Solution

Physics

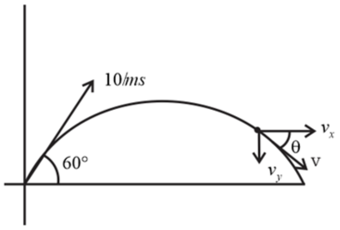
1. Dimension analysis
$I_o = KMa^2$
Now for small lamina
$I' = K\frac{M}{4}\left(\frac{a}{2}\right)^2 = \frac{kma^2}{16}$
$I' = \frac{I_o}{16}$
So moment of Inertia of remaining part
$I_L = I_o - I'$
$= I_o - \frac{I_o}{16}$
$I_L = \frac{15I_o}{16}$

2. Energy of radiation $= \frac{12500}{980} = 12.75eV$
Energy of electron in n^{th} orbit $= -\frac{13.6}{n^2}$
$\Rightarrow E_n - E_1 = -13.6\left[\frac{1}{n^2} - \frac{1}{1^2}\right]$
$\Rightarrow 12.75 = 13.6\ \left[\frac{1}{1^2} - \frac{1}{n^2}\right]$
$\Rightarrow n \approx 4$
Electron will transit to $n = 4$
New radius will be $16a_0$

3. $\Delta\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{8} = \frac{\pi}{4}$
$I = I_0 \cos^2\frac{\pi}{8}$
$= I_0\left(\frac{1 + \cos\pi/4}{2}\right)$
$= I_0\left(\frac{1 + \frac{1}{\sqrt{2}}}{2}\right) = 0.85\ I_0$

4. $K.E = \frac{1}{2}k(A^2 - x^2)$
$P.E = \frac{1}{2}kx^2$
At $t = 210$ sec
$x = A\sin\frac{7\pi}{3}$
$\frac{K.E}{P.E} = \frac{A^2}{x^2} - 1$
$= \frac{A^2}{\left(\frac{A\sqrt{3}}{2}\right)^2} - 1$
$= \frac{4}{3} - 1 = \frac{1}{3}$

5.



Velocity of particle after 1 sec
$v_x = u\cos 60° = 5\,m/s$

$v_y = u\sin 60 - a_y t = 10\frac{\sqrt{3}}{2} - 10(1)$

$= 5(\sqrt{3}-2)\,m/s$

$v = \sqrt{5^2 + [5(\sqrt{3}-2)]^2}\,m/s$

Radius of calculate $R = \frac{v^2}{a_\perp}$

$= \frac{v^2}{g\cos\theta} = \frac{v^2}{g\frac{v_x}{v}} = \frac{v^3}{g\,v_x}$

$= 2.81\,m$

6. Initially

$\frac{P}{Q} = \frac{R_1}{X}$ ...(i)

After interchanging P and Q

$\frac{Q}{P} = \frac{R_2}{X}$ ...(ii)

$1 = \frac{R_1R_2}{X^2}$

$X = \sqrt{R_1R_2}$

$= \sqrt{400 \times 405}$

$= 402.5\,\Omega$

7. $\frac{dv}{dt} = \left(\frac{f}{f+u}\right)^2 \frac{du}{dt} \Rightarrow \frac{dv}{dt} = \left(\frac{0.3}{0.3-0.2}\right)^2 \times 5$

$= 1.16 \times 10^{-3}\,m/s$ towards lens

8. $\lambda_e = 10^{-3}\lambda_p$

$f_p = 6 \times 10^{14} Hz = \frac{C}{\lambda_p}$

$\lambda_p = \frac{3 \times 10^8}{6 \times 10^{14}} = 0.5 \times 10^{-6} m$

$\lambda_e = 0.5 \times 10^{-9}, = \frac{h}{m_e v_e}$

$v_e = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 0.5 \times 10^{-9}}$

$= 1.45 \times 10^6$

9. $R_h = 2\Omega$

$\varepsilon = 0.5\,V$

Current through wire AB is $i = \frac{6}{2+4} = 1A$

$\varepsilon = \frac{4}{L}J$

When $\varepsilon = \varepsilon_2$

$\varepsilon_2 = \frac{4\left(\frac{6}{4+6}\right)J}{L}$

$\frac{\varepsilon}{\varepsilon_2} = \frac{10}{6}$

$\varepsilon_2 = \varepsilon \times \frac{6}{10} = \frac{3}{10} = 0.3\,V$

10. $\tau = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2$

$= (2\hat{\imath} + 3\hat{\jmath}) \times (F\hat{k}) + (6\hat{\jmath}) \times \left(\frac{F}{2}(-\hat{\imath}) + F\frac{\sqrt{3}}{2}(-\hat{\jmath})\right)$

$= 2F(-\hat{\jmath}) + 3F(\hat{\imath}) + 3F\hat{k}$

$= (3\hat{\imath} - 2\hat{\jmath} + 3\hat{k})$

11. $V(t) = 10[1 + 0.3\ \cos(2.2 \times 10^4 t)]\sin(5.5 \times 10^5 t)$

$= 10\sin(5.5 \times 10^5 t) + 3\cos 2.2 \times 10^4 t \sin 5.5 \times 10^5 t$

$= 10\sin(5.5 \times 10^5 t) + \frac{3}{2}[\sin(2.5 \times 10^4 t + 5.5 \times 10^5 t) - \sin(2.5 \times 10^4 t - 5.5 \times 10^5 t)]$

$= 10\sin(5.5 \times 10^5 t) + \frac{3}{2}\sin(57.5 \times 10^4 t) + \frac{3}{2}\sin(52.5 \times 10^4 t)$

Sideband frequencies are $\frac{57.5\times10^4}{2\pi}$ and $\frac{52.5\times10^4}{2\pi}$

$= 91\,kHz$ and $84\,kHz$

12. $U = n_1\frac{f_1}{2}RT + n_2\frac{f_2}{2}RT$

$= 3\left(\frac{5}{2}\right)RT + 5\left(\frac{3}{2}\right)RT$

$= 15RT$

13. Mass per unit time $= \rho Av$

Force due to momentum loss $= \frac{1}{4}\rho Av \times v$

Force due to bounce back $= \frac{1}{4}\rho Av \times 2v$

Pressure $= \frac{\frac{\rho Av^2}{4} + \frac{\rho Av^2}{2}}{A} = \frac{3}{4}\rho v^2$

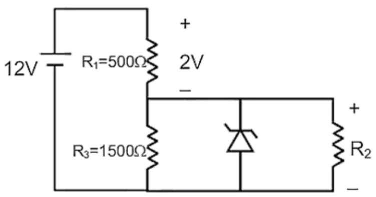
14. $F = \alpha\beta e^{\left(\frac{-x^2}{\alpha kT}\right)}$

$\frac{x^2}{\alpha} = ML^2T^{-2}$

$\alpha = M^{-1}T^2$

$\beta = \frac{MLT^{-2}}{M^{-1}T^2} = M^2LT^{-4}$

15.



