Solution

Mathematics

1. \[ x (\sin \theta - 1) - \cos \theta (x \sin \theta - 1) = 0 \]
   \[ \Rightarrow (x \sin \theta - 1) (x - \cos \theta) = 0 \]
   \[ \Rightarrow x = \csc \theta, \cos \theta \]
   Hence \( \beta = \csc \theta; \alpha = \cos \theta \)

   So, \[ \sum_{n=0}^{\infty} (\cos \theta)^n + \sum_{n=0}^{\infty} (-\sin \theta)^n = \frac{1}{1-\cos \theta} + \frac{1}{1+\sin \theta} = \frac{1}{1-\cos \theta} + \frac{1-\sin \theta}{\cos^2 \theta} \]

2. \[ (\cot^{-1}x - 5)(\cot^{-1}x - 2) > 0 \]
   \[ \Rightarrow \cot^{-1}x \in (-\infty, 2) \cup (5, \infty) \]
   \[ \Rightarrow \cot^{-1}x \in (0,2) \](Taking intersection with range of \( \cot^{-1}x \))
   \[ \Rightarrow n \in (\cot 2, \infty) \]

3. \[ b = \frac{5}{2} \]
   \[ a e = \frac{13}{2} \]
   \[ (ae)^2 = \frac{169}{4} \Rightarrow a^2 + b^2 = \frac{169}{4} \]
   \[ a^2 = \frac{169}{4} - \frac{25}{4} = \frac{144}{4} \Rightarrow a = \frac{12}{2} = 6 \]
   Hence \( e = \frac{13}{2 \times 6} = \frac{13}{12} \)

4. \[ y^2 = -4(x - a^2) \]
   
   For \( B \) and \( B' \) put \( x = 0 \) in parabola
   \[ y^2 = 4a^2 \Rightarrow y = \pm 2a \]
   
   Area = \[ \frac{1}{2} \times a^2 \times 4a = 250 \]
   \[ \Rightarrow a^3 = 125 \Rightarrow a = 5 \]
5. \[ k^2 + 4a^2 = r^2 \quad \text{(i)} \] \[ h^2 + (k - 2b)^2 = r^2 \quad \text{(ii)} \]

Hence from (i) and (ii)

\[ k^2 + 4a^2 = h^2 + (k - 2b)^2 \]
\[ \Rightarrow 4a^2 = h^2 + 4b^2 - 4bk \]
\[ \Rightarrow 4a^2 = x^2 + 4b^2 - 4by \]

Hence locus is a parabola

6. Given \( b = 2ae \)

\[ b^2 = 4(a^2 - b^2) \]
\[ \Rightarrow 4a^2 = 5b^2 \]
\[ \Rightarrow \frac{x^2}{a^2} + \frac{5y^2}{4a^2} = 1 \]

Substituting given point on the equation of ellipse, we get \( a^2 = 58 \)

Hence \((4\sqrt{3}, 2\sqrt{2})\) satisfies the required ellipse.

7. \[ \therefore AD \parallel BC \]

Hence slope of \( AD = \text{slope of } BC = \frac{4 - 3}{3 - 2} = 1 \)

Hence equation of \( AD \) is \( y - 2 = 1(x - 1) \)
\[ \Rightarrow x - y + 1 = 0 \]

8. \[ f(x) = \left| 1 - \frac{1}{x} \right| \]
\[ f(x) = \left| 1 - \frac{1}{x} \right| \]

Range of \( f(x) \) is \([0, \infty)\)

Hence function is many one and onto

9. \[ x - y = t \]
\[ 1 - \frac{dy}{dx} = \frac{dt}{dx} \]

Hence \[ 1 - \frac{dt}{dx} = t^2 \]
\[ \Rightarrow 1 - t^2 = \frac{dt}{dx} \]
\[ \Rightarrow \int \frac{dt}{1 - t^2} = \int dx \]
\[ \Rightarrow \frac{1}{2} \log \left| \frac{1 + t}{1 - t} \right| = x + c \]
\[ \frac{1}{2} \log \left| \frac{x+y+1}{x-y-1} \right| = x + c \]

At \( x = 1 \) and \( y = 1 \), we get \( c = -1 \)

Hence \[ \frac{1}{2} \log \left| \frac{x+y+1}{x-y-1} \right| = x - 1 \]

