General instructions

1. This question booklet contains five sections. Each section carries 50 marks.

2. First section is General section and it is compulsory.

3. Sections 2 to 5 are for the subjects Biology, Chemistry, Mathematics and Physics. You should attempt only three of the four subject sections.

4. Pocket calculators, cell phones, log tables etc. are NOT permitted in the examination hall.

5. Answers to the questions are to be marked in the supplied OMR sheet.

6. Please make sure that question booklet code (A or B) matches with OMR sheet code (A or B) respectively. In case of discrepancy please inform the invigilator immediately.

7. Rough work may be done on separate blank sheets provided.

8. Return the OMR sheet to the invigilator at the end of examination.

9. Read the instructions given at the beginning of each section carefully.

Instructions for writing on OMR sheet

1. Read and follow the instructions given on OMR sheet.

2. Write your name, roll number and other required information with ball-point pen in appropriate boxes provided. Sign your name with ball-point pen in the box provided.

3. In the remaining part of OMR sheet, use HB pencil only (as instructed). Make sure the bubbles are filled properly (as indicated in OMR sheet).

4. As far as possible, fill in the answers only after you are sure that you do not need to change them. In case you do have to change the answer after filling, erase the mark completely so that no black spot is left inside the bubble.

5. Ensure that you are filling the bubbles corresponding to correct sections and answers.

6. Your roll number (as given in the admit card) MUST BE ENTERED CORRECTLY. If entered wrongly or not entered, the OMR sheet will be invalid and will not be graded.
SPACE FOR ROUGH WORK
Read the following passage carefully and answer questions 1.1 to 1.3.

James Joule, in his famous experiment of a paddle wheel in water, determined how much work is required to generate a given amount of heat. His results were taken up by William Thompson later known as Lord Kelvin, who tried to find appropriate mathematical laws to express Joule’s experiment. He realised that to formulate them he needed some absolute scale of temperature rather than a scale based on freezing and boiling points of some convenient liquid. From Carnot’s published work on caloric cycle, Kelvin realised that the mechanical work done by a perfect frictionless engine depends only on amount of heat or caloric, and temperatures of heat source and heat sink. For a given temperature of the source, the lower the temperature of the sink, the greater is the amount of heat converted into mechanical work. However, establishing a universal temperature scale which is independent of working substance, needed an additional contribution from Rudolf Clausius. Clausius showed that Carnot’s ‘caloric’ was indeed the same as Joule’s ‘Heat’. Clausius’ work meant that Carnot was mistaken when he thought of an engine doing work solely because its caloric dropped in temperature. Clausius reasoned that caloric could not be destroyed but claimed it could be converted into something else; in an engine, for example, it was converted into mechanical work.

With Clausius’ work, Kelvin now had the clue he needed. He realised that in experiments with heat sinks and sources, what the heat sink does not absorb is converted into mechanical work. Thus, if the sink is at ‘zero-temperature’ and remains so, it is taking no heat from the working substance. All energy is therefore available for conversion to mechanical work. For convenience, Kelvin took his increment of 1° to be the same as the Celsius scale and deduced that his ‘absolute zero’ was at -273.1°C. This absolute zero point on Kelvin’s scale is, in fact, not attainable due to the way atoms are governed, though Kelvin did not know about this. The work of Clausius and Kelvin made it clear that heat was no mysterious weightless fluid but was a form of energy. So, too, was mechanical work. This gave deeper meaning to the principle of conservation of energy already stated by Hermann von Helmholtz in 1847.

1.1 Choose the correct statement.

(A) According to Carnot, the amount of work was proportional to change in temperature only.
According to Joule, heat could be converted to mechanical work.
According to Carnot, caloric was a form of energy.
According to Clausius, total caloric content of a body was a unchangeable quantity.

1.2 Choose the correct statement about Lord Kelvin’s universal scale of temperature.
(A) According to Kelvin, efficiency of conversion from heat to mechanical energy can never be 100% for any temperature of the heat sink.
(B) Increments in temperature on the Kelvin scale were independent of the properties of water.
(C) Kelvin chose a particular value as zero temperature based on atomic structure.
(D) Mathematical expressions like ideal gas equation hold for Kelvin scale only.