Current through $R_1 = \frac{2}{500} = \frac{1}{250}$

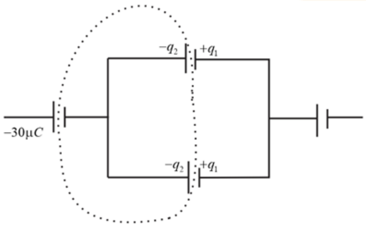
Current through zener diode= $\frac{1}{250} - \frac{100}{1500} - \frac{10}{1500}$

$= \frac{150 - 250 - 250}{250 \times 150}$

$= -ve$ not possible

Current through Zener diode is zero

16.



From parallel combination we can say

Charge on $6\mu F$ is $6 \times \frac{30}{6+4} = 18\ \mu C$

Charge on $4\mu F$ is $4 \times \frac{30}{6+4} = 12\ \mu C$

From the isolated loop,

We can say charge on $6\mu F$ is $+18\ \mu F$

17. Heat lost by water = heat gained by ice

$m_w s_2 \Delta T_w = (m_i - 20)L_i + m_i s_i \Delta T_i$

$50 \times 4.2 \times 40 = (m - 20)334 + m(2.1)20$

$m \approx 40\ g$

18. Assuming lengths of perpendicular sides as 'a'

$\frac{1}{4\pi\epsilon_0}\frac{Qq}{a} + \frac{1}{4\pi\epsilon_0}\frac{Qq}{\sqrt{2}a} + \frac{1}{4\pi\epsilon_0}\frac{qq}{a} = 0$

$Q + \frac{Q}{\sqrt{2}} + q = 0$

$Q\left(1 + \frac{1}{\sqrt{2}}\right) + q = 0$

$Q = -\frac{-q}{1+\frac{1}{\sqrt{2}}} = -\frac{\sqrt{2}q}{1+\sqrt{2}}$

19. For adiabatic process

$TV^{\gamma-1}$ = constant

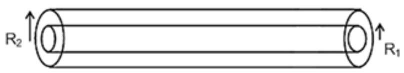
$\gamma = 1 + \frac{2}{5} = \frac{7}{5}$ (For diatomic gas)

So $x = \gamma - 1 = \frac{7}{5} - 1 = \frac{2}{5}$

20. Mutual inductance $= \mu_0 n_1 n_2 R_1^2 \ell \pi$

Self inductance of inner solenoid $= \mu_0 n_1^2 R_1^2 \ell \pi$

Ratio= $\frac{n_2}{n_1}$



21. Potential of a uniformly charged spherical shell

22. $|\Delta \vec{V}| = 2v \sin\frac{\theta}{2} = 2v \sin 30^o = 2 \times 10 \times \frac{1}{2} = 10\ m/s$

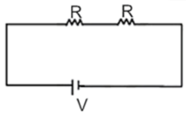
23. Growth and decay of current is of exponential nature

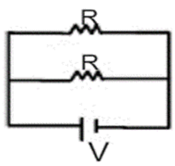
$i = i_0\left(1 - e^{-t/\tau}\right)$ →during growth
$i = i_{\max}\, e^{-t/\tau}$ → during decay
Note: - In actual paper none of the options was correct.

24. In air $\frac{E_0}{B_0} = C$
In the medium of refractive index $= n$
$\frac{E}{B} = \frac{C}{n}$
It is possible if
$E = \frac{E_0}{\sqrt{n}}$ and $B = B_0\sqrt{n}$
$\therefore \frac{E_0}{E} = \sqrt{n} \quad \frac{B_0}{B} = \frac{1}{\sqrt{n}}$

25. The velocity of 1 kg block just before striking
$V = \sqrt{2 \times 10 \times 100} = 20\sqrt{5}\ m/s$
Applying conservation of momentum, the blocks stick after collision and move with velocity v
Or $1 \times 20\sqrt{5} = 4v$
Final velocity $v = 5\sqrt{5}$,
Using energy conservation
$KE + PE = \text{const}$
$$\frac{1}{2} \times 4 \times \left(5\sqrt{5}\right)^2 + \frac{1}{2}kx_0^2 + 4 \times 10 \times x = \frac{1}{2}k(x + x_0)^2$$
On solving $x \approx 2\ cm$

26. $k = 9m^{-1}$ and $\omega = 450\ rad/s$
$$\therefore v = \frac{\omega}{k} = 50\ m/s$$
$$v = \sqrt{\frac{T}{\mu}}$$
$\therefore T = \mu v^2 = 5 \times 10^{-3} \times 50^2 = 12.5N$

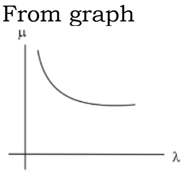
27.


$P_{\text{generated}} = 60\ W$
$$P = 60 = \frac{V^2}{2R}$$
$$\frac{V^2}{R} = 120\ watt$$


$$P_{\text{generated}} = \frac{2V^2}{R} = 2(120) = 240 \text{ watt}$$

28.
$\delta = A(\mu - 1)$ for thin prism ...(i)

From graph



Retractive index decreasing with λ

$\Rightarrow \delta$ should also decrease with λ from …(i)

29. Orbital velocity, $v_0 = \sqrt{\frac{GM}{(R+h)}}$

To escape, $v_s = \sqrt{\frac{2GM}{(R+h)}}$

Change in velocity $= \sqrt{\frac{2GM}{(R+h)}} - \sqrt{\frac{GM}{(R+h)}}$

If $h \ll R$

$\Delta v = \sqrt{\frac{2GM}{R}} - \sqrt{\frac{GM}{R}}$

$\Delta v = \sqrt{2gR} - \sqrt{gR}$

30. $r = \frac{\sqrt{2meV}}{eB}$

$= \sqrt{\frac{2meV_{ac}}{e^2B^2}}$

$= \sqrt{\frac{2\times 9\times 10^{-31}\times 500}{1.6\times 10^{-19}\times(100\times 10^{-3})^2}}$

