Mathematics

1. \[ \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)^2} \, dx \]
   \[= \int \frac{5x^8 + 7x^6}{x^{14} \left( \frac{1}{x^5} + \frac{1}{x^7} + 2 \right)} \, dx \]
   \[= \int \frac{\frac{5}{x^6} + \frac{7}{x^8}}{\left( \frac{1}{x^5} + \frac{1}{x^7} + 2 \right)^2} \, dx \]
   Put \( \frac{1}{x^5} + \frac{1}{x^7} + 2 = t \)
   \[\Rightarrow \left( -\frac{5}{x^6} - \frac{7}{x^8} \right) dx = dt \]
   \[\therefore - \int \frac{dt}{t^2} = \frac{1}{t} + c \]
   \[= \frac{x^7}{x^2 + 1 + 2x^7} + c \]

2. \( D = 121 - 24 \alpha \)
   for rotational roots \( D \) must be perfect square.
   \[\therefore \alpha = 3, 4, 5 \]
   \[\therefore \text{number of values of } \alpha = 3 \]

3. \[ \left( \frac{1-t^6}{1-t} \right)^3 = (1-t^6)^3(1-t)^{-3} \]
   \[= (1-t^{18} - 3t^6 + 3t^{12})(1-t)^{-3} \]
   \[\therefore \text{Co-efficient of } t^4 = 1 \times \binom{3+4-1}{4} = 6 \binom{4}{4} = 15 \]

4. Given \( \sum_{i=1}^{n}(x_i^2 + 2x_i + 1) = 9n \) ......(i)
   \( \sum_{i=1}^{n}(x_i^2 - 2x_i + 1) = 5n \) ........(ii)
   For \((i) + (ii)\) we get
   \[\sum_{i=1}^{n}(2x_i^2 + 2) = 14n \]
   \[\Rightarrow \sum_{i=1}^{n}x_i^2 = 6n \]
   \[\sum_{i=1}^{n}x_i = n \]
   \[\therefore \text{Variance, } r^2 = E(x^2) - \left( E(x) \right)^2 \]
   \[= \frac{6n}{n} - \left( \frac{n}{n} \right)^2 = 6 - 1 = 5 \]
   \[\therefore \text{Standard deviation, } \sigma = \sqrt{5} \]
5. 

<table>
<thead>
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<th>0, 1, 3</th>
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\[ \therefore \text{total number formed less than } 7000 = 3 \times 5 \times 5 \times 5 = 375 \]
\[ \therefore \text{total natural number formed less than } 7000 = 375 - 1 = 374 \]
(number zero has been excluded)

6. Let \( A \) is event that 2nd drawn ball is red.
\[ \therefore P(A) = P(R) \times P(A/R) + P(G) \times P(A/G) \]
\[ \begin{align*}
5 & \times \frac{4}{7} + \frac{2}{7} \times \frac{6}{7} \\
& = \frac{32}{49}
\end{align*} \]

7. 
\[ t_n = \frac{3n(1^2 + 2^2 + 3^2 + \cdots + n^2)}{2n+1} = \frac{3n(n+1)(2n+1)}{6(2n+1)} \]
\[ = \frac{1}{2} (n^3 + n^2) \]
\[ \because S_{15} = \sum_{n=1}^{15} t_n = \sum_{n=1}^{15} \frac{1}{2} (n^3 + n^2) \]
\[ = \frac{1}{2} \times \left[ \left( \frac{15 \times 16}{2} \right)^2 + \frac{15 \times 16 \times 31}{6} \right] \]
\[ = 7200 + 620 \]
\[ = 7820 \]

8. \( Z_0 = w \) or \( w^2 \)
When \( Z_0 = w \)
\[ Z = 3 + 6i \times w^{81} - 3i \times w^{93} \]
\[ = 3 + 6i - 3i = 3 + 3i \]
When \( Z_0 = w^2 \)
\[ Z = 3 + 6i \times (w^2)^{81} - 3i \times (w^2)^{93} \]
\[ = 3 + 6i - 3i = 3 + 3i \]
\[ \therefore \arg(z) = \tan^{-1} \left( \frac{3}{3} \right) = \frac{\pi}{4} \]
9. Since \((\vec{a} + \vec{b})\) is perpendicular to \(\vec{c}\), hence \((\vec{a} + \vec{b}) \cdot \vec{c} = 0\)
\[
\Rightarrow \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{c} = 0
\]
\[
\Rightarrow 5 + 1 + 2 + 5b_1 + b_2 + 2 = 0
\]
\[
\Rightarrow 5b_1 + b_2 = -10 \ldots (i)
\]
Given, \(\frac{\vec{b} \cdot \vec{a}}{|\vec{a}|} = |\vec{a}|\)
\[
\Rightarrow b_1 + b_2 + 2 = 4
\]
\[
\Rightarrow b_1 + b_2 = 2 \ldots \ldots (ii)
\]
Solving (i) & (ii), \(b_1 = -3, b_2 = 5\)
\[
\therefore |\vec{b}| = \sqrt{9 + 25 + 2} = 6
\]