Hence \(-\ln \left| \frac{1-x+y}{1+x-y} \right| = 2(x - 1)\)

10. \[ I = \int_0^\frac{\pi}{4} \frac{\sec^2 x \tan^5 x \, dx}{2 \tan x \,(\tan^{16} x + 1)} \]

Put \( \tan^5 x = t \)
\[ \Rightarrow 5 \tan^4 x \sec^2 x \, dx = dt \]
\[ \Rightarrow \frac{1}{10} \int_0^1 \frac{dt}{t^2+1} \]
\[ \Rightarrow \frac{1}{10} \left[ \tan^{-1} t \right]_0^1 = \frac{1}{10} \cdot \frac{\pi}{4} = \frac{\pi}{40} \]

11. \[ y = x^2 + 1 \]
\[ \frac{dy}{dx} = 2x \]

Hence at \( x = 2 \), \( \frac{dy}{dx} = 4 \)

\( \therefore \) equation of tangent is \( y - 5 = 4(x - 2) \)

\( \Rightarrow y = 4x - 3 \)

Required area

\[ \int_0^2 (x^2 + 1) \, dx - \int_0^4 (4x - 3) \, dx \]

\[ = \left[ \frac{x^3}{3} + x \right]_0^2 - \left[ \frac{2x^2}{2} - 3x \right]_0^4 \]

\[ = \frac{8}{3} + 2 - \left[ (8 - 6) - \left( \frac{9}{8} - \frac{9}{4} \right) \right] \]

\[ = \frac{8}{3} - \frac{9}{8} - \frac{37}{24} \]

12. Let first term of the given A.P is \( a \) and common difference \( d \).

Hence, \( a + 18d = 0 \Rightarrow a = -18d \)

\[ \frac{\ell_{49}}{\ell_{29}} = \frac{a+48d}{a+28d} = \frac{30d}{10d} = 3 \]

13. There are 30 white balls and 10 red balls

\( P \) (white ball) = \( \frac{30}{40} = \frac{3}{4} = p \), hence \( q = 1 - p = \frac{1}{4} \) and \( n = 16 \)

\[ \text{mean}(X) = \frac{np}{\sqrt{npq}} = \sqrt{\frac{np}{q}} = \sqrt{\frac{16 \times \frac{3}{4}}{\frac{1}{4}}} = 4\sqrt{3} \]

14. \( \lim_{x \to 0} \frac{x \cot 4x}{\cot^2 2x \sin^2 x} = \lim_{x \to 0} \frac{x \tan 4x}{\cos^2 2x \sin^2 x} = \lim_{x \to 0} \frac{4x}{x \cos^2 2x \sin^2 x} = 1 \)

15. \( \int \left( \frac{t^2+1}{2} \right) \frac{dt}{t} = \int \left( \frac{t^2+3}{2} \right) dt = \frac{t^3}{6} + \frac{3r}{2} + C = \frac{r}{6} (t^2 + 9) + C = \frac{r}{6} (2x - 1 + 9) + C \)

As given \( f(x)\sqrt{2x - 1} + C = \frac{x+4}{3} (x + 4) + C \)

Hence \( f(x) = \frac{x+4}{3} \)

16. Equation of angle bisector of \( OA \) and \( OB \) is \( y = x \)
Since distance of position vector $\vec{c}$ from $y = x$ is $\frac{3}{\sqrt{2}}$

Hence, $|\beta - (1-\beta)| = \frac{3}{\sqrt{2}} \Rightarrow |2\beta - 1| = 3 \Rightarrow 2\beta - 1 = \pm 3 \Rightarrow \beta = 2$ or $\beta = -1$

Hence sum of value of $\beta$ is 1

17. $\frac{x^m y^n}{(1+x^2m)(1+y^2n)} = \frac{1}{(1+x^m)(1+y^n)}$ (dividing numerator and denominator by $x^m y^n$)

As we know A.M $\geq$ G.M

Hence, $\frac{1}{x^m} + x^m \geq 2$, $\frac{1}{y^n} + y^n \geq 2$

Therefore maximum value of $\frac{x^m y^n}{(1+x^2m)(1+y^2n)} = \frac{1}{2 \times 2}$

18. Contrapositive of $p \rightarrow q$ statement is $\sim q \rightarrow \sim p$

Hence contrapositive of the statement is “If two numbers are not equal then the squares of the numbers are not equal”

