Mathematics

Single correct answer type:

1. The value of $\frac{2(\cos 75^o + i\sin 75^o)}{0.2(\cos 30^o + i\sin 30^o)}$ is

(A) $\frac{5}{\sqrt{2}}(1+i)$

(B) $\frac{10}{\sqrt{2}}(1+i)$

(C) $\frac{10}{\sqrt{2}}(1-i)$

(D) $\frac{5}{\sqrt{2}}(1-i)$

(E) $\frac{1}{\sqrt{2}}(1+i)$

Solution: (B)

$$\frac{2(\cos 75^o + i\sin 75^o)}{0.2(\cos 30^o\ i\sin 30^o)} = \frac{2\cdot e^{i\,75^o}}{0.2\cdot e^{i\,30^o}}$$

$$\left(\because \cos\theta + i\sin\theta = e^{i\theta}\right)$$

$$= 10\cdot e^{i75^o}\cdot e^{-i30^o}$$

$$= 10\cdot e^{i\,45^o}$$

$$= 10(\cos 45^o + i\sin 45^o)$$

$$\left(e^{i\theta} = \cos\theta + i\sin\theta\right)$$

$$= 10\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)$$

$$= \frac{10}{\sqrt{2}}(1+i)$$

2. If the conjugate of a complex number z is $\frac{1}{i-1}$, then z is

(A) $\frac{1}{i-1}$

(B) $\frac{1}{i+1}$

(C) $\frac{-1}{i-1}$

(D) $\frac{-1}{i+1}$

(E) $\frac{1}{i}$

Solution: (D)

$$z = \frac{1}{i-1}\times\frac{i+1}{i+1}$$

$\Rightarrow \quad z = \dfrac{i+1}{i^2 - 1^2}$

$z = -\dfrac{1}{2} \times (i+1)$

$\Rightarrow \bar{z} = -\dfrac{1}{2}(1-i) \times \dfrac{(1+i)}{(1+i)}$

$= -\dfrac{1}{2}\dfrac{(1+1)}{(1+i)} = -\dfrac{1}{(1+i)}$

3. The value of $\left(i^{18} + \left(\frac{1}{i}\right)^{25}\right)^3$ is equal to

(A) $\frac{1+i}{2}$

(B) $2 + 2i$

(C) $\frac{1-i}{2}$

(D)$\sqrt{2} - \sqrt{2}i$

(E) $2 - 2i$

Solution: (E)

$\left\{i^{18} + \left(\dfrac{1}{i}\right)^{25}\right\}^3 = \left\{(i^4)^4 \cdot i^2 + \left(\dfrac{1}{i^4}\right)^6 \cdot \dfrac{1}{i}\right\}^3$

$= \left[1 \cdot (-1) + 1.\dfrac{1}{i}\right]^3$

$= \left[\dfrac{1}{i} - 1\right]^3$

$= \dfrac{1}{i^3} - 1 + \dfrac{3}{i}\left(1 - \dfrac{1}{i}\right)$

$= i - 1 - 3i + 3$

$= 2 - 2i$

4. The modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$ is

(A) 2

(B) $\sqrt{2}$

(C) 4

(D) 8

(E) 10

Solution: (A)

$\dfrac{1+i}{1-i} - \dfrac{1-i}{1+i} = \dfrac{(1+i)^2 - (1-i)^2}{1^2 - i^2}$

$= \dfrac{1 + i^2 + 2i - 1 - i^2 + 2i}{2} = \dfrac{4i}{2}$

$= 2i = 0 + 2i$

Modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i} = |0 + 2i|$
$= \sqrt{0^2 + 2^2} = \sqrt{4} = 2$

5. If $z = e^{\frac{i4\pi}{3}}$, then $(z^{192} + z^{194})^3$ is equal to
(A) -2
(B) -1
(C) $-i$
(D) $-2i$
(E) 0

Solution: (B)
$z = e^{\frac{i\,4\pi}{3}}$
$z = \left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$
$z = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$
$z = \omega^2$
$(z^{192} + z^{194})^3 = [(\omega^2)^{192} + (\omega^2)^{194}]^3$
$= [\omega^{384} + \omega^{388}]^3$
$= [(\omega^3)^{128} + (\omega^3)^{129} \cdot \omega]^3$
$= (1 + \omega)^3$
$= 1 + \omega^3 + 3\omega^2 + 3\omega$
$= 1 + 1 + 3(\omega + \omega^2)$
$= 1 + 1 + 3(-1)$
$= 1 + 1 - 3$
$= -1$

6. If a and b are real numbers and $(a + ib)^{11} = 1 + 3i$, then $(b + ia)^{11}$ is equal to
(A) $i + 3$
(B) $1 + 3i$
(C) $1 - 3i$
(D) 0
(E) $-i - 3$

Solution: (E)
Given, $(a + ib)^{11} = 1 + 3i$
So, $(a - ib)^{11} = 1 - 3i$ …(i)
Then, $(b + ia)^{11} = (i)^{11}\left\{\frac{b}{i} + a\right\}^{11}$
$= (i)^{11}\{-bi + a\}^{11}$
$= -i(a - ib)^{11}$
From Equation (i), we get
$= -i(1 - 3i)$
$= -i + 3i^2$

$= -i - 3$

7. If $\alpha \neq \beta, \alpha^2 = 5\alpha - 3, \beta^2 = 5\beta - 3$, then the equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is

(A) $3x^2 - 19x - 3 = 0$
(B) $3x^2 + 19x - 3 = 0$
(C) $x^2 + 19x + 3 = 0$
(D) $3x^2 - 19x - 19 = 0$
(E) $3x^2 - 19x + 3 = 0$

Solution: (E)
Given, $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$
$\alpha^2 - 5\alpha + 3 = 0$

$$\Rightarrow \quad \alpha = \frac{5 \pm \sqrt{25 - 12}}{2}$$

$$= \frac{5 \pm \sqrt{13}}{2}$$

Similarly, $\beta = \frac{5 \pm \sqrt{13}}{2}$

$\because \quad \alpha \neq \beta$

$$\therefore \ \alpha = \frac{5 + \sqrt{13}}{2}, \beta = \frac{5 - \sqrt{13}}{2}$$

Now, addition of roots

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{5 + \sqrt{13}}{5 - \sqrt{13}} + \frac{5 - \sqrt{13}}{5 + \sqrt{13}} = \frac{19}{3}$$

Multiplication of roots $\frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} = 1$

$$\therefore \quad x^2 - \left(\frac{19}{3}\right)x + 1 = 0$$

$$\Rightarrow \ 3x^2 - 19x + 3 = 0$$

8. The focus of the parabola $y^2 - 4y - x + 3 = 0$ is

(A) $\left(\frac{3}{4}, 2\right)$
(B) $\left(\frac{3}{4}, -2\right)$
(C) $\left(2, \frac{3}{4}\right)$
(D) $\left(\frac{-3}{4}, 2\right)$
(E) $\left(2, \frac{-3}{4}\right)$

Solution: (D)
$y^2 - 4y - x + 3 = 0$
$(y - 2)^2 - 4 - x + 3 = 0$
$(y - 2)^2 - x - 1 = 0$
$(y - 2)^2 = (x + 1)$

Let $Y^2 = X$ ...(i)
Here, $Y = (y-2), X = (x+1)$
Vertices $(X = 0, Y = 0) = (2, -1)$
Equation (i) comparing on $y^2 = 4ax$
$4a = 1$

$$\Rightarrow \quad a = \frac{1}{4}$$

$\therefore$ Focus $= \left(\frac{1}{4} - 1, 2\right) = \left(-\frac{3}{4}, 2\right)$

9. If $f : R \to (0, \infty)$ is an increasing function and if $\lim_{x\to 2018} \frac{f(3x)}{f(x)} = 1$, then $\lim_{x\to 2018} \frac{f(2x)}{f(x)}$ is equal to
(A) $\frac{2}{3}$
(B) $\frac{3}{2}$
(C) 2
(D) 3
(E) 1

Solution: (E)
Given $f: R \to (0, \ \infty)$ is an increasing function.
And $\lim_{x\to 2018} \frac{f(3x)}{f(x)} = 1$
So, $\lim_{x\to 2018} f(3x) = \lim_{x\to 2018} f(x)$
$\Rightarrow f(x) =$ constant.
Therefore, $\lim_{x\to 2018} \frac{f(2x)}{f(x)} = 1$

10. If f is differentiable at $x = 1$, then $\lim_{x\to 1} \frac{x^2 f(1) - f(x)}{x-1}$ is
(A) $-f'(1)$
(B) $f(1) - f'(1)$
(C) $2f(1) - f'(1)$
(D) $2f(1) + f'(1)$
(E) $f(1) + f'(1)$

Solution: (C)

$$\lim_{x\to 1} \frac{x^2 f(1) - f(x)}{x - 1}$$

By L' Hospital's Rule,

$$\lim_{x\to 1} \frac{2x\, f(1) - f'(x)}{1 - 0} = 2f(1) - f'(1)$$

11. Eccentricity of the ellipse
$4x^2 + y^2 - 8x + 4y - 8 = 0$ is
(A) $\frac{\sqrt{3}}{2}$

(B) $\frac{\sqrt{3}}{4}$
(C) $\frac{\sqrt{3}}{\sqrt{2}}$
(D) $\frac{\sqrt{3}}{8}$
(E) $\frac{\sqrt{3}}{16}$

Solution: (A)
Equation $4x^2 + y^2 - 8x + 4y - 8 = 0$ is an ellipse.
$\Rightarrow 4(x-1)^2 + (y+2)^2 - 8 - 8 = 0$
$= 4(x-1)^2 + (y+2)^2 = 16$
$= \frac{(x-1)^2}{4} + \frac{(y+2)^2}{16} = 1$, where $b > a$
∴ Eccentricity $(e) = \sqrt{1 - \frac{a^2}{b^2}}$
$$= \sqrt{1 - \frac{4}{16}} = \sqrt{\frac{12}{16}} = \frac{\sqrt{3}}{2}$$

12. The focus of the parabola $(y+1)^2 = -8(x+2)$ is
(A) $(-4, -1)$
(B) $(-1, -4)$
(C) $(1, 4)$
(D) $(4, 1)$
(E) $(-1, 4)$

Solution: (A)
Given, $(y+1)^2 = -8(x+2)$
$Y^2 = -8X$
Here, $Y = y + 1, X = (x+2)$
Vertices $(X = 0, Y = 0) = (-2, -1)$
Comparing Equation (i) from $y^2 = 4ax$
$4a = -8$
$a = -2$
Focus $= (-2 - 2, -1)$
$= (-4, -1)$

13. Which of the following is the equation of a hyperbola?
(A) $x^2 - 4x + 16y + 17 = 0$
(B) $4x^2 + 4y^2 - 16x + 4y - 60 = 0$
(C) $x^2 + 2y^2 + 4x + 2y - 27 = 0$
(D) $x^2 - y^2 + 3x - 2y - 43 = 0$
(E) $x^2 + 4x + 6y - 2 = 0$

Solution: (D)

$x^2 - y^2 + 3x - 2y - 43 = 0$

$= \left(x + \frac{3}{2}\right)^2 - (y+1)^2 - \frac{5}{4} - 43 = 0$

$= \left(x + \frac{3}{2}\right)^2 - (y+1)^2 = \frac{177}{4}$

$= \frac{\left(x + \frac{3}{2}\right)^2}{\frac{177}{4}} - \frac{(y+1)^2}{\frac{177}{4}} = 1$

It is hyperbola equation.

14. Let $f(x) = px^2 + qx + r$, where p, q, r are constants and $p \neq 0$. If $f(5) = -3f(2)$ and $f(-4) = 0$, then the other root of f is

(A) 3

(B) -7

(C) -2

(D) 2

(E) 6

Solution: (A)

$f(x) = px^2 + qx + r$

$f(-4) = 0$

$\Rightarrow 16p - 4q + r = 0$ …… (i)

One root is $x = -4$

and $f(5) = -3f(2)$

$25p + 5q + r = -3(4p + 2q + r)$

$\Rightarrow 37p + 11q + 4r = 0$ …..(ii)

Equation (ii) - Equation (i), we get

$\Rightarrow -27p + 27q = 0$

$\Rightarrow p = q$

Then, equation is $px^2 + qx + r = 0$

Roots $= -4, \alpha$

Sum of roots $= -4x + \alpha = -\frac{p}{q} = -1$

So, another root $\alpha = 3$.

15. Let $f : \rightarrow$ satisfy $f(x)\,f(y) = f(xy)$ for all real numbers x and y. If $f(2) = 4$, then $f\left(\frac{1}{2}\right) =$

(A) 0

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) 1

(E) 2

Solution: (B)

Given →

$f(x)\,f(y) = f(xy)$ ...(i)
On taking $x = 1, y = 1$
$f(1)\,f(1) = f(1\cdot 1) = f(1)^2 = f(1) = f(1) = 1$
Now, $x = 2, y = \frac{1}{2}$, then from equation (i)

$f(2)\,f\left(\frac{1}{2}\right) = f\left(2\cdot\frac{1}{2}\right)$

$\Rightarrow 4\ f\left(\frac{1}{2}\right) = f(1)$ $[\because f(2) = 4]$

$\Rightarrow f\left(\frac{1}{2}\right) = \frac{1}{4}f(1)$

On putting the value of $f(1)$,

$\Rightarrow f\left(\frac{1}{2}\right) = \frac{1}{4}\cdot 1 = \frac{1}{4}$

16. Sum of last 30 coefficents in the binomial expansion of $(1+x)^{59}$ is
(A) 2^{29}
(B) 2^{59}
(C) 2^{58}
(D) $2^{59} - 2^{29}$
(E) 2^{60}

Solution: (C)
We have, $(1+x)^{59}$
Sum of last 30 coefficient of the binomial expansion
$= {}^{59}C_{30} + {}^{59}C_{31} + \ldots + {}^{59}C_{59}$
We know that,
${}^{59}C_0 + {}^{59}C_1 + {}^{59}C_2 + \cdots + {}^{59}C_{59} = 2^{59}$
$\Rightarrow\ ({}^{59}C_0 + {}^{59}C_{59}) + ({}^{59}C_1 + {}^{59}C_{58}) + \cdots + ({}^{59}C_{29} + {}^{59}C_{30}) = 2^{59}$
$\Rightarrow\ 2({}^{59}C_{59} + {}^{59}C_{58} + \cdots + {}^{59}C_{31} + {}^{59}C_{30}) = 2^{59}$ $[\because\ {}^{n}C_r = {}^{n}C_{n-r}]$

$\Rightarrow\ {}^{59}C_{30} + {}^{59}C_{31} + \cdots\ {}^{59}C_{39} + \frac{2^{59}}{2} = 2^{58}$

$\therefore$ Sum of last 30 coefficient of the binomial expansion $(1+x)^{59}$ is 2^{58}.