10. \[
\int_{0}^{\pi/3} \frac{\tan x}{\sqrt{2k} \sec x} \, dx
\]
\[
= \frac{1}{\sqrt{2k}} \int_{0}^{\pi/3} \frac{\sin x}{\cos x} \, dx
\]
Put \(t = \cos x\)
\[
dt = -\sin x \, dx, \text{ also } x = 0, t = 1; x = \frac{\pi}{3}, t = \frac{1}{2}
\]
\[
\Rightarrow \frac{1}{\sqrt{2k}} \left[ \frac{1}{2} - \frac{1}{\sqrt{2}} \right]^{1/2} \, dt = \frac{1}{\sqrt{2k}} \times 2 \left[ t^{1/2} \right]^{1}_{1/2}
\]
\[
= \frac{\sqrt{2}}{\sqrt{k}} \left( 1 - \frac{1}{\sqrt{2}} \right) = \frac{\sqrt{2} - 1}{\sqrt{k}}
\]
Given \(\frac{\sqrt{2} - 1}{\sqrt{k}} = 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2} - 1}{\sqrt{2}}\)
\[
\therefore k = 2
\]

11. \[
sin x - \sin 2x + \sin 3x = 0
\]
\[
\Rightarrow \sin x + \sin 3x - \sin 2x = 0
\]
\[
\Rightarrow 2 \sin 2x \cdot \cos x - \sin 2x = 0
\]
\[
\Rightarrow \sin 2x = 0 \text{ or } \cos x = \frac{1}{2}
\]
\[
\Rightarrow 2x = 0 \text{ or } x = \frac{\pi}{3}
\]
\[
\Rightarrow x = 0 \text{ or } x = \frac{\pi}{3}
\]
\[
\therefore \text{ number of solutions } = 2
\]
12. Area enclosed
\[ = \int_{1}^{0} (-x^2 + 1) \, dx + \int_{0}^{1} (x^2 + 1) \, dx \]
\[= \left[ -\frac{x^3}{3} + x \right]_{-1}^{0} + \left[ \frac{x^3}{3} + x \right]_{0}^{1} \]
\[= 2 \]

13. \[
\lim_{x \to 0^-} \frac{x (|x|) + |x| \sin |x|}{|x|} \quad \lim_{x \to 0^-} \frac{x(-1-x) \sin(-1)}{-x} \quad \lim_{x \to 0^-} - (1 + x) \sin 1 \quad - \sin 1
\]

14. Given relation is \( f(xy) = f(x) \cdot f(y) \forall x, y \in R \) .....(i)
On putting \( x = y = 0 \), we get \( f(0) = f^2(0) \Rightarrow f(0) = 0, 1 \) but \( f(0) \neq 0 \)
\( \Rightarrow f(0) = 1 \),
Now if we put \( y = 0 \) in (i), we get
\( f(x) = 1 \)
Hence \( \frac{dy}{dx} = 1 \Rightarrow y = x + c \)
\( \Rightarrow y = x + \frac{1}{2} \) (since \( y(0) = \frac{1}{2} \))
\( \Rightarrow y \left( \frac{1}{4} \right) + y \left( \frac{3}{4} \right) = \left( \frac{1}{4} + \frac{1}{2} \right) + \left( \frac{3}{4} + \frac{1}{2} \right) = 2 \).
15. Let the coordinates of \( C \) be \((t^2, 2t)\)

Area of \( \Delta ABC \) = \frac{1}{2} \begin{vmatrix} t^2 & 2t & 1 \\ 9 & 6 & 1 \\ 4 & -4 & 1 \end{vmatrix}

\[ \Delta = 5(6 + t - t^2) \]
\[ \frac{d\Delta}{dt} = 1 - 2t = 0 \Rightarrow t = \frac{1}{2} \]
\[ \Rightarrow c \left( \frac{1}{4}, 1 \right) \]

16. As we know, if two circles intersect each other, then \(|r_1 - r_2| < c_1 c_2 < r_1 + r_2 \) .......(i)

Now for the first circle \( c_1(8, 10) \) and \( r_1 = r \)

For the second circle \( c_2(4, 7) \) and \( r_2 = 6 \)

From (i)
\[ |r - 6| < 5 < r + 6 \]
\[ \Rightarrow r \in (1, 11) \]

17. From the given information, equation of hyperbola can be taken as
\[ \frac{x^2}{4} - \frac{y^2}{b^2} = 1, a = 2 \]

Since it passes through \((4, 2)\),
Hence \( b^2 = \frac{4}{3} \)

Now \( e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{1}{3}} = \frac{2}{\sqrt{3}} \)
18. Given \( x = 3 \tan t \) and \( y = 3 \sec t \)

\[ \frac{dx}{dt} = 3 \sec^2 t \quad \text{and} \quad \frac{dy}{dt} = 3 \sec t \tan t \]

\[ \frac{dy}{dx} = \sin t \]

\[ \Rightarrow \frac{d^2y}{dx^2} = \cos t \cdot \frac{dt}{dx} = \cos t \cdot \frac{\cos^2 t}{3} = \frac{\cos^3 t}{3} \]

At \( t = \frac{\pi}{4} \)

\[ \frac{d^2y}{dx^2} = \frac{1}{6\sqrt{2}} \]

19. Given function \( f(x) = \frac{2x}{x-1} \) and domain of \( f(x) \) is \((-\infty, 0)\)

Now \( f'(x) = \frac{-2}{(x-1)^2} < 0 \)

\[ \Rightarrow f \text{ is always decreasing in the given domain.} \]

Clears given function is one-one.