19. Only positive point of non-differentiability is $x = 0$

Checking at $x = 0$

for $x > 0$

$f(x) = \sin x - x + 2(x - \pi)\cos x$

$f'(x) = \cos x - 1 + 2\cos x - 2(x - \pi)\sin x$

$RHD = f'(0^+) = 1 - 1 + 2 - 2(-\pi).0 = 2$

for $x < 0$

$f(x) = -\sin x + x + 2(x - \pi)\cos x$

$f'(x) = -\cos x + 1 + 2\cos x - 2(x - \pi)\sin x$

$LHD = f'(0^-) = -1 + 1 + 2 - 2(-\pi).0 = 2$

$\therefore LHD = RHD$

$\therefore$ Function is differentiable at $x = 0 \Rightarrow$ function is differentiable everywhere

Hence set of non differentiable point is null set

20. $\sum_{r=1}^{101} C_r S_{r-1} = \sum_{r=1}^{101} C_r q^{r-1} = \frac{1}{q-1} \left( \sum_{r=1}^{101} C_r q^r - \sum_{r=1}^{101} C_r \right) = \frac{1}{q-1}((1 + q)^{101} - 1 - 2^{101} + 1)$

$\Rightarrow \alpha \left( \frac{(q+1)^{101}}{q} - 1 \right) = \frac{1}{q-1}((1 + q)^{101} - 2^{101})$

$\Rightarrow \alpha \left( \frac{(1+q)^{101} - 2^{101}}{q-1} \right) = \frac{1}{q-1}((1 + q)^{101} - 2^{101})$

Hence $\alpha = 2^{100}$
21. 
\[ f'(x) = \frac{\sqrt{a^2+x^2} - \frac{x^2}{a^2+x^2}}{(a^2+x^2)} + \frac{-\frac{b^2+(d-x)^2}{b^2+(d-x)^2}}{2} = \frac{a^2}{(a^2+x^2)^2} - \frac{b^2}{(b^2+(d-x)^2)^2} \]

Since \( f'(x) \) can be positive and negative both

Hence \( f(x) \) is neither increasing nor decreasing

22. 
\[ \frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b+c}{18} = k \Rightarrow a = 7k, b = 6k, c = 5k \]

\[ \cos A = \frac{36k^2 + 25k^2 - 49k^2}{2(6k)(5k)} = \frac{1}{5}, \ \cos B = \frac{49k^2 + 25k^2 - 36k^2}{2(7k)(5k)} = \frac{19}{35} \]

and \( \cos C = \frac{49k^2 + 36k^2 - 25k^2}{2(7k)(6k)} = \frac{5}{7} \)

\[ \frac{1}{5a} = \frac{19}{35\beta} = \frac{5}{7\gamma} \Rightarrow \frac{7}{35\alpha} = \frac{19}{35\beta} = \frac{25}{35\gamma} = \lambda \]

\[ \Rightarrow a = \frac{7}{35\lambda}, \beta = \frac{19}{35\lambda}, \gamma = \frac{25}{35\lambda} \]

From option possible ordered pair is \( (7, 19, 25) \)

23. 
\[ |AA'B| = 8 \text{ and } |AB^{-1}| = 8 \]

So, \( |A|^2 |B| = 8 \text{ and } |A| = 8 |B| \)

So \( 64 |B|^3 = 8 \Rightarrow |B| = \frac{1}{2} \)

\[ |BB'A^{-1}| = \frac{|B|^2}{|A|} = \frac{|B|^2}{8} = \frac{1}{16} \]

24. 
Using transformation, \( R_1 \rightarrow R_1 + R_2 + R_3 \)

\[
\begin{vmatrix}
(a + b + c) & 1 & 2b & -a - c & 2b \\
2c & 2c & c & a - b
\end{vmatrix}
\]

\[ C_3 \rightarrow C_3 - C_1 \text{ and } C_2 \rightarrow C_2 - C_1 \]

\[
\begin{vmatrix}
(a + b + c) & 1 & 0 & 0 \\
2b & -(a + b + c) & 0 & 2b \\
2c & 0 & -(a + b + c)
\end{vmatrix}
\]

\[ = (a + b + c)((a + b + c)^2 - 0) = (a + b + c)^3 \]