17. $\left(\sqrt{3}+\sqrt{2}\right)^4 - \left(\sqrt{3}-\sqrt{2}\right)^4 =$
(A) $20\sqrt{6}$
(B) $30\sqrt{6}$
(C) $5\sqrt{10}$
(D) $40\sqrt{6}$
(E) $10\sqrt{6}$

Solution: (D)
Take,
$(a+b)^4 = {}^4C_0a^4 + {}^4C_1a^3b + {}^4C_2a^2b^2 + {}^4C_3ab^3 + {}^4C_4b^4$
$= {}^4C_0a^4 + {}^4C_1a^3b + {}^4C_2a^2b^2 + {}^4C_1ab^3 + {}^4C_0b^4$ $(\because\ {}^nC_r = {}^nC_{n-r})$

$= 1 \times a^4 + 4a^3b + \frac{4 \times 3}{2} a^2b^2 + 4ab^3 + 1 \times b^4$

$\Rightarrow \ (a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$ ..... (i)

Similarly, $(a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$ ..... (ii)

On subtracting Equation (ii) from Equation (i), we get

$(a+b)^4 - (a-b)^4 = 8a^3b + 8ab^3 = 8ab(a^2+b^2)$

Now, putting $a = \sqrt{3}$ and $b = \sqrt{2}$

$\left(\sqrt{3}+\sqrt{2}\right)^4 - \left(\sqrt{3}-\sqrt{2}\right)^4 = 8\sqrt{3}\sqrt{2}\left[\left(\sqrt{3}\right)^2 + \left(\sqrt{2}\right)^2\right]$

$= 8\sqrt{6}(3+2) = 8\sqrt{6} \times 5 = 40\sqrt{6}$

18. Three players A, B and C play a game. The probability that A, B and C will finish the game are respectively $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. The probability that the game is finished is.

(A) $\frac{1}{8}$

(B) 1

(C) $\frac{1}{4}$

(D) $\frac{3}{4}$

(E) $\frac{1}{2}$

Solution: (D)

We have, $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$ and $P(C) = \frac{1}{4}$

∴ Required probability

$= 1 - P(\bar{A})\, P(\bar{B}) P(\bar{C})$

$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$

$= 1 - \frac{1}{4} = \frac{3}{4}$

19. If $z_1 = 2 - i$ and $z_2 = 1 + i$, then $\left|\frac{z_1+z_2+1}{z_1-z_2+i}\right|$ is

(A) 2

(B) $2\sqrt{2}$

(C) 3

(D) $\sqrt{3}$

(E) 1

Solution: (B)

Given, $z_1 = 2 - i$ and $z_2 = 1 + i$

Then, $\left|\frac{z_1+z_2+1}{z_1-z_2+i}\right| = \left|\frac{2-i+1+i+1}{2-i-1-i+i}\right|$

$= \left|\frac{4}{(1-i)} \times \frac{(1+i)}{(1+i)}\right|$

$= \left|\frac{4(1+i)}{2}\right|$
$= |2+2i|$
$= \sqrt{(2)^2+(2)^2}$
$= \sqrt{8} = 2\sqrt{2}$

20. If $f(x) = \sqrt{\frac{x-\sin x}{x+\cos^2 x}}$, then $\lim_{x\to\infty} f(x)$ is equal to

(A) 1
(B) 2
(C) $\frac{1}{2}$
(D) 0
(E) ∞

Solution: (A)

Given, $f(x) = \sqrt{\frac{x-\sin x}{x+\cos^2 x}}$

Now, $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \sqrt{\frac{x-\sin x}{x+\cos^2 x}}$

$$= \lim_{x\to\infty} \sqrt{\frac{1-\frac{\sin x}{x}}{1+\frac{\cos^2 x}{x}}}$$

$$= \sqrt{\frac{1-0}{1+0}} \quad \left(\because \frac{\sin x}{x} \to 0, \frac{\cos^2 x}{x} \to 0 \text{ as } x \to \infty\right)$$

$= 1$

21. The value of $\sin\frac{31}{3}\pi$ is

(A) $\frac{\sqrt{3}}{2}$
(B) $\frac{1}{\sqrt{2}}$
(C) $\frac{-\sqrt{3}}{2}$
(D) $\frac{-1}{\sqrt{2}}$
(E) $\frac{1}{2}$

Solution: (A)

$\sin\frac{31}{3}\pi = \sin\left(10\pi + \frac{\pi}{3}\right)$

$= \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

22. The sum of odd integers from 1 to 2001 is
(A) $(1121)^2$
(B) $(1101)^2$
(C) $(1001)^2$
(D) $(1021)^2$
(E) $(1011)^2$

Solution: (C)
$1+3+5+\cdots+2001$
Sum of odd integers $= n^2 = (1001)^2$

23. If $y = \frac{\sin^2 x}{1+\cot x} + \frac{\cos^2 x}{1+\tan x}$, then $y'(x)$ is equal to
(A) $2\cos^2 x$
(B) $2\cos^3 x$
(C) $-\cos 2x$
(D) $\cos 2x$
(E) $3\cos x$

Solution: (C)
$$y = \frac{\sin^2 x}{1\ +\ \cot x} + \frac{\cos^2 x}{1\ +\ \tan x}$$
$$= \frac{\sin^3 x}{\sin x\ +\ \cos x} + \frac{\cos^3 x}{\sin x\ +\ \cos x}$$
$$= \frac{\sin^3 x\ +\ \cos^3 x}{(\sin x\ +\ \cos x)}$$
$$= \sin^2 x + \cos^2 x - \sin x \cos x$$
$$= y(x) = 1 - \frac{\sin 2x}{2}$$
$$= y'(x) = 0 - \frac{\cos 2x}{x} \cdot 2$$
$$= y'(x) = -\cos 2x$$

24. The foci of the hyperbola $16x^2 - 9y^2 - 64x + 18y - 90 = 0$ are
(A) $\left(\frac{24 \pm 5\sqrt{145}}{12}, 1\right)$
(B) $\left(\frac{21 \pm 5\sqrt{145}}{12}, 1\right)$
(C) $\left(1, \frac{24 \pm 5\sqrt{145}}{2}\right)$
(D) $\left(1, \frac{21 \pm 5\sqrt{145}}{2}\right)$
(E) $\left(\frac{21 \pm 5\sqrt{145}}{2}, -1\right)$

Solution: (A)
$16x^2 - 9y^2 - 64x + 18y - 90 = 0$
$= 16(x^2 - 4x) - 9(y^2 - 2y) = 90$

$= 16(x-2)^2 - 9(y-1)^2 = 90 + 16 \times 4 - 9 \times 1$
$= 16(x-2)^2 - 9(y-1)^2 = 145$
$= \frac{(x-2)^2}{\frac{145}{16}} - \frac{(y-1)^2}{\frac{145}{9}} = 1$ ..... (i)
We know that, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ .... (ii)
Foci ⇒ $(ae, 0)$
On comparing Equations (i) and (ii), we get
$\because \ e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{16}{9}} = \frac{5}{3}$
$X = ae \Rightarrow x - 2$
$= \ +\sqrt{\frac{145}{16}} \times \frac{5}{3}$
$= \ \pm \frac{5\sqrt{145}}{12}$
$x = 2 \pm \frac{5\sqrt{145}}{12}$
$\Rightarrow x = \frac{24 \ \pm \ 5\sqrt{145}}{12}$
$y = 0 \Rightarrow y - 1 = 0 \Rightarrow y = 1$
Hence, $\left(\frac{24 \pm 5\sqrt{145}}{12}, 1\right)$

25. If the sum of the coefficients in the expansions of $(a^2x^2 - 2ax + 1)^{51}$ is zero, then a is equal to
(A) 0
(B) 1
(C) -1
(D) -2
(E) 2

Solution: (B)
$(a^2x^2 - 2ax + 1)^{51}$
For sum of coefficients put $x = 1$
$(a^2 - 2a + 1)^{51} = 0$
$\Rightarrow \ \{(a-1)^2\}^{51} = 0$
$\Rightarrow \ a = 1$

26. The mean deviation of the data $2, 9, 9, 3, 6, 9, 4$ from the mean is
(A) 2.23
(B) 3.23
(C) 2.57
(D) 3.57
(E) 1.03

Solution: (C)
Mean of the given data is

$$\bar{x} = \frac{2 + 9 + 9 + 3 + 6 + 9 + 4}{7} = \frac{42}{7} = 6$$

The deviations of the respective observations from the mean $\bar{x}$, i.e. $x_i - \bar{x}$ are
$2 - 6, 9 - 6, 9 - 6, 3 - 6, 6 - 6, 9 - 6, 4 - 6$
$\Rightarrow -4, 3, 3, -3, 0, 3, -2$
The absolute values of the deviations, i.e. $|x_i - \bar{x}|$ are $4, 3, 3, 3, 0, 3, 2$
The required mean deviation about the mean is

$$MD(\bar{x}) = \frac{\sum_{i=1}^{7}|x_i - \bar{x}|}{7}$$

$$= \frac{4 + 3 + 3 + 3 + 0 + 3 + 2}{7}$$

$$= \frac{18}{7} = 2.57$$

27. The mean and variance of a binomial distribution are 8 and 4 respectively. What is $(X = 1)$?
(A) $\frac{1}{2^8}$
(B) $\frac{1}{2^{12}}$
(C) $\frac{1}{2^6}$
(D) $\frac{1}{2^4}$
(E) $\frac{1}{2^5}$

Solution: (B)
Let n and p be the parameters of the binomial distribution.
Mean $= 8$ and variance $= 4$
$\Rightarrow np = 8$ and $npq = 4$
$\Rightarrow q = \frac{1}{2} = p$ and $n = 16$
$\therefore$ Required probability $= P(X = 1)$

$$= {}^{16}C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{15} = 16 \times \left(\frac{1}{2}\right)^{16}$$

$$= \frac{2^4}{2^{16}} = \frac{1}{2^{12}}$$

28. The number of diagonals of a polygon with 15 sides is
(A) 90
(B) 45
(C) 60
(D) 70
(E) 10

Solution: (A)
The number of diagonals of a polygon with 15 sides is
$= {}^{n}C_2 - n = {}^{15}C_2 - 15$
$= \dfrac{15 \times 14}{2} - 15$
$= 105 - 15 = 90$

29. In a class, 40% of students study Maths and Science and 60% of students study Maths. What is the probability of a students studying Science given the student is already studying Maths?
(A) $\frac{1}{3}$
(B) $\frac{1}{6}$
(C) $\frac{2}{3}$
(D) $\frac{1}{5}$
(E) $\frac{1}{4}$

Solution: (C)
Probability of Maths and Science students $= \frac{40}{100} = \frac{2}{5}$
Probability of maths students $= \frac{60}{100} = \frac{3}{5}$
P(Science/Maths) $= \frac{P(S \cap M)}{P(M)} = \frac{\frac{2}{5}}{\frac{3}{5}} = \frac{2}{3}$

30. The eccentricity of the conic $x^2 + 2y^2 - 2x + 3y + 2 = 0$ is
(A) 0
(B) $\frac{1}{\sqrt{2}}$
(C) $\frac{1}{2}$
(D) $\sqrt{2}$
(E) 1

Solution: (B)
$x^2 + 2y^2 - 2x + 3y + 2 = 0$
$= (x-1)^2 - 1 + 2\left(y^2 + \dfrac{3}{2}y + \dfrac{9}{16} - \dfrac{9}{16}\right) + 2 = 0$
$= (x-1)^2 + 2\left(y + \dfrac{3}{4}\right)^2 - \dfrac{9}{8} + 1 = 0$
$= (x-1)^2 + 2\left(y + \dfrac{3}{4}\right)^2 = \dfrac{1}{8}$
$= \dfrac{(x-1)^2}{\frac{1}{8}} + \dfrac{\left(y + \frac{3}{4}\right)^2}{\frac{1}{16}} = 1$

Ellipse : $e = \sqrt{1-\frac{b^2}{a^2}} = \sqrt{1-\frac{\frac{1}{16}}{\frac{1}{8}}}$

$$e = \sqrt{1-\frac{8}{16}} = \sqrt{1-\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

31. If the mean of a set of observations $x_1, x_2, \ldots x_{10}$ is 20, then the mean of $x_1+4, x_2+8, x_3+12, \ldots x_{10}+40$ is

(A) 34

(B) 32

(C) 42

(D) 38

(E) 40

Solution: (C)

