Mathematics

Single correct answers type:

1. $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$ is equal to
(A) 0
(B) $-\pi$
(C) $\frac{3\pi}{2}$
(D) $\frac{\pi}{2}$
(E) $\frac{\pi}{4}$

Solution: (E)
Let $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$ ....(i)

$\Rightarrow \quad I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}} dx$

$\left[\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right]$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx$ ... (ii)

On adding Equation (i) and (ii), we get

$\Rightarrow \quad 2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$

$\Rightarrow \quad 2I = \int_0^{\frac{\pi}{2}} 1dx \Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$

$\Rightarrow \quad 2I = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$

2. If (x, y) is equidistant from $(a+b, b-a)$ and $(a-b, a+b)$, then
(A) $x+y=0$
(B) $bx-ay=0$
(C) $ax-by=0$
(D) $bx+ay=0$
(E) $ax+by=0$

Solution: (B)
Let $P(x,y), A(a+b,b-a), B(a-b,a+b)$
Now, according to the question,
$PA = PB$
$\Rightarrow PA^2 = PB^2$

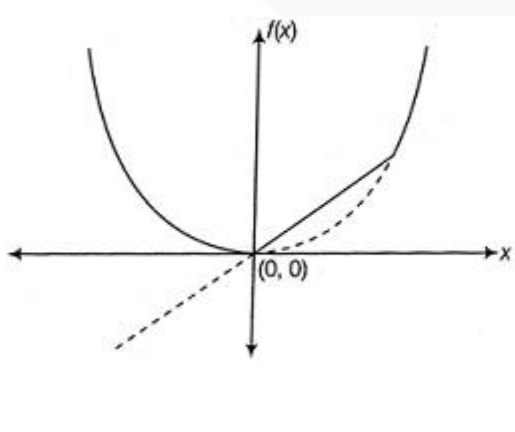
$\Rightarrow PA^2 = PB^2$
$\Rightarrow \left(x-(a+b)\right)^2 + \left(y-(b-a)\right)^2$
$= \left(x-(a-b)\right)^2 + \left(y-(a+b)\right)^2$
$\Rightarrow x^2 + (a+b)^2 - 2x(a+b) + y^2 + (b-a)^2 - 2y(b-a)$
$= x^2 + (a-b)^2 - 2x(a-b) + y^2 + (a+b)^2 - 2y(a+b)$
$\Rightarrow 2x(a-b) - 2x(a+b) + 2y(a+b) - 2y(b-a) = 0$
$\Rightarrow 2x(a-b-a-b) + 2y(a+b-b+a) = 0$
$\Rightarrow 2x(-2b) + 2y(2a) = 0$
$\Rightarrow -4xb + 4ya = 0$
$\Rightarrow bx - ay = 0$

3. If the points $(1,0)$, $(0,1)$ and $(x,8)$ are collinear, then the value of x is equal to
(A) 5
(B) -6
(C) 6
(D) 7
(E) -7

Solution: (E)
Let $A(1,0), B(0,1)$ and $C(x,8)$
Since, A, B and C are collinear, then slope of AB =Slope of BC
$\Rightarrow \dfrac{1-0}{0-1} = \dfrac{8-1}{x-0}$
$\Rightarrow -1 = \dfrac{7}{x}$
$\Rightarrow x = -7$

4. The minimum value of the function max (x, x^2) is equal to
(A) 0
(B) 1
(C) 2
(D) $\frac{1}{2}$
(E) $\frac{3}{2}$

Solution: (A)
Let $f(x) = \max\{x, x^2\}$



$\therefore$ Minimum value of $f(x) = 0$.

5. Let $f(x+y) = f(x)f(y)$ for all x and y. If $f(0) = 1, f(3) = 3$ and $f'(0) = 11$, then $f'(3)$ is equal to

(A) 11

(B) 22

(C) 33

(D) 44

(E) 55

Solution: (C)

We have,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$\Rightarrow f'(3) = \lim_{h\to 0} \frac{f(3+h) - f(3)}{h}$$

$$= \lim_{h\to 0} \frac{f(3)f(h) - f(3+0)}{h}$$

$$= \lim_{h\to 0} \frac{f(3)f(h) - f(3)f(0)}{h}$$

$$= f(3) \lim_{h\to 0} \frac{f(h) - f(0)}{h}$$

$$= f(3) \lim_{h\to 0} \frac{f(0+h) - f(0)}{h}$$

$$= f(3)f'(0)$$

$$= 3 \times 11$$

$$= 33$$

6. If $f(9) = f'(9) = 0$, then $\lim_{x\to 9} \frac{\sqrt{f(x)}-3}{\sqrt{x}-3}$ is equal to

(A) 0

(B) $f(0)$

(C) $f'(3)$

(D) $f(9)$

(E) 1

Solution: (A)

$$\lim_{x\to 9} \frac{\sqrt{f(x)}-3}{\sqrt{x}-3} \quad \left[\frac{0}{0} \text{ form}\right]$$

$$= \lim_{x\to 9} \frac{\frac{f'(x)}{2\sqrt{f(x)}}}{\frac{1}{2\sqrt{x}}}$$

$$= \lim_{x\to 9} \frac{\sqrt{x} f'(x)}{\sqrt{f(x)}}$$

$$= \frac{\sqrt{9} f'(a)}{\sqrt{f(a)}}$$

$$= \frac{3 \times 0}{3} = 0$$

7. The value of $\cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right)$ is
(A) $\sqrt{2}\sin^2 x$
(B) $\sqrt{2}\sin x$
(C) $\sqrt{2}\cos^2 x$
(D) $\sqrt{3}\cos x$
(E) $\sqrt{2}\cos x$

Solution: (E)
We have, $\cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right)$
$$= \cos\frac{\pi}{4}\cos x - \sin\frac{\pi}{4}\sin x + \cos\frac{\pi}{4}\cos x + \sin\frac{\pi}{4}\sin x$$
$$= 2\cos\frac{\pi}{4}\cos x$$
$$= 2 \times \frac{1}{\sqrt{2}}\cos x$$
$$= \sqrt{2}\cos x$$

8. Area of the triangle with vertices $(-2, 2)$, $(1, 5)$ and $(6, -1)$ is
(A) 15
(B) $\frac{3}{5}$
(C) $\frac{29}{2}$
(D) $\frac{33}{2}$
(E) $\frac{35}{2}$

Solution: (D)
Area of triangle having vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by
$$\text{Area} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$
∴ Required area $= \frac{1}{2}\begin{vmatrix} -2 & 2 & 1 \\ 1 & 5 & 1 \\ 6 & -1 & 1 \end{vmatrix}$
$$= \frac{1}{2}[-2(5+1) - 2(1-6) + 1(-1-30)]$$
$$= \frac{1}{2}[-12 + 10 - 31]$$
$$= \frac{-33}{2}$$
∴ Area $= \frac{33}{2}$sq units

9. The equation of the line passing through $(-3, 5)$ and perpendicular to the line through the points $(1, 0)$ and $(-4, 1)$ is
(A) $5x + y + 10 = 0$

(B) $5x - y + 20 = 0$
(C) $5x - y - 10 = 0$
(D) $5x + y + 20 = 0$
(E) $5y - x - 10 = 0$

Solution: (B)
E slope of the line passing through $(1, 0)$ and $(-4, 1) = \frac{1-0}{-4-1} = \frac{-1}{5}$
$\therefore$ Slope of line perpendicular to the above line
$= \frac{-1}{\left(-\frac{1}{5}\right)} = 5$
$\therefore$ Equation of required line is given by
$y - 5 = 5\big(x - (-3)\big)$
$\Rightarrow y - 5 = 5(x + 3)$
$\Rightarrow y - 5 = 5x + 15$
$\Rightarrow 5x - y + 20 = 0$

10. The coefficient of x^5 in the expansion of $(1 + x^2)^5(1 + x)^4$ is
(A) 30
(B) 60
(C) 40
(D) 10
(E) 45

Solution: (B)
We have,
$(1 + x^2)^5 = {}^5C_0(x^2)^0 + {}^5C_1(x^2)^1 + {}^5C_2(x^2)^2 + {}^5C_3(x^2)^3 + {}^5C_4(x^2)^4 + {}^5C_5(x^2)^5$
$= 1 + 5x^2 + 10x^4 + 10x^6 + 5x^8 + x^{10}(1 + x)^4 = {}^4C_0x^0 + {}^4C_1x^1 + {}^4C_2x^2 + {}^4C_3x^3 + {}^4C_4x^4$
$= 1 + 4x + 6x^2 + 4x^3 + x^4$
$\therefore$ Coefficient of x^5 in the product of
$(1 + x^2)^5(1 + x)^4$
$= (5x^2) \cdot (4x^3) + (10x^4) \cdot (4x)$
$= 20x^5 + 40x^5$
$= 60x^5$

11. The coefficient of x^4 in the expansion of $(1 - 2x)^5$ is equal to
(A) 40
(B) 320
(C) -320
(D) -32
(E) 80

Solution: (E)
General term of $(1 - 2x)^5$ is given by
$T_{r+1} = {}^5C_r(-2x)^r$
$= {}^5C_r(-2)^r x^r$
For coefficient of x^4, power of $x = 4$

$\therefore \ r = 4$
$\therefore$ Coefficient of $x^4 = \ {}^5C_4(-2)^4$
$= 5 \times 16 = 80$

12. The equation $5x^2 + y^2 + y = 8$ represents
(A) An ellipse
(B) A parabola
(C) A hyperbola
(D) A Circle
(E) A straight line

Solution: (A)
We have,
$5x^2 + y^2 + y = 8$
$\Rightarrow \ 5x^2 + \left(y + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = 8$
$\Rightarrow \ 5x^2 + \left(y + \frac{1}{2}\right)^2 = \frac{33}{8}$
$\Rightarrow \ \frac{\frac{5x^2}{33}}{8} + \frac{\left(y + \frac{1}{2}\right)^2}{\frac{33}{8}} = 1$
$\Rightarrow \ \frac{x^2}{\left(\frac{33}{40}\right)} + \frac{\left(y + \frac{1}{2}\right)^2}{\left(\frac{33}{8}\right)} = 1$
Which is an equation of ellipse.

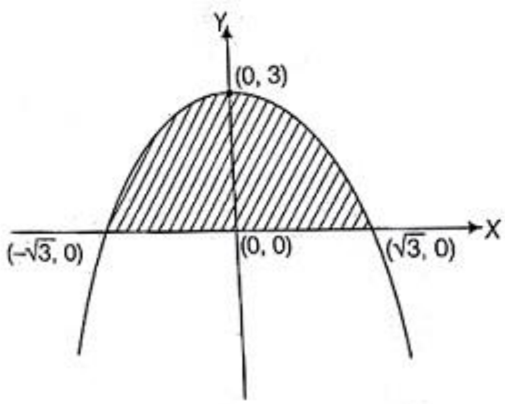
13. The centre of the ellipse $4x^2 + y^2 - 8x + 4y - 8 = 0$ is
(A) $(0, 2)$
(B) $(2, -1)$
(C) $(2, 1)$
(D) $(1, 2)$
(E) $(1, -2)$

Solution: (E)
We have,
$4x^2 + y^2 - 8x + 4y - 8 = 0$
$\Rightarrow \ (4x^2 - 8x) + (y^2 + 4y) - 8 = 0$
$\Rightarrow \ (4x^2 - 2x) + (y^2 + 4y) - 8 = 0$
$\Rightarrow \ 4[(x - 1)^2 - 1] + [(y + 2)^2 - 4] - 8 = 0$
$\Rightarrow \ 4(x - 1)^2 - 4 + (y + 2)^2 - 4 - 8 = 0$
$\Rightarrow 4(x - 1)^2 + (y + 2)^2 = 16$
$\Rightarrow \frac{(x - 1)^2}{4} + \frac{(y + 2)^2}{16} = 1$
$\therefore$ Centre $= (1, -2)$

14. The area bounded by the curves $y = -x^2 + 3$ and $y = 0$ is
(A) $\sqrt{3} + 1$

(B) $\sqrt{3}$
(C) $4\sqrt{3}$
(D) $5\sqrt{3}$
(E) $6\sqrt{3}$

Solution: (C)
We have,
$y = -x^2 + 3$
$\Rightarrow x^2 = -(y-3)$
The above curve intersect X −axis at the points
Where $y = 0$
$\therefore x^2 = 3$
$\Rightarrow x = \pm\sqrt{3}$



∴ Point of intersection with X −axis are $(\pm\sqrt{3}, 0)$
∴ Required area $= 2\int_0^{\sqrt{3}} y\,dx$

$$= 2\int_0^{\sqrt{3}} (-x^2 + 3)dx$$

$$= 2\left[\frac{-x^2}{3} + 3x\right]_0^{\sqrt{3}}$$

$$= 2\left[\frac{-3\sqrt{3}}{3} + 3\sqrt{3}\right]$$

$= 2[-\sqrt{3} + 3\sqrt{3}]$
$= 4\sqrt{3}$ sq units

15. The order of the differential equation
$\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^5 = 0$ is
(A) 3
(B) 4
(C) 1
(D) 5
(E) 6

Solution: (A)

We have,

$$\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^5 = 0$$

Since, the highest order derivative is $\frac{d^3y}{dx^3}$

∴ Order of the given differential equation is 3.