Hence, \( x = 0 \) or \( x = -2(a + b + c) \)

25. 
\[ a_2 = \frac{50c_210^{48} + 50c_210^{48}}{2 \times 10^{60}} = \frac{50c_2}{100} = \frac{50 	imes 49}{2 	imes 100} = \frac{49}{4} = 12.25 \]

26. 
\[ |z| + z = 3 + i \]

Let \( z = a + i \)

\[ |a + i| + a + i = 3 + i \]

\[ \Rightarrow \sqrt{a^2 + 1} = 3 - a \]

\[ \Rightarrow a^2 + 1 = 9 + a^2 - 6a \]
⇒ \( a = \frac{4}{3} \)

⇒ \(|z| = \sqrt{a^2 + 1} = \sqrt{\frac{16}{9} + 1} = \frac{5}{3}\)

27. \( k = \{4, 8, 12, 16, 20\} \)

\( f(k) \) will take values from set \( \{3, 6, 9, 12, 15, 18\} \). We can do this by \( ^6C_5 \times 5! \) ways = 6! Ways. For remaining 15 elements, total ways = 15!

∴ Total number of onto functions = 6! × 15!

28. \( A(7, 0, 6), B(3, 4, 2) \)

\( AB = (3 - 7)i + (4 - 0)j + (2 - 6)k = -4i + 4j - 4k \)

Direction ratio of \( AB \) is \( <1, -1, 1> \)

Plane \( P_1 \) is perpendicular to \( P_2 \). Hence direction ratio of normal of \( P_2 \) is parallel to \( P_1 \)

\( \Rightarrow <2, -5, 0> \)

Direction ratio of normal to plane \( P_1 \) is

\[
\begin{vmatrix}
i & j & k \\
1 & -1 & 1 \\
2 & -5 & 0 \\
\end{vmatrix} = i(+5) - j(-2) + k(-5 + 2) = 5i + 2j - 3k
\]

Hence equation of plane is \( 5(x - 7) + 2(y - 0) - 3(z - 6) = 0 \)

\( \Rightarrow 5x + 2y - 3z = 17 \)

Since point \( (2, \alpha, \beta) \) lies on \( P_1 \)

Hence \( 2\alpha - 3\beta = 7 \)

**Physics**

1. \( \vec{\Delta r} = \vec{u}t + \frac{1}{2} \vec{a}t^2 \)

\( \vec{r}_f = \vec{r}_i + \vec{u}t + \frac{1}{2} \vec{a}t^2 \)

\( \vec{r}_f = (2i + 4j) + (5i + 4j) \times 2 + \frac{1}{2} (4i + 4j) \times 2^2 \)

\( = 2i + 4j + 10i + 8j + 8i + 8j \)

\( = 20i + 20j \)

\( |\vec{r}_f| = 20\sqrt{2}m \)

2. \( I_0 = (M. I. of A & B) + (M. I. of C) \)
\[ I_0 = \left( \frac{mR^2}{4} + mR^2 \right) \times 2 + \frac{mR^2}{2} \]

\[ = \frac{5mR^2}{2} + \frac{mR^2}{2} = 3mR^2 \]

3. \( g = \frac{GM}{R^2} \)

\[ \frac{g_p}{g_e} = \frac{M_p}{M_e} \times \left( \frac{R_e}{R_p} \right)^2 = 3 \times \frac{1}{9} = \frac{1}{3} \]

\[ T = 2\pi \sqrt{\frac{1}{g}} \]

\[ T_p = \sqrt{3}T_e \]

Time period at earth for seconds pendulum = 2 sec

\[ \therefore T_p = 2\sqrt{3} \text{ sec} \]

4. \( V_{AB} = \frac{1+1+1}{1+1+1} = \frac{1+2+3}{3} = \frac{6}{3} = 2 \text{ volt} \)

5. \( F = \frac{dp}{dt} \Rightarrow Kt = \frac{dp}{dt} \)

\[ \int_P^{3P} dP = \int_0^t Kt \, dt \Rightarrow 3P - P = \frac{Kt^2}{2} \Rightarrow t = 2\sqrt{\frac{P}{K}} \]

6.