Mean of a set of observations

$x_1, x_2, \ldots, x_{10} = 20$

Then, according to question,

$$\frac{x_1+4+x_2+8+x_3+12+\cdots+x_{10}+40}{10}$$

$$= \frac{x_1+x_2+\cdots+x_{10}}{10} + \frac{4(1+2+\cdots+10)}{10}$$

$$= 20 + \frac{4\times 55}{10} = 20 + \frac{220}{10}$$

$$= 20+22 = 42$$

32. A letter is taken at random from the word “STATISTICS” and another letter is taken at random from the word “ASSISTANT”. The probability that they are same letters is

(A) $\frac{1}{45}$

(B) $\frac{13}{90}$

(C) $\frac{19}{90}$

(D) $\frac{5}{18}$

(E) $\frac{9}{10}$

Solution: (C) Probability of take a random from the word STATISTICS

$= {}^{10}C_1$

Probability of take a random from the word ASSISTANT

$= {}^{9}C_1$

The probability is that they are same letters T, A, I, S

$$= \frac{{}^3C_1 \times {}^3C_1 + {}^1C_1 \times {}^2C_1 + {}^2C_1 \times {}^1C_1 + {}^2C_1 \times {}^3C_1}{{}^{10}C_1 \times {}^9C_1}$$

$$= \frac{9+2+2+6}{90} = \frac{19}{90}$$

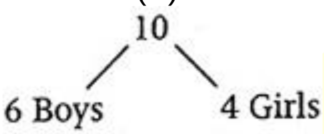
33. If $\sin\alpha$ and $\cos\alpha$ are the roots of the equation $ax^2 + bx + c = 0$, then
(A) $a^2 - b^2 + 2ac = 0$
(B) $(a - c)^2 = b^2 + c^2$
(C) $a^2 + b^2 - 2ac = 0$
(D) $a^2 + b^2 + 2ac = 0$
(E) $a + b + c = 0$

Solution: (A)
$ax^2 + bx + c = 0$
Roots are $\cos\alpha$ and $\sin\alpha$
$\therefore \quad \cos\alpha \cdot \sin\alpha = \frac{c}{a}$ .... (i)
and $\cos\alpha + \sin\alpha = -\frac{b}{a}$ ..... (ii)
$\Rightarrow (\cos\alpha + \sin\alpha)^2 = \frac{b^2}{a^2}$
Using Equation (i), we get
$\left(1 + 2\frac{c}{a}\right) = \frac{b^2}{a^2}$
$\Rightarrow \quad a^2 - b^2 + 2ac = 0$

34. If the sides of triangle are $4, 5$ and $6cm$. Then the area (in sq cm) of triangle is
(A) $\frac{\pi}{4}$
(B) $\frac{\pi}{4}\sqrt{7}$
(C) $\frac{4}{15}$
(D) $\frac{4}{15}\sqrt{7}$
(E) $\frac{15}{4}\sqrt{7}$

Solution: (E)
Given, triangle of sides $= a, b, c = 4, 5, 6\ cm$
$\therefore \quad S = \frac{a + b + c}{2}$
$= \frac{4 + 5 + 6}{2} = \frac{15}{2}$
Then, area of triangle
$= \sqrt{S(S - a)(S - b)(S - c)}$
$= \sqrt{\frac{15}{2}\left(\frac{15}{2} - 4\right)\left(\frac{15}{2} - 5\right)\left(\frac{15}{2} - 6\right)}$
$= \sqrt{\frac{15}{2} \cdot \left(\frac{7}{2}\right)\left(\frac{5}{2}\right)\left(\frac{3}{2}\right)} = \frac{15}{4}\sqrt{7}$

35. In a group of 6 boys and 4 girls, a team consisting of four children is formed such that the team has atleast one boy. The number of ways of forming a team like this is
(A) 159
(B) 209
(C) 200
(D) 240
(E) 212

Solution: (B)


The team has atleast one boy
= Total case – No anyone boy
$= {}^{10}C_4 - {}^{6}C_0$
$= \dfrac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2} - 1 = 210 - 1 = 209$

36. A password is set with 3 distinct letters from the word LOGARITHMS. How many such passwords can be formed?
(A) 90
(B) 720
(C) 80
(D) 72
(E) 120

Solution: (B)
LOGARITHMS letters are 10.
A password is set with 3 distinct letters ${}^{10}C_3 \times 3!$
$= \dfrac{10 \times 9 \times 8}{3 \times 2} \times 3 \times 2 = 720$

37. If 5^{97} is divided by 52, the remainder obtained is
(A) 3
(B) 5
(C) 4
(D) 0
(E) 1

Solution: (B) We know that, $5^4 = 625 = 3 \times 48 + 1$
$\Rightarrow 5^4 = 13\lambda + 1$, where λ is a positive integer.
$\Rightarrow (5^4)^{24} = (13\lambda + 1)^{24}$
$= {}^{24}C_0(13\lambda)^{24} + {}^{24}C_1(13\lambda)^{23} + {}^{24}C_2(13\lambda)^{22} + \cdots + {}^{24}C_{23}(13\lambda) + {}^{24}C_{24}$ (by binomial theorem)
$\Rightarrow 5^{96} = 13\left[{}^{24}C_0 13^{23}\lambda^{24} + {}^{24}C_1 13^{23}\lambda^{22} + \cdots + {}^{24}C_{23}\lambda\right] + 1$

$= (a \text{ multiple of } 13) + 1$
On multiplying both sides by 5, we get
$5^{97} = 5^{96} \cdot 5 = 5\ (\text{a multiple of } 13) + 5$
Hence, the required remainder is 5.

38. A quadratic equation $ax^2 + bx + c = 0$, with distinct coefficients is formed. It a, b, c are chosen from the numbers $2, 3, 5,$ then the probability that the equation has real roots is
(A) $\frac{1}{3}$
(B) $\frac{2}{5}$
(C) $\frac{1}{4}$
(D) $\frac{1}{5}$
(E) $\frac{2}{3}$

Solution: (A)
Total number of ways of assigning values $2, 3, 5$ to $a, b, c, = 3! = 6$
Now, for quadratic equation $ax^2 + bx + c = 0$ to have real roots $b^2 - 4ac \geq 0$. This is possible only when $a = 2, b = 5, c = 3$ or $a = 3, b = 5, c = 2$
⇒ Required probability $= \frac{2}{6} = \frac{1}{3}$

39. $\lim_{x \to \infty} \frac{3x^3 + 2x^2 - 7x + 9}{4x^3 + 9x - 2}$ is equal to
(A) $\frac{2}{9}$
(B) $\frac{1}{2}$
(C) $\frac{-9}{2}$
(D) $\frac{3}{4}$
(E) $\frac{9}{2}$

Solution: (D)

$$\lim_{x \to \infty} \frac{3x^3 + 2x^2 - 7x + 9}{4x^3 + 9x - 2}$$

$$= \lim_{x \to \infty} \frac{x^3 \left[3 + \frac{2}{x} - \frac{7}{x^2} + \frac{9}{x^3}\right]}{x^3 \left[4 + \frac{9}{x^2} - \frac{2}{x^3}\right]}$$

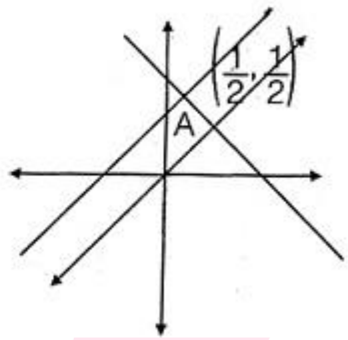
On putting $x \to \infty$, we get

$$= \frac{[3 + 0 - 0 + 0]}{[4 + 0 - 0]} = \frac{3}{4}$$

40. The minimum value of $f(x) = \max\{x, 1 + x, 2 - x\}$ is
(A) $\frac{1}{2}$

(B) $\frac{3}{2}$
(C) 1
(D) 0
(E) 2

Solution: (B)
we have,
$f(x) = \max\{x, 1 + x, 2 - x\}$
The graph of $f(x)$ is



Clearly from graph minimum value of $f(x)$ at point $A\left(\frac{1}{2}, \frac{3}{2}\right)$.
∴ Minimum value of $f(x)$ is $\frac{3}{2}$.

41. The equations of the asymptotes of the hyperbola $xy + 3y - 2y - 10 = 0$ are
(A) $x = -2, y = -3$
(B) $x = 2, y = -3$
(C) $x = 2, y = 3$
(D) $x = 4, y = 3$
(E) $x = 3, y = 4$

Solution: (B)
We have equation of hyperbola is
$xy + 3x - 2y - 10 = 0$
$xy + 3x - 2y - 6 = 4$
$(x - 2)\,(y + 3) = 4$
We know that asymptote of hyperbola
$xy = c$ is $x = 0$ and $y = 0$.
∴ Asymptote of hyperbola
$(x - 2)\,(y + 3) = 4$ is $x - 2 = 0, y + 3 = 0$
$\Rightarrow \quad x = 2, y = -3$

42. If $f(x) = x^6 + 6^x$, then $f'(x)$ is equal to
(A) $12x$
(B) $x + 4$
(C) $6x^5 + 6^x \log(6)$
(D) $6x^5 + x6^{x-1}$

(E) x^6

Solution: (C)
$f(x) = x^6 + 6^x$
$f'(x) = 6x^5 + 6^x \log(6)$ $\left[\because \frac{d}{dx}(x^n) = nx^{n-1}\right]$
$\left[\because \frac{d}{dx}(a^x) = a^x \log(a)\right]$

43. The standard deviation of the data $6, 5, 9, 13, 12, 8, 10$ is
(A) $\frac{\sqrt{52}}{7}$
(B) $\frac{52}{7}$
(C) $\frac{\sqrt{53}}{7}$
(D) $\frac{53}{7}$
(E) 6

Solution: (A)
Given data $6, 5, 9, 13, 12, 8, 10$ Mean of the given data $(\bar{x})$
$= \frac{6 + 5 + 9 + 13 + 12 + 8 + 10}{7}$
$= \frac{63}{7} = 9$
The deviation of the respective data from the mean i.e. $(x_i - \bar{x})$ are
$6-9, 5-9, 9-9, 13-9, 12-9, 8-9, 10-9$
$(x_i - \bar{x}) = -3, -4, 0, 4, 3, -1, 1$
$(x_i - \bar{x})^2 = 9, 16, 0, 16, 9, 1, 1$
$\sum_{i=i}^{7} (x_i - \bar{x})^2 = 9 + 16 + 0 + 16 + 9 + 1 + 1$
$= 52$
∴ Standard deviation (σ)
$= \sqrt{\frac{1}{n} \sum_{i=1}^{7} (x_i - \bar{x})^2} = \sqrt{\frac{52}{7}}$

44. $\lim_{x \to 0} \frac{1 - \cos mx}{1 - \cos nx} =$
(A) $\frac{m^2}{n^2}$
(B) $\frac{n^2}{m^2}$
(C) ∞
(D) $-\infty$
(E) 0

Solution: (A)

$$\lim_{x\to 0}\frac{1-\cos mx}{1-\cos nx}=\lim_{x\to 0}\left\{\frac{2\sin^2\frac{mx}{2}}{2\sin^2\frac{nx}{2}}\right\}$$

$$=\lim_{x\to 0}\left[\left\{\frac{\sin\frac{mx}{2}}{\frac{mx}{2}}\right\}\cdot\frac{m^2x^2}{4}\cdot\frac{1}{\left\{\frac{\sin\frac{nx}{2}}{\frac{nx}{2}}\right\}^2}\cdot\frac{4}{n^2x^2}\right]$$

$$=\frac{m^2}{n^2}\times 1=\frac{m^2}{n^2}$$

45. $\lim_{x\to 0}\frac{(\sqrt{1+2x})-1}{x}=$

(A) 0

(B) -1

(C) $\frac{1}{2}$

(D) 1

(E) $\frac{-1}{2}$

Solution: (D)

$$\lim_{x\to 0}\frac{\sqrt{1+2x}-1}{x}$$

Using $L'\text{Hospital}'s$ Rule,

$$\lim_{x\to 0}\frac{\frac{1}{2\sqrt{1+2x}}\cdot 2-0}{1}$$

$$\Rightarrow\quad \lim_{x\to 0}\frac{1}{\sqrt{1+2x}}$$

Using limit, we get

$$=\frac{1}{\sqrt{1+2(0)}}=1$$

46. Let f and g be differentiable functions such that $f(3)=5, g(3)=7, f'(3)=13, g'(3)=6, f'(7)=2$ and $g'(7)=0$. If $h(x)=(fog)(x)$, then $h'(3)=$

(A) 14

(B) 12

(C) 16

(D) 0

(E) 10

Solution: (B)

$h(x)=f\big(g(x)\big)$

$h'(x) = f'(g(x)) \cdot g'(x)$

$h'(3) = f'(g(3)) \cdot g'(3) \quad \begin{bmatrix} \because g(3) = 7 \\ g'(3) = 6 \end{bmatrix}$

$= f'(7) \cdot 6$

$= 2 \times 6 = 12$

47. $\frac{\sqrt{3}}{\sin(20^o)} - \frac{1}{\cos(20^o)} =$

(A) 1

(B) $\frac{1}{\sqrt{2}}$

(C) 2

(D) 4

(E) 0

Solution: (D)

$$\frac{\sqrt{3}}{\sin(20^o)} - \frac{1}{\cos(20^o)}$$

$$= \frac{\sqrt{3}\cos(20^o) - \sin(20^o)}{\sin(20^o)\cos(20^o)}$$

$$= \frac{4\left[\frac{\sqrt{3}}{2}\cos(20^o) - \frac{\sin(20)^o}{2}\right]}{2\sin(20^o)\cos(20^o)}$$