Since codomain \((-\infty, \infty) \neq \text{Range (0, 2)}, \) hence \( f \) is into.

20. Let the first terms of A.P. be \( A \) and common difference be \( d \), then

\[ a = A + 6d \]

\[ b = A = 10d \]

\[ c = A + 12d \]

Given \( a,b,c \) and in G. P. \( \Rightarrow b^2 = ac \)

\[ \Rightarrow (A + 10d)^2 = (A + 6d)(A + 12d) \]

\[ \Rightarrow A^2 + 100d^2 + 20Ad = A^2 + 18Ad + 72d^2 \]

\[ \Rightarrow A = -14d \]

Now \( \frac{a}{c} = \frac{A+6d}{A+12d} = \frac{-14d+6d}{-14d+12d} = \frac{-8d}{-2d} = 4 \)
21. Since are vertex of given triangle is $o$ (origin) and other two $A \& B$ lies on coordinate axes, hence triangle must be a right angled triangle.

Area $= \frac{1}{2} x \cdot y, \ x, y \in I$

As per given condition
\[ \frac{1}{2} x \cdot y = 50 \Rightarrow x \cdot y = 100 \]

Possible ordered pairs of $(x, y)$ when $\Delta$ formed in 1st quadrant are 9 i.e. $(1, 100), (2, 50), (4, 25), (5, 20), (10, 10), (20, 5), (25, 4), (50, 2), (100, 1)$.

$\Delta$ can be formed in any of the quadrants, hence total number of cases $9 \times 4 = 36$.

22. Given roots of $x^2 - mx + 4 = 0$ are distinct and lies in $(1,5)$, Following conditions must be true

(i) $D > 0 \Rightarrow M \in (-\infty, -4) \cup (4, \infty)$
(ii) $f(1) > 0 \Rightarrow M < 5$
(iii) $f(5) > 0 \Rightarrow M < \frac{29}{5}$
(iv) $1 < -\frac{B}{2A} < 5 \Rightarrow 2 < m < 10$

Taking intersection of all conditions, we get $m \in (4, 5)$
23. From the graph of \( y = \sin^{-1}(\sin x) \), it is clear that \( \sin^{-1}(\sin 10) = 3\pi - 10 \)

From the graph of \( y = \cos^{-1}(\cos x) \)
It is clear that \( \cos^{-1}(\cos 10) = 4\pi - 10 \)
Hence \( y - x = (4\pi - 10) - (3\pi - 10) = \pi \)

24. Since given matrix \( A \) is invertible \( \Rightarrow |A| \neq 0 \)

Now \( |A| = \begin{vmatrix} e^t & e^{-t}(\sin t - 2 \cos t) & e^{-t}(-2 \sin t - \cos t) \\ e^t & -e^{-t}(2 \sin t + \cos t) & e^{-t}(\sin t - 2 \cos t) \\ e^t & e^{-t} \cos t & e^{-t} \sin t \end{vmatrix} \)

\[ = e^t.e^{-t}.e^{-t} \begin{vmatrix} 1 & \sin t - 2 \cos t & -2 \sin t - \cos t \\ 1 & 2 \sin t + \cos t & \sin t - 2 \cos t \\ 1 & \cos t & \sin t \end{vmatrix} \]

\[ = 2e^{-t} \begin{vmatrix} 1 & \sin t - 2 \cos t & -2 \sin t - \cos t \\ 0 & \sin t & -\cos t \\ 1 & \cos t & \sin t \end{vmatrix} \]

\[ R_1 \rightarrow R_1 - R_2 \]

\[ = 2e^{-t} \begin{vmatrix} 1 & -2 \cos t & -2 \sin t \\ 0 & \sin t & -\cos t \\ 1 & \cos t & \sin t \end{vmatrix} \]

\[ R_1 \rightarrow R_1 + 2R_3 \]

\[ = 2e^{-t} \begin{vmatrix} 3 & 0 & 0 \\ 0 & \sin t & -\cos t \\ 1 & \cos t & \sin t \end{vmatrix} \]

\[ |A| = 6e^{-t} \neq 0 \ \forall \ t \in \mathbb{R} \]
Hence given matrix is always invertible.
25. DR's of the first line
\[
\begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
1 & -a & 0 \\
0 & c & -1
\end{vmatrix} = a \hat{i} + \hat{j} + c \hat{k}
\]
Hence DR's of the first line are \(a,1,c\).