1.3 Arrange the following in chronological order from older to newer.
P: Principle of conservation of energy.
Q: Carnot cycle of heat engines.
R: Celsius scale of temperature.
S: Kelvin scale of temperature.

(A) R - Q - P - S.  (B) R - Q - S - P.
(C) P - Q - R - S.  (D) S - R - Q - P.

Read the following passage carefully and answer questions 1.4 to 1.6.

If a population of individuals has to survive, reproduce and expand, it must constantly interact and adapt with its co-existing players in the same environment. Food, climate, predators as well as host-pathogen relationships collectively play a significant role in evolution. Pathogenic viruses are capable of infecting a wide range of organisms ranging from bacteria to humans. Many viruses integrate their genome in their host DNA and are capable of existing within the host cell in a dormant state. The inheritable presence of viral genome(s) within the host DNA has the potential to influence evolution of the host genome. Also, a large population of rapidly multiplying viruses makes them amenable to accumulation of higher amounts of genetic variation or mutation. This potential is used by viruses to undergo rapid adaptation and evolution to overcome host defense mechanisms and to broaden their host range. Viruses, such as the HIV-1, may be one of such biological entities that researchers can actually witness come into being, providing a real-time example of evolution in action.

1.4 Viruses can function as effective agents of evolution because
(A) they prevent occurrence of mutations in the host genome.
(B) they can integrate their genetic material in the host genome.
(C) they are capable of replicating outside a host.
(D) they do not accumulate mutations in a rapid manner.

1.5 Presence of a virus in a host need not necessarily imply manifestation of disease symptoms because

(A) the virus may remain dormant in the host.
(B) viral DNA may be rapidly degraded in the host.
(C) the virus may undergo a slow rate of replication.
(D) All of the above.

1.6 Choose the incorrect statement.

(A) Human DNA may contain traces of ancient viruses.
(B) Viruses are dependent on their host for propagation.
(C) Adaptation to drastic changes in climate conditions does not contribute to genetic modification.
(D) Viruses lose their infective potential permanently once their host develops resistance.

1.7 The distance vs. time graph on the right shows motion of two buses (A and B) between towns P and Q. Bus A goes from P to Q and bus B goes from Q to P. Which of the following statements is supported by the graph.

(A) Both the buses leave respective towns simultaneously.
(B) Excluding halting times, both the buses travel with the same average speed.
(C) Distances in the problem are referred to from town Q.
(D) Both buses take halts at the same locations during the journey.

1.8 Which of the following numbers is divisible by 5?

(A) $2^{50} + 1$  
(B) $2^{50} + 2$  
(C) $2^{50} + 3$  
(D) $2^{50} + 4$
1.9 Which of the following type of eclipses will be visible from the Moon?

(A) Total eclipse of the Earth.  
(B) Partial eclipse of the Earth.  
(C) Total eclipse of the Sun.  
(D) Annular eclipse of the Sun.  

1.10 A classic example of natural selection is the change in relative populations of the peppered moth that occurred during the industrial revolution in England. After the industrial revolution in 1870s, booming industrial cities started releasing tons of black soot, blackening tree trunks and rocks and creating an environment where moths with light colours were easily predated by birds. Peppered moths captured in 1850 and 1900 reveal that two industrial cities (X and Y) had the following percent distributions.

<table>
<thead>
<tr>
<th>Year</th>
<th>1850</th>
<th>1900</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moths</td>
<td>Light coloured</td>
<td>Dark coloured</td>
</tr>
<tr>
<td>X</td>
<td>66</td>
<td>34</td>
</tr>
<tr>
<td>Y</td>
<td>65</td>
<td>35</td>
</tr>
</tbody>
</table>

Pollution control rules were introduced in 1902 and were properly followed in city Y, but not in city X. If all other factors remain constant, then what could be the approximate expected distribution of light and dark coloured moths in 1950?

