Solution

Mathematics

1. $x\,(x\sin\theta - 1) - \cos\theta\,(x\sin\theta - 1) = 0$

$\Rightarrow (x\sin\theta - 1)\;(x - \cos\theta) = 0$

$\Rightarrow x = cosec\,\theta, \cos\theta$

Hence $\beta = cosec\theta;\ \alpha = \cos\theta$

So, $\sum_{n=0}^{\infty}(\cos\theta)^n + \sum_{n=0}^{\infty}(-\sin\theta)^n = \frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta} = \frac{1}{1-\cos\theta} + \frac{1-\sin\theta}{\cos^2\theta}$

$= \frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$

2. $(\cot^{-1}x - 5)(\cot^{-1}x - 2) > 0$

$\Rightarrow\ \cot^{-1}x \in (-\infty, 2) \cup (5, \infty)$

$\Rightarrow \cot^{-1}x \in (0,2)$ (Taking intersection with range of $\cot^{-1}x$)

$\Rightarrow \quad n \in (\cot 2, \infty)$

3. $b = \frac{5}{2}$

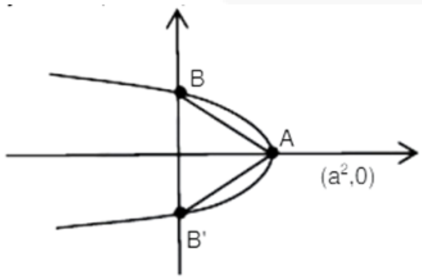
$ae = \frac{13}{2}$

$(ae)^2 = \frac{169}{4} \Rightarrow a^2 + b^2 = \frac{169}{4}$

$a^2 = \frac{169}{4} - \frac{25}{4} = \frac{144}{4} \Rightarrow a = \frac{12}{2} = 6$

Hence $e = \frac{13}{2\times 6} = \frac{13}{12}$

4. $y^2 = -4(x - a^2)$



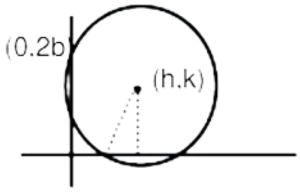
For B and B' put $x = 0$ in parabola

$y^2 = 4a^2 \qquad \Rightarrow y = \pm 2a$

Area$= \frac{1}{2} \times a^2 \times 4a = 250$

$\Rightarrow a^3 = 125\ \Rightarrow a = 5$

5.



$k^2 + 4a^2 = r^2$ ... (i) and $h^2 + (k - 2b)^2 = r^2$ ... (ii)

Hence from (i) and (ii)

$k^2 + 4a^2 = h^2 + (k - 2b)^2$

$\Rightarrow 4a^2 = h^2 + 4b^2 - 4bk$

$\Rightarrow 4a^2 = x^2 + 4b^2 - 4by$

Hence locus is a parabola

6. Given $b = 2ae$

$\Rightarrow b^2 = 4(a^2 - b^2)$

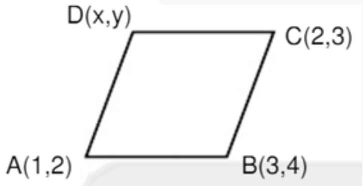
$\Rightarrow 4a^2 = 5b^2$

$\Rightarrow \frac{x^2}{a^2} + \frac{5y^2}{4a^2} = 1$

Substituting given point on the equation of ellipse, we get $a^2 = 58$

Hence $\left(4\sqrt{3}, 2\sqrt{2}\right)$ satifies the required ellipse.

7.



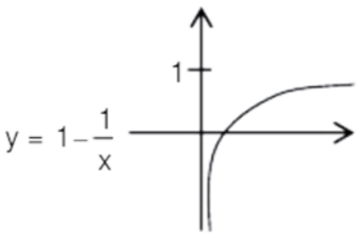
$\because AD \parallel BC$

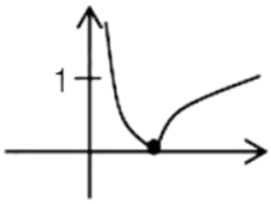
Hence slope of $AD =$ slope of $BC = \frac{4-3}{3-2} = 1$