For DR's of the second line
\[
\begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
1 & 0 & -a' \\
0 & 1 & -c'
\end{vmatrix} = a' \hat{i} + c' \hat{j} + \hat{k}
\]
Hence DR's of the 2\(^{nd}\) line = \(a',c',1\)

Since both the lines are perpendicular,
\[a.a' + c' + c = 0\]

26. To find equation of \(BC\), first we will find coordinates of \(B\) and \(C\).

Equation of \(BB^1\)
\[
\begin{align*}
y - 1 &= \frac{-2}{3} (x - 1) \\
\Rightarrow 2x + 3y &= 5 \quad \text{(i)}
\end{align*}
\]

Point of intersection of \(AB\) and \(BB^1\) \(\Rightarrow B\left(\frac{35}{2},-10\right)\)

Similarly point \(c\left(-13,-\frac{33}{2}\right)\)

Equation of \(BC\) \(y + 10 = \frac{13}{61}\left(x - \frac{35}{2}\right)\)

27. Given functional in equality is
\[
|f(x) - f(y)| \leq 2 |x - y|^{3/2}
\]
Put \(x = y + h\), we get
\[
|f(y + h) - f(y)| \leq 2 |h|^{3/2}
\]
\[
\Rightarrow \left| \frac{f(y + h) - f(y)}{h} \right| \leq 2 |h|^{1/2}
\]
\[ \lim_{h \to 0} \left| \frac{f(y + h) - f(y)}{h} \right| \leq \lim_{h \to 0} 2|h|^{1/2} \]

\[ |f'(y)| \leq 0 \]

\[ |f'(y)| = 0 \]

\[ f'(y) = 0 \]

\[ f(y) = c \]

\[ f(y) = 1 \quad \text{(since } f(0) = 1) \]

Now \[ \int_0^1 f^2(x) \, dx = \int_0^1 1 \, dx = 1 \]

**Physics**

1. From conservation of mechanical energy

\[ m g \frac{l}{2} \sin 30 = \frac{1}{2} I \omega^2 \]

\[ m g \frac{l}{2} \sin 30 = \frac{1}{2} ml^2 \frac{1}{3} \omega^2 \]

\[ \omega = \sqrt{\frac{3g}{2l}} = \sqrt{\frac{3 \times 10}{2 \times 50 \times 10^{-2}}} = \sqrt{30} \text{ rad/s} \]

2. 75 sec \( \frac{4}{10} \). Application (i) Velocity

\[ x = A \cos \omega t \]

\[ y = A \sin \omega t \]

\[ z = A \omega t \]

\[ v_x = -A \omega \sin \omega t \]

\[ v_y = A \omega \cos \omega t \]

\[ v_z = A \omega \]

\[ v = \sqrt{v_x^2 + v_y^2 + v_z^2} \]

\[ = \sqrt{(-A \omega \sin \omega t)^2 + (A \omega \cos \omega t)^2 + (A \omega)^2} \]

\[ = A \omega \sqrt{2} \]
3. 60 sec $\frac{3}{10}$, Application (i) Transformer

(ii) Efficiency of a transformer

Efficiency $y = 90$

$\Rightarrow 0.9 \times \text{power in} = \text{Power out}$

$0.9 \times 2300 \times 5 = 230 \times I_{out}$

$I_{out} = 45 \text{ A}$

4. 75 sec $\frac{4}{10}$, Application, (i) Capacitance with dielectric

![Diagram of capacitors](image)

\[ C_1 = \frac{\varepsilon_0 \frac{A}{d} K_1}{\frac{d}{2}} = \frac{\varepsilon_0 A K_1}{d} \]

\[ C_2 = \frac{\varepsilon_0 A K_2}{d} \]

\[ C_3 = \frac{\varepsilon_0 A K_3}{d} \]

\[ C_4 = \frac{\varepsilon_0 A K_4}{d} \]

\[ C_{eq} = \frac{(C_1 + C_2)(C_3 + C_4)}{C_1 + C_2 + C_3 + C_4} \]

\[ \frac{\varepsilon_0 A K_{eq}}{d} = \frac{\varepsilon_0 A (K_1 + K_2)(K_3 + K_4)}{d (K_1 + K_2 + K_3 + K_4)} \]

\[- K_{eq} = \frac{(K_1 + K_2)(K_3 + K_4)}{K_1 + K_2 + K_3 + K_4} \]

5. Electric field due to $\sqrt{10} \mu c$
\[ \vec{E}_1 = \frac{1}{4\pi \varepsilon_0} \frac{\sqrt{10} \times 10^{-6}}{r_1^3} \vec{r}_1 \]
\[ = \frac{1}{4\pi \varepsilon_0} \frac{\sqrt{10} \times 10^{-6}}{(\sqrt{10})^3} (-i + 3j) \]

Similarly electric field due to \(-25 \mu C\)
\[ \vec{E}_2 = \frac{1}{4\pi \varepsilon_0} \frac{25 \times 10^{-6}}{(5)^3} (4i - 3j) \]

Net electric field
\[ \vec{E} = 9 \times 10^9 \times 10^{-6} \left( \frac{-i + 3j}{10} + \frac{4i - 3j}{5} \right) \]
\[ = 9 \times 10^3 \left( \frac{7i}{10} - \frac{3j}{10} \right) \]
\[ = (63i - 27j) \times 10^2 \frac{N}{C} \]