\[ \frac{R_1}{R_2} = \frac{40}{60} = \frac{2}{3} \]

\[ \frac{R_1 + 10}{R_2} = \frac{50}{50} = 1 \]
\[R_1 + 10 = R_2\]
\[R_1 + 10 = \frac{3}{2}R_1\]
\[10 = \frac{1}{2}R_1R_1 = 20\Omega\]
\[\frac{(R_1+10)R}{R_1+10+R} = \frac{2}{3}\]
\[\frac{R}{R_1+10+R} = \frac{2}{3}\]
\[3R = 2R_1 + 20 + 2R\]
\[R = 2R_1 + 20\]
\[R = (40 + 20)\Omega = 60\Omega\]

7. \[\vec{\tau} = \vec{r} \times \vec{F}\]
\[\tau = rF \sin \theta\]
\[2.5 = 5 \times 1 \times \sin \theta \Rightarrow \theta = 30^\circ\]

8. Maximum kinetic energy \[\frac{1}{2}m\omega^2A^2\]
\[\omega = \sqrt{\frac{g}{L}}\]
\[A = L\theta\]
\[KE = \frac{1}{2} \frac{m}{L}g \times L^2\theta^2\]
\[KE = \frac{1}{2}mgL\theta^2\]

If length is doubled
\[K_2 = \frac{1}{2}mg(2L)\theta^2\] [Here we are assuming angular amplitude is same]
\[\frac{K_1}{K_2} = \frac{\frac{1}{2}mgL\theta^2}{\frac{1}{2}mg(2L)\theta^2} = \frac{1}{2}\]
\[K_2 = 2K_1\]
9. Considering the subscript for ball as 'b', for water as 'w' and for container as 'c' and applying principle of calorimetry (assuming final temperature = \(T^\circ C\))

\[m_b s_b (500 - T) = m_w s_w (T - 30) + m_c s_c (T - 30)\]
\[\therefore 0.1 \times 400 (500 - T) = 0.5 \times 4200 (T - 30) + 800 (T - 30)\]
\[\therefore 20000 - 40T = 2100T - 63000 + 800T - 24000\]
\[\therefore 2940T = 107000\]
\[\therefore T = \frac{107000}{294} = 36.4^\circ C\]

% rise in temperature = \(\frac{6.4}{30} \times 100\% \approx 21\%\)

10. Let \(Y = f(V,F,A)\)
\[\therefore Y = KV^x F^y a^z, K \rightarrow \text{Unit less}\]
\[\therefore [Y] = [V]^x[F]^y[a]^z\]
\[\therefore [ML^{-1}T^{-2}] = [LT^{-1}]^x[MLT^{-2}]^y[L^{-2}]^z\]
\[\therefore [M][L^{-1}] = [T^{-2}] = [M]^y[L^{x+y+z}][T^{-x-2y-2z}]\]
\[\therefore y = 1, x + y + z = -1; -x - 2y - 2z = 2\]
\[\therefore x + z = -2 \therefore x + 2y + 2z = 2\]
\[\therefore z = 2, x = -4 \therefore x + 2z = 0\]
\[\therefore [Y] = [V^{-4}F a^2]\]

11. \[I = \frac{P}{A} = \frac{1}{2} \varepsilon_0 E_0^2 C\]
\[\therefore E_0 = \sqrt{\frac{2P}{\varepsilon_0 CA}} = \sqrt{\frac{2 \times 27 \times 10^{-3} \times 36 \pi \times 10^9}{3 \times 10^8 \times 10^{-6}}}\]
\[= 100 \times 10 \times 4.5\]
\[= 4.5 \, kV/m\]

12. \[\frac{x_0 - x}{x_0 - \frac{x_0}{3}} = \frac{C - 0}{100 - 0}\]
\[
\frac{1}{4} = \frac{c}{100} \Rightarrow C = 25^\circ C
\]

13. \[\frac{1}{v} - \frac{1}{u} = \frac{1}{f}\]

\[\therefore \quad \frac{dv}{dt} + \frac{1}{u^2} \frac{du}{dt} = 0\]

\[\therefore \quad \frac{dv}{dt} = \left(\frac{v^2}{u^2}\right) \left(\frac{du}{dt}\right) = \left(-\frac{60}{-20}\right)^2 \times 5 = 9 \times 5 = 45 \text{ m/s}\]