Hence equation of AD is $y - 2 = 1(x - 1)$

$\Rightarrow x - y + 1 = 0$

8. $f(x) = \left|1 - \frac{1}{x}\right|$





$f(x) = \left|1 - \frac{1}{x}\right|$

Range of $f(x)$ is $[0, \infty)$

Hence function is many one and onto

9. $x - y = t$

$1 - \frac{dy}{dx} = \frac{dt}{dx}$

Hence $1 - \frac{dt}{dx} = t^2$

$\Rightarrow 1 - t^2 = \frac{dt}{dx}$

$\Rightarrow \int \frac{dt}{1-t^2} = \int dx$

$\Rightarrow \frac{1}{2}\log\left|\frac{1+t}{t-1}\right| = x + c$

$\frac{1}{2}\log\left|\frac{x-y+1}{x-y-1}\right| = x + c$

At $x = 1$ and $y = 1$, we get $c = -1$

Hence $\frac{1}{2}\log\left|\frac{x-y+1}{x-y-1}\right| = x - 1$

Hence $-ln\left|\frac{1-x+y}{1+x-y}\right| = 2(x-1)$

10. $I = \int_0^{\frac{\pi}{4}} \frac{\sec^2 x \tan^5 x\, dx}{2\tan x\,.(\tan^{10} x + 1)}$

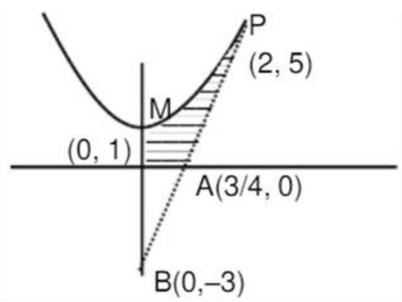
Put $\tan^5 x = t$

$\Rightarrow 5\tan^4 x \sec^2 x\ dx = dt$

$\Rightarrow \frac{1}{10}\int_0^1 \frac{dt}{t^2+1}$

$\Rightarrow \frac{1}{10}[\tan^{-1} t]_0^1 = \frac{1}{10}.\frac{\pi}{4} = \frac{\pi}{40}$

11.



$y = x^2 + 1$

$\frac{dy}{dx} = 2x$

Hence at $x = 2, \frac{dy}{dx} = 4$

$\therefore$ equation of tangent is $y - 5 = 4(x - 2)$

$\Rightarrow y = 4x - 3$

Required area $= \int_0^2 (x^2 + 1)\, dx - \int_{\frac{3}{4}}^{2} (4x - 3)\, dx$

$= \left[\frac{x^3}{3} + x\right]_0^2 - [2x^2 - 3x]_{\frac{3}{4}}^{2}$

$= \frac{8}{3} + 2 - \left[(8 - 6) - \left(\frac{9}{8} - \frac{9}{4}\right)\right]$

$= \frac{8}{3} - \frac{9}{8} = \frac{37}{24}$

12. Let first term of the given A.P is a and common difference d.

Hence, $a + 18d = 0 \Rightarrow a = -18d$

$\frac{t_{49}}{t_{29}} = \frac{a+48d}{a+28d} = \frac{30d}{10d} = 3$

13. There are 30 white balls and 10 red balls

P (white ball) $= \frac{30}{40} = \frac{3}{4} = p$, hence $q = 1 - p = \frac{1}{4}$ and $n = 16$

$\frac{mean(X)}{standard\ deviation\ (X)} = \frac{np}{\sqrt{npq}} = \sqrt{\frac{np}{q}} = \sqrt{\frac{16 \times \frac{3}{4}}{\frac{1}{4}}} = 4\sqrt{3}$

14. $\lim_{x \to 0} \frac{x \cot 4x}{\cot^2 2x . \sin^2 x} = \lim_{x \to 0} \frac{\frac{x}{\tan 4x}}{\frac{\cos^2 2x}{\sin^2 2x} . \sin^2 x} = \lim_{x \to 0} \frac{\frac{4x}{4 \tan 4x}}{\frac{\cos^2 2x}{4 \sin^2 x \cos^2 x} . \sin^2 x} = 1$

15. Put $(2x - 1) = t^2 \Rightarrow 2dx = 2tdt \Rightarrow dx = tdt$

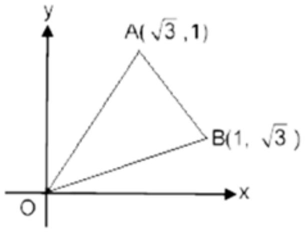
$\int \left(\frac{t^2+1}{2} + 1\right) \frac{tdt}{t} = \int \left(\frac{t^2+3}{2}\right) dt = \frac{t^3}{6} + \frac{3t}{2} + C = \frac{t}{6}(t^2 + 9) + C = \frac{t}{6}(2x - 1 + 9) + C$