16. If $f(x) = \sqrt{2x} + \frac{4}{\sqrt{2x}}$, then $f'(2)$ is equal to

(A) 0

(B) -1

(C) 1

(D) 2

(E) -2

Solution: (A)

We have,

$$f(x) = \sqrt{2x} + \frac{4}{\sqrt{2x}} = \sqrt{2x} + 4(2x)^{\frac{1}{2}}$$

$$\Rightarrow \quad f'(x) = \frac{1}{2\sqrt{2x}} - 2 + 4\left[-\frac{1}{2}(2x)^{-\frac{3}{2}}(2)\right]$$

$$= \frac{1}{\sqrt{2}\,\sqrt{x}} - \frac{4}{(2x)^{\frac{3}{2}}}$$

$$= \frac{1}{\sqrt{2}\,\sqrt{x}} - \frac{4}{2\sqrt{2}\,x^{\frac{3}{2}}}$$

$$= \frac{1}{\sqrt{2}\,\sqrt{x}} - \frac{\sqrt{2}}{x\sqrt{x}}$$

$$\therefore \quad f'(2) = \frac{1}{\sqrt{2}\,\sqrt{2}} - \frac{\sqrt{2}}{2\sqrt{2}}$$

$$= \frac{1}{2} - \frac{1}{2}$$

$$= 0$$

17. The area of the circle $x^2 - 2x + y^2 - 10y + k = 0$ is 25π. The value of k is equal to

(A) -1

(B) 1

(C) 0

(D) 2

(E) 3

Solution: (B)

We have,

$$x^2 - 2x + y^2 - 10y + k = 0$$

∴ Radius $= \sqrt{g^2 + f^2 - c}$

$$= \sqrt{(1)^2 + (5)^2 - k}$$

$$= \sqrt{1 + 25 - k}$$

$$= \sqrt{26 - k}$$

∵ Area of circle $= \pi(\text{Radius})^2$

$\therefore\ 25\pi = \pi\left(\sqrt{26-k}\right)^2$
$\Rightarrow\ 25\pi = \pi(26-k)$
$\Rightarrow\ 25 = 26-k$
$\Rightarrow\ k = 1$

18. $\int_{2016}^{2017} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4033-x}}\, dx$ is equal to
(A) $\frac{1}{4}$
(B) $\frac{3}{2}$
(C) $\frac{2017}{2}$
(D) $\frac{1}{2}$
(E) 508

Solution: (D)
Let $I = \int_{2016}^{2017} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4033-x}}\, dx$ .....(i)

$$\therefore I = \int_{2016}^{2017} \frac{\sqrt{4033-x}}{\sqrt{4033-x} + \sqrt{4033-(4033-x)}}\, dx$$

$$\left[\because \int_a^b f(x)dx = \int_b^a f(a+b-x)dx\right]$$

$\Rightarrow I = \int_{2016}^{2017} \frac{\sqrt{4033-x}}{\sqrt{4033-x}+\sqrt{x}}\, dx$ ....(ii)
On adding Equations (i) and (ii), we get

$$2I = \int_{2016}^{2017} dx$$

$\Rightarrow\ 2I = [x]_{2016}^{2017}$
$\Rightarrow\ 2I = 1$
$\Rightarrow\ I = \frac{1}{2}$

19. The solution of $dy/dx + y\tan x = \sec x, y(0) = 0$ is
(A) $y\sec x = \tan x$
(B) $y\tan x = \sec x$
(C) $\tan x = y\tan x$
(D) $x\sec x = \tan y$
(E) $y\cot x = \sec x$

Solution: (A)
We have,

$$\frac{dy}{dx} + y\tan x = \sec x$$

Which is a linear differential equation.
$\therefore\ I.F = e^{\int \tan x\, dx} = e^{\log\sec x} = \sec x$
$\therefore$ The solution is given by

$$y.\sec x = \int \sec x.\sec x\, dx + C$$

$y \sec x = \tan x + C$ .....(i)
Now, $y = 0$, when $x = 0$,
$\therefore \quad 0 = 0 + c$ [From equation (i)]
$\Rightarrow \quad c = 0$
Putting $c = 0$ in Equation (i), we get
$y sec\ x = \tan x$

20. If the vectors $2\hat{\imath} + 2\hat{\jmath} + 6\hat{k}, 2\hat{\imath} + \lambda\hat{\jmath} + 6\hat{k}$ and $2\hat{\imath} - 3\hat{\jmath} + \hat{k}$ are coplanar, then the value of λ is
(A) -10
(B) 1
(C) 0
(D) 10
(E) 2

Solution: (E)

Since, the vectors $2\hat{\imath} + 2\hat{\jmath} + 6\hat{k}, 2\hat{\imath} + \lambda\hat{\jmath} + 6\hat{k}$ and $2\hat{\imath} - 3\hat{\jmath} + \hat{k}$ are coplanar

$$\therefore \begin{vmatrix} 2 & 2 & 6 \\ 2 & \lambda & 6 \\ 2 & -3 & 1 \end{vmatrix} = 0$$

$\Rightarrow 2(\lambda + 18) - 2(2 - 12) + 6(-6 - 2\lambda) = 0$

$\Rightarrow 2\lambda + 36 + 20 - 36 - 12\lambda = 0$

$\Rightarrow \ -10\lambda + 20 = 0$

$\Rightarrow \lambda = 2$

21. The distance between $(2, 1, 0)$ and $2x + y + 2z + 5 = 0$ is
(A) 10
(B) $10/3$
(C) $10/9$
(D) 5
(E) 1

Solution: (B)
The distance of a point(x_1, y_1, z_1) from the plane $ax + by + cz + d = 0$ is given by
$$= \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$$
$\therefore$ Distance of the point $(2, 1, 0)$ from the plane $2x + y + 2z + 5 = 0$ is equal to
$$= \left|\frac{2 \times 2 + 1 \times 1 + 2 \times 0 + 5}{\sqrt{(2)^2 + (1)^2 + (2)^2}}\right| = \left|\frac{4 + 1 + 5}{\sqrt{4 + 1 + 4}}\right| = \frac{10}{3}$$

22. The equation of the hyperbola with vertices $(0, \pm 15)$ and foci $(0, \pm 20)$ is
(A) $\frac{x^2}{175} - \frac{y^2}{225} = 1$
(B) $\frac{x^2}{625} - \frac{y^2}{125} = 1$
(C) $\frac{y^2}{225} - \frac{x^2}{125} = 1$

(D) $\frac{y^2}{65} - \frac{x^2}{65} = 1$
(E) $\frac{y^2}{225} - \frac{x^2}{175} = 1$

Solution: (E)
We have,
Vertices and foci of hyperbola at $(0, \pm 15)$ and $(0, \pm 20)$
Since, both foci and vertices lies on Y −axis, then equation of hyperbola will be
$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$
Now, vertices $= (0, \pm b)$
$\therefore \ b = 15$
Again, foci $= (0, \ \pm be)$
$\therefore \ be = 20$
$$\Rightarrow \ e = \frac{20}{15} = \frac{4}{3}$$
$$\Rightarrow \ \sqrt{1 + \frac{a^2}{b^2}} = \frac{4}{3} \qquad \left[\because e = \sqrt{1 + \frac{a^2}{b^2}}\right]$$
$$\Rightarrow \ 1 + \frac{a^2}{b^2} = \frac{16}{9}$$
$$\Rightarrow \ \frac{a^2}{b^2} = \frac{7}{9}$$
$$\Rightarrow \ a^2 = \frac{7}{9} \times b^2 = \frac{7}{9} \times 225$$
$$\Rightarrow \ a = \frac{\sqrt{7}}{3} \times 15 = 5\sqrt{7}$$
$\therefore$ Equation of hyperbola is
$$\frac{y^2}{225} - \frac{x^2}{175} = 1$$

23. The value of $\frac{15^3 + 6^3 + 3.6.15.21}{1 + 4(6) + 6(36) + 4(216) + 1296}$ is equal to
(A) 29/7
(B) 7/19
(C) 6/17
(D) 21/19
(E) 27/7

Solution: (E)
We have, $\frac{15^3 + 6^3 + 3 \cdot 6 \cdot 15 \cdot 21}{1 + 4(6) + 6(36) + 4(216) + 1296}$
$$= \frac{(15)^3 + (6)^2 + 3 \times 6 \times 15(6 + 15)}{{}^4C_0(6)^0 + \ {}^4C_1(6)^1 + \ {}^4C_2(6)^2 + \ {}^4C_3(6)^3 + \ {}^4C_4(6)^4}$$
$$= \frac{(15 + 6)^3}{(1 + 6)^4} = \frac{(21)^3}{(7)^4}$$
$$= \frac{21 \times 21 \times 21}{7 \times 7 \times 7 \times 7} = \frac{27}{7}$$

24. The equation of the plane that passes through the points $(1, 0, 2), (-1, 1, 2), (5, 0, 3)$ is
(A) $x + 2y - 4z + 7 = 0$
(B) $x + 2y - 3z + 7 = 0$
(C) $x - 2y + 4z + 7 = 0$
(D) $2y - 4z - 7 + x = 0$
(E) $x + 2y + 3z + 7 = 0$

Solution: (A)
Equation of the plane passing through $(1, 0, 2), (-1, 1, 2), (5, 0, 3)$ is given by

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x - 1 & y - 0 & z - 2 \\ -1 - 1 & 1 - 0 & 2 - 2 \\ 5 - 1 & 0 - 0 & 3 - 2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x - 1 & y & z - 2 \\ -2 & 1 & 0 \\ 4 & 0 & 1 \end{vmatrix} = 0$$

$\Rightarrow \ x + 2y - 4z + 7 = 0$

25. The vertex of the parabola $y^2 - 4y - x + 3 = 0$ is
(A) $(-1, 3)$
(B) $(-1, 2)$
(C) $(2, -1)$
(D) $(3, -1)$
(E) $(1, 2)$

Solution: (B)
We have,
$y^2 - 4y - x + 3 = 0$
$\Rightarrow \ (y - 2)^2 - 4 - x + 3 = 0$
$\Rightarrow \ (y - 2)^2 = (x + 1)$
$\therefore$ Vertex of the parabola $= (-1, 2)$

26. If a, b, c are vectors such that $a + b + c = 0$ and $|a| = 7, |b| = 5, |c| = 3$, then the angle between c and b is
(A) $\pi/3$
(B) $\pi/6$
(C) $\pi/4$
(D) π
(E) 0

Solution: (A)
We have,
$a + b + c = 0$
$\Rightarrow \ b + c = -a$
$\Rightarrow \ |b + c| = |-a|$
$\Rightarrow \ |b + c| = |a|$

$\Rightarrow \quad |b+c|^2 = |a|^2$
$\Rightarrow \quad (b+c)\cdot(b+c) = |a|^2$
$\Rightarrow \quad |b|^2 + |c|^2 + 2|b||c|\cos\theta = |a|^2$
$\Rightarrow \quad (5)^2 + (3)^2 + 2\times 5\times 3\cos\theta = (7)^2$
$\Rightarrow \quad 25 + 9 + 30\cos\theta = 49$
$\Rightarrow \quad 30\cos\theta = 15$
$\Rightarrow \quad \cos\theta = \dfrac{1}{2}$
$\Rightarrow \quad \theta = 60^o$ or $\pi/3$
$\therefore$ Angle between b and c is $\pi/3$.