6. \[ E_1 + \left( \frac{-GmM}{R} \right) = \frac{GmM}{R+h} \]
\[ E_2 = \frac{1}{2} mV^2 \text{ and } \frac{mu^2}{R+h} = \frac{GmM}{(R+h)^2} \]
\[ \Rightarrow E_2 = \frac{1}{2} \left( \frac{GmM}{R+h} \right) \]
\[ E_1 = E_2 \Rightarrow \frac{GmM}{R} - \frac{GmM}{R+h} = \frac{GmM}{2(R+h)} \]
\[ \frac{h}{R(R+h)} = \frac{1}{2(R+h)} \]
\[ h = \frac{R}{2} = 3.2 \times 10^3 \text{ km} \]

7. \[ B = B_0 \left[ \sin(3.14 \times 10^7 Ct) + \sin(6.28 \times 10^7 Ct) \right] \]

8. \[ \rho(r) = \frac{A}{r^2} e^{-\frac{2r}{a}} \]

Charge enclosed between \( r \) and \( r + dr \) is
\[ dq = \rho(r) 4\pi r^2 dr \]
To get total charge 'Q',

\[ Q = \int_{0}^{R} \frac{A}{r^2} e^{-2r/a} 4\pi r^2 dr \]

\[ = -4\pi A \frac{a}{2} \left[e^{-2r/a}\right]^{R}_{0} \]

\[ Q = -4\pi A \frac{a}{2} \left[e^{-2R/a} - 1\right] \]

\[ R = -\ln \left[1 - \frac{Q2}{4\pi Aa}\right] \frac{a}{2} \]

\[ = \frac{a}{2} \ln \left[\frac{2\pi QA}{2\pi Aa - Q}\right] \]

9.

\[ W = Q_1 - Q_2 = Q_2 - Q_3 \]

\[ \frac{Q_1}{Q_2} + \frac{Q_3}{Q_2} = 2 \]

\[ \frac{T_1}{T_2} + \frac{T_3}{T_2} = 2 \]

\[ T_2 = \frac{T_1 + T_3}{2} = 500K \]

10. \[ S = 3t^2 + 5 \]

Velocity \( V = 6t + 0 \)

From work energy theorem

Work done = \( \Delta KE \)

\[ = \frac{1}{2} \times 2 \times (6 \times 5)^2 - \frac{1}{2} \times 2 \times (6 \times 0)^2 \]

\[ = 900 J \]
\[ T \cos 45 = mg \]
\[ T \sin 45 = F \]
\[ \tan 45 = \frac{F}{mg} \]
\[ F = mg = 100 \, N \]

12. Current through \( R_4 \) (as well as \( R_3 \))

is \( i_1 = \frac{V}{R} = \frac{5}{500} = 10 \, mA \)

Potential drop across 400 \( \Omega \) is

\[ V_{400} = 18 - 10 \times 10^{-3} \times 600 \]
\[ = 12 \, V \]

Current through battery

\[ I = \frac{12}{400} = 30 \, mA \]

\( \therefore \) Current through \( R_2 \) \( i_2 = 30 - 10 \)
\[ = 20 \, mA \]

\( \therefore \) \( R_2 = \frac{6}{20 \times 10^{-3}} = 300 \, \Omega \)

13.

\[ 90 - \alpha + \theta + x \]
\[ x = 90 + \alpha - \theta \]
\[ 90 + \alpha - \theta = \theta \]
\[ \text{and} \quad 90 - \alpha = \theta \]
\[ 180 - \theta = 2\theta \]
\[ \theta = 60^\circ \]

14. \[ \frac{1}{2} a_1 t_0^2 = \frac{1}{2} a_2 (t_0 + t)^2 \]
\[ v_1 = a_1 t_0 \quad v_2 = a_2 (t_0 + t) \]
\[ v = v_1 - v_2 = (a_1 - a_2)t_0 - a_2 t \]
\[ \sqrt{\frac{a_1}{a_2}} t_0 = t_0 + t \]
\[ t_0 = \frac{t}{\sqrt{\frac{a_1}{a_2}}} - 1 \]
\[ v = (a_1 - a_2) \frac{t}{\sqrt{\frac{a_1}{a_2}}} - a_2 t \]
\[ = t \left( \frac{a_1 \sqrt{a_2}}{\sqrt{a_1} - \sqrt{a_2}} - \frac{a_2 \sqrt{a_2}}{\sqrt{a_1} - \sqrt{a_2}} - a_2 \right) \]
\[ = t \left( \frac{a_1 \sqrt{a_2} - a_2 \sqrt{a_1}}{\sqrt{a_1} - \sqrt{a_2}} \right) \]
\[ = t(\sqrt{a_1} \sqrt{a_2}) \]

15. \[ KE = \frac{1}{2} kx^2 \quad PE = \frac{1}{2} K(A^2 - x^2) \]
\[ \frac{1}{2} kx^2 = \frac{1}{2} K(A^2 - x^2) \]
\[ x^2 = \frac{A^2}{2} \]
\[ x = \frac{A}{\sqrt{2}} \]