$= \sqrt{\frac{2\times 9\times 10^{-31}\times 500}{1.6\times 10^{-19}\times 10^{-2}}}$

$= \sqrt{\frac{2\times 9.1\times 5\times 10^{-31+23}}{1.6}}$

$= \sqrt{\frac{9.1}{1.6}\times 10^{-7}}$

$= \sqrt{\frac{91}{16}\times 10^{-7}}$

$= \sqrt{\frac{910}{16}\times 10^{-8}}$

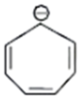
$= \sqrt{56.875}\times 10^{-4}$

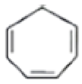
$= 7.54\times 10^{-4}m$

Chemistry

1. The necessary conditions for aromaticity are: molecule should be planar, cyclic, have conjugation and follow the Huckel's rule $(4n+2)\ \pi\ e^-$ where $n = 0,1,2\ldots.$

 cyclic conjugation, planar, 4 $\pi\ e^-$ it is anti aromatic

 cyclic conjugation, planar, 8 π e^- it is anti aromatic

 no cyclic conjugation, non-planar, it is non-aromatic

2. Nitric oxide (NO) and Nitrogen dioxide (NO_2) are emitted from the combination of fossil fuels, along with being naturally emitted from things such as volcanos and forest Fires. When exposed to ultraviolet radiations, NO_2 goes through a complex series of reaction with hydrocarbons to produce the components of photochemical smog which is a mixture of Ozone, nitric acid, aldehydes, peroxyacylnitrates ($PANs$) and other secondary pollutants.

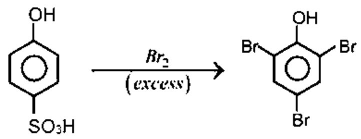
3. At 20°C (room temperature) and standard atmospheric pressure (sea level), the maximum amount of oxygen that can be dissolved in a fresh water is 9 – 10 ppm.

4. Sugar + water → Sugar solution can be separated using crystallization process in which the solution is allowed to cool and water is evaporated, leaving behind the sugar crystals.

   Toluene + water → The two form an immiscible mixture. The components can be separated using extraction distillation or separting funnel.

   Aniline + water → Aniline (intramolecular hydrogen bonding) being more volatile than water (intermolecular hydrogen bonding) can easily be separated using steam distillation.

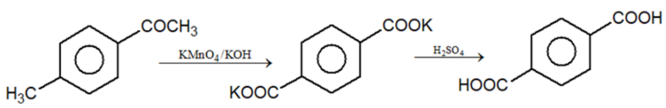
5.



→ OH group is an electron releasing group. It makes Benzene ring ortho-para directing towards electrophilic substitution reaction.

→ SO_3H being a good leaving group, departs in presence of excess of bromine.

6.



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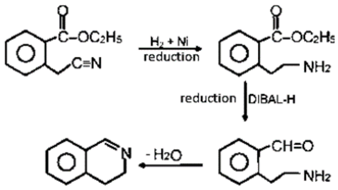

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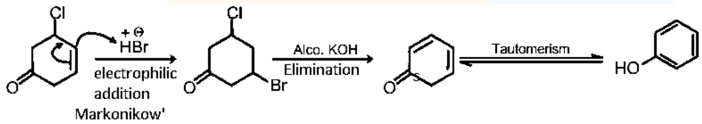


9. Nitrogenous bases are present in nucleic acid (DNA & RNA)

- DNA contains →Adenine, Guanine, Cytosine & Thymine

-RNA contains → Adenine, Guanine, Cytosine & Uracil

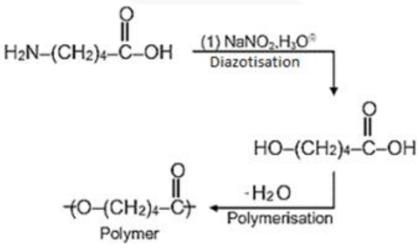
- Adenine & Guanine are bicyclic compounds →No option

- Option (A) is Thymine

- Option (C) is Uracil

- Option (B) & (D) are not nitrogenous bases of nucleic acid.

10.



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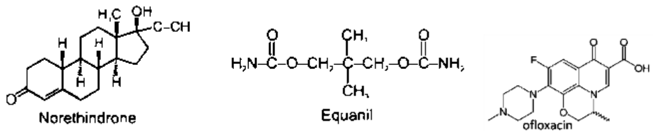

Norethindrone is used for birth control (contraception) to prevent pregnancy. Norethindrone is also used to treat menstrual disorders, endometriosis, or abnormal vaginal bleeding caused by a hormone imbalance.

Ofloxacin medication is used to treat a variety of bacterial infections. Ofloxacin belongs to a class of drugs called quinolone antibiotics. It works by stopping the growth of bacteria. This antibiotic treats only bacterial infections.

Equanil medication is used in short-term to treat symptoms of anxiety and nervousness. It acts on certain centers of the brain to help calm our nervous system.

12. Sc has the highest tendency to adopt Noble gas configuration among all d-block elements. It has Argon (Ar) as the nearest noble gas, so by loosing $3e^{-}s$, it attains a noble gas configuration of Ar or $3s^2 3p^6$. Thus, Sc shows mostly +3 oxidation state. It doesn't exhibit +1 oxidation state as 3d orbitals are further inside the atom than 4s orbitals. Thus, the 4s orbital loses 2 electrons first followed by the lose of 3d electron giving it a +3-oxidation state.

13. $K = Ae^{-E_a/RT} \Rightarrow ln\ K = ln\, A - \frac{E_a}{RT}$

Comparing with $y = c + mx$

$-Ea = -y$

Or $Ea = y$

(Energy required to start reaction i.e. activation energy)

14. C in CCl_4 does not have vacant d-orbital. It will undergo partial hydrolysis and form product that will be unstable. While other compounds have vacant d-orbitals, so they're able to undergo hydrolysis.