27. Let $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where $a > 0$. The minimum of f is attained at a point q and the maximum is attained at a point p. If $p^3 = q$, then a is equal to
(A) 1
(B) 3
(C) 2
(D) $\sqrt{2}$
(E) $\frac{1}{2}$

Solution: (A)
$f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$
$f'(x) = 6x^2 - 18ax + 12a^2$
For maximum or minimum, $f'(x) = 0$
$\Rightarrow 6x^2 - 18ax + 12a^2 = 0$
$x = \dfrac{18a \pm \sqrt{324a^2 - 288a^2}}{2\times 6}$
$= \dfrac{18a \pm \sqrt{36a^2 - 288a^2}}{2\times 6}$
$= \dfrac{18a \pm \sqrt{36a^2}}{12} = \dfrac{18a \pm 6a}{12} = 2a, a$
Now, $f''(x) = 12x - 18a$
At $x = 2a$,
$f''(x) = 24a - 18a$
$= 6a > 0$, maxima
$\therefore \quad p = f(2a) = 2\times 8a^3 - 36a^3 + 24a^3 + 1 = 4a^3 + 1$
At $x = a$,
$f''(x) = 12\times a - 18a$
$= -6a < a$, minima
$\therefore \quad q = f(a)$
$= 2a^3 - 9a^3 + 12a^3 + 1 = 5a^3 + 1$
Also given $p^3 = q$
$\therefore \; (4a^3+1)^3 = (5a^3+1)$
$\Rightarrow \quad a = 0$, but $a > 0$ (given)

28. For all rest numbers x and y, it is known as the real valued function f satisfies $f(x) + f(y) = f(x+y)$. If $f(1) = 7$, then $\sum_{r=1}^{100} f(r)$ is equal to
(A) $7\times 51\times 102$

(B) $6 \times 50 \times 102$
(C) $7 \times 50 \times 102$
(D) $6 \times 25 \times 102$
(E) $7 \times 50 \times 101$

Solution: (E)
We have,
$f(x+y) = f(x) + f(y)$ ......(i)
Put, $x = y = 1$ in Equation (i),
We get, $f(2) = f(1) + f(1) = 2f(1) = 2 \times 7$
Again, put $x = 1, y = 2$ in Equation (i), we get
$f(3) = f(1) + f(2) = 7 + 2 \times 7 = 3 \times 7$
$\therefore \quad f(n) = n \times 7$
Now, $\sum_{r=1}^{100} f(r) = f(1) + f(2) + f(3) + \cdots + f(100)$
$= 7 + 2 \times 7 + 3 \times 7 + \cdots + 100 \times 7$
$= 7[1 + 2 + 3 + \cdots + 100]$
$= 7 \times \frac{100(100+1)}{2} \left[\because \Sigma n = \frac{n(n+1)}{2}\right]$
$= 7 \times 50 \times 101$

29. The eccentricity of the ellipse $\frac{(x-1)^2}{2} + \left(y + \frac{3}{4}\right)^2 = \frac{1}{16}$ is
(A) $1/\sqrt{2}$
(B) $1/2\sqrt{2}$
(C) $1/2$
(D) $1/4$
(E) $1/4/\sqrt{2}$

Solution: (A)
We have,
$$\frac{(x-1)^2}{2} + \left(y + \frac{3}{4}\right)^2 = \frac{1}{16}$$
$$\Rightarrow \quad 8(x-1)^2 + 16\left(y + \frac{3}{4}\right)^2 = 1$$
$$\Rightarrow \quad \frac{(x-1)^2}{\frac{1}{8}} + \frac{\left(y + \frac{3}{4}\right)^2}{\frac{1}{16}} = 1$$
$\therefore \quad a^2 = \frac{1}{8}$ and $b^2 = \frac{1}{16}$
$\Rightarrow \quad a = \frac{1}{2\sqrt{2}}$ and $b = \frac{1}{4}$
$$\therefore e = \sqrt{1 - \frac{b^2}{a^2}} \qquad [\because a > b]$$
$$= \sqrt{1 - \frac{\frac{1}{16}}{\frac{1}{8}}} = \sqrt{1 - \frac{1}{2}} = \frac{1}{\sqrt{2}}$$

30. $\int_{-1}^{1} \max\{x, x^3\} dx$ is equal to
(A) $3/4$
(B) $1/4$
(C) $1/2$
(D) 1
(E) 0

Solution: (B)
Let $I = \int_{-1}^{1} \max\{x, x^3\} dx$
$= \int_{-1}^{0} \max\{x, x^3\} dx + \int_{0}^{1} \max\{x, x^3\} dx$
$= \int_{-1}^{0} x^3 dx + \int_{0}^{1} x dx$
$= \left[\frac{x^4}{4}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{1}$
$= \left[0 - \left(\frac{1}{4}\right)\right] + \left[\frac{1}{2} - 0\right]$
$= -\frac{1}{4} + \frac{1}{2} = \frac{1}{4}$

31. If $x \in \left[0, \frac{\pi}{2}\right], y \in \left[0, \frac{\pi}{2}\right]$ and $\sin x + \cos y = 2$, then the value of $x + y$ is equal to
(A) 2π
(B) π
(C) $\pi/4$
(D) $\pi/2$
(E) 0

Solution: (D)
We have,
$\sin x + \cos y = 2$
Since, $x \in \left[0, \frac{\pi}{2}\right]$
And $y \in \left[0, \frac{\pi}{2}\right]$
$\therefore \sin x = 1$ and $\cos y = 1$
$\Rightarrow x = \frac{\pi}{2}$ and $y = 0$
$\therefore \ x + y = \frac{\pi}{2} + 0 = \frac{\pi}{2}$

32. Let $a, a + r$ and $a + 2r$ be positive real number such that their product is 64. Then the minimum value of $a + 2r$ is equal to
(A) 4
(B) 3
(C) 2
(D) $1/2$
(E) 1

Solution: (A)

We know $AM \geq GM$

$$\frac{a+(a+r)+(a+2r)}{3} \geq \left(a(a+r)(a+2r)\right)^{\frac{1}{3}}$$

$$\Rightarrow \frac{3(a+r)}{3} \geq (64)^{\frac{1}{3}}$$

$\Rightarrow (a+r) \geq 4$

Also, $64 = a(a+r)(a+2r)$

$\Rightarrow 64 \geq (4-r) \times 4(r+4)$

$\Rightarrow 16 \geq 16 - r^2$

$\Rightarrow r^2 \leq 0$

$\therefore r = 0$

Now, $a + 2r = 4 + 0 = 4$

33. The sum $S = \frac{1}{9!} + \frac{1}{3!7!} + \frac{1}{5!5!} + \frac{1}{7!3!} + \frac{1}{9!}$ is equal to

(A) $2^{10}/8!$

(B) $2^9/10!$

(C) $2^7/10!$

(D) $2^6/10!$

(E) $2^5/8!$

Solution: (B)

Let $S = \frac{1}{9!} + \frac{1}{3!7!} + \frac{1}{5!5!} + \frac{1}{7!3!} + \frac{1}{9!}$

$$= \frac{1}{10!}\left[\frac{10!}{9!} + \frac{10!}{3!\,7!} + \frac{10!}{5!\,5!} + \frac{10!}{7!\,3!} + \frac{10!}{9!}\right]$$

$$= \frac{1}{10!}\left[{}^{10}C_1 + {}^{10}C_3 + {}^{10}C_5 + {}^{10}C_7 + {}^{10}C_9\right]$$

$$= \frac{1}{10!}(2^{10-1})$$

$[\because C_1 + C_3 + C_5 + \cdots 2^{n-1}]$

$$= \frac{2^9}{10!}$$

34. If $f(x) = \begin{vmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{vmatrix}$, then $f'(x)$ is equal to

(A) $x^3 + 6x^2$

(B) $6x^3$

(C) $3x$

(D) $6x^2$

(E) 0

Solution: (D)

We have,

$$f(x) = \begin{vmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{vmatrix}$$

On taking x from R_1, we get

$= x\begin{vmatrix} 1 & x & x^2 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{vmatrix}$

On taking x common from C_3, we get

$= x^2\begin{vmatrix} 1 & x & x \\ 1 & 2x & 3x \\ 0 & 2 & 6 \end{vmatrix}$

On applying $R_2 \to R_2 - R_1$, we get

$= x^2\begin{vmatrix} 1 & x & x \\ 0 & x & 2x \\ 0 & 2 & 6 \end{vmatrix} = x^2 \cdot 1(6x - 4x)$

[On expanding along C_1]

$= x^2(2x) = 2x^3$

$\therefore \; f'(x) = 6x^2$

35. $\int \frac{x^2}{1+(x^3)^2} dx$ is equal to

(A) $\tan^{-1} x^2 + C$

(B)$2/3 \tan^{-1} x^3 + C$

(C) $1/3 \tan^{-1}(x^3) + C$

(D) $1/2 \tan^{-1} x^2 + C$

(E) $\tan^{-1} x^3 + C$

Solution: (C)

Let $I = \int \frac{x^2}{1+(x^3)^2} dx$

Put $x^3 = t$

$\therefore \; 3x^2 dx = dt$

$\therefore \; I = \frac{1}{3}\int \frac{dt}{1 + t^2} \frac{1}{3} \tan^{-1} t + C$

$= \frac{1}{3}\tan^{-1}(x^3) + C$

36. Let $f_n(x)$ be the nth derivative of $f(x)$. The least value of n so that $f_n = f_{n+1}$, where $f(x) = x^2 + e^x$ is

(A) 4

(B) 5

(C) 2

(D) 3

(E) 6

Solution: (D)

We have,

$f(x) = x^2 + e^x$

$\Rightarrow \; f_1(x) = 2x + e^x$

$\Rightarrow \; f_2(x) = 2 + e^x$

$\Rightarrow \; f_3(x) = e^x \Rightarrow f_4(x) = e^x$

Since, $f_3(x) = f_4(x)$

$\therefore$ Last value of n is 3.

37. $\sin 765^o$ is equal to
(A) 1
(B) 0
(C) $\sqrt{3}/2$
(D) $1/2$
(E) $1/\sqrt{2}$

Solution: (E)
$\sin 765^o$
$= \sin(720^o + 45^o) = \sin 45^o$
$= \frac{1}{\sqrt{2}}$

38. The distance of the point $(3, -5)$ from the line $3x - 4y - 26 = 0$ is
(A) $3/7$
(B) $2/5$
(C) $7/5$
(D) $3/5$
(E) 1

Solution: (D)
Distance of a point (x_1, y_1) from the line $Ax + By + C = 0$ is given by
$= \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}\right|$
∴ Distance of the point $(3, -5)$ from the line $3x - 4y - 26 = 0$ is equal to
$= \left|\frac{3 \times 3 - 4 \times (-5) - 26}{\sqrt{(3)^2 + (-4)^2}}\right|$
$= \left|\frac{9 + 20 - 26}{\sqrt{9 + 16}}\right|$
$= \left|\frac{3}{5}\right| = \frac{3}{5}$ unit

39. The difference between the maximum and minimum value of the function $f(x) = \int_0^x (t^2 + t + 1)\, dt$ on $[2, 3]$ is
(A) $39/6$
(B) $49/6$
(C) $59/6$
(D) $69/6$
(E) $79/6$

Solution: (C)
Given $f(x) = \int_0^x (t^2 + t + 1)\, dt$
$f'(x) = (x^2 + x + 1) \times 1 - 0$
$= x^2 + x + 1$
For $x \in [2, 3)$
$f'(x) > 0$
∴ Minimum is at $x = 2$ and maximum is at $x = 3$.

Now, minimum value $= \int_0^x (t^2 + t + 1)\, dt$

$= \left[\frac{t^3}{3} + \frac{t^2}{2} + t\right]_0^2$

$= \frac{8}{3} + \frac{4}{2} + 2$

$= \frac{8}{3} + 4 = \frac{20}{3}$

And maximum value $= \int_0^3 (t^2 + t + 1)\, dt$

$= \left[\frac{t^3}{3} + \frac{t^2}{2} + t\right]_0^3$

$= \frac{9}{2} + 12 = \frac{33}{2}$

$\therefore$ Difference between maximum and minimum value $= \frac{33}{2} - \frac{20}{3} = \frac{99-40}{6} = \frac{59}{6}$

40. If a and b are the non−zero distinct roots of $x^2 + ax + b = 0$, then the minimum value of $x^2 + ax + b$ is

(A) $2/3$

(B) $9/4$

(C) $-9/4$

(D) $-2/3$

(E) 1

Solution: (C)

Given, $x^2 + ax + b = 0$

For distinct now zero roots

$D > 0$

$\Rightarrow\ a^2 - 4b > 0$

Now, $x^2 + ax + b$

$= \left(x + \frac{a}{2}\right)^2 + \left(b - \frac{a^2}{4}\right)$

$= \left(x + \frac{a}{2}\right)^2 - \left(a^2 - \frac{4b}{4}\right)$

We know, sum of roots $a + b = -a$ ......(i)

$\Rightarrow\ 2a + b = 0$

Product of roots $a \times b = b$

$\Rightarrow\ b(a-1) = 0$

$\Rightarrow\ a = 1, b \neq 0$

From Equation (i),

$2a + b = 0$

$2(1) + b = 0$

$b = -2$

Now, $\left(x + \frac{a}{2}\right)^2 - \left(\frac{1^2 + 4 \times 2}{4}\right)$

$= \left(x + \frac{a}{2}\right)^2 - \frac{9}{4}$

$\therefore$ Maximum value $= -\frac{9}{4}$

41. If the straight line $y = 4x + c$ touches the ellipse $\frac{x^2}{4} + y^2 = 1$, then c is equal to
(A) 0
(B) $\pm\sqrt{65}$
(C) $\pm\sqrt{62}$
(D) $\pm\sqrt{2}$
(E) ± 13

Solution: (B)
We have,
$y = 4x + c$ .....(i)
And $\frac{x^2}{4} + y^2 = 1$ ......(ii)
Put value of y from Equations (i) into (ii), we get
$\frac{x^2}{4} + (4x + c)^2 = 1$
$\Rightarrow \ x^2 + 4(4x + c)^2 = 4$
$\Rightarrow \ x^2 + 4(16x^2 + 8cx + c^2) = 4$
$\Rightarrow \ x^2 + 64x^2 + 32cx + 4c^2 = 4$
$\Rightarrow \ 65x^2 + 32cx + 4(c^2 - 1) = 0$
Since, given line is a tangent to the ellipse.
$\therefore$ Discriminant $= 0$
$\Rightarrow \ (32c)^2 - 4 \times 65 \times 4(c^2 - 1) = 0$
$\Rightarrow \ 1024c^2 - 1040(c^2 - 1) = 0$
$\Rightarrow \ 1024c^2 - 1040c^2 + 1040 = 0$
$\Rightarrow \ 16c^2 = 1040$
$\Rightarrow \ c^2 = 65$
$\Rightarrow \ c = \pm\sqrt{65}$