14. Taking torque about the point of contact

\[40 \times [1] = [mr^2 + mr^2] \alpha\]

\[40 = 2m \times \frac{1}{4} \alpha\]

\[40 = 2 \times 5 \times \frac{1}{4} \alpha\]

\[\therefore \alpha = 16 \text{ rad/s}^2\]

15. 1st condition:

\[\Delta \theta_g = \Delta \theta_i\]

\[100 \times S_A \times (100 - 90) = 50 \times S_B \times (90 - 75) \quad \ldots (i)\]

2nd condition:

\[\Delta \theta_g = \Delta \theta_{\theta_1}\]

\[100 \times S_A(100 - \theta) = 50 \times S_B \times (\theta - 50) \quad \ldots (ii)\]

dividing (ii) by (i)

\[\frac{100-\theta}{100-90} = \frac{\theta-50}{90-75}\]

\[300 - 3\theta = 2\theta - 100\]

\[\theta = 80^\circ C\]

16. \[V = I_g(R_g + R)\]

\[15 = 15 \times 10^{-2}(20 + R)\]

\[R = 80 \Omega\]
17. From $M$ orbit to $L$ orbit:

$$\frac{hc}{\lambda^2} = (13.6eV)Z^2\left(\frac{1}{4} - \frac{1}{9}\right) \quad \text{(i)}$$

From $N$ orbit to $L$ orbit:

$$\frac{hc}{\lambda^2} = (13.6eV)Z^2\left(\frac{1}{4} - \frac{1}{16}\right) \quad \text{(ii)}$$

dividing (i) by (ii)

$$\frac{\lambda_2}{\lambda_1} = \frac{5}{36} \times \frac{64}{12} = \frac{20}{27} \Rightarrow \lambda_2 = \frac{20}{27}\lambda_1$$

18. Since the second diode is reverse biased the simplified circuit is as shown in the figure

![Diode Circuit](image)

$$I = \frac{6}{300} = 0.02 \, A$$

19.

![Circuit Diagram](image)

$$C_{12} = 6 \mu F$$

$$C_{23} = \frac{2}{4} = \frac{1}{2} \mu F$$

$$C_{13} = \frac{6 \times \frac{1}{2}}{6 + \frac{1}{2}} = \frac{3}{13} = \frac{6}{13} \mu F$$

20. Electric potential energy of dipole is

$$U = -pE \cos \theta$$

$$U = -10^{-29} \times 1000 \cos 45^o$$

$$= -\frac{1}{\sqrt{2}} \times 10^{-26} = -5\sqrt{2} \times 10^{-27} J \approx -7 \times 10^{-27} J$$
21. \[ \Delta L_1 = \Delta L_2 \]

\[ L\alpha_1 \times 60 = L\alpha_2 \times \Delta T_2 \]

\[ \Delta T_2 = \frac{\alpha_1}{\alpha_2} \times 60 = \frac{4}{3} \times 60 = 80^\circ C \]

22. \[ n = \frac{\sin\left(\frac{A+\delta m}{2}\right)}{\sin\left(\frac{A}{2}\right)} \]

\[ \sqrt{3} = \frac{\sin\left(\frac{60^\circ + \delta m}{2}\right)}{\sin\left(\frac{60^\circ}{2}\right)} \]

\[ \frac{\sqrt{3}}{2} = \sin\left(\frac{60^\circ + \delta m}{2}\right) \]

\[ 60^\circ = \frac{60^\circ + \delta_{\text{min}}}{2} \Rightarrow \delta_{\text{min}} = 60^\circ \]

\[ i = \frac{A+\delta_{\text{min}}}{2} = \frac{60^\circ + 60^\circ}{2} = 60^\circ \]

23. Self-inductance \( L \propto \ell \)

24.

Radius of curvature

\[ R = \frac{mv}{qB} = 2d \]

\[ \cos \theta = \frac{d}{2d} \Rightarrow \theta = 60^\circ \]

\[ a = \frac{qvB}{m} \]

\[ \ddot{a} = -\sin \theta \dot{i} - \cos \theta \dot{j} \]

\[ \ddot{a} = \left( \frac{-\sqrt{3}}{2} \right) \frac{qvB}{m} \]
25. \( B_{net} = \mu_0(H + I) \)

\[ B_{net} = m_0(H + \chi)H \]

\[
(1 + \chi) = \frac{B_{net}}{\mu_0 H} = \frac{B_{net}}{B_{ext}} = \frac{B_{ext} + B_{int}}{B_{ext}}
\]

\[
(1 + \chi) = 1 + \frac{\mu_0 I}{B_{ext}}
\]

\[
\chi = \frac{\mu_0 \times 20 \times 10^{-6}}{10^{-6} \times 50 \times 10^{-6}} = \frac{2}{5} \times 10^6 \times 4\pi \times 10^{-7}
\]

\[
= \frac{8\pi}{50} = \frac{16\pi}{100}
\]

\[
= 16\pi \times 10^{-2} = 50.264 \times 10^{-2}
\]

\[
= 0.50
\]