15. A gemstone is a type of colloid, in which the solid phase is dispersion medium and solid itself is dispersed phase.

16. For an FCC lattice, number of atoms per unit cell = 4

Edge length = $200\sqrt{2}pm$ = $200\sqrt{2}$ X 10^{-12}meter

$$d = \frac{ZM}{N_A \times a^3}$$

$$9 \times 10^3 = \frac{4 \times \frac{M_0}{6 \times 10^{23}}}{\left(200 \times \sqrt{2} \times 10^{-12}\right)^3}$$

$$9 \times 10^3 = \frac{4 \times \frac{M_0}{6 \times 10^{23}}}{2^3 \times 2 \times \sqrt{2} \times 10^{-30}}$$

$$M_0 = \frac{9 \times 10^3 \times 6 \times 10^{23} \times 2^4 \times \sqrt{2} \times 10^{-30}}{4}$$

$= 9 \times 6 \times 10^{-4} \times 4 \times \sqrt{2}$

$= 0.0305 \quad kg/mol$

17. $2NH_3 \rightleftharpoons N_2 + 3H_2 \qquad K'_p = \frac{1}{k_p}$

$P_{Total} = P = P_{N_2} + P_{H_2} + P_{NH_3}$

$P_{N_2} = \frac{P}{4}; P_{H_2} = \frac{3P}{4}$ ($\because P_{NH_3} << P_T$, its contribution in total pressure can be neglected)

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$$\frac{1}{K_p} = \frac{\left(\frac{P}{4}\right)\left(\frac{3P}{4}\right)^3}{\left(P_{NH_3}\right)^2}$$

$$\Rightarrow \left(P_{(NH_3)^2}\right)^2 = \frac{p}{4} \times \left(\frac{3p}{4}\right)^3 \times K_p$$

$$P_{NH_3} = \frac{3^{\frac{3}{2}} P^2}{16} \left(K_p\right)^{1/2}$$

18. Saline hydrides, also called ionic hydrides, are the compounds formed between hydrogen and most active metals. For example: LiH, NaH, KH etc. Saline hydrides react violently with water producing H_2 gas.

$NaH_{(s)} + H_2O_{(aq)} \rightarrow NaOH_{(aq)} + H_{2(g)}$

19. Be forms amphoteric hydroxide. The amphoteric nature of Aluminium is well known as it can react with both acids and bases to give neutralization reactions. Beryllium shares this nature with Aluminium due to its diagonal relationship.

20. $T_F = \frac{T_1+T_2}{2}$

$\Delta S_i = C_p ln\left(\frac{T_f}{T_1}\right)$

$\Delta S_{ii} = C_P ln\left(\frac{T_f}{T_2}\right)$

$\Delta S = \Delta S_i + \Delta$

$$= C_P\ ln\left(\frac{{T_f}^2}{T_1T_2}\right) = C_P\ ln\left[\frac{\left(\frac{T_1+T_2}{2}\right)^2}{T_1T_2}\right] = Cp\ \ ln\left[\frac{(T_1+T_2)^2}{(4T_1T_2)}\right]$$

21. (i) Siderite $FeCO_3$

(ii) Kaolinite $-Al_2Si_2O_5(OH)_4$

(iii) Malachite $-CuCO_3Cu(OH)_2$

(iv) Calamine $-ZnCO_3$

22. Trend of atomic radius in periodic table:

→ On moving across the period ($L \rightarrow R$): atomic radius decreases generally

→ On moving down the group ($T \rightarrow B$): atomic radius increases generally

C being in 2^{nd} period is smallest, while Cs is present in period 6 and thus, the largest. S is smaller than Al as it is placed to the right of Al in period 3.

23. $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$$\frac{1}{\lambda} = 10^7\left(\frac{1}{(3)^2} - \frac{1}{\infty}\right)$$

$\lambda = 9 \times 10^{-7}$

$\lambda = 900\ nm$

24. Freezing point of milk = $-0.5°C \quad \therefore\ \Delta T_f = 0.5°C$

Freezing point of diluted milk $-0.2°C \quad \therefore\ \Delta T_f = 0.2°C$

$$\frac{(\Delta T_f)_i}{(\Delta T_f)_{ii}} = \frac{0.5}{0.2} = \frac{K_f m}{k_f m}$$

Both has same amount of solute. Let that be x mole.

$$\frac{0.5}{0.2} = \frac{x \text{ mole} \times W_2}{W_1 \times x \text{ mole}}$$

$$\frac{5}{2} = \frac{W_2}{W_1}$$

$W_2 = \frac{5}{2}W_1, \frac{W_2}{W_1} = \frac{5}{2}$

$\therefore$ 5 cup of solution has 3 cups of water and 2 cups of pure milk.

25. $2NaHCO_3 + H_2C_2O_4 \rightarrow Na_2C_2O_4 + 2CO_2 + 2H_2O$

Let mass of $NaHCO_3$ be x mg

$$n = \frac{0.25}{25000} = 10^{-5}$$

$W = \text{moles } \times \text{molecular weight} = 84 \times 10^{-5} g$

$$\% = \frac{84 \times 10^{-5}}{10^{-2}} \times 100 = 8.4\%$$

26. The half cell which has the highest reduction potential per electron will form the cell with maximum value of $E°$

$E°_{red}/\text{electron}$ of $Ag^+/Ag = 0.8$

$E°_{red}$/electron of $Au^{3+}/Au = 1.4/3 = 0.467$

$E°_{red}$/electron of $Fe^{3+}/Fe^{2+} = 0.77$

$E°_{red}$/electron for Fe^{2+}/Fe is negative hence, can be neglected.

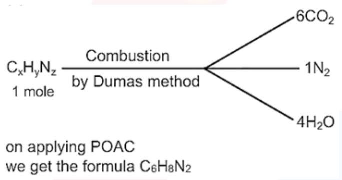
$\therefore E°_{red}$/electron for Ag^{+}/Ag is highest, its cathode will give highest E°_{cell}

27. $\Delta G = 0$, at equilibrium

For equilibrium, $120 - \frac{3}{8}T = 0$ or $T = 320$

Hence for $T < 320, \Delta G$ is positive and more reactants will be pressure. Hence, major component is at $T = 315\,K$

28.