42. The equations $\lambda x - y = 2, 2x - 3y = -\lambda$ and $3x - 2y = -1$ are consistent for
(A) $\lambda = -4$
(B) $\lambda = 1, 4$
(C) $\lambda = 1, -4$
(D) $\lambda = -1, 4$
(E) $\lambda = -1$

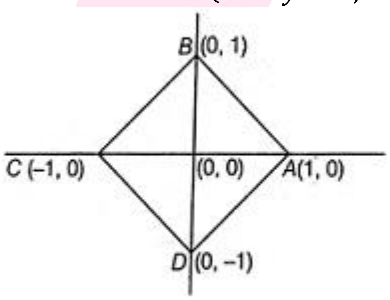
Solution: (D)
In a consistent, the intersection point of two lines, satisfy the third line.
Consider $\lambda = -1$, then given equation become
$-x - y = 2$
$2x - 3y = 1$
$\Rightarrow \ x = -1, y = -1$
Third equation is $3x - 2y = -1$
Put $x = -1, y = -1$
$\therefore \ -3 + 2 = -1$
$\Rightarrow \ -1 = -1$, true
Consider $\lambda = 4$, then given equation become
$4x - y = 2$

$2x-3y=-4$
$\Rightarrow \quad y=2, x=1$
Third equation is $3x-2y=-1$
Put $y=2, x=1$
$\therefore \quad 3-4=-1$
$\Rightarrow \quad -1=-1$, true

43. The set $\{(x, y): |x|+|y|=1\}$ in the xy −plane represents
(A) A square
(B) A circle
(C) An ellipse
(D) A rectangle which is not a square
(E) A rhombus which is not a square

Solution: (A)
We have,

$$|x|+|y|=1=\begin{cases} x+y=1, & x, y \in \text{I quadran} \\ -x+y=1, & x, y \in \text{II quadrant} \\ -x-y=1, & x, y \in \text{III quadrant} \\ x-y=1, & x, y \in \text{IV quadrant} \end{cases}$$



Clearly, $ABCD$ is a square.

44. The value of $\cos\left(\tan^{-1}\left(\frac{3}{4}\right)\right)$ is
(A) $\frac{4}{5}$
(B) $\frac{3}{5}$
(C) $\frac{3}{4}$
(D) $\frac{2}{5}$
(E) 0

Solution: (A)
We have,

$$\cos\left(\tan^{-1}\frac{3}{4}\right)$$

$$=\cos\left(\cos^{-1}\frac{1}{\sqrt{1+\left(\frac{3}{4}\right)^2}}\right)$$

$\left[\because \tan^{-1} x = \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right]$

$= \cos\cos^{-1}\frac{4}{5}$

$= \frac{4}{5} \quad [\because \cos\cos^{-1} x = x$

45. Let $A(6,-1)$ $B(1,3)$ and $C(x,8)$ be three points such that $AB = BC$. The values of x are
(A) $3, 5$
(B) $-3, 5$
(C) $3, -5$
(D) $4, 5$
(E) $-3, -5$

Solution: (B)
We have, $A(6,-1), B(1,3), C(x,8)$
Also, $AB = BC$
$\Rightarrow AB^2 = BC^2$
$\Rightarrow (1-6)^2 + (3+1)^2 = (x-1)^2 + (8-3)^2$
$\Rightarrow 25 + 16 = (x-1)^2 + 25$
$\Rightarrow (x-1)^2 = 16$
$\Rightarrow x - 1 = \pm 4$
$\Rightarrow x = 1 \pm 4$
$\Rightarrow x = 5, -3$

46. In an experiment with 15 observations on x, the following results were available $\Sigma x^2 = 2830, \Sigma x = 170$ On observation that was 20, was found to be wrong and was replaced by the correct value 30. Then, the corrected variance is
(A) 9.3
(B) 8.3
(C) 188.6
(D) 177.3
(E) 78

Solution: (E)
We have $N = 15$, Incorrect $\Sigma x_1^2 i = 2830$, Incorrect $\Sigma x_i = 170$
$\therefore$ Correct $\Sigma x_i =$ Incorrect $\Sigma x_i - 20 + 30 = 170 + 10 = 180$
$\therefore$ Correct mean $= \ \bar{x} = \frac{\Sigma x_i}{N}$ (correct)
$= \frac{180}{15}$
$= 12$
Also, correct $\Sigma_1^2 =$ Incorrect $\Sigma_1^2 - (20)^2 + (30)^2$
$= 2830 - 400 + 900$
$= 2830 + 500$
$= 3330$
$\therefore$ Correct variance $= \frac{\Sigma_1^2(\text{correct})}{N} - (\bar{x})^2$
$= \frac{330}{15} - (12)^2$

$= 222 - 144$
$= 78$

47. The angle between the pair of lines $\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$ is

(A) $\cos^{-1}\left(\frac{21}{9\sqrt{38}}\right)$

(B) $\cos^{-1}\left(\frac{23}{9\sqrt{38}}\right)$

(C) $\cos^{-1}\left(\frac{24}{9\sqrt{38}}\right)$

(D) $\cos^{-1}\left(\frac{25}{9\sqrt{38}}\right)$

(E) $\cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$

Solution: (E)

Given lines are

$\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$

dr' of above lines are $< 2, 5, -3 >$ and $< -1, 8, 4 >$ respectively.

$$\therefore \quad \cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{(2)(-1) + (5)(8) + (-3)(4)}{\sqrt{(2)^2 + (5)^2 + (-3)^2}\,\sqrt{(-1)^2 + (8)^2 + (4)^2}}$$

$$= \frac{-2 + 40 - 12}{\sqrt{4 + 25 + 9}\,\sqrt{1 + 64 + 16}}$$

$$= \frac{26}{\sqrt{38}\,\sqrt{81}}$$

$$= \frac{26}{9\sqrt{38}}$$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$$

48. Let $\vec{a}$ be a unit vector. If $(x - \vec{a}) \cdot (x + \vec{a}) = 12$, then the magnitude of x is

(A) $\sqrt{8}$

(B) $\sqrt{9}$

(C) $\sqrt{10}$

(D) $\sqrt{13}$

(E) $\sqrt{12}$

Solution: (D)

We have,

$|\vec{a}| = 1$

Now, $(x - \vec{a}) \cdot (x + \vec{a}) = 12$

$\Rightarrow \quad x \cdot x + x \cdot \vec{a} - \vec{a} \cdot x - \vec{a} \cdot \vec{a} = 12$

$\Rightarrow \quad |x|^2 - |\vec{a}|^2 = 12]$

$\Rightarrow \quad |x|^2 - 1 = 12$

$\Rightarrow \quad |x|^2 = 13$

$\Rightarrow \quad |x| = \sqrt{13}$

49. The area of triangular region whose sides are $y = 2x + 1, y = 3x + 1$ and $x = 4$ is
(A) 5
(B) 6
(C) 7
(D) 8
(E) 9

Solution: (D)
We have,
$y = 2x + 1, y = 3x + 1, x = 4$
Intersecting points of above lines are $(0, 1), (4, 9), (4, 13)$
∴ Area of triangle
$$= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_1 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$
$$= \frac{1}{2}\begin{vmatrix} 0 & 1 & 1 \\ 4 & 9 & 1 \\ 4 & 13 & 1 \end{vmatrix}$$
$$= \frac{1}{2}[0(9 - 13) - 1(4 - 4) + 1(52 - 36)]$$
$$= \frac{1}{2} \times 16 = 8$$

50. If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then the value of r is
(A) 9
(B) 3
(C) 4
(D) 5
(E) 6

Solution: (B)
We have
${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$
$$\therefore \quad \frac{{}^nC_{r-1}}{{}^nC_r} = \frac{36}{84}$$
$$\Rightarrow \quad \frac{\frac{n!}{(n-r+1)!\,(r-1)!}}{\frac{n!}{(n-r)!\,r!}} = \frac{3}{7}$$
$$\Rightarrow \quad \frac{r}{n-r+1} = \frac{3}{7}$$
$$\Rightarrow \quad 7r = 3n - 3r + 3$$
$$\Rightarrow \quad 10r = 3n + 3 \qquad \text{...(i)}$$
Again,
$$\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{84}{126}$$

$$\Rightarrow \quad \frac{\frac{n!}{(n-r)!\,r!}}{\frac{n!}{(n-r-1)!\,(r+1)!}} = \frac{2}{3}$$

$$\Rightarrow \quad \frac{r+1}{n-r} = \frac{2}{3}$$

$\Rightarrow \quad 3r + 3 = 2n - 2r$

$\Rightarrow \quad 5r = 2n - 3$ .....(ii)

On solving Equations (i) and (ii), we get

$n = 9, r = 3$

51. Let $f(x+y) = f(x)\,f(y)$ and $f(x) = 1 + \sin(3x)\,g(x)$, where g is differentiable. The $f'(x)$ is equal to

(A) $3f(x)$

(B) $g(0)$

(C) $f(x)\,g(0)$

(D) $3g(x)$

(E) $3f(x)\,g(0)$

Solution: (C)

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x)f(h) - f(x)}{h}$$

$$= f(x) \lim_{h\to 0} \left(\frac{1 + \sin 3h\big(g(h)\big) - 1}{h} \right)$$

$$= f(x) \lim_{h\to 0} \frac{\sin 3h}{3h} \lim_{h\to 0} g(h)$$

$$= f(x) \times 1 \times g(0) = f(x)g(0)$$

52. The roots of the equation

$\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$ are

(A) $1, 2$

(B) $-1, 2$

(C) $-1, -2$

(D) $1, -2$

(E) $1, 1$

Solution: (B)

We have,

$$\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$$

On applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} x+1 & 1 & 1 \\ x+1 & x-1 & 1 \\ x+1 & 1 & x-1 \end{vmatrix} = 0$$

On taking $(x+1)$ common from C_1, we get

$$(x+1)\begin{vmatrix}1 & 1 & 1\\ 1 & x-1 & 1\\ 1 & 1 & x-1\end{vmatrix}=0$$

On applying, $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$, we get

$$\Rightarrow \quad (x+1)\begin{vmatrix}0 & 2-x & 0\\ 0 & x-2 & 2-x\\ 1 & 1 & x-1\end{vmatrix}=0$$

$$\Rightarrow \quad (x+1)\cdot 1[(2-x)^2-0]=0$$

$$\Rightarrow \quad (x+1)(2-x)^2=0$$

$$\Rightarrow \quad x=-1,2$$

53. If the 7th and 8th term of the binomial expansion $(2a-3b)^n$ are equal, then $\frac{2a+3b}{2a-3b}$ is equal to

(A) $\frac{13-n}{n+1}$

(B) $\frac{n+1}{13-n}$

(C) $\frac{6-n}{13-n}$

(D) $\frac{n-1}{13-n}$

(E) $\frac{2n-1}{13-n}$

Solution: (A)

We have, $(2a-3b)^n$

$$\Rightarrow \quad {}^nC_6(2a)^{n-6}(-3b)^6 = {}^nC_7(2a)^{n-7}(-3b)^7$$

$$\Rightarrow \quad {}^nC_6(2a) = {}^nC_7(-3b)$$

$$\Rightarrow \quad \frac{2a}{3b} = -\frac{{}^nC_7}{{}^nC_6}$$

$$\Rightarrow \quad \frac{2a}{3b} = -\frac{\frac{n!}{(n-7)!\,7!}}{\frac{n!}{(n-6)!\,6!}}$$

$$\Rightarrow \quad \frac{2a}{3b} = -\frac{n-6}{7}$$

$$\Rightarrow \quad \frac{2a}{3b} = \frac{6-n}{7}$$

On applying componendo and dividend, we get

$$\frac{2a+3b}{2a-3b} = \frac{6-n+7}{6-n-7}$$

$$= \frac{13-n}{-(n+1)}$$

$$= -\left[\frac{13-n}{n+1}\right]$$

54. Standard deviation of first n odd natural numbers is

(A) $\sqrt{n}$

(B) $\sqrt{\frac{(n+2)(n+1)}{3}}$

(C) $\sqrt{\frac{n^2-1}{3}}$

(D) n

(E) $2n$

Solution: (C)