$= \frac{\sqrt{2x-1}}{3}(x + 4) + C$

As given $f(x)\sqrt{2x - 1} + C = \frac{\sqrt{2x-1}}{3}(x + 4) + C$

Hence $f(x) = \frac{x+4}{3}$

16. Equation of angle bisector of OA and OB is $y = x$



Since distance of position vector $\vec{c}$ from $y = x$ is $\frac{3}{\sqrt{2}}$

Hence, $\left|\frac{\beta-(1-\beta)}{\sqrt{2}}\right| = \frac{3}{\sqrt{2}} \Rightarrow |2\beta - 1| = 3 \Rightarrow 2\beta - 1 = \pm 3 \Rightarrow \beta = 2$ or $\beta = -1$

Hence sum of value of β is 1

17. $\frac{x^m y^n}{(1+x^{2m})(1+y^2 n)} = \frac{1}{\left(\frac{1}{x^m}+x^m\right)\left(\frac{1}{y^n}+y^n\right)}$ (dividing numerator and denominator by $x^m y^n$)

As we know $\mathrm{A.M} \geq \mathrm{G.M}$

Hence, $\frac{1}{x^m} + x^m \geq 2$, $\frac{1}{y^n} + y^n \geq 2$

Therefore maximum value of $\frac{x^m y^n}{(1+x^{2m})(1+y^{2n})} = \frac{1}{2\times 2}$

18. Contrapositive of $p \rightarrow q$ statement is $\sim q \rightarrow \sim p$

Hence contrapositive of the statement is "If two numbers are not equal then the squares of the numbers are not equal"

19. Only positive point of non-differentiability is $x = 0$

Checking at $x = 0$

for $x > 0$

$f(x) = \sin x - x + 2(x - \pi)\cos x$

$f'(x) = \cos x - 1 + 2\cos x - 2(x - \pi)\sin x$

$RHD = f'(0^+) = 1 - 1 + 2 - 2(-\pi).0 = 2$

for $x < 0$

$f(x) = -\sin x + x + 2(x - \pi)\cos x$

$f'(x) = -\cos x + 1 + 2\cos x - 2(x - \pi)\sin x$

$LHD = f'(0^-) = -1 + 1 + 2 - 2(-\pi).0 = 2$

$\because LHD = RHD$

$\therefore$ Function is differentiable at $x = 0 \Rightarrow$ function is differentiable everywhere

Hence set of non differentiable point is null set

20. $\sum_{r=1}^{101} {}^{101}C_r S_{r-1} = \sum_{r=1}^{101} {}^{101}C_r \frac{q^r-1}{q-1} = \frac{1}{q-1}\left(\sum_{r=1}^{101} {}^{101}C_r q^r - \sum_{r=1}^{101} {}^{101}C_r\right) = \frac{1}{q-1}((1+q)^{101} - 1 - 2^{101} + 1)$

$\Rightarrow \alpha \frac{\left(\frac{q+1}{2}\right)^{101} - 1}{\frac{q+1}{2} - 1} = \frac{1}{q-1}((1+q)^{101} - 2^{101})$

$\Rightarrow \frac{\alpha}{2^{100}}\left(\frac{(1+q)^{101} - 2^{101}}{q-1}\right) = \frac{1}{q-1}((1+q)^{101} - 2^{101})$

Hence $\alpha = 2^{100}$

21. $f'(x) = \frac{\sqrt{a^2+x^2} - \frac{x^2}{\sqrt{a^2+x^2}}}{(a^2+x^2)} + \frac{-\sqrt{b^2+(d-x)^2} + \frac{(d-x)^2}{\sqrt{b^2+(d-x)^2}}}{b^2+(d-x)^2}$

$= \frac{a^2}{(a^2+x^2)^{\frac{3}{2}}} - \frac{b^2}{(b^2+(d-x)^2)^{\frac{3}{2}}}$

Since $f'(x)$ can be positive and negative both

Hence $f(x)$ is neither increasing nor decreasing

22. $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13} = \frac{a+b+c}{18} = k \Rightarrow a = 7k, b = 6k, c = 5k$

$\cos A = \frac{36k^2+25k^2-49k^2}{2(6k)(5k)} = \frac{1}{5}, \cos B = \frac{49k^2+25k^2-36k^2}{2(7k)(5k)} = \frac{19}{35}$

and $\cos C = \frac{49k^2+36k^2-25k^2}{2(7k)(6k)} = \frac{5}{7}$

$\frac{1}{5\alpha} = \frac{19}{35\beta} = \frac{5}{7\gamma} \Rightarrow \frac{7}{35\alpha} = \frac{19}{35\beta} = \frac{25}{35\gamma} = \lambda$

$\Rightarrow \alpha = \frac{7}{35\lambda}, \beta = \frac{19}{35\lambda}, \gamma = \frac{25}{35\lambda}$