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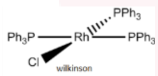
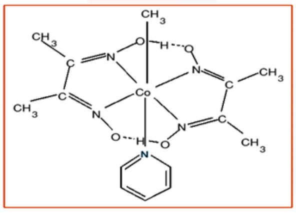
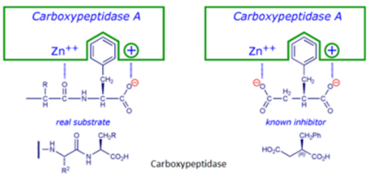
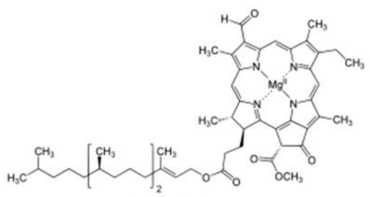







Chlorophyll

30. All statement except 4 are correct. The calorific values of liquid hydrogen and LPG are 142 kJ and 50 kJ respectively.

Mathematics

1. ${}^{20}C_0\,{}^{20}C_r + {}^{20}C_1\,{}^{20}C_{r-1} + {}^{20}C_2\,{}^{20}C_{r-2} + \ldots\ldots + {}^{20}C_r\,{}^{20}C_0$

$=$ coefficient of x^r in

$({}^{20}C_0 + {}^{20}C_1x + {}^{20}C_2x^2 + \ldots. + {}^{20}C_{20}x^{20}) \times ({}^{20}C_0x^{20} + {}^{20}C_1x^{19} + \ldots + {}^{20}C_{19}x + {}^{20}C_{20})$

$=$ coefficient of x^r in $(1+x)^{20}(x+1)^{20}$

$=$ coefficient of x^r in $(1+x)^{40}$

$= {}^{40}C_r$

Which is maximum for $r = 20$

2. $\left(-2-\frac{i}{3}\right)^3 = \frac{-1}{27}(6+i)^3$

$= \frac{-1}{27}[6^3 + i^3 + 18i(6+i)]$

$= \frac{-1}{27}[216 - i + 108i - 18]$

$= \frac{-1}{27}[198 + 107i]$

$= \frac{x+iy}{27}$

$\Rightarrow x = -198 \;\; y = -107$

$\Rightarrow y - x = 91$

3. $I = \int_{-2}^{0} \frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}} dx + \int_{0}^{2} \frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}} dx$

$= \int_{-2}^{0} -2\sin^2 x dx + \int_0^2 2\sin^2 x dx$

Put $x = -t$ in first integral

$dx = -dt$

$\therefore \int_2^0 2\sin^2 t\, dt + \int_0^2 2\sin^2 x\, dx = -\int_0^2 2\sin^2 x\, dx + \int_0^2 2\sin^2 x\, dx = 0$

4. $f_4(x) - f_6(x) = \frac{1}{4}(\sin^4 x + \cos^4 x) - \frac{1}{6}(\sin^6 x + \cos^6 x)$

$= \frac{1}{4}(1 - 2\sin^2 x.\cos^2 x) - \frac{1}{6}(1 - 3\sin^2 x \cos^2 x)$

$= \frac{1}{4} - \frac{1}{2}\sin^2 x.\cos^2 x - \frac{1}{6} + \frac{1}{2}\sin^2 x.\cos^2 x$

$= \frac{1}{4} - \frac{1}{6} = \frac{3-2}{12} = \frac{1}{12}$

Alternate Method:

Put $x = 0$ we get,

$f_4(x) - f_6(x) = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$

5. $f(x) = \frac{x}{1+x^2} = y$

$x = y + yx^2$

$yx^2 - x + y = 0$

For $x \in R, D \geq 0$

$1 - 4y^2 \geq 0$

$y^2 \leq \frac{1}{4}$

$y \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

So the range of $f(x) \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

6. Let first term of $GP = a$, common ratio $= r, (r \in (-1,0) \cup (0,1)$

Given $\frac{a}{1-r} = 3 \quad \dots(i)$ and $\frac{a^3}{1-r^3} = \frac{27}{19} \qquad (ii)$

$\Rightarrow a^3 = 27(1-r)^3$

$\therefore$ from (ii)

$\frac{27(1-r)^3}{1-r^3} = \frac{27}{19}$

$\Rightarrow 19(1-r)^3 = (1-r)(1+r+r^2)$

$\Rightarrow 19(1-r)^2 = 1 + r + r^2 \ (r \neq 1)$

$\Rightarrow 19(1-2r+r^2) = 1 + r + r^2$

$\Rightarrow 19 - 38r + 19r^2 = 1 + r + r^2$

$\Rightarrow 18r^2 - 39r + 18 = 0$

$\Rightarrow 6r^2 - 13r + 6 = 0$

$\Rightarrow 6r^2 - 9r - 4r + 6 = 0$

$\Rightarrow 3r(2r-3) - 2(2r-3) = 0$

$\Rightarrow (2r-3)(3r-2) = 0$

$\Rightarrow r = \frac{3}{2}, \frac{2}{3}$ (but $r \neq \frac{3}{2}$)

So only possible common ratio $= \frac{2}{3}$

7. Roots are α, α^3

$\therefore \ \alpha \times \alpha^3 = \frac{256}{81} = \left(\frac{4}{3}\right)^4$

$\because \alpha = \frac{4}{3}$ or $\frac{-4}{3}$

$\therefore \alpha + \alpha^3 = -\frac{k}{81}$

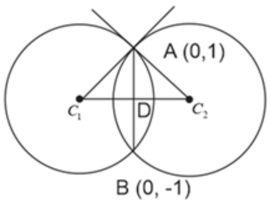
$\Rightarrow \frac{4}{3} + \frac{64}{27} = \frac{-k}{81}$ or $\frac{-4}{3} - \frac{64}{27} = \frac{-k}{81}$

$\Rightarrow \frac{100}{27} = \frac{-k}{81}$ or $\frac{-100}{27} = \frac{-k}{81}$

$\Rightarrow k = 300$ or -300

8. Given circles intersect at $A(0,1)$ and $B(0,-1)$, hence length of common chord $AB = 2$. Also, this common chord bisects the line joining centres of two circles