26. \( \omega_s = \frac{2\pi}{100 \times 10^{-6}} = 2\pi \times 10^4 s^{-1} \)

\[
\omega_c = \frac{2\pi}{8 \times 10^{-6}} = 2.5 \pi \times 10^6 s^{-1}
\]

\[
V_{\text{max}} = V_c + V_s = 10
\]

\[
V_{\text{min}} = V_c - V_s = 10
\]

\[
\therefore V_c = 9mV
\]

\[
V_s = 1mV
\]

Equation of AM wave

\[
V_{\text{AM}} = (V_c + V_s \sin \omega_s t) \sin \omega_c t
\]

\[
= \{9 + \sin(2\pi \times 10^4 t)\} \times \sin(2.5\pi \times 10^6 t)(\text{mV})
\]

27. \( ML^{-1}T^{-2} = v^x a^y F^z \)

\[
ML^{-1}T^{-2} = [LT^{-1}]^x [LT^{-2}]^y [MLT^{-2}]^z
\]

\[
ML^{-1}T^{-2} = M^x L^{x+y+z} T^{-x-2y-2z}
\]

\[
z = 1 \ldots (i)
\]

\[
x + y + z = -1 \quad \ldots (ii)
\]
\[-x - 2y - 2z = -2 \quad \text{(iii)}
\]
\[
2x + 2y + 2z = -2
\]
\[
-x - y - 2z = -2
\]
\[
x = -4
\]
\[-4 + y - 1 = -1
\]
\[y = 2\]

**(Chemistry)**

1. \(\frac{t_1}{2} = \frac{C_0}{2k} \Rightarrow 6 = \frac{0.2}{2k} \Rightarrow k = \frac{1}{60}\)
   
   now, \(C_t = C_0 - Rk\)
   
   \[
   0.2 = 0.5 - \frac{1}{60} \times t
   \]
   
   \[
   0.3 = \frac{1}{60} \Rightarrow 1 = 18 \text{ hrs}
   \]

2. \(K_2[HgI_4] \rightarrow 2K^+ + [HgI_4]^{-2};\)
   
   \[
   i = (1 - \alpha) + 2\alpha + \alpha
   \]
   
   \[
   = 1 + 2 \times 0.4
   \]
   
   \[
   = 1 + 2 \times 0.4 = 1.8
   \]

3. Gallium belongs to the group 13 whereas Germanium belongs to group 14. As we move from left to right, the electronegativity increases.

4. The inert pair effect is the tendency of the two electrons in the outermost atomic s-orbital to remain unionized or unshared in compounds of post-transition metals. It explains the increasing stability of oxidation states that are two less than the group valency for the heavier elements of groups 13, 14, 15 and 16

5. Cheese: Dispersed Phase = liquid, Dispersion medium = solid
   
   Milk: Dispersed Phase = liquid, Dispersion medium = liquid
   
   Smoke: Dispersed Phase = solid, Dispersion medium = gas

6. \(E_{\text{cell}}^o = \frac{0.0591}{n} \log K_c\)
   
   \[
   = \frac{0.0591}{2} \log(1 \times 10^{16}) = 0.4728 \text{ V}
   \]

7. SiH₄ has a complete octet and thus isn't electron deficient. B₂H₆ appears also to have completed its octet by dimerizing, but the 3 centre 2 electron bond is not as fulfilling as a 2 centre 2 electron bond, and thus still has electron deficient nature.

8. \(C. No. = 4 \times 2 + 2 \times 1 = 10\)
   
   \(\text{OH}^-\) is monodentate whereas \(C_2O_4^{2-}\) is bidentate.