From option possible ordered pair is $(7, 19, 25)$

23. $|AA'B| = 8$ and $|AB^{-1}| = 8$

So, $|A|^2|B| = 8$ and $|A| = 8|B|$

So $\quad 64|B|^3 = 8 \Rightarrow |B| = \frac{1}{2}$

$|BB'A^{-1}| = \frac{|B|^2}{|A|} = \frac{|B|^2}{8|B|} = \frac{|B|}{8} = \frac{1}{16}$

24. Using transformation, $R_1 \rightarrow R_1 + R_2 + R_3$

$$(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-a-c & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$C_3 \rightarrow C_3 - C_1$ and $C_2 \rightarrow C_2 - C_1$

$$(a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 0 & -(a+b+c) \end{vmatrix}$$

$= (a+b+c)((a+b+c)^2 - 0) = (a+b+c)^3$

Hence, $x = 0$ or $x = -2(a+b+c)$

25. $\frac{a_2}{a_0} = \frac{{}^{50}C_2 10^{48} + {}^{50}C_2 10^{48}}{2\times 10^{50}} = \frac{{}^{50}C_2}{100} = \frac{50\times 49}{2\times 100} = \frac{49}{4} = 12.25$

26. $|z| + z = 3 + i$

Let $z = a + i$

$|a+i| + a + i = 3 + i$

$\Rightarrow \sqrt{a^2+1} = 3 - a$

$\Rightarrow a^2 + 1 = 9 + a^2 - 6a$

$\Rightarrow a = \frac{4}{3}$

$\Rightarrow |z| = \sqrt{a^2+1} = \sqrt{\frac{16}{9}+1} = \frac{5}{3}$

27. $k = \{4, 8, 12, 16, 20\}$

$f(k)$ will take values from set $\{3, 6, 9, 12, 15, 18\}$. We can do this by ${}^6C_5 \times 5!$ ways $= 6!$ Ways. For remaining 15 elements, total ways $= 15!$

$\therefore$ Total number of onto functions $= 6! \times 15!$

28. $A(7, 0, 6), B(3, 4, 2)$

$AB = (3-7)\hat{\imath} + (4-0)\hat{\jmath} + (2-6)\hat{k} = -4\hat{\imath} + 4\hat{\jmath} - 4\hat{k}$

Direction ratio of AB is $< 1, -1, 1 >$

Plane P_1 is perpendicular to P_2. Hence direction ratio of normal of P_2 is parallel to P_1

$\Rightarrow < 2, -5, 0 >$

Direction ratio of normal to plane P_1 is

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 1 \\ 2 & -5 & 0 \end{vmatrix} = \hat{\imath}(+5) - \hat{\jmath}(-2) + \hat{k}(-5+2) = 5\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$$

Hence equation of plane is $5(x-7) + 2(y-0) - 3(z-6) = 0$

$\Rightarrow 5x + 2y - 3z = 17$

Since point $(2, \alpha, \beta)$ lies on P_1

Hence $2\alpha - 3\beta = 7$

Physics

1. $\overrightarrow{\Delta r} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$\vec{r_f} = \vec{r_i} + \vec{u}t + \frac{1}{2}\vec{a}t^2$

$\vec{r_f} = (2\hat{\imath} + 4\hat{\jmath}) + (5\hat{\imath} + 4\hat{\jmath}) \times 2 + \frac{1}{2}(4\hat{\imath} + 4\hat{\jmath}) \times 2^2$

$= 2\hat{\imath} + 4\hat{\jmath} + 10\hat{\imath} + 8\hat{\jmath} + 8\hat{\imath} + 8\hat{\jmath}$

$= 20\hat{\imath} + 20\hat{\jmath}$

$|\vec{r_f}| = 20\sqrt{2}m$

2. $I_0 = (M.I.\text{ of } A \,\&\, B) + (M.I.\text{ of } C)$

$$I_0 = \left(\frac{mR^2}{4} + mR^2\right) \times 2 + \frac{mR^2}{2}$$

$$= \frac{5mR^2}{2} + \frac{mR^2}{2} = 3mR^2$$

3. $g = \frac{GM}{R^2}$

$$\frac{g_p}{g_e} = \frac{M_p}{M_e} \times \left(\frac{R_e}{R_p}\right)^2 = 3 \times \frac{1}{9} = \frac{1}{3}$$

$$T = 2\pi\sqrt{\frac{1}{g}}$$

$$\frac{T_p}{T_e} = \sqrt{\frac{g_e}{g_p}} = \sqrt{3}$$

$$T_p = \sqrt{3}T_e$$

Time period at earth for seconds pendulum = 2 sec

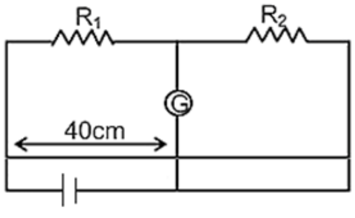
$\therefore T_p = 2\sqrt{3}$ sec

4. $V_{AB} = \frac{\frac{1}{1}+\frac{2}{1}+\frac{3}{1}}{\frac{1}{1}+\frac{1}{1}+\frac{1}{1}} = \frac{1+2+3}{3} = \frac{6}{3} = 2$ volt