$\Rightarrow C_1D = C_2D$



Since $\angle C_1AC_2 = 90° \Rightarrow \angle C_1AD = 45°$

Also $AC_1 = AC_2 \Rightarrow \angle AC_1C_2 = \angle AC_2C_1 = 45°$

$\Rightarrow C_1D = AD = 1$

$\Rightarrow C_1C_2 = 2$

9. Let common ratio $= r$

$\therefore \frac{a_3}{a_1} = \frac{a_1r^2}{a_1} = 25$

$\Rightarrow r^2 = 25 = 5^2$

Now, $\frac{a_9}{a_5} = \frac{a_1r^8}{a_1r^4} = r^4 = 5^4$

10. In the given integral $\int \frac{\sqrt{1-x^2}}{x^4}dx$, if we put $x = \sin\theta$,

we get $\int \frac{\sqrt{1-x^2}}{x^4}dx = \int \frac{\cos\theta\cos\theta}{\sin^4\theta}d\theta (dx = \cos\theta d\theta)$

$= \int \cot^2\theta . cosec^2\theta \, d\theta$

Put $\cot\theta = t$, we get $-cosec^2\theta \, d\theta = dt$

$= -\int t^2 dt = \frac{-t^3}{3}$+c

$= -\frac{(\cot\theta)^3}{3} \quad \left(\sin\theta = x \Rightarrow \cot\theta = \frac{\sqrt{1-x^2}}{x}\right)$

$= -\frac{1}{3x^3}.(1-x^2)^{\frac{3}{2}}$+c

Comparing, we get $A(x) = -\frac{1}{3x^3}$ and $m = 3$

$\Rightarrow (A(x))^m = \left(-\frac{1}{3x^3}\right)^3 = -\frac{1}{27x^9}$.

11. Equation of tangent to $y^2 = 4x$ is

$y = mx + \frac{1}{m} \quad \dots (i)$

$\because$ equation (i) is tangent to $xy = 2$

$\Rightarrow x\left(mx + \frac{1}{m}\right) = 2$

$\Rightarrow mx^2 + \frac{x}{m} = 2$

$\Rightarrow m^2x^2 + x - 2m = 0$

Now discriminant has to be zero

$\therefore D = (1)^2 - 4m^2 \times (-2m) = 0$

$\Rightarrow 1 + 8m^3 = 0$

$\Rightarrow m^3 = -\frac{1}{8}$

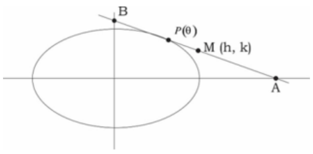
$\Rightarrow m = -\frac{1}{2}$

$\therefore$ equation of tangent is

$y = -\frac{1}{2}x - 2$

$\Rightarrow x + 2y + 4 = 0$

12. Given ellipse $\frac{x^2}{2} + y^2 = 1$ Let $P(\theta)$ be $\left(\sqrt{2}\cos\theta, \sin\theta\right)$



A general tangent on the given ellipse can be written as $\frac{x}{\sqrt{2}}\cos\theta + y\sin\theta = 1 \quad \dots (i)$

It will intersect the coordinate axes at $A = (\sqrt{2}\sec\theta, 0)$ and $B = (0, cosec\theta)$

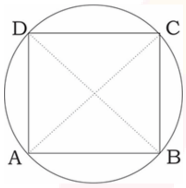
Let the midpoint of AB be $M(h, k)$ then

$h = \frac{\sqrt{2}\sec\theta}{2}, k = \frac{\operatorname{cosec}\theta}{2}$

$\Rightarrow \left(\frac{1}{\sqrt{2}h}\right)^2 + \left(\frac{1}{2k}\right)^2 = 1\ (\because \cos^2\theta + \sin^2\theta = 1)$

$\Rightarrow$ the required locus is $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$

13. Centre of the given circle is $0\ (3, -4)$ and radius is $8\sqrt{2}$



Since sides of the squares are parallel to coordinate axes, slope of diagonals will be 1 or -1.

For vertex A and C, parametric equation of AC, through the point O is

$\frac{x-3}{\frac{1}{\sqrt{2}}} = \frac{y+4}{\frac{1}{\sqrt{2}}} = r$ put $r = 8\sqrt{2}$ and $-8\sqrt{2}$

$A(-5, -12)$

$C(11, 4)$

Similarly, $B(11, -12)$, $D(-5, 4)$ using $\frac{x-3}{\frac{-1}{\sqrt{2}}} = \frac{y+4}{\frac{1}{\sqrt{2}}} = 8\sqrt{2}$ or $-8\sqrt{2}$

For minimum distance, we will consider vertex $D(-5, 4)$.

Hence minimum distance $= \sqrt{25 + 16} = \sqrt{41}$.

14. $LHL = \lim_{x \to 0^-} \frac{\tan(\pi \sin^2 x) + \{|x| - \sin(x[x])^2\}}{x^2}$

$= \lim_{x \to 0^-} \frac{\tan(\pi \sin^2 x) + \{-x - \sin(-x)\}}{x^2}$

$= \lim_{x \to 0^-} \frac{\tan(\pi \sin^2 x) + \sin x - x}{x^2}$

$= \lim_{x \to 0^-} \frac{\tan(\pi \sin^2 x)}{x^2(\pi \sin^2 x)} \times (\pi \sin^2 x) + \lim_{x \to 0^-} \frac{\sin x - x}{x^2}$

$= 1.\pi.1 + \lim_{x \to 0^-} \frac{\cos x - 1}{2x}$

$= \pi + \lim_{x \to 0^-} \frac{-\sin x}{2}$

$= \pi + 0 = \pi$

$RHL = \lim_{x\to 0^+} \frac{\tan(\pi\sin^2 x)+\{|x|-\sin(x[x])\}}{x^2}$

$= \lim_{x\to 0^+} \frac{\tan(\pi\sin^2 x)+\{x-\sin(x.0)\}}{x^2}$

$= \lim_{x\to 0^+} \frac{\tan(\pi\sin^2 x)+x}{x^2}$

$= \lim_{x\to 0^+} \frac{\sec^2(\pi\sin^2 x)(\pi\sin 2x)+1}{2x}$

$\to \infty$

$\Rightarrow$ Hence limit does not exist

15. Mean $\bar{x} = \frac{\sum f_i x_i}{N}$

$= \frac{10\left(\frac{1}{2}-d\right)+10\left(\frac{1}{2}\right)+10\left(\frac{1}{2}+d\right)}{30}$