$[\because 2\sin A \cos A = \sin 2A]$

$$= \frac{4(\sin 60^o \cos 20^o - \cos 60^o \sin 20^o)}{\sin 40^o}$$

$$= \frac{4\sin(60^o - 20^o)}{\sin 40^o}$$

$[\because \sin(A - B) = \sin A \cos B - \cos A \sin B]$

$$= \frac{4\sin 40^o}{\sin 40^o} = 4$$

48. A poison variate X satisfies

$P(X - 1) = P(X = 2). P(X = 6)$ is equal to

(A) $\frac{4}{45}e^{-2}$

(B) $\frac{1}{45}e^{-1}$

(C) $\frac{1}{9}e^{-2}$

(D) $\frac{1}{4}e^{-2}$

(E) $\frac{1}{45}e^{-2}$

Solution: (A)

Given that,

$P(X = 1) = P(X = 2)$

$= \frac{e^{-\lambda}\lambda^1}{1!} = \frac{e^{-\lambda}\lambda^2}{2!}$
$\Rightarrow \ \lambda = 2$
$\therefore \ \ P(X = 6) = \frac{e^{-2}(2)^6}{6!}$
$= \frac{e^{-2} \times 4 \times 2^4}{6 \times 5 \times 4 \times 3 \times 2}$

$= \frac{4 \times e^{-2} \times 2^4}{45 \times 2^4} = \frac{4e^{-2}}{45}$

49. Let a and b be 2 consecutive integers selected from the first 20 natural numbers. The probability that $\sqrt{a^2 + b^2 + a^2b^2}$ is an odd positive integer is
(A) $\frac{9}{19}$
(B) $\frac{10}{19}$
(C) $\frac{13}{19}$
(D) 1
(E) 0

Solution: (D)
a and b are two consecutive number.
Let $a = n, b = n + 1$
Now, $\sqrt{a^2 + b^2 + a^2b^2}$
$= \sqrt{n^2 + (n + 1)^2 + n^2(n + 1)^2}$
$= \sqrt{n^2 + n^2 + 2n + 1 + n^2(n^2 + 2n + 1)}$
$= \sqrt{n^2 + n^2 + 2n + 1 + n^4 + 2n^3 + n^2}$
$= \sqrt{n^4 + n^2 + 1 + 2n^3 + 2n^2 + 2n + 1}$
$= \sqrt{(n^2 + n + 1)^2}$
$= n^2 + n + 1 = n(n + 1) + 1$
It is always odd.
$\because$ Probability of $\sqrt{a^2 + b^2 + a^2b^2}$ is an odd integer is 1

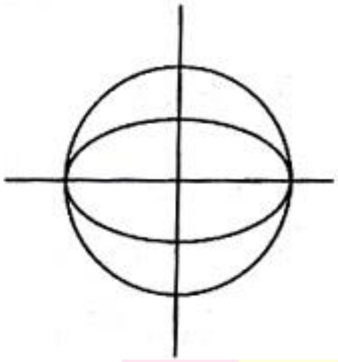
50. An ellipse of eccentricity $\frac{2\sqrt{2}}{3}$ is inscribed in a circle. A point is chosen inside the circle at random. The probability that the point lies outside the ellipse is
(A) $\frac{1}{3}$
(B) $\frac{2}{3}$
(C) $\frac{1}{9}$
(D) $\frac{2}{9}$
(E) $\frac{1}{27}$

Solution: (B)

Given, $e = \frac{2\sqrt{2}}{3}$

$e^2 = 1 - \frac{b^2}{a^2}$

$\Rightarrow \frac{b^2}{a^2} = 1 - \frac{8}{9}$ ....(i)



$P(x) = \frac{\pi a^2 - \pi ab}{\pi a^2}$

$= 1 - \frac{b}{a}$

$= 1 - \frac{1}{3}$ [using Equation (i)]

$P(x) = \frac{2}{3}$

51. If the vectors $4\hat{i} + 11\hat{j} + m\hat{k}, 7\hat{i} + 2\hat{j} + 6\hat{k}$ and $\hat{i} + 5\hat{j} + 4\hat{k}$ are coplanar, then m is equal to

(A) 38

(B) 0

(C) 10

(D) -10

(E) 25

Solution: (C)

Given vectors $4\hat{i} + 11\hat{j} + m\hat{k}, 7\hat{i} + 2\hat{j} + 6\hat{k}$ and $\hat{i} + 5\hat{j} + 4\hat{k}$ are coplanar.

Then, $\begin{vmatrix} 1 & 5 & 4 \\ 4 & 11 & m \\ 7 & 2 & 6 \end{vmatrix}$

$\Rightarrow \; 1(66 - 2m) - 5(24 - 7m) + 4(8 - 77)$

$= 66 - 2m - 120 + 35m + 32 - 308$

$= 33m - 330 = 0$

$\Rightarrow \; m = 10$

52. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} + 3\hat{j} + 5\hat{k}$ and $\vec{c} = 7\hat{i} + 9\hat{j} + 11\hat{k}$. Then, the area of the parallelogram with diagonals $\vec{a} + \vec{b}$ and $\vec{b} + \vec{c}$ is

(A) $4\sqrt{6}$

(B) $\frac{1}{2}\sqrt{21}$
(C) $\frac{\sqrt{6}}{2}$
(D) $\sqrt{6}$
(E) $\frac{1}{\sqrt{6}}$

Solution: (A)
Given, $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$
$\vec{b} = \hat{\imath} + 3\hat{\jmath} + 5\hat{k}$
$\vec{c} = 7\hat{\imath} + 9\hat{\jmath} + 11\hat{k}$
Diagonals: $D_1 = \vec{a} + \vec{b}$ and $D_2 = \vec{b} + \vec{c}$
Area of parallelogram $= \frac{1}{2}[D_1 \times D_2]$ … (i)
$D_1 = \vec{a} + \vec{b} = 2\hat{\imath} + 4\hat{\jmath} + 6\hat{k}$
$D_2 = \vec{b} + \vec{c} = 8\hat{\imath} + 12\hat{\jmath} + 16\hat{k}$
From Equation (i),

$$|\mathrm{Area}| = \frac{1}{2}\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 4 & 6 \\ 8 & 12 & 16 \end{vmatrix}$$

$$= \frac{1}{2}\left[\hat{\imath}(64 - 72) + \hat{\jmath}(48 - 32) + \hat{k}(24 - 32)\right]$$

$$= \frac{1}{2}\left|-8\hat{\imath} + 16\hat{\jmath} - 8\hat{k}\right|$$

$$= \frac{1}{2}\sqrt{64 + 256 + 64} = \frac{1}{2} \cdot 8\sqrt{6} = 4\sqrt{6}$$

53. If $|\vec{a}| = 3, |\vec{b}| = 1, |\vec{c}| = 4$ and $\vec{a} + \vec{b} + \vec{c} = 0$, then the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ is equal to
(A) 13
(B) 26
(C) -29
(D) -13
(E) -26

Solution: (D)
$|\vec{a}| = 3, |\vec{b}| = 1, |\vec{c}| = 4,$
$\vec{a} + \vec{b} + \vec{c} = 0$

$$\left(\vec{a} + \vec{b} + \vec{c}\right)^2 = |a|^2 + |b|^2 + |c|^2 + 2\left(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}\right)$$

$$\Rightarrow \quad 0 = (3)^2 + (1)^2 + (4)^2 + 2\left(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}\right)$$

$$\Rightarrow \quad \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -\frac{26}{2} = -13$$

54. If $\left|\vec{a} - \vec{b}\right| = |\vec{a}| = \left|\vec{b}\right| = 1$, then the angle between $\vec{a}$ and $\vec{b}$ is equal to

(A) $\frac{\pi}{3}$
(B) $\frac{3\pi}{4}$
(C) $\frac{\pi}{2}$
(D) 0
(E) π

Solution: (A)
$|\vec{a}-\vec{b}| = |\vec{a}| = |\vec{b}| = 1$
$|\vec{a}-\vec{b}|^2 = a^2 + b^2 - 2\vec{a}\cdot\vec{b}$
$1 = 1 + 1 - 2|\vec{a}||\vec{b}|\cos\theta$
$\cos\theta = \frac{1}{2} \Rightarrow \cos\frac{\pi}{3}$
$\theta = \frac{\pi}{3}$

55. If the vectors $\vec{a} = \hat{\imath} - \hat{\jmath} + 2\hat{k}, \vec{b} = 2\hat{\imath} + 4\hat{\jmath} + \hat{k}$ and $\vec{c} = \lambda\hat{\imath} + 9\hat{\jmath} + \mu\hat{k}$ are mutually orthogonal, then $\lambda + \mu$ is equal to
(A) 5
(B) -9
(C)-1
(D) 0
(E) -5

Solution: (B)
Given,
$\vec{a} = \hat{\imath} - \hat{\jmath} + 2\hat{k}, \vec{b} = 2\hat{\imath} + 4\hat{\jmath} + \hat{k}$
$\vec{c} = \lambda\hat{\imath} + 9\hat{\jmath} + \mu\hat{k}$
Vectors are mutually orthogonal, so
$\vec{a}\cdot\vec{b} = 0 = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a}$
$\vec{a}\cdot\vec{b} = 2 - 4 + 2$
$= 2\lambda + 36 + \mu$
$\Rightarrow \quad 2\lambda + 36 + \mu = 0$ .....(i)
$\lambda - 9 + 2\mu = 0$ .....(ii)
On solving Equations (i) and (ii), we get
$\mu = 18, \lambda = -27$
$\therefore \quad \lambda + \mu = 18 - 27 = -9$

56. The solution of $x^{\frac{2}{5}} + 3x^{\frac{1}{5}} - 4 = 0$ are
(A) $1, 1024$
(B) $-1,1024$
(C) $1,1031$
(D) $-1024,1$
(E) $-1,1031$

Solution: (D)

We have, $x^{\frac{2}{5}} + 3x^{\frac{1}{5}} - 4 = 0$

Let $x^{\frac{1}{5}} = y$

$y^2 + 3y - 4 = 0$

$\Rightarrow \ y^2 + 4y - y - 4 = 0$

$\Rightarrow \ y(y+4) - 1(y+4) = 0$

$\Rightarrow \ (y+4)\,(y-1) = 0$

$\Rightarrow \ y = -4, 1$

$\therefore \ x^{\frac{1}{5}} = -4$ or $x^{\frac{1}{5}} = 1$

$\Rightarrow \ x = (-4)^5$ or $x = 1$

$\Rightarrow \ x = -1024$ or $x = 1$

57. If the equations $x^2 + ax + 1 = 0$ and $x^2 - x - a = 0$ have a real common root b, then the value of b is equal to

(A) 0

(B) 1

(C) -1

(D) 2

(E) 3

Solution: (C)

Given equations, $x^2 + ax + 1 = 0$

$x^2 - x - a = 0$

$\because b$ is common root, so b satisfied both equations.

$b^2 + ab + 1 = b^2 - b - a$

$= ab + b = -a - 1$

$\Rightarrow \ b(a+1) = -(a+1)$

$\Rightarrow b = -1$

58. If $\sin\theta - \cos\theta = 1$, then the value of $\sin^3\theta - \cos^3\theta$ is equal to

(A) 1

(B) -1

(C) 0

(D) 2

(E) -2

Solution: (A)

Given, $\sin\theta - \cos\theta = 1$

$\sin^3\theta - \cos^3\theta = (\sin\theta - \cos\theta)\ (\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)$

$= 1\ (1 + \sin\theta\cos\theta)$

$= 1 + \sin\theta\cos\theta$ .....(i)

$[\because \sin\theta - \cos\theta = 1]$

On squaring both sides,

$(\sin\theta - \cos\theta)^2 = (1)^2$

$\Rightarrow \quad (\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta) = 1$
$\Rightarrow \quad 1 - 2\sin\theta\cos\theta = 1$
$\Rightarrow \quad \sin\theta\cos\theta = 0 \quad \ldots\ldots$ (ii)
From Equations (ii) and (i), we get $\sin^3\theta - \cos^3\theta = 1$

59. Two dice of different colours are thrown at a time. The probability that the sum is either 7 or 11 is
(A) $\frac{7}{36}$
(B) $\frac{2}{9}$
(C) $\frac{2}{3}$
(D) $\frac{5}{9}$
(E) $\frac{6}{7}$

Solution: (B)
Probability of sum of 7
$= (6,1), (5,2), (4,3), (3,4), (2,5), (1,6)$ and probability of sum of 11
$= (6,5), (5,6)$

$$P(x) = \frac{6}{36} + \frac{2}{36}$$
$$= \frac{8}{36} = \frac{2}{9}$$

60. $\frac{1}{9!} + \frac{1}{3!7!} + \frac{1}{5!5!} + \frac{1}{7!3!} + \frac{1}{9!}$ Is equal to
(A) $\frac{2^9}{10!}$
(B) $\frac{2^{10}}{8!}$
(C) $\frac{2^{11}}{9!}$
(D) $\frac{2^{10}}{7!}$
(E) $\frac{2^8}{9!}$

Solution: (A)

$$\frac{1}{9!} + \frac{1}{3!\,7!} + \frac{1}{5!\,7!} + \frac{1}{7!\,3!} + \frac{1}{9!}$$
$$= \frac{1}{10!}\left[\frac{10!}{9!\,1!} + \frac{10!}{3!\,7!} + \frac{10!}{5!\,7!} + \frac{10!}{7!\,3!} + \frac{10!}{9!\,1!}\right]$$
$$= \frac{1}{10!}\left[{}^{10}C_1 + {}^{10}C_3 + {}^{10}C_5 + {}^{10}C_7 + {}^{10}C_9\right]$$
$$= \frac{1}{10!} \cdot 2^9 = \frac{2^9}{10!}$$

61. The order and degree of the differential equation $(y''')^2 + (y'')^3 - (y')^4 + y^5 = 0$ is

(A) 3 and 2
(B) 1 and 2
(C) 2 and 3
(D) 1 and 4
(E) 3 and 5

Solution: (A)
The given differential equation is $(y''')^2 + (y'')^3 - (y')^4 + y^5 = 0$
Clearly, its order is 3 and degree is 2. Hence, option 3 and 2 is correct.