Standard deviation, $\sigma = \sqrt{\frac{\Sigma x_i^2}{N} - (\bar{x})^2}$

$\therefore\ \bar{x} = \dfrac{\Sigma x_i}{N}$

$= \dfrac{1 + 3 + 5 + \cdots (2n-1)}{n}$

$= \dfrac{\frac{n}{2}[1 + 2n - 1]}{n}$

$= \dfrac{n^2}{n} = n$

Again, $\Sigma x_i^2 = 1^2 + 3^2 + 5^2 + \cdots (2n-1)^2$

$= \Sigma(2n-1)^2$

$= \Sigma(4n^2 - 4n + 1)$

$= 4\Sigma n^2 - 4\Sigma n + \Sigma 1$

$= \dfrac{4n(n+1)(2n+1)}{6} - \dfrac{4n(n+1)}{2} + n$

$= n\left[\dfrac{2}{3}(n+1)2n+1) - 2(n+1) + 1\right]$

$= \dfrac{n}{3}[2(2n^2 + 3n + 1) - 6(n+1) + 3]$

$= \dfrac{n}{3}[4n^2 + 6n + 2 - 6n - 6 + 3]$

$= \dfrac{n}{3}[4n^2 - 1]$

$\therefore\ \ \sigma = \sqrt{\dfrac{n(4n^2-1)}{3n} - n^2}$

$= \sqrt{\dfrac{4n^2-1}{3} - n^2}$

$= \sqrt{\dfrac{4n^2 - 1 - 3n^2}{3}}$

$= \sqrt{\dfrac{n^2-1}{3}}$

55. Let $S = \{1, 2, 3, \ldots .10\}$. The number of subsets of S containing only odd numbers is

(A) 15

(B) 31

(C) 63

(D) 7

(E) 5

Solution: (B)

Given set is $\{1, 2, 3, \ldots 10\}$

The odd numbers in the given set are $1, 3, 5, 7, 9$.
$\therefore$ The number of subsets of 5 containing only number $= 2^5 - 1$
$= 32 - 1 = 31$

56. The area of the parallelogram with vertices $(0,0), (7,2), (5,9)$ and $(12,11)$ is
(A) 50
(B) 54
(C) 51
(D) 52
(E) 53

Solution: (E)
Let $A(0,0), B(7,2), C(5,9), D(12,11)$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 7 & 2 & 1 \\ 5 & 9 & 1 \end{vmatrix}$

$= \frac{1}{2} \cdot 1(63 - 10) = \frac{53}{2}$sq unit

$\therefore$ Area of $\Delta ACD = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 5 & 9 & 1 \\ 12 & 11 & 1 \end{vmatrix}$

$= \frac{1}{2} \cdot 1(55 - 108)$

$= \frac{53}{2}$sq unit

$\therefore$ Area of parallelogram $ABCD$
$=$ Area of $\Delta ABC +$ Area of ΔACD

$= \frac{53}{2} + \frac{53}{2}$

$= 53$ sq unit

57. $\begin{vmatrix} 1 & 1 & 1 \\ p & q & r \\ p & q & r+1 \end{vmatrix}$ is equal to
(A) $q - p$
(B) $q + p$
(C) q
(D) p
(E) 0

Solution: (A)
We have,

$= \begin{vmatrix} 1 & 1 & 1 \\ p & q & r \\ p & q & r+1 \end{vmatrix}$

On applying, $C_1 \to C_1 - C_2, C_2 \to C_3$, we get

$= \begin{vmatrix} 0 & 0 & 1 \\ p-q & q-r & r \\ p-q & q-r-1 & r+1 \end{vmatrix}$

On taking common $(p - q)$ from C_1, we get

$$= (p-q)\begin{vmatrix} 0 & 0 & 1 \\ 1 & q-r & r \\ 1 & q-r-1 & r+1 \end{vmatrix}$$
$$= (p-q)\cdot 1[q-r-1-q+r]$$
$$= q-p$$

58. Let $A = \begin{bmatrix} 5 & 0 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 20 & 5 \\ -1 & 0 \end{bmatrix}$. If $4A + 5B - C = 0$, then C is

(A) $\begin{bmatrix} 5 & 25 \\ -1 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 20 & 5 \\ -1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 5 & -1 \\ 0 & 25 \end{bmatrix}$

(D) $\begin{bmatrix} 5 & 25 \\ -1 & 5 \end{bmatrix}$

(E) $\begin{bmatrix} 0 & 5 \\ 5 & 25 \end{bmatrix}$

Solution: (B)

We have,

$A = \begin{bmatrix} 5 & 0 \\ 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 20 & 5 \\ -1 & 0 \end{bmatrix}$

Now,

$4A + 5B - C = 0$

$\Rightarrow C = 4A + 5B$

$= 4\begin{bmatrix} 5 & 0 \\ 1 & 0 \end{bmatrix} + 5\begin{bmatrix} 20 & 5 \\ -1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 20 & 0 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 100 & 25 \\ -5 & 0 \end{bmatrix}$

$= \begin{bmatrix} 120 & 25 \\ -1 & 0 \end{bmatrix}$

59. If $U = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$, then U^{-1} is

(A) U^T

(B) U

(C) I

(D) 0

(E) U^2

Solution: (A)

We have,

$$U = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$\therefore \quad U^{-1} = \frac{1}{|U|}\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

$\left[\because \text{If } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \text{then } A^{-1} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\right]$

$= \frac{1}{1}\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \quad [\because |U| = 1]$

$= \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

Again, $U^T = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

$\therefore \quad U^{-1} = U^T$

60. If $A = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$, then A^{-1} is

(A) A^T

(B) A^2

(C) A

(D) I

(E) 0

Solution: (A)

We have,

$A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

$\therefore \quad |A| = -(-1)[(1)(-1) - 0] = -1$

Now, cofactors are

$C_{11} = 0, C_{12} = 1, C_{13} = 0$

$C_{21} = -1, C_{22} = 0, C_{23} = 0$

$C_{31} = 0, C_{32} = 0, C_{33} = 1$

$\therefore A^{-1} = \frac{1}{|A|} \text{adj } A$

$= \frac{1}{-1}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T$

$= -\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

$= A^T$

61. If $\begin{pmatrix} x+y & x-y \\ 2x+z & x+z \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$, then the values of x, y and z are respectively

(A) $0, 0, 1$

(B) $1, 1, 0$

(C) $-1, 0, 0$

(D) $0, 0, 0$

(E) $1, 1, 1$

Solution: (A)

We have

$\begin{bmatrix} x+y & x-y \\ 2x+z & x+z \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow \;\; x+y=0, x-y=0, 2x+z=1, x+z=1$

On solving above equations, we get

$x=y=0, z=1$

62. $\begin{pmatrix} 7 & 1 & 5 \\ 8 & 0 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + 5\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is equal to

(A) $\begin{pmatrix} 16 \\ 27 \end{pmatrix}$

(B) $\begin{pmatrix} 27 \\ 16 \end{pmatrix}$

(C) $\begin{pmatrix} 15 \\ 16 \end{pmatrix}$

(D) $\begin{pmatrix} 16 \\ 15 \end{pmatrix}$

(E) $\begin{pmatrix} 16 \\ 16 \end{pmatrix}$

Solution: (B)

We have,

$\begin{bmatrix} 7 & 1 & 5 \\ 8 & 0 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} + 5\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} 7\times 2 + 1\times 3 + 5\times 1 \\ 8\times 2 + 0\times 3 + 0\times 1 \end{bmatrix} + \begin{bmatrix} 5 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} 22 \\ 16 \end{bmatrix} + \begin{bmatrix} 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 27 \\ 16 \end{bmatrix}$

63. If $\begin{pmatrix} 1 & 2 & 4 \\ 1 & 3 & 5 \\ 1 & 4 & a \end{pmatrix}$ is singular, then the value of a is

(A) $a=-6$

(B) $a=5$

(C) $a=-5$

(D) $a=6$

(E) $a=0$

Solution: (D)

Let $A = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 3 & 5 \\ 1 & 4 & a \end{bmatrix}$

Since, A is a singular matrix.

$\therefore\ |A| = 0$

$\Rightarrow \begin{vmatrix} 1 & 2 & 4 \\ 1 & 3 & 5 \\ 1 & 4 & a \end{vmatrix} = 0$

$\Rightarrow\ 1[3a - 20] - 2[a - 5] + 4[4 - 3] = 0$

$\Rightarrow\ 3a - 20 - 2a + 10 + 4 = 0$

$\Rightarrow\ a - 6 = 0$

$\Rightarrow\ a = 6$

64. If $\begin{pmatrix} 1 & 2 & -3 \\ 0 & 4 & 5 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, then (x, y, z) is equal to

(A) $(1, 6, 6)$

(B) $(1, -6, 1)$

(C) $(1, 1, 6)$

(D) $(6, -1, 1)$

(E) $(-1, 6, 1)$

Solution: (D)

We have,

$\begin{bmatrix} 1 & 2 & -3 \\ 0 & 4 & 5 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x + 2y - 3z \\ 4y + 5z \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\Rightarrow\ x + 2y - 3z = 1, 4y + 5z = 1, z = 1$

On solving above equations, we get

$x = 6, y = -1, z = 1$

65. If $A = \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}$, then

(A) $A^2 - 2A + 2l = 0$

(B) $A^2 - 3A + 2l = 0$

(C) $A^2 - 5A + 2l = 0$

(D) $2A^2 - A + l = 0$

(E) $A^2 + 3A + 2l = 0$

Solution: (B)

We have,

$A = \begin{bmatrix} 1 & 5 \\ 0 & 2 \end{bmatrix}$

$\therefore\ A^2 = A \cdot A = \begin{bmatrix} 1 & 5 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 15 \\ 0 & 4 \end{bmatrix}$

$\therefore\ A^2 - 3A + 2I = \begin{bmatrix} 1 & 15 \\ 0 & 4 \end{bmatrix} - 3\begin{bmatrix} 1 & 5 \\ 0 & 2 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow A^2 - 3A + 2I = 0$

66. If $\begin{pmatrix} 2x + y & x + y \\ p - q & p + q \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$, then (x, y, p, q) equals

(A) $0, 1, 0, 0$
(B) $0, -1, 0, 0$
(C) $1, 0, 0, 0$
(D) $0, 1, 0, 1$
(E) $1, 0, 1, 0$

Solution: (A)
We have,
$\begin{bmatrix} 2x+y & x+y \\ p-q & p+q \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$
$\therefore\ 2x + y = 1$ .....(i)
$x + y = 1$ .....(ii)
$p - q = 0$ ......(iii)
$p + q = 0$ .....(iv)
On solving Equations (i) and (ii), we get
$x = 0, y = 1$
And on solving Equations (iii) and (iv), we get
$p = q = 0$

67. The value of $\left|\sqrt{4+2\sqrt{3}}\right| - \left|\sqrt{4-2\sqrt{3}}\right|$ is
(A) 1
(B) 2
(C) 4
(D) 3
(E) 5

Solution: (B)
We have,
$\left|\sqrt{4+2\sqrt{3}}\right| - \left|\sqrt{4-2\sqrt{3}}\right|$
$= \left|\sqrt{3+1+2\sqrt{3}}\right| - \left|\sqrt{3+1-2\sqrt{3}}\right|$
$= \left|\sqrt{\left(\sqrt{3}\right)^2 + (1)^2 + 2\cdot\sqrt{3}\cdot 1}\right| - \left|\sqrt{\left(\sqrt{3}\right)^2 + (1)^2 - 2\cdot\sqrt{3}\cdot 1}\right|$
$= \left|\sqrt{\left(\sqrt{3}+1\right)^2}\right| - \left|\sqrt{\left(\left(\sqrt{3}\right)-1\right)^2}\right|$
$= |\sqrt{3}+1| - |\sqrt{3}-1|$
$= \left(\sqrt{3}+1\right) - \left(\sqrt{3}-1\right) = 2$

68. The value of $8^{2/3} - 16^{1/4} - 9^{1/2}$ is
(A) -1
(B) -2
(C) -3
(D) -4
(E) -5

Solution: (A)
We have,
$8^{\frac{2}{3}} - 16^{\frac{1}{4}} - 9^{\frac{1}{2}}$
$= (2^3)^{\frac{2}{3}} - (2^4)^{\frac{1}{4}} - (3^2)^{\frac{1}{2}}$
$= 2^2 - 2^1 - 3^1$
$= 4 - 2 - 3 = -1$

69. Let $x = 2$ be a root of $y = 4x^2 - 14x + q = 0$. Then y is equal to
(A) $(x-2)(4x-6)$
(B) $(x-2)(4x+6)$
(C) $(x-2)\ (-4x-6)$
(D) $(x-2)(-4x+6)$
(E) $(x-2)\ (4x+3)$

Solution: (A)
We have
$y = 4x^2 - 14x + q = 0$
Since, $x = 2$ is the root
$\therefore 4(2)^2 - 14(2) + q = 0$
$\Rightarrow 16 - 28 + q = 0$
$\Rightarrow q = 12$
$\therefore y = 4x^2 - 14x + 12$
$= 4x - 8x - 6x + 12$
$= 4x(x-2) - 6(x-2)$
$= (x-2)(4x-6)$

70. If $x_1$and x_2 are the roots of $3x^2 - 2x - 6 = 0$, then $x_1^2 + x_2^2$ is equal to
(A) $\frac{50}{9}$
(B) $\frac{40}{9}$
(C) $\frac{30}{9}$
(D) $\frac{20}{9}$
(E) $\frac{10}{9}$

