = \frac{6-3}{2} = 1.5$

Having 1- unpaired e^- so paramagnetic bond order decrease

$(D)\ N_2 = \sigma 1s^2 < \overset{*}{\sigma} 1s^2 < \sigma 2s^2 < \overset{*}{\sigma} 2s^2 < \pi 2py^2 = \pi 2p_y^2 < \sigma 2p_x^2$

$\text{B.O.} = \frac{6-0}{2} = 3$

No unpaired e^- so diamagnetic

$N_2^+ \rightarrow \sigma 1s^2 < \overset{*}{\pi} 1s^2 < \sigma 2s^2 < \overset{*}{\sigma} 2s^2 < \pi 2p_y^2 = \pi 2p_z^2 < \sigma 2p_x^1$

$\text{B.O.} = \frac{5-0}{2} = 2.5$

Having one unpaired e^- so paramagnetic

Bond angle increases N_2 to N_2^+

3.

$\lambda_{\text{emission}}$	Blue < Green < Red
	$L_1 > L_2 > L_3$
Δ_{emission}	$L_1 > L_2 > L_3$
$\Delta_{\text{Absorption}}$	$L_1 > L_2 > L_3$

Strength of ligand $\propto \Delta_{\text{Absorption}}$

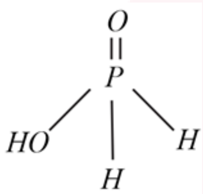
4.

Elements	V	Cu	Zn	Fe
Enthalpy of atomization	$515\ KJ/mol$	$339\ KJ/mol$	$216\ KJ/mol$	$416\ KJ/mol$

5. According to Hardy's -schulz rule, greater the valency of the active ion or flocculating ion greater will be its coagulating power. The cation having more $+ve$ charge more will coagulating power.

Hence Al^{+3} having maximum coagulating power

6.



The hydrogen atoms in OH bond are ionizable and are acidic whereas the $P-H$ bonds have reducing property. There are two such $P-H$ bonds in Hypophosphours aicd.

7. Crystal structure is FCC so number of atom per unit all $(N) = 4$

Edge length $= x.A^o = x10^{-8}cm^3$

Density $\frac{zm}{N_A a^3} = \frac{4\times63.5}{(6.023\times10^{23})(x\times10^{-8})^3} gm/cm^3$

$$= \frac{4 \times 63.5}{(6.023) \times x^3 \times 10^{-1}} = \frac{421.716}{x^3} gm/cm^3$$

8. $\Delta G = \Delta G^o + RTlnQ$

At equilibrium $\Delta G = 0, Q = k$

$0 = \Delta G^o + RT\ lnK$

$\Delta G^o = -RT\ lnK\ (\therefore \Delta G^o = -nFE^o)$

So $-nFE^o = -RT\ lnk$

$'n'$ factor for the above cell $= 2$

So $-2 \times 96500 \times 2 = -8 \times 300\ lnk$

$lnk = 160$

$k = e^{160}$

9. $2A$

$2A + B \rightarrow C$

Initial $r_1 = k[A]^x[B]^y = 0.3\ m/\text{sec}$ ...(i)

$r_2 = k[2A]^x[2B]^y = 2.4$ ...(ii)

$r_3 = k[2A]^x[B]^y = 0.6$ ...(iii)

From (i) $\div$ (iii)

$$\left(\frac{1}{2}\right)^x (1)^y = \frac{0.3}{0.6}$$

$$\left(\frac{1}{2}\right)^x = \frac{1}{2}$$

$x = 1$

(ii) $\div$ (iii)