(A) Light X = 8, Y = 28 and Dark X = 92, Y = 72  
(B) Light X = 8, Y = 62 and Dark X = 92, Y = 38  
(C) Light X = 18, Y = 23 and Dark X = 82, Y = 77  
(D) Light X = 18, Y = 50 and Dark X = 82, Y = 50  

1.11 An electronics store conducted a survey by asking every fourth person entering their store if they already owned a television set. On a given day out of 100 total respondents, 80 answered the question in the affirmative. Next day the same survey was conducted by choosing every fifth person entering the store and the number of respondents was again 100. Which of the following is the most likely number of respondents answering in the affirmative?

(A) 64  
(B) 78  
(C) 100  
(D) 92  

1.12 Choose the correct statement.

(A) Detergents are sodium salts of carboxylic acids.  
(B) Soaps are sodium salts of sulfonic acids.  
(C) Detergents form precipitate with calcium and magnesium ions.  
(D) Soaps are generally biodegradable.

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*For rest of the questions in this section, each correct answer will earn 2 marks.*
1.13 Which of the following date has been declared as “National Mathematics Day”?  
(A) 28 February.   (B) 22 June.  
(C) 23 September.  (D) 22 December.

1.14 Which of the following wildlife sanctuaries is not involved in “Project Tiger”?  
(A) Corbett.   (B) Gir.   (C) Kanha. (D) Bandipur.

1.15 Who is known as father of ‘white revolution’?  
(A) Verghese Kurien.   (B) Sundar Lal Bahuguna.  
(C) M. S. Swaminathan.  (D) Birbal Sahni.

1.16 Which of the following is a primary source of gelatin used in ice-cream?  
(A) Jellyfish.  (B) Marine algae.  
(C) Fruit skin. (D) Animal bones.

1.17 Hand-held GPS units in modern phones identify your location by  
(A) transmitting their location and time to GPS satellites.  
(B) receiving location data of GPS satellites.  
(C) receiving time data from GPS satellites.  
(D) exchanging location and time data with GPS satellites.

1.18 Which of the following is a strong smelling agent that is added to LPG to detect gas leakage?  
(A) Ethanethiol.   (B) Dimethyl sulfide.  
(C) Dimethyl sulfone. (D) Hydrogen sulfide.

1.19 What is the principle behind the light emission by a firefly?  
(A) Chemiluminescence.  (B) Fluorescence.  
(C) Phosphorescence. (D) Electroluminescence.
Section 2: BIOLOGY

Marks for Section 2: 50

This section contains 14 questions.
For questions 2.1 to 2.10 only one of the 4 options is correct. A correct answer will earn 3 marks, a wrong answer will earn \((-1)\) mark, and an unattempted question will earn 0 marks.

2.1 Which one of the following occurs during osmosis?
   (A) The movement of water from a concentrated solution to a dilute solution across a semi-permeable membrane.
   (B) The movement of a dissolved substance from a concentrated solution to a dilute solution across a semi-permeable membrane.
   (C) The movement of water from a dilute solution to a concentrated solution across a semi-permeable membrane.
   (D) The movement of a solute from a dilute solution across a semi-permeable membrane.

2.2 The umbilical artery carries blood from the fetus to the placenta and the umbilical vein carries blood from the placenta to the fetus. Which is the correct statement regarding differences in blood composition between the umbilical artery and vein?
   (A) Blood in the umbilical artery contains less glucose than blood in the umbilical vein.
   (B) Blood in the umbilical artery contains less carbon dioxide than blood in the umbilical vein.
   (C) Blood in the umbilical artery contains more deoxygenated than blood in the umbilical vein.
   (D) Blood in the umbilical artery contains less urea than blood in the umbilical vein.

2.3 Antibiotics are routinely used for controlling bacterial infections, but are ineffective against most viral infections. Why?
   (A) Viruses are too small to be affected by antibiotics.
   (B) Viruses have mechanisms to block the action of antibiotics.
   (C) Antibiotics stimulate the immune system against bacteria but not viruses.
   (D) Viruses do not have an independent metabolism.
2.4 Programmed cell death (PCD) is a tightly regulated process that is important in the growth and development of plants. In plants there are several tissues which become functional after undergoing PCD. Which of the following cells/tissues represent “functional corpses” arising due to PCD in plants?

(i) Cork cells in bark.
(ii) Sieve elements of phloem.
(iii) Companion cells of phloem.
(iv) Tracheary elements of xylem.