Solution: (B)
We have,
$3x^2 - 2x - 6 = 0$
Since, x_1 and x_2 are the roots of above equation
$\therefore \quad x_1 + x_2 = \dfrac{-(-2)}{3} = \dfrac{2}{3}$
And $x_1\ x_2 = \frac{-6}{3} = -2$
Now,
$(x_1 + x_2)^2 = x_1^2 + x_2^2 + 2x_1x_2$
$\Rightarrow x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2$
$= \left(\dfrac{2}{3}\right)^2 - 2\ (-2)$

$= \frac{4}{9} + 4 = \frac{40}{9}$

71. Let x_1 and x_2 be the roots of the equations $x^2 + px - 3 = 0$. If $x_1^2 + x_2^2 = 10$, then the value of p is equal to

(A) -4 or 4
(B) -3 or 3
(C) -2 or 2
(D) -1 or 1
(E) 0

Solution: (C)
We have,
$x^2 - px - 3 = 0$
Since, x_1 and x_2 are the roots of above equation.
$\therefore x_1 + x_2 = p$ and $x_1 x_2 = -3$
Now, we have
$x_1^2 + x_2^2 = 10$
$\Rightarrow \ (x_1 + x_2)^2 - 2x_1 x_2 = 10$
$\Rightarrow \ p^2 + 6 = 10$
$\Rightarrow \ p^2 = 4$
$\Rightarrow \ p = \pm 2$

72. If the product of roots of the equation $mx^2 + 6x + (2m - 1) = 0$ is -1, then the value of m is

(A) $\frac{1}{3}$
(B) 1
(C) 3
(D) -1
(E) -3

Solution: (A)
We have,
$mx^2 + 6x + (2m - 1) = 0$
$\therefore$ Product of roots $= \frac{2m-1}{m}$
$\Rightarrow \ \frac{2m-1}{m} = -1$ [$\because$ product of roots $= -1$]
$\Rightarrow \ 2m - 1 = -m$
$\Rightarrow \ 3m = 1$
$\Rightarrow \ m = \frac{1}{3}$

73. If $f(x) = \frac{1}{x^2+4x+4} - \frac{4}{x^4+4x^3+4x^2} + \frac{4}{x^3+2x^2}$, then $f\left(\frac{1}{2}\right)$ is equal to

(A) 1
(B) 2
(C) -1
(D) 3
(E) 4

Solution: (E)
We have,

$$f(x) = \frac{1}{x^2 + 4x + 4} - \frac{4}{x^4 + 4x^3 + 4x^2} + \frac{4}{x^3 + 2x^2}$$

$$= \frac{1}{(x+2)^2} - \frac{4}{x^2(x+2)^2} + \frac{4}{x^2(x+2)}$$

$$= \frac{x^2 - 4 + 4(x+2)}{(x+2)^2 \cdot x^2}$$

$$= \frac{x^2 - 4 + 4x + 8}{(x+2)^2 \cdot x^2}$$

$$= \frac{x^2 + 4x + 4}{(x+2)^2 \cdot x^2}$$

$$= \frac{(x+2)^2}{(x+2)^2 \cdot x^2}$$

$$= \frac{1}{x^2}$$

$$\therefore f(x) = \frac{1}{x^2}$$

$$\Rightarrow f\left(\frac{1}{2}\right) = \frac{1}{\left(\frac{1}{2}\right)^2} = 4$$

74. If x and y are the roots of the equation $x^2 + bx + 1 = 0$, then the value of $\frac{1}{x+b} + \frac{1}{y+b}$ is

(A) $\frac{1}{b}$
(B) b
(C) $\frac{1}{2b}$
(D) $2b$
(E) 1

Solution: (B)
We have, given that x, y are the roots of the equation $x^2 + bx + 1 = 0$
$\therefore \ x + y = -b$ and $xy = 1$
Now, $\frac{1}{x+b} + \frac{1}{(y+b)} = \frac{y+b+x+b}{(x+b)(y+b)}$

$$= \frac{(x+y) + 2b}{xy + b(x+y) + b^2}$$

$$= \frac{-b + 2b}{1 + b(-b) + b^2}$$

$$= \frac{b}{1 - b^2 + b^2} = b$$

75. The equations $x^5 + ax + 1 = 0$ and $x^6 + ax^2 + 1 = 0$ have a common root. Then a is equal to

(A) -4
(B) -2
(C) -3
(D) -1
(E) 0

Solution: (B)
We have,
$x^5 + ax + 1 = 0$
And $x^6 + ax^2 + 1 = 0$
Or $x^6 + ax^2 + x = 0$
And $x^6 + ax^2 + 1 = 0$
∴ Common root is given by
$(x^6 + ax^2 + x) - (x^6 + ax^2 + 1) = 0$
$\Rightarrow \quad x = 1$
$\therefore \quad x = 1$ is the common root.
$\therefore \quad (1)^5 + a(1) + 1 = 0$
$\Rightarrow \quad a = -2$

76. The root $ax^2 + x + 1 = 0$, where $a \neq 0$, are in the ratio $1 : 1$. Then a is equal to
(A) $\frac{1}{4}$
(B) $\frac{1}{2}$
(C) $\frac{3}{4}$
(D) 1
(E) 0

Solution: (A)
We have, $ax^2 + x + 1 = 0$
Since, roots are in the ratio $1 : 1$, thus roots are equal
∴ Discriminant $= 0$
$\Rightarrow \quad (1)^2 - 4(a)(1) = 0$
$\Rightarrow \quad 1 - 4a = 0$
$\Rightarrow \quad a = \frac{1}{4}$

77. If $z^2 + z + 1 = 0$ where z is a complex number, then the value of $\left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right)^2 + \left(z^3 + \frac{1}{z^3}\right)^2$ equal

(A) 4
(B) 5
(C) 6
(D) 7
(E) 8

Solution: (C)
We have,
$z^2 + z + 1 = 0$
$\Rightarrow \quad z = \dfrac{-1 \pm \sqrt{1-4}}{2}$
$= \dfrac{-1 \pm \sqrt{3}i}{2}$

$\therefore \quad z = w$ or w^2
Let $z = w$, then
$\left(z+\frac{1}{2}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2+\left(z^3+\frac{1}{z^3}\right)^2$
$=\left(\omega+\frac{1}{\omega}\right)^2+\left(\omega^2+\frac{1}{\omega^2}\right)^2+\left(\omega^3+\frac{1}{\omega^3}\right)^2$
$=(\omega+\omega^2)^2+(\omega^2+\omega)^2+(\omega^3+1)^2 \quad [\because \omega^3=1]$
$=(-1)^2+(-1)^2+(1+1)^2 \; [\because 1+\omega+\omega^2=0]$
$=1+1+4=6$
The value will be same when $z=\omega^2$.

78. Let $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-w^2 & w^2 \\ 1 & w & w^4 \end{vmatrix}$, where $w \neq 1$ is a complex number such that $w^3 = 1$. Then Δ equals
(A) $3w+w^2$
(B) $3w^2$
(C) $3(w=w)^2$
(D) $-3w^2$
(E) $3w^2+1$

Solution: (B)
We have,
$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-w^2 & w^2 \\ 1 & w & w^4 \end{vmatrix}$
$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & w & w^2 \\ 1 & w & w \end{vmatrix}$
$[\because 1+w+w^2=0, w^3=1]$
$=1(w^2-w^3)-1(w-w^2)+1(w-w)$
$=w^2-1-w-w^2$
$=2w^2-(1+w)$
$=2w^2-(-w^2)$
$=3w^2$

79. If $\begin{vmatrix} 3i & -9i & 1 \\ 2 & 9i & -1 \\ 10 & 9 & i \end{vmatrix} = x+iy$, then
(A) $x=1, y=1$
(B) $x=0, y=1$
(C) $x=1, y=0$
(D) $x=0, y=0$
(E) $x=-1, y=0$

Solution: (D)
We have,
$\begin{vmatrix} 3i & -9i & 1 \\ 2 & 9i & -1 \\ 10 & 9 & i \end{vmatrix} = x+iy$

$\Rightarrow \begin{vmatrix} 3i+2 & 0 & 0 \\ 2 & 9i & -1 \\ 10 & 9 & i \end{vmatrix} = x + iy \ \ [\because R_1 \to R_1 + R_2]$

$\Rightarrow \ (3i+2)[9i^2+9] = x + iy$

$\Rightarrow \ (3i+2)(-9+9) = x + iy \ \ [\because i^2 = -1]$

$\Rightarrow \ 0 = x + iy$

$\Rightarrow \ x = 0, y = 0$

80. If $z = \cos\left(\frac{\pi}{3}\right) - i\sin\left(\frac{\pi}{3}\right)$, then $z^2 - z + 1$ is equal to

(A) 0

(B) 1

(C) -1

(D) $\frac{\pi}{2}$

(E) π

Solution: (A)

We have

$z = \cos\frac{\pi}{3} - i\sin\frac{\pi}{3}$

$= \frac{1}{2} - \frac{i\sqrt{3}}{2}$

$= \frac{1-\sqrt{3}i}{2}$

$= -\left[\frac{-1+\sqrt{3}i}{2}\right]$

$= -w \qquad \left[\because w = \frac{-1+\sqrt{3}i}{2}\right]$

Now, $z^2 - z + 1 = (-w)^2 - (-w) + 1$

$= w^2 + w + 1$

$= 0 \ \ [\because 1 + w + w^2 = 0]$

81. $\left(\frac{1+\cos\left(\frac{\pi}{12}\right)+i\sin\left(\frac{\pi}{12}\right)}{1+\cos\left(\frac{\pi}{12}\right)-i\sin\left(\frac{\pi}{12}\right)}\right)^{72}$ is equal to

(A) 0

(B) -1

(C) 1

(D) $\frac{1}{2}$

(E) $\frac{-1}{2}$

Solution: (C)

Let $z = \left(\frac{1+\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}}{1+\cos\frac{\pi}{12}-i\sin\frac{\pi}{12}}\right)^{72}$

$$= \left(\frac{2\cos^2\frac{\pi}{24} + 2i\sin\frac{\pi}{24}\cos\frac{\pi}{24}}{2\cos^2\frac{\pi}{24} - 2i\sin\frac{\pi}{24}\cos\frac{\pi}{24}}\right)^{72}$$

$$= \left(\frac{\cos\frac{\pi}{24} + i\sin\frac{\pi}{24}}{\cos\frac{\pi}{24} - i\sin\frac{\pi}{24}}\right)^{72}$$

$$= \frac{\cos\frac{72\pi}{24} + i\sin\frac{72\pi}{24}}{\cos\frac{72\pi}{24} - i\sin\frac{72\pi}{24}}$$

$[\because (\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta]$

$$= \frac{\cos 3\pi + i\sin 3\pi}{\cos 3\pi - i\sin 3\pi}$$

$$= \frac{-1+0}{-1-0}$$

$= 1$

82. If $A = \begin{vmatrix} 4 & k & k \\ 0 & k & k \\ 0 & 0 & k \end{vmatrix}$ and $det\ (A) = 256$, then $|k|$ equals

(A) 4
(B) 5
(C) 6
(D) 7
(E) 8

Solution: (E)
We have,

$$A = \begin{bmatrix} 4 & k & k \\ 0 & k & k \\ 0 & 0 & k \end{bmatrix}$$

$$\therefore\ \ |A| = \begin{vmatrix} 4 & k & k \\ 0 & k & k \\ 0 & 0 & k \end{vmatrix}$$

$\Rightarrow\ \ 256 = 4(k^2 - 0)$

$\Rightarrow\ \ 64 = k^2$

$\Rightarrow\ \ k = \pm 8$

$\therefore\ \ |k| = 8$

83. If $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$, then $A^n + nI$ is equal to

(A) I
(B) nA
(C) $I + nA$
(D) $I - nA$
(E) $nA - I$

Solution: (C)
We have,

$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$\therefore \quad A^2 = A \cdot A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

$\therefore \quad A^n = \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix}$

Now,

$A^n + nI = \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix} + n \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix} + \begin{bmatrix} n & 0 \\ 0 & n \end{bmatrix}$

$= \begin{bmatrix} 1+n & 0 \\ n & 1+n \end{bmatrix}$

Again, $I + nA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + n \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} n & 0 \\ n & n \end{bmatrix}$

$= \begin{bmatrix} 1+n & 0 \\ n & 1+n \end{bmatrix}$

$\therefore \quad A^n + nI = I + nA$

84. If $|z| = 5$ and $w = \frac{z-5}{z+5}$, then the $Re(w)$ is equal to

(A) 0

(B) $\frac{1}{25}$

(C) 25

(D) 1

(E) -1

Solution: (A)

Let $\; z = x + iy$

$\therefore \quad |z| = \sqrt{x^2 + y^2}$

$\Rightarrow \sqrt{x^2 + y^2} = 5$

$\Rightarrow \quad x^2 + y^2 = 25$ ....(i)

Now, $w = \frac{z-5}{z+5} = \frac{x+iy-5}{x+iy+5}$

$= \dfrac{(x-5)+iy}{(x+5)+iy}$

$= \dfrac{(x-5)+iy}{(x+5)+iy} \times \dfrac{(x+5)-iy}{(x+5)-iy}$

$= \dfrac{x^2 - 25 + i\,y(x+5) - y(x-5)i + y^2}{(x+5)^2 + y^2}$

$= \dfrac{(x^2 + y^2 - 25) + i[xy + 5y - xy + 5y]}{(x+5)^2 + y^2}$

$= \frac{0+10y\,i}{(x+5)^2+y^2}$ [$\because$ from Equation (i)]