5. $F = \frac{dp}{dt} \Rightarrow Kt = \frac{dp}{dt}$

$$\int_P^{3P} dP = \int_0^t Ktdt \Rightarrow 3P - P = \frac{Kt^2}{2} \Rightarrow t = 2\sqrt{\frac{P}{K}}$$

6.



$$\frac{R_1}{R_2} = \frac{40}{60} = \frac{2}{3}$$

$$\frac{R_1+10}{R_2} = \frac{50}{50} = 1$$

$R_1 + 10 = R_2$

$R_1 + 10 = \frac{3}{2}R_1$

$10 = \frac{1}{2}R_1\ R_1 = 20\Omega$

$\frac{\frac{(R_1+10)R}{R_1+10+R}}{R_2} = \frac{2}{3}$

$\frac{R}{R_1+10+R} = 1\frac{2}{3}$

$3R = 2R_1 + 20 + 2R$

$R = 2R_1 + 20$

$R = (40 + 20)\Omega = 60\Omega$

7. $\vec{\tau} = \vec{r} \times \vec{F}$

$\tau = rF\sin\theta$

$2.5 = 5 \times 1 \times \sin\theta \Rightarrow \theta = 30^o$

8. Maximum kinetic energy $= \frac{1}{2}m\omega^2 A^2$

$\omega = \sqrt{\frac{g}{L}}$

$A = L\theta$

$KE = \frac{1}{2}\ m\frac{g}{L} \times L^2\theta^2$

$KE = \frac{1}{2}mgL\theta^2$

If length is doubled

$K_2 = \frac{1}{2}mg(2L)\theta^2$ [Here we are assuming angular amplitude is same]

$\frac{K_1}{K_2} = \frac{\frac{1}{2}mgl\theta^2}{\frac{1}{2}mg(2L)\theta^2} = \frac{1}{2}$

$K_2 = 2K_1$

9. Considering the subscript for ball as $'b'$, for water as $'w'$ and for container as $'c'$

and applying principle of

calorimetry (assuming final temperature = T^oC)

$m_b s_b(500 - T) = m_w s_w(T - 30) + m_c s_c(T - 30)$

$\therefore 0.1 \times 400(500 - T) = 0.5 \times 4200(T - 30) + 800(T - 30)$

$\therefore 20000 - 40T = 2100T - 63000 + 800T - 24000$

$\therefore 2940T = 107000$

$\therefore T = \dfrac{10700}{294} = 36.4^oC$

% rise in temperature $= \frac{6.4}{30} \times 100\% \simeq 21\%$

10. Let $Y = f(V, F, A)$

$\therefore Y = KV^xF^ya^z, K \to$ Unit less

$\therefore [Y] = [V]^x[F]^y[a]^z$

$\therefore [ML^{-1}T^{-2}] = [LT^{-1}]^x[MLT^{-2}]^y[LT^{-2}]^z$

$\therefore [M][L^{-1}] = [T^{-2}] = [M]^y[L^{x+y+z}][T^{-x-2y-2z}]$

$\therefore y = 1, x + y + z = -1; \ -x - 2y - 2z = 2$

$\therefore x + z = -2 \therefore x + 2y + 2z = 2$

$\therefore z = 2, x = -4 \ \therefore x + 2z = 0$

$\therefore [Y] = [V^{-4}Fa^2]$

11. $I = \frac{P}{A} = \frac{1}{2}\varepsilon_0 E_0^2 C$

$\therefore E_0 = \sqrt{\frac{2P}{\varepsilon_0 CA}} = \sqrt{\frac{2\times27\times10^{-3}\times36\pi\times10^9}{3\times10^8\times10^{-6}}}$

$= 100 \times 10 \times 4.5$

$= 4.5 \ kV/m$

12. $\frac{\frac{X_0}{2} - \frac{X_0}{3}}{X_0 - \frac{X_0}{3}} = \frac{C-0}{100-0}$