(A) (i) and (ii) only.  
(B) (ii), (iii) and (iv) only.  
(C) (i) only.  
(D) (i) and (iv) only.

2.5 With reference to the nervous system, pick the correct statement from the options given below.

(A) A ganglion is a collection of cell bodies.
(B) A ganglion is a collection of axons.
(C) A nerve is a collection of cell bodies.
(D) A nerve is a collection of dendrites.

2.6 Unlike animal cells, plant cells are characterised by the presence of a large central vacuole. The central vacuole performs a variety of functions during plant growth and development. One of these functions is similar to that performed by lysosomes in animal cells. Which of the following represent functions of vacuoles in plants?

(i) Storage of water and salts.
(ii) Cell expansion.
(iii) Storage of metabolic waste products.
(iv) Degradation.

(A) (i) and (ii) only.  
(B) (i), (iii) and (iv) only.  
(C) (i), (ii), (iii) and (iv).  
(D) (i), (ii) and (iii) only.

2.7 A short peptide has a sequence of amino acids valine-serine-methionine-proline, and the t-RNAs used in its synthesis have the following corresponding anticodons:

3’-CAG-5’, 3’-UCG-5’, 3’-UAC-5’, 3’-UUU-5’. What is the sequence of DNA that encodes the peptide?

(A) 5’ GACGCTCATT TT 3’  
(B) 5’ UUUCAUGCUGAC 3’  
(C) 5’ CAGTCGTACTTTT 3’  
(D) 5’ TTTCATGCTGAC 3’
2.8 Any population comprises of three categories of individuals based on their ages: (i) pre-reproductive (ii) reproductive and (iii) post-reproductive individuals. A certain population has the following numbers of individuals in each category:

- Reproductive individuals: 3.0 million
- Post-reproductive individuals: 0.5 million
- Pre-reproductive individuals: 4.3 million

Such a population would be termed as

(A) expanding. (B) declining. (C) stable. (D) aging.

2.9 Two different DNA samples were isolated from biological specimens. Further experiments demonstrated that one of these (X) was composed of 30% A, 30% G, 20% T and 20% C but could not be cut by an exonuclease (an enzyme that cuts nucleotides from the ends of DNA). The second DNA sample (Y) could be cut by the exonuclease and was found to be composed of 30% A, 30% T, 20% G and 20% C. Which of the following statements can be correctly deduced from the above?

(A) DNA X has a double-stranded, linear structure.
(B) DNA X has a single-stranded, circular structure.
(C) DNA Y has a double-stranded, circular structure.
(D) DNA Y has a single-stranded, linear structure.

2.10 Increased application of pesticides and insecticides has been shown to increase the occurrence of resistant varieties of pests and pathogens. What could be the probable reasons for the rapid evolution of resistant varieties of insects?

(i) Mutations in the existing population of insects/pests.
(ii) Preferential multiplication of resistant individuals.
(iii) Rapid degradation of the insecticide/pesticide.
(iv) Migration of insects/pests away from the sprayed areas.

(A) (iii) and (iv) only. (B) (i) and (ii) only.
(C) (i) and (iii) only. (D) (ii) and (iv) only.

For questions 2.11 to 2.14 one or more than one of the 4 options may be correct. Your answer is regarded correct only if you choose all the correct option(s) and no incorrect option(s). A correct answer will earn 5 marks, a wrong answer or an unattempted question will earn 0 marks.

2.11 Neurons were kept in a physiological solution. During the resting phase, the membrane potential in the axoplasm of neurons was negative compared to the extracellular space and a potential difference of -70 mV was observed in this phase. Neurons were then
treated in two different experiments with either gamma-amino butyric acid (GABA; an inhibitory neurotransmitter) or glutamate (an excitatory neurotransmitter) and the membrane potentials were recorded. Choose the correct statement/s:

(A) The resting membrane potential of -70 mV would not change with either GABA or glutamate treatments.

(B) The membrane potential would be even more negative than resting phase with GABA treatment.

(C) The membrane potential would be positive when the neuron was exposed to glutamate.

(D) The membrane potential would be more negative than resting potential after glutamate treatment.