9. \(\Delta G^o = -2.303 \times RT \log K\)
   
   \[
   = -2.303 \times 8.314 \times 298 \log(10^{-14})
   \]
\[ = +80 \frac{kJ}{mol} \]

10. \( A \rightarrow R, MgHCO_3 \) is responsible for temporary hardness
\( B \rightarrow P, \) Dicalcium silicate is a component of Portland cement
\( C \rightarrow S, NaOH \) is produced in the Castner-Keller Cell
\( D \rightarrow Q, \) Solvay process is the process for industrial preparation of sodium carbonate.

11. \( \Delta H - T\Delta S < 0 \)
\( T > \frac{\Delta H}{\Delta S} = \frac{491.1 \times 1000}{198} \)
\( T > 2480.3030 \text{ K} \)

12. \[
\begin{array}{ccc}
\text{n = 4} & \lambda' & \text{N} \\
\text{n = 3} & \lambda & \text{M} \\
\text{n = 2} & \lambda' & \text{L} \\
\text{n = 1} & \lambda & \text{K}
\end{array}
\]
\[
\frac{1}{\lambda} = RZ^2 \left( \frac{1}{2^2} - \frac{1}{3^2} \right)
= RZ^2 \left( \frac{5}{36} \right)
\]
\[
\frac{1}{\lambda'} = RZ^2 \left( \frac{1}{2^2} - \frac{1}{4^2} \right)
= RZ^2 \left( \frac{3}{16} \right)
\]
\[
\frac{\lambda'}{\lambda} = \frac{5}{9} \times \frac{4}{3} = \frac{20}{27} \Rightarrow \lambda' = \frac{20}{27} \lambda
\]

13. Equivalents of \( HCl = \) equivalents of \( Na_2CO_3 \)
\( 25 \times N = 2 \times 0.1 \times 30 \)
\( HCl = 0.24 \text{ N} \)
Equivalents of \( HCl = \) equivalents of \( NaOH \)
\( 0.24 \times V = 30 \times 0.2 \)
\( V = 25 \text{ mL} \)

14. \( Cu_2S + O_2 \rightarrow Cu_2O + 2SO_2 \) is roasting process

15. \( h\nu = h\nu_0 + \frac{1}{2}mV^2 \)
\[ h(v - v_0) = \frac{1}{2}mV^2; V = \left( \frac{2h(v - v_0)}{m} \right)^{1/2} \]
\[ \lambda = \frac{h}{mV} = \frac{h}{m \sqrt{2h(v - v_0)}} = \frac{h}{m(v - v_0)}; \lambda \propto \frac{1}{(v - v_0)^{1/2}} \]

16. \( \Delta G = A - BT \)
\( = \Delta H - T\Delta S \)
\[ A = \Delta H \]
\[ \Delta H = A = +ve \text{ endothermic} \]

17. 
\[ \text{Number of } Co - Co \text{ bond} = 1 \]
\[ \text{Number of bridging carbonyl} = 2 \]

18. 
\[ MnO_2(A) + KOH + O_2 \rightarrow K_2MnO_4(B) \text{ (Green)} \rightarrow KMnO_4(C) \text{ (purple)} \rightarrow MnO_2(A) + KIO_3(D) \]

19. 
[Chemical reaction diagram]

20. Acid rain has been recognised as the primary cause for decolouration of Taj Mahal. The acid reacts with the marble and brings about a dull yellow colour.

21. The stiffening of flower buds is caused by \( SO_2 \) gas.

22. 
[Chemical reaction diagram]

23. 
[Chemical reaction diagram]

Intermediate carbocation is resonance stabilized and aromatic.

25. \( LiAlH_4 \) reduces to carboxylic acid, ketone, nitro group but it does not reduce alkenes.

26. 
\[ CH_3CH_2CH_2CONH_2(A) \xrightarrow{Br_2+NaOH} B(CH_3CH_2CH_2NH_2) \]

27. Nitrogen (\( c \)) is most basic in adenine and hence, gets protonated most easily.

28. Phthalein test is given by phenolic ring, that is present in Tyrosine.

29. 
[Chemical reaction diagram]

PHB is biodegradable polymer.
30. When enzymes bind to a site other than the active site, the site is called allosteric site.
Poison binds at the active site through covalent bonds.
Receptors are important for communication.
In competitive inhibition, an inhibitor that resembles the normal substrate binds to the enzyme, usually at the active site, and prevents the substrate from binding.