62. $\int_{-2}^{2} |x| dx$ is equal to
(A) 0
(B) 1
(C) 2
(D) 4
(E) $\frac{1}{2}$

Solution: (D) Given that,

$$I = \int_{-2}^{2} |x| dx$$

$$= -\int_{-2}^{0} x\, dx + \int_{0}^{2} dx$$

$$= -\left[\frac{x^2}{2}\right]_{-2}^{0} + \left[\frac{x^2}{2}\right]_{0}^{2}$$

$$= -(-2) + (2) = 4$$

63. $\int_{-1}^{0} \frac{dx}{x^2+2x+2}$ is equal to
(A) 0
(B) $\frac{\pi}{4}$
(C) $\frac{-\pi}{4}$
(D) $\frac{\pi}{2}$
(E) $\frac{-\pi}{2}$

Solution: (B)
Given that,

$$I = \int_{-1}^{0} \frac{dx}{x^2 + 2x + 2}$$

$$= \int_{1}^{0} \frac{dx}{(x+1)^2 + 1} = [\tan^{-1}(x+1)]_{-1}^{0}$$

$$= [\tan^{-1}(1) - \tan^{-1}(0)] = \frac{\pi}{4}$$

64. If $\int_{-1}^{4} f(x)dx = 4$ and $\int_{2}^{4}\left(3 - f(x)\right)dx = 7$, then $\int_{-1}^{2} f(x)dx$ is
(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

Solution: (E)
We know, $\int_{2}^{4}[3 - f(x)]dx = 7$
$\Rightarrow \int_{2}^{4} 3dx - \int_{2}^{4} f(x)dx = 7$
$\Rightarrow (3x)_{2}{}^{4} - \int_{2}^{4} f(x)dx = 7$
$\Rightarrow (12 - 6) - \int_{2}^{4} f(x)dx = 7$
$\Rightarrow 6 - \int_{2}^{4} f(x)dx = 7$
$\Rightarrow \int_{2}^{4} f(x)dx = -1$ ……(i)
Now, $\int_{-1}^{4} f(x)dx = 4$
$\Rightarrow \int_{-1}^{2} f(x)dx + \int_{2}^{4} f(x)dx = 4$
$\Rightarrow \int_{-1}^{2} f(x)dx - 1 = 4$ [from Equation (i)]
$\Rightarrow \int_{-1}^{2} f(x)dx = 5$

65. $\int \frac{xe^x}{(1+x)^2}dx =$
(A) $\frac{e^x}{1+x} + C$
(B) $\frac{e^x}{1+e^x} + C$
(C) $\frac{e^{2x}}{1+e^x} + C$
(D) $\frac{e^{-x}}{1+x} + C$
(E) $\frac{e^{-2x}}{1+x} + C$

Solution: (A)
Given that,
$$I = \int \frac{xe^x}{(1 + x)^2}dx = \int \frac{(x + 1 - 1)}{(1 + x)^2}e^x\, dx$$
$$= \int e^x\left(\frac{1}{1 + x} - \frac{1}{(1 + x)^2}\right)dx$$

$= \dfrac{e^x}{1 + x} + C$

66. The remainder when 2^{2000} is divided by 17 is
(A) 1
(B) 2
(C) 8
(D) 12
(E) 4

Solution: (A)
$2^{2000} = (2^4)^{500}$
$= (16)^{500} = (17 - 1)^{500}$
When divided by 17, then remainder $= (-1)^{500} = 1$
Hence, remainder $= 1$

67. The coefficient of x^5 in the expansion of $(x + 3)^8$ is
(A) 1542
(B) 1512
(C) 2512
(D) 12
(E) 4

Solution: (B)
$T_{r+1} = {}^8C_r x^{8-r} 3^r$
For the coefficient of x^5,
$8 - r = 5 \Rightarrow r = 3$
$\therefore$ Coefficient of $x^5 = {}^8C_3 \cdot 3^3$
$= \dfrac{8!}{3!\,5!} \times 3^3 = \dfrac{8 \times 7 \times 6}{6} \times 3^3$
$= 8 \times 7 \times 3^3 = 56 \times 27$
$= 1512$

68. The maximum value of 5
$\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$ is
(A) 5
(B) 11
(C) 10
(D) -1
(E) 2

Solution: (C)
$5\cos\theta + 3\cos\left(\theta + \dfrac{\pi}{3}\right) + 3$
$= 5\cos\theta + 3[\cos\theta\cos 60^o - \sin\theta\sin 60^o] + 3$

$= 5\cos\theta + 3\left[\frac{\cos\theta}{2} - \frac{\sqrt{3}}{2}\sin\theta\right] + 3$

$= \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$

Let $\frac{13}{2} = a$ and $\frac{3\sqrt{3}}{2} = b$

Then expression becomes,

$a\cos\theta - b\sin\theta + 3$

Maximum value of this type of expression is equal to $[a^2 + b^2]^{\frac{1}{2}} + 3 =$ Maximum value

After putting values of a and b, we get $[49]^{\frac{1}{2}} + 3 =$ Max value

$10 =$ Max value

69. The area of the triangle in the complex plane formed by z, iz and $z + iz$ is

(A) $|z|$

(B) $|\bar{z}|^2$

(C) $\frac{1}{2}|z|^2$

(D) $\frac{1}{2}|z + iz|^2$

(E) $|z + iz|$

Solution: (C)

Let $z = x + iy; z + iz = (x - y) + i(x + y)$ and $iz = -y + ix$. If A is the area of triangle formed by $z, z + iz$ and iz, then

$$A = \frac{1}{2}\begin{vmatrix} x & y & 1 \\ x - y & x + y & 1 \\ -y & x & 1 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1 - R_3$

$$A = \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 0 & 0 & -1 \\ -y & x & 0 \end{vmatrix}$$

$= \frac{1}{2}(x^2 + y^2) = \frac{1}{2}|z|^2$

70. Let $f : f(-x) \to f(x)$ be a differentiable function. If f is even, then $f'(0)$ is equal to

(A) 1

(B) 2

(C) 0

(D) -1

(E) $\frac{1}{2}$

Solution: (C)

$\because f(-x) = f(x)$

$-f'(-x) = f'(x) \Rightarrow -f'(0) = f'(0)$

$\Rightarrow \; 2f'(0) = 0 \Rightarrow f'(0) = 0$

71. The coordinate of the point dividing internally the line joining the points $(4,-2)$ and $(8,6)$ in the ratio $7:5$ is

(A) $(16,18)$

(B) $(18,16)$

(C) $\left(\frac{19}{3},\frac{8}{3}\right)$

(D) $\left(\frac{8}{3},\frac{19}{3}\right)$

(E) $(7,3)$

Solution: (C)

Here, $x_1 = 4, y_1 = -2, x_2 = 8, y_2 = 6$ and $m : n = 7:5$

$$\therefore \quad x = \frac{mx_2 + nx_1}{m + n} = \frac{7 \times 8 + 5 \times 4}{12}$$

$$= \frac{56 + 20}{12} = \frac{76}{12} = \frac{19}{3}$$

and $y = \frac{my_2+ny_1}{m+n}$

$$= \frac{7 \times 6 + 5 \times (-2)}{7 + 5}$$

$$= \frac{42 - 10}{12} = \frac{32}{12} = \frac{8}{3}$$

$$\therefore \quad (x,y) = \left(\frac{19}{3},\frac{8}{3}\right)$$

72. The area of the triangle formed by the points $(a,b+c),(b,c+a),(c,a+b)$ is

(A) abc

(B) $a^2+b^2+c^2$

(C) $ab+bc+ca$

(D) 0

(E) $a(ab+bc+ca)$

Solution: (D)

Area of triangle $= \frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$

$$= \frac{1}{2}\begin{vmatrix} a & a+b+c & 1 \\ b & a+b+c & 1 \\ c & a+b+c & 1 \end{vmatrix}$$

[Applying $c_2 \to c_2 + c_1$]

$$= \frac{a + b + c}{2}\begin{vmatrix} a & 1 & 1 \\ b & 1 & 1 \\ c & 1 & 1 \end{vmatrix} = 0$$

73. If (x,y) is equidistant from $(a+b,b-a)$ and $(a-b,a+b)$, then

(A) $ax+by=0$

(B) $ax-by=0$

(C) $bx+ay=0$

(D) $bx - ay = 0$
(E) $x = y$

Solution: (D)
According to question,
$\{x-(a+b)\}^2 + \{y-(b-a)\}^2$
$= \{x-(a-b)\}^2 + \{y-(a+b)\}^2$
$\Rightarrow x^2 + (a+b)^2 - 2x(a+b) + y^2 + (b-a)^2 - 2y(b-a)$
$= x^2 + (a+b)^2 - 2x(a-b) + y^2 + (a+b)^2 - 2y(a+b)$
By solving, we get
$\Rightarrow \quad bx - ay = 0$

74. The equation of the line passing through (a, b) and parallel to the line $\frac{x}{a} + \frac{y}{b} = 1$ is
(A) $\frac{x}{a} + \frac{y}{b} = 3$
(B) $\frac{x}{a} + \frac{y}{b} = 2$
(C) $\frac{x}{a} + \frac{y}{b} = 0$
(D) $\frac{x}{a} + \frac{y}{b} + 2 = 0$
(E) $\frac{x}{a} + \frac{y}{b} = 4$

Solution: (B)
Given equation of line is
$\frac{x}{a} + \frac{y}{b} = 1 \qquad \ldots\ldots$(i)
$\Rightarrow \; bx + ay = ab$
$\Rightarrow \; bx + ay - ab = 0$
$\therefore \; m = -\dfrac{b}{a}$
So, equation of line passing through (a, b) and parallel to Equation (i) is
$y - b = -\dfrac{b}{a}(x - a)$
$ay - ab = -bx + ab$
$ay + bx = 2ab$
$\dfrac{y}{b} + \dfrac{x}{a} = 2$
$\Rightarrow \quad \dfrac{x}{a} + \dfrac{y}{b} = 2$

75. If the points $(2a, a), (a, 2a)$ and (a, a) enclose a triangle of area $18\ sq$ units, then the centroid of the triangle is equal to
(A) $(4, 4)$
(B) $(8, 8)$
(C) $(-4, -4)$
(D) $(4\sqrt{2}, 4\sqrt{2})$
(E) $(6, 6)$

Solution: (B)
Given, that Area of triangle $= 18$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 2a & a & 1 \\ a & 2a & 1 \\ a & a & 1 \end{vmatrix} = \pm 18$$

$$\Rightarrow \begin{vmatrix} 2a & a & 1 \\ a & 2a & 1 \\ a & a & 1 \end{vmatrix} = \pm 36$$

$\Rightarrow 2a(2a - a) - a(a - a) + 1(a^2 - 2a^2) = \pm 36$
$\Rightarrow 2a^2 - a^2 = \pm 36$
$\Rightarrow a^2 = \pm 36$
$\Rightarrow a^2 = 36$
$\Rightarrow a = \pm 6$
Now, centroid of the given triangle will be

$$= \left(\frac{2a + a + a}{3}, \frac{a + 2a + a}{3}\right) = \left(\frac{4a}{3}, \frac{4a}{3}\right)$$

When $a = 6$, centroid $= \left(\frac{4 \times 6}{3}, \frac{4 \times 6}{3}\right) = (8, 8)$

76. The area of a triangle is $5\, sq$ units. Two of its vertices are $(2, 1)$ and $(3, -2)$. The third vertex lies on $y = x + 3$. The coordinates of the third vertex can be
(A) $\left(\frac{-3}{2}, \frac{-3}{2}\right)$
(B) $\left(\frac{3}{4}, \frac{-3}{2}\right)$
(C) $\left(\frac{7}{2}, \frac{13}{2}\right)$
(D) $\left(\frac{-1}{4}, \frac{1}{2}\right)$
(E) $\left(\frac{3}{2}, \frac{3}{2}\right)$

Solution: (C)
Let the coordinates of third vertex be (x, y). Given that, area of a triangle $= 5$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 2 & 1 & 1 \\ 3 & -2 & 1 \\ x & y & 1 \end{vmatrix} = 5$$

$\Rightarrow 2(-2 - y) - 1(3 - x) + 1(3y + 2x) = 10$
$\Rightarrow -4 - 2y - 3 + x + 3y + 2x = 10$
$\Rightarrow 3x + y = 17$ .....(i)
Since, third vertex lies as $y = x + 3$ .....(ii)
By solving Equations (i) and (ii), we get

$$x = \frac{7}{2}, y = \frac{13}{2}$$

77. If $x^2 + y^2 + 2gx + 2fy + 1 = 0$ represents a pair of straight lines, then $f^2 + g^2$ is equal to
(A) 0
(B) 1

(C) 2
(D) 4
(E) 3

Solution: (B)
Given equation of pair of straight lines is $x^2 + y^2 + 2gx + 2fy + 1 = 0$
Since, the necessary and sufficient condition for pair of straight lines is

$$\begin{vmatrix} a & h & g \\ h & b & f \\ h & f & c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & g \\ 0 & 1 & f \\ g & f & 1 \end{vmatrix} = 0$$

$\Rightarrow\ 1(1 - f^2) + g(0 - g) = 0$
$\Rightarrow\ 1 - f^2 - g^2 = 0$
$\Rightarrow\ f^2 + g^2 = 1$