2.12 *Agrobacterium*-mediated transformation is routinely used for the development of transgenic plants. In these experiments, a suitable plant part (explant) is incubated with *Agrobacterium* cells containing the gene of interest (passenger gene) that is to be introduced into the plant. In addition, a selectable marker gene is also used to ensure growth of only the transformed cells on regeneration media containing the selection agent (for e.g., antibiotics such as Kanamycin or Hygromycin). Based on such strategies, six independent transformation experiments of tobacco leaf explants were carried out using two different constructs - (1) construct I containing only a Hygromycin-resistance gene (*HYG*R) as a selection marker and (2) construct II with the *HYG*R gene as a selection marker and a passenger gene encoding a dephosphorylase enzyme (*DEP*) that inhibits phosphorylation in cells in which it is expressed. The marker as well as passenger genes were placed under transcriptional control of a constitutive promoter (CaMV35S), which expresses in all cells/tissues. The results obtained are tabulated below.

<table>
<thead>
<tr>
<th>Expt. No.</th>
<th>Construct I (CaMV35S- <em>HYG</em>R)</th>
<th>Construct II (CaMV35S-<em>DEP</em> + CaMV35S-<em>HYG</em>R)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No. of explants used</td>
<td>No. of transgenic plants obtained</td>
</tr>
<tr>
<td>1</td>
<td>200</td>
<td>45</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td>200</td>
<td>48</td>
</tr>
</tbody>
</table>

Which of the following statements could represent probable explanations of the above results?

(A) Regeneration medium for use of the selection marker in construct II was not optimised.

(B) Expression of the *DEP* gene is lethal to the transformed cells.

(C) Degradation of Hygromycin occurred in experiments with construct II.

(D) The *HYG*R gene used in construct II lost its functionality probably due to mutation(s) either in the gene or in the promoter.
2.13 Secretin and cholecystokinin (CCK) are two hormones acting on the pancreas. While hydrochloric acid (HCl) from the stomach predominantly stimulates release of secretin, peptones are potent stimulators of CCK secretion. Fatty acids however, stimulate release of both secretin as well as CCK. Secretin results in pancreatic secretion of bicarbonate ions which helps to set the pH for optimal functioning of pancreatic digestive enzymes. CCK stimulates secretion of pancreatic digestive enzymes. A graphical representation of the effects of HCl, fatty acids and peptones on the pancreas is shown below. Based on the this data, which of the following statement/s is/are correct?

(A) HCl would be required for optimal digestion of peptones.
(B) Compared to HCl and fatty acids, peptones are less effective in stimulating release of secretin.
(C) Compared to HCl, fatty acids are less potent stimulators of sodium bicarbonate secretion.
(D) HCl and peptone are equally potent stimulators of digestive enzymes.

2.14 In order to improve local cattle breeds in India, male cattle were procured from Australia. All imported male cattle had white skin (genotype bb). These were further bred with indigenous Indian female cattle having black skin (genotype BB or Bb). It was found that approximately 25% cattle of the F1 generation died soon after birth due to heart defects. The heart defects were found to be associated with either of two lethal genes, (D and Z), which cause this problem under recessive homozygous condition (dd or zz). The cattle that survived, as well as those that died, were black in colour. Which statement correctly describes the genetic make-up of the parent cattle?

(A) Genotype of Indian breed is (BBDDZZ) and Australian breed is (bbDDZz) where gene B and D are linked genes.
(B) Genotype of Indian breed is (BBDDzz) and Australian breed is (bbDDzz) where gene B and D are linked genes.
(C) Genotype of Indian breed is (BBDDZz) and Australian breed is (bbDDZz) where gene B and D are linked genes.
(D) Genotype of Indian breed is (BBddZZ) and Australian breed is (bbDdZZ) where gene B and D are linked genes.
Section 3: Chemistry

Marks for Section 3: 50

This section contains 14 questions.
For questions 3.1 to 3.10 only one of the 4 options is correct. A correct answer will earn 3 marks, a wrong answer will earn \((-1)\) mark, and an unattempted question will earn 0 marks.