$= \dfrac{10y}{(x+5)^2 + y^2} i$

$\therefore \quad Re(w) = 0$

85. If $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, then A^{2017} is equal to
(A) $2^{2015}A$
(B) $2^{2016}A$
(C) $2^{2014}A$
(D) $2^{2017}A$
(E) $2^{2020}A$

Solution: (B)
We have,
$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
$\therefore\ A^2 = A \cdot A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
$= \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = 2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 2A$
Again, $A^3 = A^2 \cdot A$
$= \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
$= \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$
$= 4\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 2^2 \cdot A$
$\therefore A^n = 2^{n-1}A$
$\therefore A^{2017} = 2^{2016}A$

86. If $a = e^{i\theta}$, then $\frac{1+a}{1-a}$ is equal to
(A) $\cot\frac{\theta}{2}$
(B) $\tan\theta$
(C) $i\cot\frac{\theta}{2}$
(D) $i\tan\frac{\theta}{2}$
(E) $2\tan\theta$

Solution: (C)
We have,
$a = e^{i\theta}$
$= \cos\theta + i\sin\theta$
Now, $\frac{1+a}{1-a} = \frac{1+(\cos\theta+i\sin\theta)}{1-(\cos\theta+i\sin\theta)}$
$$= \frac{(1+\cos\theta) + i\sin\theta}{(1-\cos\theta) - i\sin\theta}$$
$$= \frac{2\cos^2\frac{\theta}{2} + i2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2} - i2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$
$$= \frac{2\cos\frac{\theta}{2}\left[\cos\frac{\theta}{2} + i\sin\ \frac{\theta}{2}\right]}{2\sin\frac{\theta}{2}\left[\sin\frac{\theta}{2} - i\cos\ \frac{\theta}{2}\right]}$$

$$= \frac{\cot\frac{\theta}{2}\left[\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right]}{-i\left[\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right]}$$
$$= \frac{\cot\frac{\theta}{2}}{-i}$$
$$= i\cot\frac{\theta}{2}$$

87. Three numbers x, y and z are in arithmetic progression. If $x + y + z = -3$ and $xyz = 8$, then $x^2 + y^2 + z^2$ is equal to
(A) 9
(B) 10
(C) 21
(D) 20
(E) 1

Solution: (C)
Let $x = a - r, y = a, z = a + r$
Now, we have
$x + y + z = -3$
$\therefore\ a - r + a + a + r = -3$
$\Rightarrow\ 3a = -3$
$\Rightarrow\ a = -1$
Again, $xyz = 8$
$\therefore\ (a - r)(a)(a + r) = 8$
$\Rightarrow a(a^2 - r^2) = 8$
$\Rightarrow\ -1(1 - r^2) = 8$
$\Rightarrow\ -1 + r^2 = 8$
$\Rightarrow r^2 = 9$
$\Rightarrow r = \pm 3$
$\therefore x, y, z$ are $-4, -1, 2$ or $2, -1, -4$
$\therefore x^2 + y^2 + z^2 = (-4)^2 + (-1)^2 + (2)^2$
$= 16 + 1 + 4 = 21$

88. The 30th term of the arithmetic progression $10, 7, 4$ is
(A) -97
(B) -87
(C) -77
(D) -67
(E) -57

Solution: (C)
We have, $10, 7, 4$
Which is an $A.P.$
$\therefore a = 10, d = -3$
$\therefore a_{30} = a + 29d\quad [\because a_n = a + (n - 1)d]$
$= 10 + 29(-3)$
$= 10 - 87$

$= -77$

89. The arithmetic mean of two numbers x and y is 3 and geometric mean is 1. Then $x^2 + y^2$ is equal to
(A) 30
(B) 31
(C) 32
(D) 33
(E) 34

Solution: (E)
We have,
$AM = 3$ and $GM = 1$
$\therefore \frac{x+y}{2} = 3$ and $\sqrt{xy} = 1$
$\Rightarrow x + y = 6$ and $xy = 1$
Now, $x^2 + y^2 = (x+y)^2 - 2xy$
$= (6)^2 - 2(1)$
$= 36 - 2 = 34$

90. The solution of $3^{2x-1} = 81^{1-x}$ is
(A) $\frac{2}{3}$
(B) $\frac{1}{6}$
(C) $\frac{7}{6}$
(D) $\frac{5}{6}$
(E) $\frac{1}{3}$

Solution: (D)
We have,
$3^{2x-1} = 81^{1-x}$
$\Rightarrow \ 3^{2x-1} = (3^4)^{1-x}$
$\Rightarrow \ 3^{2x-1} = 3^{4-4x}$
$\therefore \ 2x - 1 = 4 - 4x$
$\Rightarrow \ 6x = 5$
$$\Rightarrow \ x = \frac{5}{6}$$

91. The sixth term in the sequence is $3, 1, \frac{1}{3}, \ldots$ is
(A) $\frac{1}{27}$
(B) $\frac{1}{9}$
(C) $\frac{1}{81}$
(D) $\frac{1}{17}$
(E) $\frac{1}{7}$

Solution: (C)

We have,

$3, 1, \frac{1}{3}, \dots..$

Which is a $G.P.$ with

$a = 3, r = \frac{1}{3}$

$\therefore\ a_6 = ar^5\ \ [\because a_n = ar^{n-1}]$

$\Rightarrow a_6 = 3\left(\frac{1}{3}\right)^5$

$= 3 \times \frac{1}{3^5}$

$= \frac{1}{3^4}$

$= \frac{1}{81}$

92. Three numbers are in arithmetic progression. Their sum is 21 and the product of the first number and the third number is 45. Then the product of these three number is

(A) 315

(B) 90

(C) 180

(D) 270

(E) 450

Solution: (A)

Let the numbers be $a - d, a, a + d$

$\therefore a + d + a + a - d = 21$

$\Rightarrow 3a = 21$

$\Rightarrow a = 7$

Again, $(a - d)(a + d) = 45$

$\Rightarrow\ \ a^2 - d^2 = 45$

$\Rightarrow\ \ (7)^2 - d^2 = 45$

$\Rightarrow\ \ 49 - d^2 = 45$

$\Rightarrow\ \ d^2 = 4$

$\Rightarrow\ \ d = \pm 2$

$\therefore$ Numbers are $5, 7, 9$ or $9, 7, 5$

$\therefore$ Products of three numbers $= 5 \times 7 \times 9$

$= 315$

93. If $a + 1, 2a + 1, 4a - 1$ are in arithmetic progression, then the value of a is

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

Solution: (B)

We have,

$a + 1, 2a + 1;\ 4a - 1$ are in AP

$\therefore 2(2a+1) = (4a-1) + (a+1)$
[$\because$ If a, b, c are in AP, then $2b = a + c$]
$\Rightarrow 4a + 2 = 5a$
$\Rightarrow a = 2$

94. Two numbers x and y have arithmetic mean 9 and geometric mean 4. Then, x and y are the roots of
(A) $x^2 - 18x - 16 = 0$
(B) $x^2 - 18x + 16 = 0$
(C) $x^2 + 18x - 16 = 0$
(D) $x^2 + 18x + 16 = 0$
(E) $x^2 - 17x + 16 = 0$

Solution: (B)
We have,
AM of $x, y = 9$ and GM of $x, y = 4$
$\therefore \frac{x+y}{2} = 9$ and $\sqrt{xy} = 4$
$\Rightarrow x + y = 18$ and $xy = 16$
$\Rightarrow y = 18 - x$ and $xy = 16$
$\therefore x(18 - x) = 16$
$\Rightarrow 18x - x^2 = 16$
$\Rightarrow x^2 - 18x + 16 = 0$
$\therefore x$ and y are the roots of the equation
$x^2 - 18x + 16 = 0$

95. Three unbiased coins are tossed. The probability of getting atleast 2 tails is
(A) $\frac{3}{4}$
(B) $\frac{1}{4}$
(C) $\frac{1}{2}$
(D) $\frac{1}{3}$
(E) $\frac{2}{3}$

Solution: (C)
Total numbers of outcomes when three coins are tossed $= 2 \times 2 \times 2$
$= 8$
$\therefore n(S) = 8$
Let $E =$ Event getting at least 2 tails
$= \{TTH, THT, TTH, TTT\}$
$\therefore n(E) = 4$
$\therefore$ Required probability $= P(E)$
$= \frac{n(E)}{n(S)}$
$= \frac{4}{8}$
$= \frac{1}{2}$

96. A single letter is selected from the word $TRICKS$. The probability that it is either T or R is
(A) $\frac{1}{36}$
(B) $\frac{1}{4}$
(C) $\frac{1}{2}$
(D) $\frac{2}{3}$
(E) $\frac{1}{3}$

Solution: (E)
Number of ways of selecting one letter from the word TRICKS $n(S) = {}^6C_1 = 6$
Let E be the event of selecting T or R
$\therefore\ E = \{T, R\}$
$\therefore\ n(E) = 2$
$\therefore$ Required probability $= p(E)$
$= \frac{n(E)}{n(S)}$
$= \frac{2}{6}$
$= \frac{1}{3}$

97. From 4 red balls, 2 white balls and 4 black balls, four balls are selected. The probability of getting 2 red balls is
(A) $\frac{7}{21}$
(B) $\frac{8}{21}$
(C) $\frac{9}{21}$
(D) $\frac{10}{21}$
(E) $\frac{11}{21}$

Solution: (C)
We have, 4 red, 2 white and 4 black balls
$\therefore$ Total balls $= 4 + 2 + 4 = 10$
Number of ways of selecting 4 balls from 10 balls $= {}^{10}C_4$
$\therefore n(S) = {}^{10}C_4$
Let $E =$ Event getting 2 red balls
$\therefore\ n(E) = {}^4C_2 \times {}^6C_2$
$\therefore$ Required probability $= p(E)$
$= \frac{n(E)}{n(S)}$
$= \frac{{}^4C_2 = {}^6C_2}{{}^{10}C_4}$
$= \frac{\frac{4 \times 3}{2 \times 1} \times \frac{6 \times 5}{2 \times 1}}{\frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1}}$

$= \frac{6 \times 15}{10 \times 3 \times 7}$
$= \frac{3}{7}$
$= \frac{9}{21}$

98. In a class, 60% of the students know lesson $I, 40\%$ know lesson II and 20% know I and II. A student is selected of random. The probability that the student does not know lesson I and lesson II is
(A) 0
(B) $\frac{4}{5}$
(C) $\frac{3}{5}$
(D) $\frac{1}{5}$
(E) $\frac{2}{5}$

Solution: (D)
Let E_1 = Event that student know lesson I
E_2 = Event that student know lesson II
Now, according to the question,
$P(E_1) = 0.60, P(E_2) = 0.40,$
$P(E_1 \cap E_2) = 0.20$
$\therefore$ Required probability $= P(E_1' \cap E_2')$
$= P(E_1 \cup E_2)'$
$= 1 - P(E_1 \cup E_2)$
$= 1 - [P(E_1) + P(E_2) - P(E_1 \cap E_2)]$
$= 1 - [0.60 + 0.40 - 0.20]$
$= 1 - [0.80]$
$= 0.20$
$= \frac{20}{100}$
$= \frac{1}{5}$

99. Two distinct numbers x and y are chosen from $1, 2, 3, 4, 5$. The probability that the arithmetic mean of x and y is an inter is
(A) 0
(B) $\frac{1}{5}$
(C) $\frac{3}{5}$
(D) $\frac{2}{5}$
(E) $\frac{4}{5}$

Solution: (D)
Let S: Event that two numbers are selected from $1, 2, 3, 4, 5$
$\therefore n(S) = {}^5C_2 = 10$
E: Event that two numbers selected have integer mean.
$\therefore E = \{(1,3), (1,5), (2,4), (3,5)\}$

$\therefore n(E) = 4$
$\therefore$ Required probability $= P(E)$
$= \frac{n(E)}{n(S)}$
$= \frac{4}{10}$
$= \frac{2}{5}$

100. The number of 3×3 matrices with entries -1 or $+1$ is
(A) 2^{-4}
(B) 2^5
(C) 2^6
(D) 2^7
(E) 2^9

Solution: (E)
In 3×3 matrix, total number of elements $= 3 \times 3 = 9$
$\therefore$ Total number of 3×3 matrices with entries either -1 or $1 = 2^9$

101. Let S be the set of all 2×2 symmetric matrices whose entries are either zero or one. A matrix X is chosen from S. The probability that the determinant of X is not zero is
(A) $\frac{1}{3}$
(B) $\frac{1}{2}$
(C) $\frac{3}{4}$
(D) $\frac{1}{4}$
(E) $\frac{2}{9}$

Solution: (B)
$S =$ {2×2 symmetric matrices whose entries are either zero or one}
$= \left\{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\1 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}\right\}$
$\therefore \quad n(s) = 8$
Let $x =${matrix whose determinant is non$-$zero}
$= \left\{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\1 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}\right\}$
$\therefore \quad n(x) = 4$
$\therefore \quad P(x) = \frac{n(x)}{n(s)}$
$= \frac{4}{8} = \frac{1}{2}$

102. The number of words that can be formed by using all the letters of the word PROBLEM only one is
(A) $5!$
(B) $6!$
(C) $7!$
(D) $8!$

(E) 9!