$$\frac{r_2}{r_3} = \frac{2.4}{0.6} = (2)^y$$

$4 = (2)^y$

$y = 2$

The order of reaction w.r.t. $A = 1$ and w.r.t. $B = 2$

Total order of reaction $= 1 + 2 = 3$

10. $C_{57}H_{110}O_6 + \frac{163}{2}O_2 \rightarrow 57CO_2 + 55H_2O$

Molecular weight of $C_{57}H_{110}O_6 = 890$

Moles of $C_{57}H_{110}O_6 = \frac{\text{weight(gm)}}{\text{Molecular weight}}$

$$= \frac{445}{890} = \frac{1}{2}$$

According to stoichiometry

$\because$ 1 *mole* of $C_{57}H_{110}O_6$ gives $= 55$ *mole* of O_2

$\because \frac{1}{2}$ *mole* of $C_{57}H_{110}O_6$ gives $= 55 \times \frac{1}{2}$ *mole* of H_2O

Wt. of H_2O =mole of $H_2O \times M$ Wt. of H_2O

$= \frac{1}{2} \times 55 \times 18 = 495\ gm$

11. $A_2 + B_2 \rightleftharpoons 2AB\ k_1$

$$k_1 = \frac{[AB]^2}{[A_2][B_2]}$$

$6AB \rightleftharpoons 3A_2 + 3B_2$

$k_2 = \frac{[A_2]^3[B_2]^3}{[AB]^6}$

So, $\frac{1}{k_1^3} = k_2$

12. Cl^- ions is cause of permanent hardness & HCO_3^- is cause of temporary Hardness.

13. Strong field ligands have more $CFSE$ value while weak field ligand have less $CFSE$ value.
CN^- is strongest ligand among all so $[CO(CN)_6]^{-3}$ has maximum $CFSE$ & value.

14. Molecular weight of ethylene glycol
$(C_2H_6O_2) = 62$
Moles of ethylene glycol $= \frac{Wt}{M.Wt}$
$= \frac{62}{62} = 1$
$\Delta T_f = i \times k_f \times m$
$i = 1$ for ethylene glycol
$\Delta T_f = 1 \times 8.6 \times \frac{1}{\frac{250}{1000}}$
$\Delta T_f = 1.86 \times 4 = 7.44$
As $\Delta T_f = 10$ it implies that some amount of water has frozen and this has led a greater depression of freezing point
$10 = 1.86 \times \frac{\frac{62}{62}}{\frac{Wt\ of\ H_2O}{1000}}$
$W_{H_2O} = 186$
So weight of water freeze $= 250 - 186$
$= 64\ gm$

15. $\Delta G = \Delta H - T\Delta S$
For reaction feasibility $\Delta G = -ve$
So Zn can be extracted by ZnO
By using Al at 500^oC

16.

(A) 

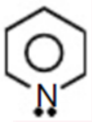
Cyclic conjugation and by Huckel rule $= 4\pi e^-$

So Anti aromatic

(B) 

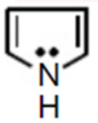
Cyclic conjugation and by Huckel rule $= 6\pi e^-$

So aromatic

(C) 

Cyclic conjuration lp of $-N$ is not involved in delocalization

Having $= 6\pi e^-$ Aromatic

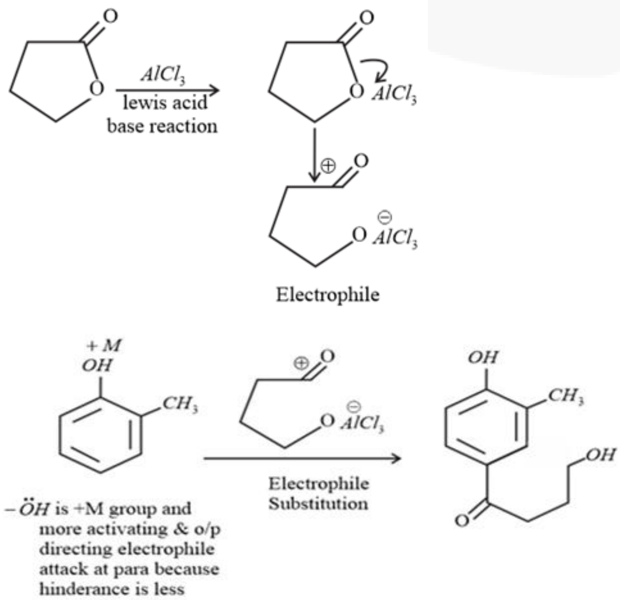
(D) 