16. \[ T = 2\pi \sqrt{\frac{I}{C}} \]
\[ f = \frac{1}{2\pi} \sqrt{\frac{C}{I}} \]
\[ f_{\text{initial}} = \frac{1}{2\pi} \sqrt{\frac{C_3}{MC^2}} \]
\[ f_{\text{final}} = \frac{1}{2\pi} \sqrt{\frac{C}{\left(\frac{MC^2}{3} + 2mL^2/4\right)}} \]
\[ 0.2 \frac{1}{2\pi} \sqrt{\frac{3C}{ML^2}} = \frac{1}{2\pi} \sqrt{\frac{C}{\frac{MC^2}{3} + \frac{2MC^2}{4}}} \]
\[ \frac{3}{M} = \frac{12}{4M + 6M} \]

\[ 4M = 0.16M + 0.24M \]

\[ \frac{m}{M} = \frac{3.84}{0.24} = 16 \]

17. \[ V_{rms} = \sqrt{\frac{3RT}{M}} \]

\( T \) must be increased 4 times to get \( V_{rms} \) doubled.

\[ \Delta Q = nC_v\Delta T \]

\[ = \frac{15}{28} \times \frac{5}{2} \times 8.314 \times (1200 - 300) \]

\[ = 10021.3 J = 10 kJ \]

19. Frequency of flute \( f = 2 \times \frac{V}{2l} \)

\[ = 660 \text{ Hz} \]

From Doppler effect, observed frequency \( f = f_0 \frac{(V + V_0)}{V} \)

\[ = 660 \left( \frac{330 + 10 \times \frac{5}{18}}{330} \right) \]

\[ = 665 \text{ Hz} \]

20. For circular loop, \( r = \frac{L}{2\pi} \)

Magnetic field at centre \( B_2 = \frac{\mu_0 i}{2r} \)

\[ B_C = \frac{\mu_0 i}{2L} \]

For square, magnetic field

\[ B_3 = \frac{4\mu_0 i}{4\pi \left( \frac{2}{8} \right) \sqrt{2}} \]

\[ B_2 = \frac{2\pi \pi \left( \frac{2}{8} \right)}{2L \sqrt{2}} \]
\[ = \frac{\pi^2}{8 \sqrt{2}} \]

21. **Color** | **Value**
---|---
Green & 5 
Orange & 3 
Yellow & 4 

\[ R = 53 \times 10^4 \pm 58 \]

22. For particle to move in circle under magnetic field
\[ qVB = \frac{mv^2}{R} \ldots (i) \]

For particle to move in straight line
In both magnetic field and electric field
\[ qVB = 2E \]
\[ \Rightarrow V = \frac{E}{B} \ldots (ii) \]

From (i) to (ii)
\[ qB = \frac{mv}{R} \]
\[ = \frac{mE}{BR} \]
\[ \Rightarrow m = \frac{qE^2R}{E} = \frac{1.6 \times 10^{-13} \times 0.5^2 \times 10^{-2}}{0.15} kg = \frac{4}{3} \times 10^{-20} kg \]

23. \[ V_0 = 12V - V_{Ge} \]
\[ V = (12 - 0.3)V \]
\[ = 11.7 V \]
\[ V_0 = 12V - 0.7V \]
\[ = 11.3V \]
\[ \therefore \Delta V_0 = 11.7 - 11.3 = 0.4V \]

25. Let \([T] = [G]^x[L]^y[C]^z\]
\[ = [M^{-1}L^3T^{-2}]^9 \cdot [ML^2T^{-1}]^\beta [LT^{-1}]^\gamma \]
\[ \Rightarrow -\alpha + \beta = 0 \]
\[ 3 \alpha + 2\beta + \gamma = 0 \]
\[ -2\alpha - \beta - \gamma = 1 \]
On solving we get
\[ \alpha = \frac{1}{2}, \beta = \frac{1}{2}, \gamma = \frac{5}{2} \]

Chemistry

1. Among the alkali metals, only lithium is able to form a stable nitride.

\[ 6Li + N_2 \rightarrow 2Li_3N \]

Lithium nitride

2. According to molecular orbital theory

Bond order: \[ \frac{(e^- \text{ in bonding molecular orbital}) - (e^- \text{ in Anti bonding Molecular orbital})}{2} \]

Molecular orbital configuration.

\( A \): \( O_2 \)  \[ \sigma 1s^2 < \sigma 1s^2 < \pi 2s^2 < \pi 2p_x^2 < \pi 2p_y^2 = \pi 2p_z^2 < \sigma 2p_y^1 = \sigma 2p_y^1 \]

B.O. = \[ \frac{6 - 2}{2} = 2 \]

\( O_2^+ \)  \[ \sigma 1s^2 < \sigma 1s^2 < \sigma 2s^2 < \sigma 2p_x^2 < \pi 2p_y^2 < \pi 2p_z^2 < \pi 2p_y^1 \]

B.O. = \[ \frac{6 - 3}{2} = 2.5 \]