$= \frac{1}{2}$

Variance of $(\sigma^2) = \frac{1}{N}\sum f_i (x_i - \bar{x})^2$

$= \frac{1}{30}\left[10\left[\left(\frac{1}{2}-d\right)-\frac{1}{2}\right]^2 + 10\left(\frac{1}{2}-\frac{1}{2}\right)^2 + 10\left[\left(\frac{1}{2}+d\right)-\frac{1}{2}\right]^2\right]$

$= \frac{20}{30}d^2 = \frac{4}{3}$ (given)

$\Rightarrow d^2 = 2 \Rightarrow |d| = \sqrt{2}$

16. The middle term in the expansion $\left(\frac{x^3}{3}+\frac{3}{x}\right)^8$

$= T_{\frac{8}{2}+1} = T_5 = {}^8C_4\left(\frac{x^3}{3}\right)^4 . \left(\frac{3}{x}\right)^4$

$= {}^8C_4 . x^8$

$= 70x^8 = 5670$ (given)

$\Rightarrow x^8 = \frac{5670}{70} = 81$

$\Rightarrow x^8 - 81 = 0 \Rightarrow x^8 = \left(\sqrt{3}\right)^8 \Rightarrow x = \pm\sqrt{3}$ (Rest all values are not real)

$\Rightarrow$ Sum $= \sqrt{3} - \sqrt{3}$

$= 0$

17. The given set $\{1, 2, 3, \ldots, 11\}$ has

Odd numbers $= 1, 3, \ldots, 11$: 6 numbers

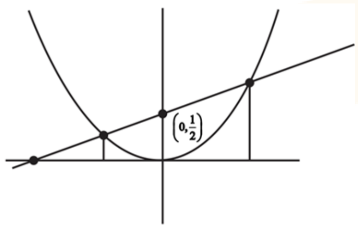
Even numbers $= 2, 4, 6, \ldots, 10$: 5 numbers

Now for their sum to be even, both must be even or both must be odd.

$P\left(\frac{\text{both even}}{\text{sum is even}}\right) = \frac{\text{n(both even)}}{\text{n(both even)+n(both odd)}}$

$= \frac{{}^5C_2}{{}^5C_2 + {}^6C_2} = \frac{10}{10+15} = \frac{10}{25} = \frac{2}{5}$

18.



Shaded portion is required Area.

$\left.\begin{matrix} x^2 = 4y \\ x = 4y - 2 \end{matrix}\right\} \Rightarrow x^2 = x + 2$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow x = \frac{1 \pm \sqrt{1+8}}{2} = 2 \text{ or } -1$

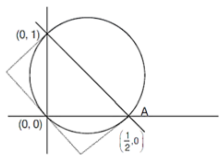
Area $= \int_{-1}^{2} \left(\frac{x+2}{4} - \frac{x^2}{4}\right) dx$

$= \left[\frac{x^2}{8} + \frac{x}{2} - \frac{x^3}{12}\right]_{-1}^{2}$

$= \frac{(2)^2 - (-1)^2}{8} + \frac{(2)-(-1)}{2} - \frac{(2)^3 - (-1)^3}{12}$

$= \frac{9}{8}$

19.



Equation of circle is $(x-0)(x-1) + \left(y - \frac{1}{2}\right)(y-0) = 0$

$\Rightarrow x^2 + y^2 - x - \frac{y}{2} = 0$

Equation of tangent to circle at $(0,0)$ is $2x + y = 0$ ....(i) distance of $(1,0)$ from (i) is $\frac{2}{\sqrt{5}}$

distance of $\left(0,\frac{1}{2}\right)$ from (i) is $\frac{1}{2\sqrt{5}}$

$\Rightarrow$ required sum $= \frac{2}{\sqrt{5}} + \frac{1}{2\sqrt{5}} = \frac{5}{2\sqrt{5}} = \frac{\sqrt{5}}{2}$

20. $(p \wedge q) \leftrightarrow r$ is true

Case-I $p \wedge q$ is true and r is true

It is not possible as q is false

Case-II $p \wedge q$ is false and r is false

$\Rightarrow p = T$ or F, $q = F, r = F$

Hence, $p \vee r$, may be true or false

$p \wedge r$, is always false

$(p \wedge r) \rightarrow (p \vee r)$ is $(F) \rightarrow (T \text{ or } F)$ which is always true

$(p \vee r) \rightarrow (p \wedge r)$ is $(T$ or $F) \rightarrow (F)$ which may be T or F

21. Differentiate w.r.t. x

$$\frac{x}{\log_e x} \times \frac{1}{x} + \log_e(\log_e x) - 2x + 2yy' = 0$$

$\Rightarrow \frac{1}{\log_e x} + \log_e(\log_e x) + 2yy' = 2x$ ....(i)

When $x = e$ the original curve gives $0 - e^2 + y^2 = 4$

$\Rightarrow y = \pm\sqrt{4 + e^2}$

so (i) becomes $1 + 0 + 2yy' = 2e$

$$y' = \frac{2e - 1}{2y} = \pm\frac{2e - 1}{2\sqrt{4 + e^2}}$$

22. $AA^T = I \Rightarrow \begin{bmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{bmatrix} \begin{bmatrix} 0 & p & p \\ 2q & q & -q \\ r & -r & r \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 4q^2 + r^2 & 2q^2 - r^2 & -2q^2 + r^2 \\ 2q^2 - r^2 & p^2 + q^2 + r^2 & p^2 - q^2 - r^2 \\ -2q^2 + r^2 & p^2 - q^2 - r^2 & p^2 + q^2 + r^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$= p^2 + q^2 + r^2 = 4q^2 + r^2 = 1$ and $2q^2 - r^2 = 0, p^2 - q^2 - r^2 = 0$

Now $r^2 = 2q^2$ and $r^2 + 4q^2 = 1 \Rightarrow q^2 = \frac{1}{6}, r^2 = \frac{1}{3}$

Hence $p^2 = \frac{1}{2} \Rightarrow |p| = \frac{1}{\sqrt{2}}$

23. Let a, b, c be the three sides, given

$a + b = x, ab = y, (a + b)^2 - c^2 = ab$

here $\frac{a^2+b^2-c^2}{2ab} = -\frac{1}{2} \Rightarrow \cos C = -\frac{1}{2}$

$\frac{c}{\sin C} = 2R \Rightarrow \frac{2c}{\sqrt{3}} = 2R \Rightarrow R = \frac{c}{\sqrt{3}}$