78. If θ is the angle between the pair of straight lines $x^2 - 5xy + 4y^2 + 3x - 4 = 0$, then $\tan^2\theta$ is equal to
(A) $\frac{9}{16}$
(B) $\frac{16}{25}$
(C) $\frac{9}{25}$
(D) $\frac{21}{25}$
(E) $\frac{25}{9}$

Solution: (C)
Given equation of straight line is $x^2 - 5xy + 4y^2 + 3x - 4 = 0$

$$\therefore\ \tan\theta = \left|\frac{2\sqrt{\left(-\frac{5}{2}\right)^2 - 4}}{5}\right|$$

$$= \left|\frac{2\sqrt{\frac{25}{4} - 4}}{5}\right| = \frac{2}{5} \times \sqrt{\frac{9}{4}} = \frac{2}{5} \times \frac{3}{2} = \frac{3}{5}$$

$$= \tan^2\theta = \frac{9}{25}$$

79. If $3\hat{\imath} + 2\hat{\jmath} - 5\hat{k} = x(2\hat{\imath} - \hat{\jmath} + \hat{k}) + y(\hat{\imath} + 3\hat{\jmath} - 2\hat{k}) + z(-2\hat{\imath} + \hat{\jmath} - 3\hat{k})$, then
(A) $x = 1, y = 2, z = 3$
(B) $x = 2, y = 3, z = 1$
(C) $x = 3, y = 1, z = 2$
(D) $x = 1, y = 3, z = 2$

(E) $x = 2, y = 2, z = 3$

Solution: (C)
Given that,
$3\hat{\imath} + 2\hat{\jmath} - 5\hat{k} = x(2\hat{\imath} - \hat{\jmath} + \hat{k}) + y(\hat{\imath} + 3\hat{\jmath} - 2\hat{k}) + 2(-2\hat{\imath} + \hat{\jmath} - 3\hat{k})$
$\Rightarrow \; 3\hat{\imath} + 2\hat{\jmath} - 5\hat{k} = i(2x + y - 2z) + \hat{\jmath}(-x + 3y + z) + \hat{k}(x - 2y - 3z)$
By equating the coefficients of $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$, we get
$\Rightarrow \; 2x + y - 2z = 3$ .....(i)
$-x + 3y + z = 2$ .....(ii)
$x - 2y - 3z = -5$ ... (iii)
By solving Equations (i), (ii) and (iii), we get
$x = 3, y = 1, z = 2$

80. $\sin 15^o =$
(A) $\frac{\sqrt{3}-1}{2\sqrt{2}}$
(B) $\frac{\sqrt{3}+1}{2\sqrt{2}}$
(C) $\frac{1-\sqrt{3}}{2\sqrt{2}}$
(D) $\frac{1+\sqrt{3}}{\sqrt{2}}$
(E) $\frac{-(1+\sqrt{3})}{2\sqrt{2}}$

Solution: (A)
$\because \sin 15^o = \sin(45^o - 30^o)$
$= \sin 45^o \cos 30^o - \sin 30^o \cos 45^o$
$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$
$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$
$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

81. If $\bar{a}$ and $\bar{b} = 3\hat{\imath} + 6\hat{\jmath} + 6\hat{k}$ are collinear and $\bar{a} \cdot \bar{b} = 27$, then $\bar{a}$ is equal to
(A) $3(\hat{\imath} + \hat{\jmath} + \hat{k})$
(B) $\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$
(C) $2\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$
(D) $\hat{\imath} + 3\hat{\jmath} + 3\hat{k}$
(E) $\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$

Solution: (B)
Since, $\vec{a}$ and $\vec{b}$ are collinear vector. Therefore,
$\vec{a} = \lambda\vec{b}$ .....(i)

$\because \ \vec{a} \cdot \vec{b} = 27$

$\Rightarrow \ |\vec{a}||\vec{b}| \cos 0^o = 27$

$\Rightarrow \ |\vec{b}| \cdot \sqrt{9 + 36 + 36} = 27$

$\Rightarrow \ |\vec{a}| = \frac{27}{9} = 3$

By Equation (i),

$\vec{a} = \lambda \vec{b}$

$\Rightarrow \ |\vec{a}| = |\lambda||\vec{b}|$

$3 = |\lambda| \cdot 9$

$\Rightarrow \ |\lambda| = \pm \frac{1}{3}$

$\therefore \ \vec{a} = \pm \frac{1}{3}(3\hat{\imath} + 6\hat{\jmath} + 6\hat{k})$

$\vec{a} = \pm(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$

82. If $|\vec{a}| = 13, |\vec{b}| = 5$ and $\vec{a} \cdot \vec{b} = 30$, then $|\vec{a} \times \vec{b}|$ is equal to

(A) 30

(B) $\frac{30}{25}\sqrt{233}$

(C) $\frac{30}{33}\sqrt{193}$

(D) $\frac{65}{23}\sqrt{493}$

(E) $\frac{65}{13}\sqrt{133}$

Solution: (E)

Given that,

$|\vec{a}| = 13, |\vec{b}| = 5$ and $\vec{a} \cdot \vec{b} = 30$

$\because \ \vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

$\Rightarrow \ 30 = 13.5 \cos\theta$

$\Rightarrow \ \cos\theta = \frac{30}{13.5} = \frac{6}{13}$

$\Rightarrow \ \sin^2\theta = 1 - \frac{36}{169}$

$\Rightarrow \ \sin^2\theta = \frac{133}{169}$

$\Rightarrow \ \sin\theta = \frac{\sqrt{133}}{13}$

$\therefore \ |\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$

$= 13.5 \cdot \frac{\sqrt{133}}{13}$

$= \frac{65}{13}\sqrt{133}$

83. If ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$, then r is equal to

(A) 69
(B) 41
(C) 51
(D) 61
(E) 49

Solution: (B)
Given that, ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$

$$\frac{56!}{(50-r)!} \times \frac{(51-r)!}{54!} = \frac{30800}{1}$$

$\Rightarrow \quad 56 \times 55 \times (51-r) = 30800$
$\Rightarrow \quad r = 41$

84. Distance between two parallel lines $y = 2x + 4$ and $y = 2x - 1$ is
(A) 5
(B) $5\sqrt{5}$
(C) $\sqrt{5}$
(D) $\frac{1}{5}$
(E) $\frac{3}{\sqrt{5}}$

Solution: (C)
Distance between parallel lines $y = 2x + 4$
or $2x - y + 4 = 0$ and $y = 2x - 1$
or $2x - y - 1 = 0$ is

$$= \frac{4+1}{\sqrt{(2)^2 + (1)^2}} = \frac{5}{\sqrt{5}} = \sqrt{5}$$

85. $\left({}^7C_0 + {}^7C_1\right) + \left({}^7C_2 + {}^7C_3\right) + \cdots + \left({}^7C_6 + {}^7C_7\right) =$
(A) $2^8 - 2$
(B) $2^7 - 1$
(C) 2^7
(D) $2^8 - 1$
(E) $2^7 - 2$

Solution: (C)
$\left({}^7C_0 + {}^7C_1\right) + \left({}^7C_2 + {}^7C_3\right) + \cdots + \left({}^7C_6 + {}^7C_7\right)$
$= {}^7C_0 + {}^7C_1 + \cdots + {}^7C_7$
$= 2^7 [\because C_0 + C_1 + C_2 + \cdots + C_n = 2^n]$

86. The coefficient of x in the expansion of $(1 - 3x + 7x^2)(1-x)^{16}$ is
(A) 17
(B) 19
(C) -17

(D) -19
(E) 20

Solution: (D)
$(1-3x+7x^2)\,(1-x)^{16}$
$=(1-3x+7x^2)$
$=\left({}^{16}C_0+{}^{16}C_1x^1+{}^{16}C_2x^2+\cdots+{}^{16}C_{16}x^{16}\right)$
$=(1-3x+7x^2)\,(1-16x+120x^2+\cdots)$
$\therefore$ Coefficient of $x=-16-3=-19$

87. The equation of the circle with centre $(2,2)$ which passes through $(4,5)$ is
(A) $x^2+y^2-4x+4y-77=0$
(B) $x^2+y^2-4x-4y-5=0$
(C) $x^2+y^2+2x+2y-59=0$
(D) $x^2+y^2-2x-2y-23=0$
(E) $x^2+y^2+4x-2y-26=0$

Solution: (B)
Radius of circle is $\sqrt{(4-2)^2+(5-2)^2}=\sqrt{13}$ So, equation of circle is
$(x-2)^2+(y-2)^2=13$
$\Rightarrow\ x^2+4-4x+y^2+4-4y=13$
$\Rightarrow\ x^2+y^2-4x-4y-5=0$

88. The point in the xy −plane which is equidistant from $(2,0,3),(0,3,2)$ and $(0,0,1)$ is
(A) $(1,2,3)$
(B) $(-3,2,0)$
(C) $(3,-2,0)$
(D) $(3,2,0)$
(E) $(3,2,1)$

Solution: (D)
Let the points are $A\,(2,0,3), B(0,3,2)$ and $D(0,0,1)$.
We know that Z-coordinate of every point an xy-plane is zero so let $p(x,y,0)$ be a point on xy-plane such that $PA=PB=PC$.
Now, $PA=PB$
$\Rightarrow PA^2=PB^2$
$\Rightarrow\ (x-2)^2+(y-0)^2+(0-3)^2=(x-0)^2+(y-3)^2+(0-2)$
$\Rightarrow 4x-6y=0 \Rightarrow 2x-3y=0$ ……(i)
and, $PB=PC$
$\Rightarrow\ PB^2=PC^2$
$\Rightarrow (x-0)^2+(y-3)^2+(0-2)^2=(x-0)^2+(y-0)^2+(0-1)^2$
$\Rightarrow\ -6y+12=0$
$\Rightarrow\ \ y=2$ ….. (ii)
Putting $y=2$ in Equation (i), we get $x=3$
Hence, the required point is $(3,2,0)$.

89. Let $f: x \to y$ be such that $f(1) = 2$ and $f(x + y) = f(x)f(y)$ for all natural numbers x and y. If $\sum_{k=1}^{n} f(a + k) = 16(2^n - 1)$, then a is equal to

(A) 3
(B) 4
(C) 5
(D) 6
(E) 7

Solution: (A)
We have,
$f(1) = 2$ and $f(x + y) = f(x) \cdot f(y)$
Now, $f(2) = f(1 + 1) = f(1) \cdot f(1) = 2 \cdot 2 = 2^2$
$f(3) = f(2 + 1) + f(2) \cdot f(1) = 2^2 \cdot 2 = 2^3$
and so on
$\therefore \ f(x) = 2^n$ ....(i)
Now, we have
$$\sum_{k=1}^{n} f(a + k) = 16(2^n - 1)$$
$\Rightarrow \ f(a + 1) + f(a + 2) + \cdots + f(a + n) = 16(2^n - 1)$
$\Rightarrow \ f(a) \cdot f(1) + f(a) \cdot f(2) + \cdots + f(a) \cdot f(n) = 16(2^n - 1)$
$\Rightarrow \ f(a) = [f(1) + f(2) + \cdots + f(n)] = 16(2^n - 1)$
$\Rightarrow f(a)[2 + 2^2 + \cdots + 2^n] = 16(2^n - 1)]$
$$\Rightarrow f(a) \cdot \left[2\frac{(2^n - 1)}{2 - 1}\right] = 16(2^n - 1)$$
$\Rightarrow 2f(a) \cdot (2^n - 1) = 16 \cdot (2^n - 1) \Rightarrow f(a) = 8$
$\Rightarrow 2^a = 8 \ [\because f(x) = 2^n \Rightarrow f(a) = 2^a]$
$\Rightarrow 2^a = 2^3 = a = 3$

90. If ${}^{n}C_{r-1} = 36$, ${}^{n}C_{r} = 84$ and ${}^{n}C_{r+1} = 126$, then $n =$
(A) 3
(B) 4
(C) 8
(D) 9
(E) 10

Solution: (D)
Given that,
${}^{n}C_{r-1} = 36$, ${}^{n}C_{r} = 84$
and ${}^{n}C_{r+1} = 126$
Here, $\frac{{}^{n}C_{r-1}}{{}^{n}C_{r}} = \frac{36}{84}$ and $\frac{{}^{n}C_{r}}{{}^{n}C_{r+1}} = \frac{84}{126}$
$\Rightarrow 3n - 10r = -3$ and $4n - 10r = 6$
By solving these equations, we get $n = 9, r = 3$

91. Let $f:(-1,1)\to(-1,1)$ be continuous, $f(x)=f(x)^2$ for all $x\in(-1,1)$ and $f(0)=\frac{1}{2}$, then the value of $4f\left(\frac{1}{4}\right)$ is

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

Solution: (B)
$\because f$ is continuous
$\therefore f(0)=f(0+h)=f(0-h)$

$$f\left(\frac{1}{4}\right)=f\left(\frac{1}{4}+h\right)$$

$$f(0)=f\left(0+\frac{1}{4}\right)$$

Given, $f\left(\frac{1}{2}\right)=f\left(\frac{1}{2^2}\right)$

$\therefore\ f(0)=f\left(0+\frac{1}{2^2}\right)=\frac{1}{2}$ … (i)

Therefore, $4f\left(\frac{1}{4}\right)$
$=4\cdot\frac{1}{2}$ [using Equation (i)]
$=2$

92. $\lim_{x\to\infty}\sqrt{x^2+1}-\sqrt{x^2-1}=$

(A) -1
(B) 1
(C) 0
(D) 2
(E) 4

Solution: (C)

$$\lim_{x\to\infty}\left(\sqrt{x^2+1}-\sqrt{x^2-1}\right)$$

$$=\lim_{x\to\infty}\left(\sqrt{x^2+1}-\sqrt{x^2-1}\right)\frac{\left(\sqrt{x^2+1}+\sqrt{x^2-1}\right)}{\left(\sqrt{x^2+1}+\sqrt{x^2-1}\right)}$$ (by rationalization)

$$=\lim_{x\to\infty}\frac{x^2+1-x^2+1}{\sqrt{x^2+1}+\sqrt{x^2-1}}=\lim_{x\to\infty}\frac{2}{\sqrt{x^2+1}+\sqrt{x^2-1}}$$

$$=\lim_{x\to\infty}\frac{2}{x\left(\sqrt{1+\frac{1}{x^2}}+\sqrt{1-\frac{1}{x^2}}\right)}=0$$

93. If f is differentiable at $x=1$ and $\lim_{h\to0}\frac{1}{h}f(1+h)=5, f'(1)=$

(A) 0

(B) 1
(C) 3
(D) 4
(E) 5

Solution: (E)
$f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$; function is differentiable.
$f(1) = 0$ and $\lim_{h\to 0} \frac{f(1+h)}{h} = 5$; Given function is continuous.
Hence, $f'(1) = \lim_{h\to 0} \frac{f(1+h)}{h} = 5$

94. The maximum value of the function $2x^3 - 15x^2 + 36x + 4$ is attained at
(A) 0
(B) 3
(C) 4
(D) 2
(E) 5

Solution: (D)
We have, $f(x) = 2x^3 - 15x^2 + 36x + 4$
$\Rightarrow f'(x) = 6x^2 - 30x + 36$
and $f'(x) = 12x - 30$
At point of local maximum as minimum, we must have
$f'(x) = 0 \Rightarrow 6(x^2 - 5x + 6) = 0 \Rightarrow x = 2, 3$
Clearly, $f'(2) = 24 - 30 = -6 < 0$
and $f'(3) = 36 - 30 = 6 > 0$
So, $f(x)$ has local maximum at $x = 2$.