Both are paramagnetic because having unpaired \( e^- \)
bond order increase $O_2 \rightarrow O_2^+$

\[(B) NO = \sigma 1s^2 < \sigma 1s^2 < \sigma 2s^2 < \sigma 2p_x^2 < \sigma 2p_y^2 < \pi 2p_x^2 < \pi 2p_y^2 = \pi 2p_z^0\]

B.O. = $\frac{6 - 1}{2} = 2.5$

1 Unpaired $e^-$ so paramagnetic

$NO^+ \Rightarrow \sigma 1s^2 < \sigma 1s^2 < \sigma 2s^2 < \sigma 2p_x^2 < \pi 2p_y^2 = \pi 2p_z^0$

B.O. = $\frac{6 - 0}{2} = 3$

doesn’t have unpaired $e^-$ so diamagnetic

$NO \rightarrow NO^+$ {Bond order increases and paramagnetic to diamagnetic character change}

\[(C) O_2 \rightarrow \sigma 1s^2 < \sigma 1s^2 < \sigma 2s^2 < \sigma 2p_x^2 < \sigma 2p_y^2 < \pi 2p_x^2 < \pi 2p_y^2 = \pi 2p_z^1\]

B.O. = $\frac{6 - 2}{2} = 2$

Having 2 – unpaired $e^-$ so paramagnetic

$O_2^- \rightarrow \sigma 1s^2 < \sigma 1s^2 < \sigma 2s^2 < \sigma 2p_x^2 < \sigma 2p_y^2 < \sigma 2p_z^2 < \pi 2p_x^2 < \pi 2p_y^2 = \pi 2p_z^0$

B.O. = $\frac{6 - 3}{2} = 1.5$

Having 1- unpaired $e^-$ so paramagnetic bond order decrease

\[(D) N_2 = \sigma 1s^2 < \sigma 1s^2 < \sigma 2s^2 < \sigma 2p_y^2 < \pi 2p_y^2 = \pi 2p_z^0 < \sigma 2p_x^1\]

B.O. = $\frac{6 - 0}{2} = 3$

No unpaired $e^-$ so diamagnetic

$N_2^+ \rightarrow \sigma 1s^2 < \pi 1s^2 < \sigma 2s^2 < \sigma 2p_y^2 < \pi 2p_y^2 = \pi 2p_z^2 < \sigma 2p_x^1$

B.O. = $\frac{5 - 0}{2} = 2.5$

Having one unpaired $e^-$ so paramagnetic

Bond angle increases $N_2$ to $N_2^+$

3. 

$\lambda_{\text{emission}}$ Blue < Green < Red

$L_1 > L_2 > L_3$

$\Delta_{\text{emission}}$ $L_1 > L_2 > L_3$

$\Delta_{\text{Absorption}}$ $L_1 > L_2 > L_3$

Strength of ligand $\propto \Delta_{\text{Absorption}}$
4. Elements

<table>
<thead>
<tr>
<th></th>
<th>V</th>
<th>Cu</th>
<th>Zn</th>
<th>Fe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enthalpy of atomization</td>
<td>515 KJ/mol</td>
<td>339 KJ/mol</td>
<td>216 KJ/mol</td>
<td>416 KJ/mol</td>
</tr>
</tbody>
</table>

5. According to Hardy’s -schulz rule, greater the valency of the active ion or flocculating ion greater will be its coagulating power. The cation having more +ve charge more will coagulating power.

Hence $Al^{+3}$ having maximum coagulating power.

6. The hydrogen atoms in $OH$ bond are ionizable and are acidic whereas the $P-H$ bonds have reducing property. There are two such $P-H$ bonds in Hypophosphhous acid.

7. Crystal structure is FCC so number of atom per unit all ($N$) = 4

Edge length = $x \cdot A^0 = x \times 10^{-8} cm^3$

Density $\frac{zn}{{NA}a^3} = \frac{4 \times 63.5}{(6.023 \times 10^{23})(x \times 10^{-8})^3} gm/cm^3$

$= \frac{4 \times 63.5}{(6.023) \times x^3 \times 10^{-1}} = \frac{421.716}{x^3} gm/cm^3$

8. $\Delta G = \Delta G^o + RT \ln Q$

At equilibrium $\Delta G = 0$, $Q = k$

$0 = \Delta G^o + RT \ln K$

$\Delta G^o = -RT \ln K$ (∴ $\Delta G^o = -nFE^o$)

So $-nFE^o = -RT \ln k$

'\text{'n'}$ factor for the above cell = 2

So $-2 \times 96500 \times 2 = -8 \times 300 \ln k$

$\ln k = 160$

$k = e^{160}$
9. \(2A + B \rightarrow C\)

Initial \(r_1 = k[A]^x[B]^y = 0.3 \text{ m/sec} \)  \(\ldots \) (i)

\(r_2 = k[2A]^x[2B]^y = 2.4 \)  \(\ldots \) (ii)

\(r_3 = k[2A]^x[B]^y = 0.6 \)  \(\ldots \) (iii)

From (i) ÷ (iii)

\[
\left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^y = \frac{0.3}{0.6}
\]

\[
\frac{x}{y} = \frac{1}{2}
\]

\(x = 1\)