24. $\frac{dy}{dx} + \left(2 + \frac{1}{x}\right) y = e^{-2x}$

$I.F. = e^{\int \left(2+\frac{1}{x}\right)dx} = e^{2x+\ln(x)} = xe^{2x}$

Solution is $y(xe^{2x}) = \frac{x^2}{2} + C$, since $y(1) = \frac{1}{2e^2}$

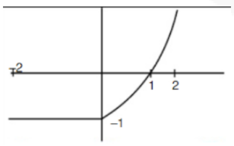
Hence, $\frac{1}{2e^2} \times 1 \times e^2 = \frac{1}{2} + C \Rightarrow C = 0$

hence $y = \frac{xe^{-2x}}{2}$

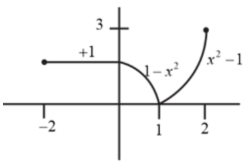
$\frac{dy}{dx} = \frac{e^{-2x}}{2} + \frac{x\, e^{-2x}(-2)}{2} = e^{-2x} \left[\frac{1}{2} - x\right] < 0 \Rightarrow x > \frac{1}{2}$

Hence $y(x)$ is decreasing in $\left(\frac{1}{2}, 1\right)$

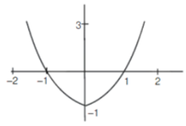
25. $y = f(x)$



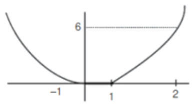
$y = |f(x)|$



$y = f(|x|)$



$y = g(x)$



Hence $g(x)$ has one non – differentiable point, $x = 1$

26. $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 4\hat{k}$

$\vec{b} = \hat{\imath} + \lambda\hat{\jmath} + 4\hat{k}$

$\vec{c} = 2\hat{\imath} + 4\hat{\jmath} + (\lambda^2 - 1)\hat{k}$

$$[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} 1 & 2 & 4 \\ 1 & \lambda & 4 \\ 2 & 4 & \lambda^2 - 1 \end{vmatrix} = \lambda(\lambda^2 - 1) - 16 - 2\,(\lambda^2 - 9) + 4\,(4 - 2\lambda)$$

$= \lambda^3 - 2\lambda^2 - 9\lambda + 18 = \lambda(\lambda^2 - 9) - 2(\lambda^2 - 9)$

$[\vec{a}\vec{b}\vec{c}] \Rightarrow (\lambda - 3)\,(\lambda + 3)\,(\lambda - 2)$

for $\lambda = \pm 3, \vec{c} = 2\vec{a} \Rightarrow \vec{a} \times \vec{c} = \vec{0}$

Hence, for $\lambda = 2$

$$\vec{a} \times \vec{c} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 4 \\ 2 & 4 & 3 \end{vmatrix} = -10\hat{\imath} + 5\hat{\jmath}$$

27. Equation of line passing through $(0, -1, 0)$ & $(0, 0, 1)$ is $\frac{x-0}{0} = \frac{y+1}{1} = \frac{z-0}{1} = \lambda$

Let the plane be $(z - y - 1) + ax = 0$

Now $\left|\frac{\bar{n}_1.\bar{n}_2}{|n_1||n_2|}\right| = \cos\frac{\pi}{4}$

$$\left|\frac{-1-1}{\sqrt{2}\sqrt{a^2+2}}\right| = \frac{1}{\sqrt{2}}$$

$2 = \sqrt{a^2 + 2} \Rightarrow a^2 + 2 = 4 \Rightarrow a = \pm\sqrt{2}$

So direction ratios: $(\sqrt{2}, -1, 1)$ or $(-\sqrt{2}, -1, 1)$

Direction ratio $(-\sqrt{2}, -1, 1)$ can be written as $(\sqrt{2}, 1, -1)$

28. System of equations have more than one solutions

$2x + 2y + 3z = a$

$3x - y + 5z = b$

$x - 3y + 2z = c$

$D = D_1 = D_2 = D_3 = 0$

$D = \begin{vmatrix} 2 & 2 & 3 \\ 3 & -1 & 5 \\ 1 & -3 & 2 \end{vmatrix} = 26 - 2 - 24 = 0$

$D_1 = \begin{vmatrix} a & 2 & 3 \\ b & -1 & 5 \\ c & -3 & 2 \end{vmatrix} = 0 \Rightarrow a(-2 + 15) - b(4 + 9) + c(10 + 3) = 0$

$\Rightarrow 13a - 13b + 13c = 0$

$\Rightarrow a - b + c = 0$

$D_2 = \begin{vmatrix} 2 & a & 3 \\ 3 & b & 5 \\ 1 & c & 2 \end{vmatrix} = 0 \ \Rightarrow -a(6 - 5) + b(4 - 3) - c\,(10 - 9) = 0$

$\Rightarrow -a + b - c = 0$

$\Rightarrow a - b + c = 0$

$D_3 = \begin{vmatrix} 2 & 2 & a \\ 3 & -1 & b \\ 1 & -3 & c \end{vmatrix} = 0 \ \Rightarrow a(-9 + 1) - b(-6 - 2) + c\,(-2 - 6) = 0$

$\Rightarrow a - b + c = 0$

Hence the required condition is $b - c - a = 0$

29. $f(x) = 3x^3 - 18x^2 + 27x - 40$

$x^2 - 11x + 30 \leq 0 \Rightarrow x \in [5, 6]$

$f'(x) = 9x^2 - 36x + 27 = 9\,(x^2 - 4x + 3) = 9\,(x - 1)\,(x - 3)$

for $x \in [5, 6]$ $f(x)$ is increasing function

hence maximum value in this interval occurs at $x = 6$

so $f(6) = 648 - 648 + 162 - 40 = 122$

30. Direction vector of normal to the required plane is $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 2 & 3 & -1 \end{vmatrix} = -8\hat{i} + 8\hat{j} + 8\hat{k}$

Hence, direction ratio of required plane is $< -1, 1, 1 >$

Hence, equation of plane is

$-(x - 3) + (y + 2) + (z - 1) = 0$

$\Rightarrow x-y-z=4$

Hence, point $(2,0,-2)$ satisfy this plane.