95. If $\int f(x) \cos x dx = \frac{1}{2}\{f(x)\}^2 + C$, then $f\left(\frac{\pi}{2}\right)$ is
(A) C
(B) $\frac{\pi}{2} + C$
(C) $C + 1$
(D) $2\pi + C$
(E) $C + 2$

Solution: (C)
We have,
$$\int f(x) \cos x dx = \frac{1}{2}\{f(x)\}^2 + C$$
On differentiating both sides, we get
$f(x) \cos x = f(x) f'(x)$
$\because \; f'(x) = \cos x$
$\Rightarrow \; f(x) = \sin x + C$
$$\because \; f\left(\frac{\pi}{2}\right) = \sin\frac{\pi}{2} + C = 1 + C$$

96. $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x} dx =$

(A) $\pi(\sqrt{2}-2)$

(B) $\pi(\sqrt{2}+1)$

(C) $2\pi(\sqrt{2}-1)$

(D) $2\pi(\sqrt{2}+1)$

(E) $\frac{\pi}{\sqrt{2}+1}$

Solution: (E)

Let $I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x} dx$ ....(i)

$$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\left(\frac{3\pi}{4}+\frac{\pi}{4}-x\right)}{1+\sin\left(\frac{3\pi}{4}+\frac{\pi}{4}-x\right)}$$

$= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{(\pi-x)dx}{1+\sin x}$ ....(ii)

$$\left[\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right]$$

By adding Equations (i) and (ii), we get

$$2I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\pi\, dx}{1+\sin x}$$

$$\Rightarrow 2I = \pi \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1-\sin x}{(1+\sin x)(1-\sin x)} dx$$

$$\Rightarrow 2I = \pi \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1-\sin x}{\cos^2 x} dx$$

$$\Rightarrow 2I = \pi \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} [\sec^2 x - \sec x \tan x] dx$$

$\Rightarrow 2I = \pi[\tan x - \sec x]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$

$\Rightarrow 2I = \pi\left[-1-(-\sqrt{2})-(1-\sqrt{2})\right]$

$\Rightarrow 2I = \pi\left[-1+\sqrt{2}-1+\sqrt{2}\right]$

$\Rightarrow 2I = \pi\left[-2+2\sqrt{2}\right]$

$\therefore I = \pi[\sqrt{2} - 1]$

$= \dfrac{\pi(\sqrt{2} - 1)}{(\sqrt{2} + 1)}(\sqrt{2} + 1)$

$= \dfrac{\pi}{\sqrt{2} + 1}$

97. $\int_0^{\frac{\pi}{2}} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}} dx =$

(A) 2

(B) π

(C) $\frac{\pi}{4}$

(D) 2π

(E) 0

Solution: (C)

Let $I = \int_0^{\frac{\pi}{2}} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}} dx$ .....(i)

$$\Rightarrow I = \int_2^{\frac{\pi}{2}} \frac{2^{\sin\left(\frac{\pi}{2} - x\right)}}{2^{\sin\left(\frac{\pi}{2} - x\right)} + 2^{\cos\left(\frac{\pi}{2} - x\right)}}$$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{2^{\cos x}}{2^{\cos x} + 2^{\sin x}}$ .....(ii)

By adding Equations (i) and (ii), we get

$$2I = \int_0^{\frac{\pi}{2}} \frac{2^{\sin x} + 2^{\cos x}}{2^{\sin x} + 2^{\cos x}} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

98. $\lim_{x \to 0} \left(\frac{\int_0^{x^2} \sin\sqrt{t}\, dt}{x^2} \right) =$

(A) $\frac{2}{3}$

(B) $\frac{2}{9}$

(C) $\frac{1}{3}$

(D) 0

(E) $\frac{1}{6}$

Solution: (D)

$$\lim_{x\to 0}\frac{\int_0^{x^2}\sin\sqrt{t}\,dt}{x^2}$$

Where, $f(0)=0, g(0)=0$

$$\therefore\ I=\lim_{x\to 0}\frac{f'(x)}{g'(x)}$$

Where, $f'(x)=\sin\sqrt{x^2}\frac{d}{dx}(x^2)-0$

$=2x\sin x$

$$\therefore I=\lim_{x\to 0}\frac{2x\sin x}{2x}$$

$=\lim_{x\to 0}\sin x=0$

99. The area bounded by $y=\sin^2 x, x=\frac{\pi}{2}$ and $x=\pi$ is

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{8}$

(D) $\frac{\pi}{16}$

(E) 2π

Solution: (B)

Required area $=\int_{\frac{\pi}{2}}^{\pi}\sin^2 x\,dx$

$$=\int_{\frac{\pi}{2}}^{\pi}\left[\frac{1-\cos 2x}{2}\right]dx$$

$$=\frac{1}{2}\int_{\frac{\pi}{2}}^{\pi}(1-\cos 2x)\,dx$$

$$=\frac{1}{2}\left[x-\frac{\sin 2x}{2}\right]_{\frac{\pi}{2}}^{\pi}$$

$$=\frac{1}{2}\left[(\pi-0)-\left(\frac{\pi}{2}-0\right)\right]$$

$$=\frac{1}{2}\left[\frac{\pi}{2}\right]=\frac{\pi}{4}$$

100. The differential equation of the family of curves $y=e^x(A\cos x+B\sin x)$, where A and B are arbitrary constants is

(A) $y''-2y'+2y=0$

(B) $y''+2y'-2y=0$

(C) $y''+y'^2+y=0$

(D) $y''+2y'-y=0$

(E) $y''-2y'-2y=0$

Solution: (A)
Given, system of equation is
$y = e^x(A\cos x + B\sin x)$
$\Rightarrow \frac{dy}{dx} = e^x(-A\cos x + B\cos x) + y$ .....(i)
$\Rightarrow \frac{d^2y}{dx^2} = e^x(-A\sin x + B\cos x) + e^x$
$[-A\cos x - B\sin x] + \frac{dy}{dx}$
$\Rightarrow \frac{d^2y}{dx^2} = \left(\frac{dy}{dx} - y\right)y + \frac{dy}{dx}$ [by Equation (i)]
$\Rightarrow \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$
$\Rightarrow y'' - 2y' + 2y = 0$
This is required differential equation.

101. The real part of $\left(i - \sqrt{3}\right)^{13}$ is
(A) 2^{-10}
(B) 2^{12}
(C) 2^{-12}
(D) -2^{-12}
(E) 2^{10}

Solution: (B)
$\left(i - \sqrt{3}\right)^{13}$
$= 2^{13} \times i^{13}\left[\frac{1 + \sqrt{3}i}{2}\right]^{13}$
$= 2^{13}i^{13}(-1)^{13}\left[\frac{-1 - \sqrt{3}i}{2}\right]^{13}$
$= -2^{13} \cdot 1^{13}w^{13} = -2^{13} \cdot i \cdot \left[\frac{-1 + \sqrt{3}i}{2}\right]$
$= -2^{13}\left[-i - \sqrt{3}\right] = -i2^{13} + 2^{13}\sqrt{3}$
Hence, real part is $2^{13}\sqrt{3}$.

102. $\lim_{x\to 0}\frac{1 + x - e^x}{x^2} =$
(A) $\frac{1}{2}$
(B) $\frac{-1}{2}$
(C) 1
(D) -1
(E) 0

Solution: (B) $\lim_{x\to 0} \frac{1+x-e^x}{x^2}$

$= \lim_{x\to 0} \frac{1-e^x}{2x}$ [by L' Hospital's rule]

$= \lim\limits_{x\to 0} \frac{-e^x}{2} = -\frac{e^0}{2} = -\frac{1}{2}$

103. $\int \frac{(\sin x + \cos x)(2 - \sin 2x)}{\sin^2 2x} dx =$

(A) $\frac{\sin x + \cos x}{\sin 2x} + C$

(B) $\frac{\sin x - \cos x}{\sin 2x} + C$

(C) $\frac{\sin x}{\sin x + \cos x} + C$

(D) $\frac{\sin x}{\sin x - \cos x} + C$

(E) $\frac{\sin x - \cos x}{\sin x + \cos x} + C$

Solution: (B)

We have,

$$I = \int \frac{(\sin x + \cos x)(2 - \sin 2x)}{\sin^2 2x} dx$$

Put $\sin x - \cos x = 1 \Rightarrow (\sin x + \cos x)\, dx = dt$ and $(\sin x - \cos x)^2 = t^2 \Rightarrow 1 - \sin 2x = t^2$

$\Rightarrow \sin 2x = 1 - t^2$

$$\therefore I = \int \frac{\left(2 - (1 - t^2)\right)dt}{(1 - t^2)^2}$$

$$\Rightarrow I = \int \frac{(1 + t^2)dt}{(1 - t^2)^2}$$

$$\Rightarrow I = \int \frac{1 + t^2}{1 - 2t^2 + t^4} dt$$

$$\Rightarrow I = \int \frac{1 + \frac{1}{t^2}}{\frac{1}{t^2} + t^2 - 2} dt$$

$$\Rightarrow I = \int \frac{1 + \frac{1}{t^2}}{\left(t - \frac{1}{t}\right)^2} dt$$

Put $t - \frac{1}{t} = y \Rightarrow \left(1 + \frac{1}{t^2}\right) dt = dy$

$$\therefore I = \int \frac{dy}{y^2} = -\frac{1}{y} + C$$

$$\Rightarrow I = \frac{-1}{t - \frac{1}{t}} + C$$

$$\Rightarrow I = \frac{t}{1 - t^2} + C$$

$\Rightarrow I = \dfrac{\sin x \; - \; \cos x}{\sin 2x} + C$

104. A plane is at a distance of 5 units from the origin and perpendicular to the vector $2\hat{\imath} + \hat{\jmath} + 2\hat{k}$. The equation of the plane is
(A) $\vec{r} \cdot (2\hat{\imath} + \hat{\jmath} - 2\hat{k}) = 15$
(B) $\vec{r} \cdot (2\hat{\imath} + \hat{\jmath} - \hat{k}) = 15$
(C) $\vec{r} \cdot (2\hat{\imath} + \hat{\jmath} + 2\hat{k}) = 15$
(D) $\vec{r} \cdot (\hat{\imath} + \hat{\jmath} + 2\hat{k}) = 15$
(E) $\vec{r} \cdot (2\hat{\imath} - \hat{\jmath} + 2\hat{k}) = 15$

Solution: (C)
Equation of plane whose distance from origin is P and normal is $\hat{n}$ is
$P = \vec{r} \cdot \hat{n}$
Given that, $P = 5$

$\therefore \hat{n} = \dfrac{2\hat{\imath} + \hat{\jmath} + 2\hat{k}}{\sqrt{2^2 + 1^2 + 2^2}}$

$= \dfrac{2\hat{\imath} + \hat{\jmath} + 2\hat{k}}{3}$

By formula,

$5 = \vec{r} \cdot \dfrac{2\hat{\imath} + \hat{\jmath} + 2\hat{k}}{3}$

$\Rightarrow \vec{r} \cdot (2\hat{\imath} + \hat{\jmath} + 2\hat{k}) = 15$

105. $\frac{\sin A - \sin B}{\cos A + \cos B}$ is equal to
(A) $\sin\left(\frac{A+B}{2}\right)$
(B) $2\tan(A + B)$
(C) $\cot\left(\frac{A-B}{2}\right)$
(D) $\tan\left(\frac{A-B}{2}\right)$
(E) $2\cot(A + B)$

Solution: (D
Given that, $\frac{\sin A - \sin B}{\cos A + \cos B}$

$= \dfrac{2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}$

$= \dfrac{\sin\left(\frac{A-B}{2}\right)}{\cos\left(\frac{A-B}{2}\right)} = \tan\left(\dfrac{A-B}{2}\right)$

106. If $x = A\cos 4t + B\sin 4t$, then $\frac{d^2x}{dt^2} =$

(A) x

(B) $-16x$

(C) $15x$

(D) $16x$

(E) $-15x$

Solution: (B)

Given that,

$x = A\cos 4t + B\sin 4t$ ……(i)