Solution: (C)
The word 'PROBLEM' has 7 letters viz. P, R, O, B, L, E, M
$\therefore$ Total number of words that can be formed by using all the letters only one $=$ Number of arranging the seven letters $= 7!$

103. The number of diagonals in a hexagon is
(A) 8
(B) 9
(C) 10
(D) 11
(E) 12

Solution: (B)
The number of diagonals in a n-side polygon
$$= \frac{n(n-3)}{2}$$
$\therefore$ Number of diagonals in a hexagon
$$= \frac{6(6-3)}{2}$$
$= 9$

104.The sum of odd integers from 1 to 2001 is
(A) 1001^2
(B) 1000^2
(C) 1002^2
(D) 1003^2
(E) 999^2

Solution: (A)
The odd integers from 1 to 2001 are $1, 3, 5, \dots, 1999, 2001$.
They forms an AP with $a = 1, d = 2$ and $a_n = 2001$.
$\therefore \ a_n = a + (n-1)d$
$\Rightarrow \ 2001 = 1 + (n-1)2$
$\Rightarrow \ 2000 = (n-1)2$
$\Rightarrow \ n = 1001$
$$\therefore \ S_n = \frac{n}{2}[2a + (n-1)d]$$
$$= \frac{1001}{2}[2 \times 1 + (1001-1) \times 2]$$
$$= \frac{1001}{2}[2 + 2000]$$
$$= \frac{1001}{2} \times 2002$$
$= 1001 \times 1001$
$= (1001)^2$

105. Two balls are selected from two black and two red balls. The probability that the two balls will have no black balls is
(A) $\frac{1}{7}$
(B) $\frac{1}{5}$
(C) $\frac{1}{4}$
(D) $\frac{1}{3}$
(E) $\frac{1}{6}$

Solution: (E)
We have, 2 black and 2 red balls.
$S =$ Selecting two balls
$\therefore n(S) = {}^4C_2$
$E =$ Event that two balls will have no black balls
$=$ Selecting 2 red balls
$\therefore n(E) = {}^2C_2$
$\therefore$ Required probability $= P(E)$
$= \dfrac{n(E)}{n(S)}$
$= \dfrac{{}^2C_2}{{}^4C_2} = \dfrac{1}{6}$

106. If $z - i^9 + i^{19}$, then z is equal to
(A) $0 + 0i$
(B) $1 + 0i$
(C) $0 + i$
(D) $1 + 2i$
(E) $1 + 3i$

Solution: (A)
We have
$z - i^9 + i^{19}$,
$= (1^4)^2 \cdot i + (i^4)^4 \cdot i^3$
$= i + i^3$ $\quad [\because i^4 = 1]$
$= i - i$ $\quad [\because i^3 = -i]$
$= 0$
$= 0 + 0i$

107. The mean for the data $6, 7, 10, 12, 13, 4, 8, 12$ is
(A) 9
(B) 8
(C) 7
(D) 6
(E) 5

Solution: (A)

We have,
$x_i = 6, 7, 10, 12, 13, 4, 8, 12$
$\therefore \text{Mean} = \frac{\text{Sum of all the observations}}{\text{Total number of observations}}$
$= \frac{6+7+10+12+13+4+8+12}{8}$
$= \frac{72}{8} = 9$

108. The set of all real numbers satisfying the inequality $x - 2 < 1$ is
(A) $(3, \infty)$
(B) $[3, \infty)$
(C) $[-3, \infty)$
(D) $(-\infty, -3)$
(E) $(-\infty, 3)$

Solution: (E)
We have,
$x - 2 < 1$
$\Rightarrow x - 2 + 2 < 1 + 2$
$\Rightarrow x < 3$
$\therefore x \in (-\infty, 3)$

109. If $\frac{|x-3|}{x-3} >$, then
(A) $x \in (-3, \infty)$
(B) $x \in (3, \infty)$
(C) $x \in (2, \infty)$
(D) $x \in (1, \infty)$
(E) $x \in (-\infty, 3)$

Solution: (B)
We have
$\frac{|x-3|}{x-3} > 0$
Now, $\frac{|x-3|}{x-3} = \begin{cases} \frac{x-3}{x-3} = 1, & x \geq 3 \\ \frac{-(x-3)}{x-3} = -1, & x < 3 \end{cases}$
$\therefore$ $\frac{|x-3|}{x-3} > 0$ only holds when $x \in (3, \infty)$

110. The mode of the data $8, 11, 9, 8, 11, 9, 7, 8, 7, 3, 2$ is
(A) 11
(B) 9
(C) 8
(D) 3
(E) 7

Solution: (C)

We have,
Observation $= 8, 11, 9, 8, 11, 9, 7, 8, 7, 3, 2$
Since, 8 is occurring highest time
$\therefore$ Mode $= 8$

111. If the mean of six numbers is 41, then the sum of these numbers is
(A) 246
(B) 236
(C) 226
(D) 216
(E) 206

Solution: (A)
We know that,
$$\bar{x} = \frac{\Sigma x_i}{N}$$
$$\Rightarrow \Sigma x_i = \bar{x} \times N = 41 \times 6 = 246$$

112. If $\int_0^x f(t)dt = x^2 + e^x (x > 0)$, then $f(1)$ is equal to
(A) $1 + e$
(B) $2 + e$
(C) $3 + e$
(D) e
(E) 0

Solution: (B)
We have
$$\int_0^x f(t)dt = x^2 + e^x$$
Using Leibnitz Rule,
$f(x) = 2x + e^x$
$\therefore f(1) = 2 + e$

113. $\int \frac{x+1}{x^{\frac{1}{2}}} dx =$
(A) $-x^{\frac{3}{2}} + x^{\frac{1}{2}} + C$
(B) $x^{\frac{1}{2}}$
(C) $x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$
(D) $x^{\frac{3}{2}} + x^{\frac{1}{2}} + C$
(E) $x^{\frac{3}{2}}$

Solution: (A)
Let $I = \int \frac{x+1}{x^{\frac{1}{2}}} dx$
$$= \int \left(x^{\frac{1}{2}} + \frac{1}{x^{\frac{1}{2}}} \right) dx$$

$= \frac{2x^{\frac{3}{2}}}{3} + 2 \cdot x^{\frac{1}{2}} + C$

114. In a flight 50 people speak Hindi, 20 speak English and 10 speak both English and Hindi. The number of people who speak atleast one of the two languages is
(A) 40
(B) 50
(C) 20
(D) 80
(E) 60

Solution: (E)
Let $H =$ People who speak Hindi
$E =$ People who speak English
According to the questions,
$n(H) = 50, n(E) = 20, n(H \cap E) = 10$
$\therefore$ Number of people who speak atleast two language $= n(H \cup E)$
$= n(H) + n(E) - n(H \cap E)$
$= 50 + 20 - 10 = 60$

115. If $f(x) = \frac{x+1}{x-1}$, then the value of $f\left(f(x)\right)$ is equal to
(A) x
(B) 0
(C) $-x$
(D) 1
(E) 2

Solution: (A)
We have,
$f(x) = \frac{x+1}{x-1}$
$\therefore f\left(f(x)\right) = f\left(\frac{x+1}{x-1}\right)$
$= \frac{\frac{x+1}{x-1} + 1}{\frac{x+1}{x-1} - 1}$
$= \frac{x+1+x-1}{x+1-x+1}$
$= \frac{2x}{2}$
$= x$

116. Two dice are thrown simultaneously. What is the probability of getting two numbers whose product is even?
(A) $\frac{3}{4}$
(B) $\frac{1}{4}$

(C) $\frac{1}{2}$
(D) $\frac{2}{3}$
(E) $\frac{1}{16}$

Solution: (A)
Total number of outcomes when two dice are thrown $= 6 \times 6$
$\therefore n(S) = 36$
Let $E =$ outcomes in which product of two number is even
{(1, 2), (1, 4), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 2), (3, 4), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 2), (5, 4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
$\therefore\ n(E) = 27$
$\therefore$ Required probability $= P(E)$
$= \dfrac{n(E)}{n(S)}$
$= \dfrac{27}{36} = \dfrac{3}{4}$

117. $\lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2-x}}{x}$ is equal to
(A) $\frac{1}{\sqrt{2}}$
(B) $\sqrt{2}$
(C) 0
(D) Does not exist
(E) $\frac{1}{2\sqrt{2}}$

Solution: (A)
We have,
$$\lim_{x\to 0} \frac{\sqrt{2\ +\ x}-\sqrt{2\ -\ x}}{x}$$
$$= \lim_{x\to 0} \frac{\sqrt{2\ +\ x}-\sqrt{2\ -\ x}}{x} \times \frac{\sqrt{2+x}+\sqrt{2-x}}{\sqrt{2+x}+\sqrt{2-x}}$$
$$= \lim_{x\to 0} \frac{(2+x)-(2-x)}{x\left[\sqrt{2\ +\ x}+\sqrt{2\ -\ x}\right]}$$
$$= \lim_{x\to 0} \frac{2x}{x\sqrt{2\ +\ x}+\sqrt{2\ -\ x}}$$
$$= \lim_{x\to 0} \frac{2}{\sqrt{2\ +\ x}+\sqrt{2\ -\ x}}$$
$$= \frac{2}{\sqrt{2\ +\ 0}+\sqrt{2\ -\ 0}}$$
$$= \frac{2}{2\sqrt{2}}$$
$$= \frac{1}{\sqrt{2}}$$

118. $\int \frac{dx}{e^x + e^{-x} + 2}$ is equal to

(A) $\frac{1}{e^x+1}+C$
(B) $\frac{-1}{e^x+1}+C$
(C) $\frac{1}{1+e^{-x}}+C$
(D) $\frac{1}{e^{-x}-1}+C$
(E) $\frac{1}{e^x-1}+C$

Solution: (B)
Let $I=\int\frac{dx}{e^x+e^{-x}+2}$
$=\int\frac{e^x}{e^{2x}+2e^x+1}dx$
Put $e^x=t$
$\Rightarrow\ e^x dx=dt$
$\therefore\ I=\int\frac{dt}{t^2+2t+1}$
$=\int\frac{dt}{(t+1)^2}$
$=\frac{-1}{t+1}+C$
$=\frac{-1}{e^x+1}+C$

119. $\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)+\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$ is equal to
(A) $\sec\theta$
(B) $2\sec\theta$
(C) $\sec\frac{\theta}{2}$
(D) $\sin\theta$
(E) $\cos\theta$

Solution: (B)
We have,
$\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)+\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$
$=\frac{\tan\frac{\pi}{4}+\tan\frac{\theta}{2}}{1-\tan\frac{\pi}{4}\tan\frac{\theta}{2}}+\frac{\tan\frac{\pi}{4}-\tan\frac{\theta}{2}}{1+\tan\frac{\pi}{4}\tan\frac{\theta}{2}}$
$=\frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}}+\frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}$
$=\frac{\left(1+\tan\frac{\theta}{2}\right)^2+\left(1-\tan\frac{\theta}{2}\right)^2}{1-\tan^2\frac{\theta}{2}}$
$=\frac{1+\tan^2\frac{\theta}{2}+2\tan\frac{\theta}{2}+1+\tan^2\frac{\theta}{2}-2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$

$$= 2\left[\frac{1+\tan^2\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}\right]$$

$$= \frac{2}{\cos\theta} \qquad \left[\because \cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}\right]$$

$$= 2\sec\theta$$

120. $\int_{-1}^{0}\frac{dx}{x^2+x+2}$ is equal to

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) π

(D) 0

(E) $-\pi$

Solution: (A)

Let $I = \int_{-1}^{0}\frac{dx}{x^2+x+2}$

$$= \int_{-1}^{0}\frac{dx}{\left(x+\frac{1}{2}\right)^2+2-\frac{1}{4}}$$

$$= \int_{-1}^{0}\frac{dx}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2}$$

$$= \left[\frac{1}{\left(\frac{\sqrt{7}}{2}\right)}\tan^{-1}\frac{x+\frac{1}{2}}{\frac{\sqrt{7}}{2}}\right]_{-1}^{0}$$

$$= \frac{2}{\sqrt{7}}\left[\tan^{-1}\frac{2x+1}{\sqrt{7}}\right]_{-1}^{0}$$

$$= \frac{2}{\sqrt{7}}\left[\tan^{-1}\frac{1}{\sqrt{7}} - \tan^{-1}\left(\frac{1}{\sqrt{7}}\right)\right]$$

$$= \frac{2}{\sqrt{7}}\left[\tan^{-1}\frac{2}{\sqrt{7}} + \tan^{-1}\frac{2}{\sqrt{7}}\right]$$

$$= \frac{4}{\sqrt{7}}\tan^{-1}\frac{1}{\sqrt{7}}$$