$\frac{1}{4} = \frac{C}{100} \Rightarrow C = 25^oC$

13. $\frac{1}{V} - \frac{1}{u} = \frac{1}{f}$

$\therefore = \frac{1}{v^2}\frac{dV}{dt} + \frac{1}{u^2}\frac{du}{dt} = 0$

$\therefore \frac{dv}{dt} = \left(\frac{V^2}{u^2}\right)\left(\frac{du}{dt}\right) = \left(-\frac{60}{-20}\right)^2 \times 5 = 9 \times 5 = 45\, m/s$

14. Taking torque about the point of contact

$40 \times [1] = [mr^2 + mr^2]\alpha$

$40 = 2m \times \frac{1}{4}\alpha$

$40 = 2 \times 5 \times \frac{1}{4}\alpha$

$\therefore \alpha = 16\ rad/s^2$

15. 1^{st} condition:

$\Delta\theta_g = \Delta\theta_l$

$100 \times S_A \times (100 - 90) = 50 \times S_B \times (90 - 75) \quad \ldots(i)$

2^{nd} condition:

$\Delta\theta_g = \Delta\theta_{\theta_l}$

$100 \times S_A(100 - \theta) = 50 \times S_B \times (\theta - 50) \quad \ldots(ii)$

dividing (ii) by (i)

$\frac{100-\theta}{100-90} = \frac{\theta-50}{90-75}$

$300 - 3\theta = 2\theta - 100$

$\theta = 80^oC$

16. $V = I_g(R_g + R)$

$15 = 15 \times 10^{-2}(20 + R)$

$R = 80\Omega$

17. From M orbit to L orbit:

$$\frac{hc}{\lambda_2} = (13.6eV)Z^2\left(\frac{1}{4}-\frac{1}{9}\right) \quad \ldots(i)$$

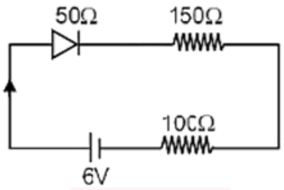
From N orbit to L orbit:

$$\frac{hc}{\lambda_2} = (13.6eV)Z^2\left(\frac{1}{4}-\frac{1}{16}\right) \quad \ldots(ii)$$

dividing (i) by (ii)

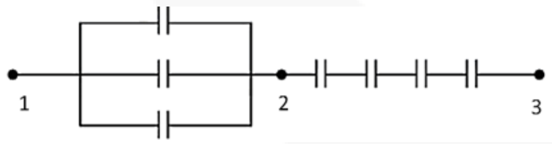
$$\frac{\lambda_2}{\lambda_1} = \frac{5}{36}\times\frac{64}{12} = \frac{20}{27} \Rightarrow \lambda_2 = \frac{20}{27}\lambda_1$$

18. Since the second diode is reverse biased the simplified circuit is as shown in the figure



$$I = \frac{6}{300} = 0.02\ A$$

19.



$$C_{12} = 6\mu F$$

$$C_{23} = \frac{2}{4} = \frac{1}{2}\mu F$$

$$C_{13} = \frac{6\times\frac{1}{2}}{6+\frac{1}{2}} = \frac{3}{\frac{13}{2}} = \frac{6}{13}\mu F$$

20. Electric potential energy of dipole is

$$U = -pE\cos\theta$$

$$U = -10^{-29}\times 1000\cos 45^o$$

$$= -\frac{1}{\sqrt{2}}\times 10^{-26} = -5\sqrt{2}\times 10^{-27} J \approx -7\times 10^{-27}\ J$$

21. $\Delta L_1 = \Delta L_2$

$L\alpha_1 \times 60 = L\alpha_2 \times \Delta T_2$

$\Delta T_2 = \dfrac{\alpha_1}{\alpha_2} \times 60 = \dfrac{4}{3} \times 60 = 80^o C$

22. $n = \dfrac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$\sqrt{3} = \dfrac{\sin\left(\frac{60^o+\delta_m}{2}\right)}{\sin\left(\frac{60^o}{2}\right)}$

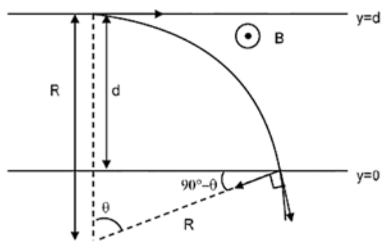
$\frac{\sqrt{3}}{2} = \sin\left(\frac{60^o+\delta_m}{2}\right)$

$60^o = \frac{60^o+\delta_{\min}}{2} \Rightarrow \delta_{\min} = 60^o$

$i = \frac{A+\delta_{\min}}{2} = \frac{60^o+60^o}{2} = 60^o$

23. Self-inductance $L \propto \ell$

24.