Differentiating $w.r.t$ to t,

$$\frac{dx}{dt} = 4 \cdot A(-\sin 4t) + 4 \cdot B\cos 4t$$

$$\Rightarrow \quad \frac{dx}{dt} = 4[-A\sin 4t + B\cos 4t]$$

Again differentiating $w.r.t$ to t,

$$\Rightarrow \frac{d^2x}{dt^2} = 4[-4 \cdot A\cos 4t + (-4)\,B\sin 4t]$$

$$= \; -16\,[A\cos 4t + B\sin 4t]$$

$\Rightarrow \frac{d^2x}{dt^2} = -16x$ [by Equation (i)]

107. The arithmetic mean of ${}^nC_0, {}^nC_1, {}^nC_2 \ldots, {}^nC_n$ is

(A) $\frac{2^n}{n+1}$

(B) $\frac{2^n}{n}$

(C) $\frac{2^{n-1}}{n+1}$

(D) $\frac{2^{n-1}}{n}$

(E) $\frac{2^{n+1}}{n}$

Solution: (A)

$\because (1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2 \cdot x^2 + \cdots + {}^nC_n \cdot x^n$

Take $x = 1$

$(1+1)^n = {}^nC_0 + {}^nC_1 \cdot (1) + {}^nC_2 \cdot (1)^2 + \cdots + {}^nC_0 \cdot (1)^n$

$2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n$

Now, arithmetic mean

$\bar{X} = \frac{\sum_{i=1}^{n} x_i}{N}$, where $N = (n+1)$

$$\Rightarrow \; \bar{X} = \frac{{}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n}{n+1}$$

$$\Rightarrow \bar{X} = \frac{2^n}{n\, +\, 1}$$

108. The variance of first 20 natural numbers is

(A) $\frac{399}{2}$
(B) $\frac{379}{12}$
(C) $\frac{133}{2}$
(D) $\frac{133}{4}$
(E) $\frac{169}{2}$

Solution: (D)
Since, variance of first n natural number is
$(S.D.)^2 = \frac{n^2 - 1}{12}$
∴ Variance of first 20 natural number is
$(S.D.)^2 = \frac{(20)^2 - 1}{12}$
$= \frac{400 - 1}{12}$
$= \frac{399}{12} = \frac{133}{4}$

109. If S is a set with 10 elements and $A = \{(x, y): x, y \in S, x \neq y\}$, then the number of elements in A is
(A) 100
(B) 90
(C) 80
(D) 150
(E) 45

Solution: (B)
Total numbers of elements in the set $A =$ The selection of two distinct elements from given 10 elements.
$\Rightarrow\ n(A) = {}^{10}C_1 \times {}^{9}C_1 = 10 \times 9 = 90$

110. A coin is tossed and a die is rolled. The probability that the coin shows head and the die shows 3 is
(A) $\frac{1}{6}$
(B) $\frac{1}{12}$
(C) $\frac{1}{9}$
(D) $\frac{11}{12}$
(E) $\frac{1}{11}$

Solution: (B)
$P(E_1) = \frac{1}{2}$ and $P(E_2) = \frac{1}{6}$

So, required probability $= \left(\frac{1}{2}\right)\left(\frac{1}{6}\right) = \frac{1}{12}$

111. If $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$, then the sum of all the diagonal entries of A^{-1} is
(A) 2
(B) 3
(C) -3
(D) -4
(E) 4

Solution: (E)
$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

$\Rightarrow |A| = 0(2-3) - 1(1-9) + 2(1-6) = 8 - 10 = -2$
$\therefore\ C_{11} = (2-3) = -1, C_{12} = -(1-9) = 8$
$C_{13} = (1-6) = -5, C_{21} = -(1-2) = 1$
$C_{22} = (0-6) = -6, C_{23} = -(0-3) = -3$
$C_{31} = (3-4) = -1, C_{32} = -(0-2) = 2$
$C_{33} = (0-1) = -1$
$\therefore\ adj|A| = \begin{bmatrix} -1 & 8 & -5 \\ 1 & -6 & 3 \\ -1 & 2 & -1 \end{bmatrix}$

$A^{-1} = \frac{adj\ [A]}{|A|} = \frac{\begin{bmatrix} -1 & 8 & -5 \\ 1 & -6 & 3 \\ -1 & 2 & -1 \end{bmatrix}}{-2}$

$A^{-1} = \begin{bmatrix} \frac{1}{2} & 4 & \frac{5}{2} \\ -\frac{1}{2} & 3 & -\frac{3}{2} \\ \frac{1}{2} & -1 & \frac{1}{2} \end{bmatrix}$

$\therefore$ Sum of all diagonal entries of A^{-1}
$= \frac{1}{2} + 3 + \frac{1}{2} = 4$

112. Let $f(x) = \begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$. If $x = -9$ is a root of
$f(x) = 0$, then the other roots are
(A) 2 and 7
(B) 3 and 6
(C) 7 and 3

(D) 6 and 2
(E) 6 and 7

Solution: (A)

Given, $f(x) = \begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$

$= \begin{vmatrix} x+9 & x+9 & x+9 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$

[applying $R_1 \to R_1 + R_2 + R_3$]

$= (x+9) \begin{vmatrix} 1 & 1 & 1 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$

$= (x+9) \begin{vmatrix} 0 & 0 & 1 \\ 2-x & x-2 & 2 \\ 1 & 6-x & x \end{vmatrix}$

[applying $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$]

$= (x+9)[(2-x)(6-x) - (x-2)]$

$= (x+9)(x-2)[x-6-1]$

$f(x) = (x+9)\,(x-2)(x-7)$

at $f(x) = 0$

$(x+9)\,(x-2)(x-7) = 0$

$\Rightarrow \quad x = -9, 2, 7$

Hence, other roots are 2 and 7.

113. If $[1 \times 1] \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$, then x can be

(A) -1
(B) 2
(C) 14
(D) -14
(E) 0

Solution: (D)

Given that, $[1 \times 1] \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} 1+2x+15 \\ 3+5x+3 \\ 2+x+2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} 2x+16 \\ 5x+6 \\ x+4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$

$\Rightarrow 2x + 16 + 2(5x+6) + x(x+4) = 0$

$\Rightarrow 2x + 16 + 10x + 12 + x^2 + 4x = 0$

$\Rightarrow x^2 + 16x + 28 = 0$

$\Rightarrow x^2 + 2x + 14x + 28 = 0$
$\Rightarrow x(x+2) + 14(x+2) = 0$
$\Rightarrow (x+2)(x+14) = 0$
$\Rightarrow x = -14$

114. If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = A = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, then $x =$
(A) 2
(B) $\frac{1}{2}$
(C) 1
(D) 3
(E) 0

Solution: (B)
We have,
$A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$
$A^{-1} = \frac{1}{2x^2}\begin{bmatrix} x & 0 \\ -x & 2x \end{bmatrix}$
$\left\{\because A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\right\}$
$\Rightarrow A^{-1} = \begin{bmatrix} \frac{1}{2x} & 0 \\ -\frac{1}{2x} & \frac{1}{x} \end{bmatrix}$
Now, it is given that
$A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$
$\Rightarrow \begin{bmatrix} \frac{1}{2x} & 0 \\ -\frac{1}{2x} & \frac{1}{x} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$
$\therefore x = \frac{1}{2}$

115. If $\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = ax^4 + bx^3 + cx^2 + dx + e$, then $5a + 4b + 3c + 2d + e$ is equal to
(A) 11
(B) -11
(C) 12
(D) -12
(E) 13

Solution: (B)

Given that, $\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = ax^4 + bx^3 + cx^2 + dx + e,$

$\Rightarrow x(6x - 6x) - 2(6x^2 - 6x) + x(x^3 - x^2)$

$= ax^4 + bx^3 + cx^2 + dx + e$

$\Rightarrow -12x^2 + 12x + x^4 - x^3 = ax^4 = bx^3 + cx^2 + dx + e$

$\Rightarrow x^4 - x^3 - 12x^2 + 12x = ax^4 + bx^3 + cx^2 + dx + e$

On equating the coefficient of both sides, we get

$a = 1, b = -1, c = -12, d = 12, e = 0$

$\therefore 5a + 4b + 3c + 2d + e = 5 \times 1 + 4 \times (-1) + 3(-12) + 2(12) + 0$

$= 5 - 4 - 36 + 24$

$= 29 + 40 = -11$

116. $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} =$

(A) 1

(B) 0

(C) $(1-a)(1-b)(1-c)$

(D) $a + b + c$

(E) $2(a + b + c)$

Solution: (B)

Given that,

$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = \left|\begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}\right|$$

[applying $C_3 \to C_3 + C_2$]

$= (a + b + c) \begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$

$= (a + b + c) \times 0$ [$\because C_1$ and C_3 are equal]

$= 0.$

117. If $f(x) = \begin{vmatrix} 1 & 1 & 1 \\ 2x & x-1 & x \\ 3x(x-1) & (x-1)(x-2) & x(x-1) \end{vmatrix}$, then $f(50) =$

(A) 0

(B) 2

(C) 4

(D) 1

(E) 3

Solution: (A)

Given,

$$f(x)=\begin{vmatrix}1 & 1 & 1\\ 2x & x-1 & x\\ 3x(x-1) & (x-1)(x-2) & x(x-1)\end{vmatrix}$$

$$=\begin{vmatrix}0 & 0 & 1\\ x+1 & -1 & x\\ 2(x+1)(x-1) & -2(x-1) & x(x-1)\end{vmatrix}$$

[applying $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$]

$$=(x-1)\begin{vmatrix}0 & 0 & 1\\ (x+1) & -1 & x\\ 2(x+1) & -2 & x\end{vmatrix}$$

$=(x-1)[-2(x+1)+2(x+1)]$

$\Rightarrow f(x)=0$

$\therefore f(50)=0$

118. If $\Delta(x)=\begin{vmatrix}1 & \cos x & 1-\cos x\\ 1+\sin x & \cos x & 1+\sin x-\cos x\\ \sin x & \sin x & 1\end{vmatrix}$, then

$\int_0^{\frac{\pi}{2}} \Delta(x)\,dx =$

(A) $\frac{-1}{2}$

(B) $\frac{1}{2}$

(C) 1

(D) -1

(E) 0

Solution: (A)

Given,

$$\Delta(x)=\begin{vmatrix}1 & \cos x & 1-\cos x\\ 1+\sin x & \cos x & 1+\sin x-\cos x\\ \sin x & \sin x & 1\end{vmatrix}$$

$$=\begin{vmatrix}1 & \cos x & 1-\cos x\\ 0 & -\sin x & \sin x-1\\ \sin x & \sin x & 1\end{vmatrix}$$

[applying $R_1 \to R_2 - (R_1 + R_3)$]

$$=\begin{vmatrix}1 & \cos x & 1\\ 0 & -\sin x & -1\\ \sin x & \sin x & 1+\sin x\end{vmatrix}$$

[applying $C_3 \to C_3 + C_2$]

$=1(0+\sin^2 x)+1(\sin x-\sin x\cos x)+(1+\sin x)(-\sin x-0)$

$=\sin^2 x+\sin x-\sin x\cos x-\sin x-\sin^2 x$

$= -\sin x\cos x$

$$\Delta x=-\frac{\sin 2x}{2}$$

$$\therefore \int_0^{\frac{\pi}{2}} \Delta(x)dx=-\frac{1}{2}\int_0^{\frac{\pi}{2}} \sin 2x\,dx$$

$= -\frac{1}{2}\left[-\frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{2}}$

$= \ -\frac{1}{4}[\cos\pi - \cos 0]$

$-\frac{1}{4}[-1-1]$

$= \ -\frac{2}{4} = -\frac{1}{2}$

119. The equation of the plane passing through the points $(1,2,3), (-1,4,2)$ and $(3,1,1)$ is

(A) $5x + y + 12z = 23$

(B) $5x + 6y + 2z = 23$

(C) $5x - 6y + 2z = 23$

(D) $x + y + z = 13$

(E) $2x + 6y + 5z = 7$

Solution: (B)

Given that,

$x_1 = 1, y_1 = 2, z_1 = 3$

$x_2 = -1, y_2 = 4, z_2 = 2$

and $x_3 = 3, y_3 = 1, z_3 = 1$

Equation of plane passing through these points is

$\begin{vmatrix} x-1 & y-2 & z-3 \\ -2 & 2 & -1 \\ 2 & -1 & -2 \end{vmatrix} = 0$

$\Rightarrow (x-1)(-4-1) - (y-2)(y+2) + (z-3)(2-4) = 0$

$\Rightarrow (x-1)(-5) - (y-2)(6) + (z-3)(-2) = 0$

$\Rightarrow -5x + 5 - 6y + 12 - 2z + 6 = 0$

$\Rightarrow -5x - 6y - 2z + 23 = 0$

$\Rightarrow 5x + 6y + 2z - 23 = 0$

$\Rightarrow 5x + 6y + 2z = 23$

120. In an arithmetic progression, if the kth term is $5k + 1$, then the sum of first 100 terms is

(A) $50(507)$

(B) $51(506)$

(C) $50(506)$

(D) $51(507)$

(E) $52(506)$

Solution: (A)

Let a be the first term of an AP and d is the common difference.

$\therefore \quad a_k = a + (n-1)d$

Since, $a_k = 5k + 1$

$a + (k-1)d = 5(k-1) + 6$

$\Rightarrow a + (k-1)d = 6 + (k-1)6$
Equating both sides, we get
$a = 6$ and $d = 5$
$\therefore S_{100} = \frac{n}{2}[2a + (n-1)d]$
$= \frac{100}{2}[2 \times 6 + 99 \times 5]$
$= 50[12 + 495] = 50(507)$