3.1 The basicity of the following amines is measured in water and in chlorobenzene.

(i) \(\text{CH}_3\text{NH}_2\)  (ii) \((\text{CH}_3)_2\text{NH}\)  (iii) \((\text{CH}_3)_3\text{N}\)

The correct statement regarding the most basic compound in respective solvent is

(A) (ii) in water, (i) in chlorobenzene.
(B) (iii) in water, (i) in chlorobenzene.
(C) (iii) in water, (ii) in chlorobenzene.
(D) (ii) in water, (iii) in chlorobenzene.

3.2 Compound X contains C=62.0% and H=10.4%. X reacts with PCl\(_5\) to give Y which reacts with bromine under normal conditions. X is

(A) \(\text{CH}_3\text{CH}_2\text{CHO}\)  (B) \(\text{O}\)  (C) \(\text{OH}\)  (D) \(\text{O}^–\)

3.3 Benzene is reacted with limited amount of methyl chloride in the presence of anhydrous aluminium chloride to obtain X. X on reaction with bromine in the presence of iron powder gives a mixture of two compounds. The major product is reacted with 1-bromopropane in the presence of sodium/ether to obtain Y. The compound Y is

(A) \(\text{CH}_3\text{CH}_2\text{CH}_3\)  (B) \(\text{CH}_2\text{CH}_2\text{CH}_2\text{CH}_3\)

(C) \(\text{CH}_3\text{CH}_2\text{CH}_3\)  (D) \(\text{Br}\)
3.4 The molecule that has the smallest bond angle (X-M-X), where X is a halogen and M is the central atom, is

(A) PI₃    (B) AsI₃    (C) SbI₃    (D) SbBr₃

3.5 A crystal of KCl containing CaCl₂ as an impurity has a lower density than that of pure KCl, because it has

(A) vacant Cl⁻ sites.    (B) vacant K⁺ sites.
(C) K⁺ ions in the interstitial sites.    (D) Cl⁻ ions in the interstitial sites.

3.6 Pb₃O₄ is a mixed oxide of PbO and PbO₂ and reacts with hydrochloric and nitric acid in different ways. Select the correct statement with respect to this difference.

(A) PbO₂ does not react with HCl    (B) PbO does not react with HNO₃
(C) PbO₂ does not react with HNO₃    (D) PbO does not react with HCl

3.7 Atoms A and B are interacting to form bonds (σ, π). If z-axis is taken as the internuclear axis, the correct statement regarding the formation of a bond is

(A) pₓₐ and pₓₐ atomic orbitals contribute to π bond.
(B) dₓ²−y² and dₓ²−y² atomic orbitals contribute to π bond.
(C) pᵧₐ and pᵧₐ atomic orbitals contribute to σ bond.
(D) dₓ² and dᵧ² atomic orbitals contribute to σ bond.

3.8 The change in Gibbs free energy of vapourisation of 1 mol of H₂O at 1 atm and 100°C is (q_p is the heat change at constant P)

(A) q_p    (B) −TΔS    (C) 0    (D) 2q_p

3.9 Consider the cell Pt|Cl₂(P_L)|HCl(aq)|Cl₂(P_R)|Pt, where P_L and P_R are the Cl₂ pressures at the left and right electrodes. The standard electrode potential of Cl₂ + 2e⁻ → 2Cl⁻ is 1.36 eV. The correct expression for electrode potential (E) is (F is the Faraday constant)

(A) E = 1.36 − \frac{RT}{2F} \ln \frac{P_L}{P_R}
(B) E = −\frac{RT}{2F} \ln \frac{P_L}{P_R}
(C) E = −1.36 − \frac{RT}{2F} \ln \frac{P_L}{P_R}
(D) E = −\frac{RT}{F} \ln \frac{P_L}{P_R}
3.10 The paramagnetic species is

(A) KO₂  (B) N₂  (C) Na₂O₂  (D) CO

For questions 3.11 to 3.14 one or more than one of the 4 options may be correct. Your answer is regarded correct only if you choose all the correct option(s) and no incorrect option(s). A correct answer will earn 5 marks, a wrong answer or an unattempted question will earn 0 marks.