(ii) ÷ (iii)

\[
\frac{r_2}{r_3} = \frac{2.4}{0.6} = (2)^y
\]

\(4 = (2)^y\)

\(y = 2\)

The order of reaction w.r.t. \(A = 1\) and w.r.t. \(B = 2\)

Total order of reaction = 1 + 2 = 3

10. \( C_{57}H_{110}O_6 + \frac{163}{2}O_2 \rightarrow 57CO_2 + 55H_2O\)

Molecular weight of \(C_{57}H_{110}O_6 = 890\)

Moles of \(C_{57}H_{110}O_6 = \frac{\text{weight(gm)}}{\text{Molecular weight}}\)

\[
= \frac{445}{890} = \frac{1}{2}
\]

According to stoichiometry

\(\because 1 \text{ mole of } C_{57}H_{110}O_6 \text{ gives } = 55 \text{ mole of } O_2\)

\(\because \frac{1}{2} \text{ mole of } C_{57}H_{110}O_6 \text{ gives } = 55 \times \frac{1}{2} \text{ mole of } H_2O\)

Wt. of \(H_2O = \text{mole of } H_2O \times M \text{ Wt. of } H_2O\)

\[
= \frac{1}{2} \times 55 \times 18 = 495 \text{ gm}
\]

11. \(A_2 + B_2 \rightleftharpoons 2AB \ k_1\)

\[
k_1 = \frac{[AB]^2}{[A_2][B_2]}
\]

\(6AB \rightleftharpoons 3A_2 + 3B_2\)
\[ k_2 = \frac{[A_2]^3[B_2]^3}{[AB]^6} \]

So, \( \frac{1}{k_1} = k_2 \)

12. \( Cl^{-} \) ions is cause of permanent hardness & \( HCO_3^{-} \) is cause of temporary Hardness.

13. Strong field ligands have more \( CFSE \) value while weak field ligand have less \( CFSE \) value.

\( CN^{-} \) is strongest ligand among all so \( [CO(CN)_6]^{-3} \) has maximum \( CFSE \) & value.

14. Molecular weight of ethylene glycol
\( (C_2H_6O_2) = 62 \)

Moles of ethylene glycol = \( \frac{Wt}{M.Wt} \)

\[ = \frac{62}{62} = 1 \]

\( \Delta T_f = i \times k_f \times m \)

\( i = 1 \) for ethylene glycol

\[ \Delta T_f = 1 \times 8.6 \times \frac{1}{\frac{250}{1000}} \]

\[ \Delta T_f = 1.86 \times 4 = 7.44 \]

As \( \Delta T_f = 10 \) it implies that some amount of water has frozen and this has led a greater depression of freezing point

\[ 10 = 1.86 \times \frac{62}{62} \times \frac{Wt \ of \ H_2O}{1000} \]

\( W_{H_2O} = 186 \)

So weight of water freeze = 250 – 186
= 64 gm

15. \( \Delta G = \Delta H - T \Delta S \)

For reaction feasibility \( \Delta G = -ve \)

So \( Zn \) can be extracted by \( ZnO \)

By using \( Al \) at 500°C
16. 

(A)  
Cyclic conjugation and by Huckel rule = $4\pi e^-$  
So Anti aromatic

(B)  
Cyclic conjugation and by Huckel rule = $6\pi e^-$  
So aromatic

(C)  
Cyclic conjuration lp of $-N$ is not involved in delocalization  
Having = $6\pi e^-$ Aromatic

(D)  
Cyclic conjugation  
Lone pair involved in Aromatically  
Having = $6\pi e^-$ Aromatic

17. 

[Chemical reactions and structures]

- $\text{AlCl}_3$ base reaction
- Electrophile

$-\text{H}$ is $-M$ group and more activating $\alpha/p$ directing electrophile attack at para because hindrance is less
18. \[ \text{SN}_1 \text{ TH reaction} \]

\[ \text{NH}_2 > \text{OH} \]

is the order of nucleophilicity

19. Basic character \( \propto +M \propto +H \propto +I \)

In case of polar protic solvent two factor decide the basic character according to \( +I \) effect of \(-CH_3\)

\( 3^o > 2^o > 1^o \) Amine

& According to \( H \)-bonding the basic character is \( 1^o > 2 > 3^o \)

& the resultant basic character is determined by taking the above two factors into account.

So order will be \( 2^o > 1^o > 3^o \)

In aromatic & Aliphatic: Aliphatic Amine are more basic than aromatic because the lone pair of \( N \) is involved in delocalization.

So basic character is in the order \( 2 > 1^o > 3^o > \text{Aromatic Amine} \)

So (iii) > (i) > (iv) > (ii)

20. Normal clean rain has a \( pH \) value of around 5.6, which is slightly acidic.

21.

22. Methemoglobinemia disease occur due to \( NO_3^- \), when it’s concentration is higher than 50 ppm in water.
Ketones with a methyl group gives positive iodoform test as well as 2,4 DNP: Test which is an identification for carbonyl groups. Azo dye: For Azo dye H should be present with $\text{−N}$, or there should be a primary and secondary amines.