Radius of curvature

$R = \dfrac{mv}{qB} = 2d$

$\cos\theta = \dfrac{d}{2d} \Rightarrow \theta = 60^o$

$a = \frac{QvB}{m}$

$\vec{a} = -\sin\theta\,\hat{\imath} - \cos\theta\,\hat{\jmath}$

$\vec{a} = \left(\frac{-\sqrt{3}\hat{\imath}-\hat{\jmath}}{2}\right)\frac{QvB}{m}$

25. $B_{net} = \mu_0(H + I)$

$B_{net} = m_0(H + \chi)H$

$(1 + \chi) = \frac{B_{net}}{\mu_0 H} = \frac{B_{net}}{B_{ext}} = \frac{B_{ext} + B_{int}}{B_{ext}}$

$(1 + \chi) = 1 + \frac{\mu_0 I}{B_{ext}}$

$\chi = \frac{\mu_0 \times 20 \times 10^{-6}}{10^{-6} \times 50 \times 10^{-6}} = \frac{2}{5} \times 10^6 \times 4\pi \times 10^{-7}$

$= \frac{8\pi}{50} = \frac{16\pi}{100}$

$= 16\pi \times 10^{-2} = 50.264 \times 10^{-2}$

$= 0.50$

26. $\omega_s = \frac{2\pi}{100\times10^{-6}} = 2\pi \times 10^4 s^{-1}$

$\omega_c = \frac{2\pi}{8 \times 10^{-6}} = 2.5\,\pi \times 10^6 s^{-1}$

$V_{\max} = V_c + V_s = 10$

$V_{\min} = V_c - V_s = 10$

$\therefore V_c = 9mV$

$V_s = 1mV$

Equation of AM wave

$V_{AM} = (V_c + V_s \sin \omega_s t) \sin \omega_c t$

$= \{9 + \sin(2\pi \times 10^4)t\} \times \sin(2.5\pi \times 10^6 t)(\text{In } mV)$

27. $ML^{-1}T^{-2} = v^x a^y F^z$

$ML^{-1}T^{-2} = [LT^{-1}]^x [LT^{-2}]^y [MLT^{-2}]^z$

$ML^{-1}T^{-2} = M^z L^{x+y+z}\, T^{-x-2y-2z}$

$z = 1 \ldots (i)$

$x + y + z = -1 \qquad \ldots (ii)$

$-x-2y-2z=-2 \quad \ldots(iii)$

$$\begin{array}{r} 2x+2y+2z=-2 \\ -x-y-2z=-2 \\ \hline x=-4 \end{array}$$

$-4+y-1=-1$

$y=2$

$(v^{-4}a^2F)$

Chemistry

1. $t_{\frac{1}{2}}=\frac{C_0}{2k} \Rightarrow 6=\frac{0.2}{2k} \Rightarrow k=\frac{1}{60}$
now, $C_t=C_0-Rk$
$0.2=0.5-\frac{1}{60}\times t$
$0.3=\frac{1}{60} \Rightarrow 1=18$ hrs

2. $K_2[HgI_4] \rightarrow 2K^+ + [HgI_4]^{-2}$;
$i=(1-\alpha)+2\alpha+\alpha$
$i=1+2\times 0.4$
$=1+2\times 0.4=1.8$

3. Gallium belongs to the group 13 whereas Germanium belongs to group 14. As we move from left to right, the electronegativity increases.

4. The inert pair effect is the tendency of the two electrons in the outermost atomic s-orbital to remain unionized or unshared in compounds of post-transition metals. It explains the increasing stability of oxidation states that are two less than the group valency for the heavier elements of groups 13, 14, 15 and 16

5. Cheese: Dispersed Phase = liquid, Dispersion medium = solid
Milk: Dispersed Phase = liquid, Dispersion medium = liquid
Smoke: Dispersed Phase = solid, Dispersion medium = gas

6. $E^o_{\text{cell}}=\frac{0.0591}{n}\log K_c$
$=\frac{0.0591}{2}\log(1\times 10^{16})=0.4728\,V$

7. SiH_4 has a complete octet and thus isn't electron deficient. B_2H_6 appears also to have completed its octet by dimerizing, but the 3 centre 2 electron bond is not as fulfilling as a 2 centre 2 electron bond, and thus still has electron deficient nature.

8. $C.No.=4\times 2+2\times 1=10$
OH^- is monodentate whereas $C_2O_4^{2-}$ is bidentate.

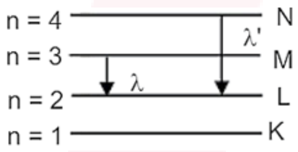
9. $\Delta G^o=-2.303\,RT\log K$
$=-2.303\times 8.314\times 298\log(10^{-14})$

$= +80\frac{kJ}{mol}$

10. $A \to R$, $MgHCO_3$ is responsible for temporary hardness
$B \to P$, Dicalcium silicate is a component of Portland cement
$C \to S$, $NaOH$ is produced in the Castner-Keller Cell
$D \to Q$, Solvay process is the process for industrial preparation of sodium carbonate.