3.11 A sequence of reactions is shown below starting from an optically active compound P.

\[
\begin{align*}
P & \xrightarrow{\text{SOCl}_2} Q \xrightarrow{\text{C}_2\text{H}_5\text{NH}_2} R \\
\text{C}_4\text{H}_8\text{O}_3 & \xrightarrow{\text{C}_4\text{H}_6\text{Cl}_2\text{O}} \text{C}_6\text{H}_12\text{ClNO}
\end{align*}
\]

P does not react with 2,4-dinitrophenylhydrazine (2,4-DNP) to form a hydrazone. Q reacts very rapidly with one molar equivalent of ethylamine at low temperature to form R. Select the most appropriate statement(s).

(A) Q as well as R are optically active.

(B) Q on heating with an excess of ethylamine in the presence of a base gives a basic compound.

(C) R is a basic compound and can react with an acid to form a salt.

(D) An optically inactive isomer of P on heating can form compound S (C₄H₆O₂) which does not react with thionyl chloride.

3.12 The plot of solubility of some salts against temperature is given on the right. A salt like NaCl can be recovered fully, when dissolved in a solvent and the solvent is removed by evaporation. Choose the correct statement(s).

(A) No heat is absorbed or released when NaCl is dissolved in water.

(B) More heat is released when the ions of Yb₂(SO₄)₃ are solvated than that absorbed when the ions are separated.

(C) The heat of solution is negative for H₃BO₃.

(D) When AlCl₃ is dissolved in water and the water is evaporated, solid AlCl₃ is recovered.
3.13 The nitration of toluene takes place at the ortho, para and meta positions, giving ortho-, para-, and meta- nitrotoluenes, respectively. Assume that the meta product is formed in negligible amount and that the ortho and para substitutions are irreversible first-order reactions. Let the initial toluene concentration be $[A]_0$ and $k_1$ and $k_2$ be the ortho- and para- rate constants, respectively. Choose the correct statement(s).

(A) The rate of formation of the ortho substituted product is given by $k_1[A]_0 e^{-(k_1+k_2)t}$, where $t$ is the time.

(B) If the rate constant for the ortho product formation is twice that for the para product formation, then $[o\text{-nitrotoluene}] = 2[p\text{-nitrotoluene}]$, at any time.

(C) Nitration at the ortho- and para- positions is favorable compared to that at the meta position as the ortho- and para-intermediates are more stabilised than the meta-intermediate.

(D) If the methyl group of toluene is replaced by tertiary butyl group, more ortho product is expected.

3.14 The addition of hydrogen bromide to an unsymmetrical alkene in the presence of a peroxide does not follow Markovnikov rule. The mechanism is a radical mechanism and is illustrated below

(i) $\text{R-O-O-R} \rightarrow 2\text{R-O}$ $\Delta H = +146 \text{ kJ mol}^{-1}$
(ii) $\text{RO} + \text{HBr} \rightarrow \text{ROH} + \text{Br}$ $\Delta H = -96 \text{ kJ mol}^{-1}$
(iii) $\text{CH}_3\text{CH}=\text{CH}_2 + \text{Br} \rightarrow \text{CH}_3\text{CH}−\text{CH}_2−\text{Br}$ $\Delta H = -21 \text{ kJ mol}^{-1}$
(iv) $\text{CH}_3\text{CH}−\text{CH}_2−\text{Br} + \text{HBr} \rightarrow \text{CH}_3\text{CH}−\text{CH}_2−\text{CH}_2−\text{Br} + \text{Br}$ $\Delta H = -46 \text{ kJ mol}^{-1}$

Combination of any two radicals is a chain termination step and gives rise to a neutral molecule. Addition of HCl and HF under all conditions follows Markovnikov rule. (Given the bond energies: HF = 565 kJ mol$^{-1}$, HCl = 431 kJ mol$^{-1}$, HBr = 366 kJ mol$^{-1}$ and HI = 297 kJ mol$^{-1}$.) Choose the correct statement(s).

(A) In step (iii), the attack of bromine atom on propene takes place at the first carbon atom as it is less hindered.

(B) Step (ii) is endothermic if HF is used instead of HBr and exothermic if HI is used.

(C) Bromotrichloromethane adds to 1-hexene on heating in the presence of a peroxide to form $\text{CH}_3\text{(CH}_2)_3\text{CH(Br)}−\text{CH}_2\text{CCl}_3$.