Cyclic conjugation

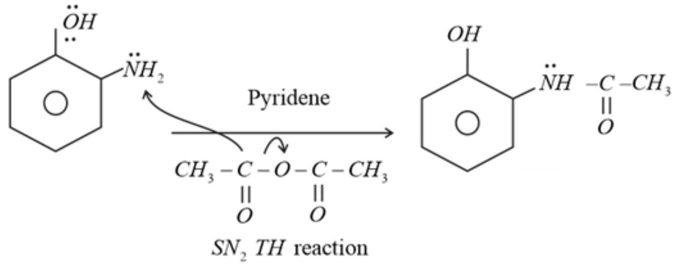
Lone pair involved in Aromatically

Having $= 6\pi e^-$ Aromatic

17.



18.



$\ddot{N}H_2 > \ddot{O}H$ is the order of nucleophilicity

19. Basic character $\propto +M \propto +H \propto +I$

In case of polar protic solvent two factor decide the basic character according to $+I$ effect of $-CH_3$

$3^o > 2^o > 1^o$ Amine

& According to H –bonding the basic character is $1^o > 2 > 3^o$

& the resultant basic character is determined by taking the above two factors into account.

So order will be $2^o > 1^o > 3^o$

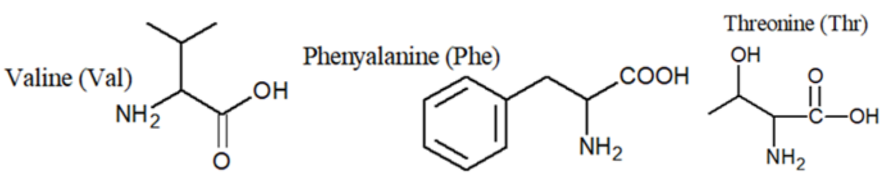
In aromatic & Aliphatic: Aliphatic Amine are more basic than aromatic because the lone pair of N is involved in delocalization.

So basic character is in the order $2 > 1^o > 3^o >$ Aromatic Amine

So (iii) > (i) > (iv) > (ii)

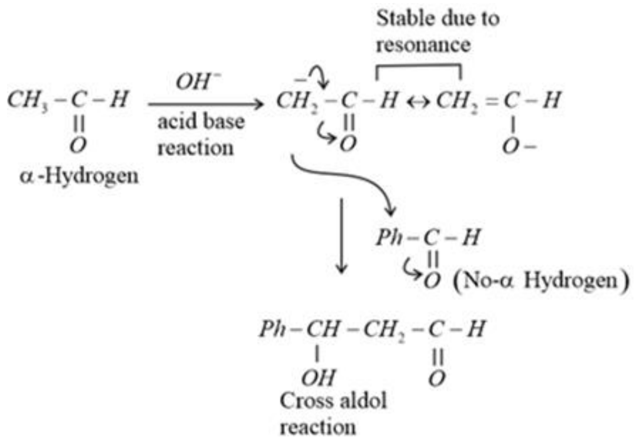
20. Normal clean rain has a pH value of around 5.6, which is slightly acidic.

21.



22. Methemoglobinemia disease occur due to NO_3^-, when it's concentration is higher than 50 ppm in water.

23.



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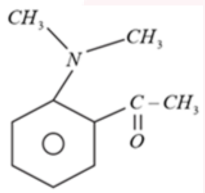

Ketones with a methyl group gives positive iodoform test as well as 2, 4 DNP: Test which is an identification for carbonyl groups. Azo dye: For Azo dye H should be present with $-N$, or there should be a primary and secondary amines

25.