11. $\Delta H - T\Delta S < O$
$T > \frac{\Delta H}{\Delta S}$
$T > \frac{491.1 \times 1000}{198}$
$T > 2480.3030\ K$

12.



$\frac{1}{\lambda} = RZ^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$
$= RZ^2\left(\frac{5}{36}\right)$
$\frac{1}{\lambda'} = RZ^2\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$
$= RZ^2\left(\frac{3}{16}\right)$
$\frac{\lambda'}{\lambda} = \frac{5}{9} \times \frac{4}{3} = \frac{20}{27} \Rightarrow \lambda' = \frac{20}{27}\lambda$

13. Equivalents of HCl = equivalents of Na_2CO_3
$25 \times N = 2 \times 0.1 \times 30$
$HCl = 0.24\ N$
Equivalents of HCl = equivalents of $NaOH$
$0.24 \times V = 30 \times 0.2$
$V = 25\ mL$

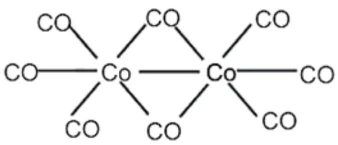
14. $Cu_2S + O_2 \to Cu_2O + 2SO_2$ is roasting process

15. $hv = hv_0 + \frac{1}{2}mV^2$
$h(v - v_0) = \frac{1}{2}mV^2 ; V = \left(\frac{2h(v - v_0)}{m}\right)^{1/2}$
$\lambda = \frac{h}{mV} = \frac{h}{m}\frac{\sqrt{m}}{\sqrt{2h(v - v_0)}} = \sqrt{\frac{h}{m(v - v_0)}}; \lambda \propto \frac{1}{(v - v_0)^{1/2}}$

16. $\Delta G = A - BT$
$= \Delta H - T\Delta S$

$A = \Delta H$
$\Delta H = A = +ve$ endothermic

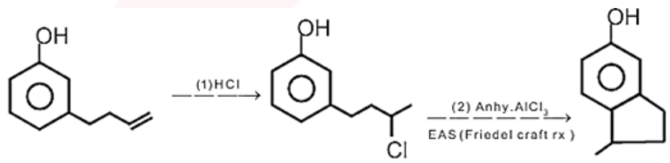
17.



Number of $Co - Co$ bond = 1
Number of bridging carbonyl = 2

18. $MnO_2(A) + KOH + O_2 \rightarrow K_2MnO_4(B)$ (Green) $\xrightarrow{[H^+]} KMnO_4(C)$(purple) $\xrightarrow{Kl + KOH} MnO_2(A) + KIO_3(D)$

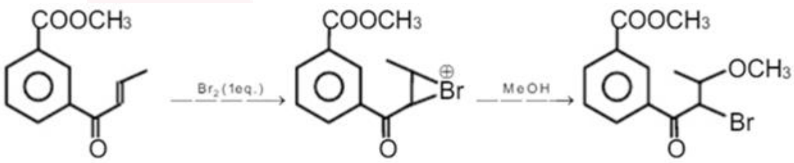
19.



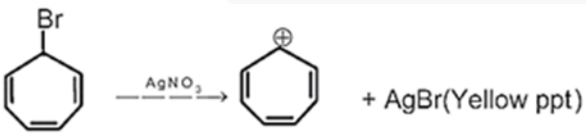
20. Acid rain has been recognised as the primary cause for decolouration of Taj Mahal. The acid reacts with the marble and brings about a dull yellow colour.

21. The stiffening of flower buds is caused by SO_2 gas.

22.



23.



Intermediate carbocation is resonance stabilized and aromatic.

25. $LiAlH_4$ reduces to carboxylic acid, ketone, nitro group but it does not reduce alkenes.

26. $CH_3CH_2CH_2CONH_2(A) \xrightarrow{Br_2+NaOH} B(CH_3CH_2CH_2NH_2)$

27. Nitrogen (c) is most basic in adenine and hence, gets protonated most easily.

28. Phthalein test is given by phenolic ring, that is present in Tyrosine.

29.

PHB is biodegradable polymer

30. When enzymes bind to a site other than the active site, the site is called allosteric site.
Poison binds at the active site through covalent bonds.
Receptors are important for communication.
In competitive inhibition, an inhibitor that resembles the normal substrate binds to the enzyme, usually at the active site, and prevents the substrate from binding.