(D) Endothermicity of step (i) makes the overall process very slow.
Section 4: Mathematics

This section contains 14 questions.
For questions 4.1 to 4.10 only one of the 4 options is correct. A correct answer will earn 3 marks, a wrong answer will earn \((-1)\) mark, and an unattempted question will earn 0 marks.

4.1 Let \(\omega\) be the complex cube root of unity. Consider the set

\[ E = \{(a, b, c) \in \mathbb{R}^3 : \det \begin{pmatrix} \omega & a & 1 \\ 0 & c & 1 \\ 1 & -b & \omega \end{pmatrix} = 0\} \]

Then \(E\) is

(A) an empty set.  
(B) a nonempty finite set.  
(C) a plane in \(\mathbb{R}^3\).  
(D) a straight line in \(\mathbb{R}^3\).

4.2 If \(\int_0^1 e^{x^2}(2x - a)dx = 0\), where \(a\) is any real number, then

(A) \(a > 2\)  
(B) \(a\) is negative.  
(C) \(a = 1/e^2\)  
(D) \(a\) lies between \(1/3\) and 2.

4.3 Let \(A\) be a \(2 \times 2\) matrix with real entries and \(\det(A) > 0\). If \(f\) is a real valued function on \(\mathbb{R}\), given by \(f(t) = \det ((t^2 - 2t + 2)A)\) then the range of \(f\) is

(A) \((-\infty, \infty)\)  
(B) \([0, \infty)\)  
(C) \((0, \infty)\)  
(D) a bounded interval in \(\mathbb{R}\).

4.4 In triangle \(PQR\), \(PQ = PR\) and the lengths of the sides are integers. Let \(E\) be the midpoint of \(QR\). If the lengths of \(QE, PE\) and \(PQ\) are in arithmetic progression, then the least possible value of the perimeter of \(PQR\) is

(A) 8 units.  
(B) 16 units.  
(C) 32 units.  
(D) 14 units.

4.5 If a line segment is the latus rectum of the parabola \(y^2 = 4cx\) and also a latus rectum of the ellipse \(x^2/a^2 + y^2/b^2 = 1\), \((a, b, c > 0)\), then the eccentricity of the ellipse is

(A) \(\sqrt{2} - 1\)  
(B) \(\frac{1}{\sqrt{2}}\)  
(C) \(\frac{1}{\sqrt{c}}\)  
(D) not obtainable from the data.

4.6 Let \(\vec{a}, \vec{b}, \vec{c}\) be three non-coplanar vectors in the three dimensional \(xyz\)-space. Which of the following vector(s) lies (lie) in the plane containing \(\vec{a}\) and \(\vec{b}\)?

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(i) \((\vec{a} \times \vec{b}) \times \vec{a}\); (ii) \((\vec{a} \times \vec{b}) \times \vec{b}\); (iii) \((\vec{a} \times \vec{b}) \times \vec{c}\).

(A) (i) only 
(B) (i) and (ii) only 
(C) (iii) only 
(D) (i), (ii) and (iii)

4.7 Let \(a, b, c, d\) be four integers (not necessarily distinct) in the set \(\{1, 2, 3, 4, 5\}\). The number of polynomials \(x^4 + ax^3 + bx^2 + cx + d\) which is divisible by \(x + 1\) is

(A) between 55 and 65. 
(B) between 66 and 85. 
(C) between 86 and 105. 
(D) more than 105.

4.8 Let \(a, b, c\) be distinct real numbers. Then the number of real solutions of

\[(x - a)^5 + (x - b)^3 + (x - c)\]

is

(A) 1 
(B) 3 
(C) 5 
(D) dependent on \(a, b,\) and \(c\).

4.9 The probability that a randomly chosen relation from a set \(A = \{1, 2, \ldots, n\}, n \geq 1\) to itself is a symmetric relation is

(A) \(2^{-\frac{n^2+n}{2}}\) 
(B) \(2^{-\frac{n^2}{2}}\) 
(C) \(2^{-\frac{n^2}{n}}\) 
(D) \(2^{\frac{n+1-n^2}{2}}\)

4.10 Let \(A\) and \(B\) be two fixed points on a fixed straight line. Two circles touch this line at \(A\) and \(B\) respectively and tangent to each other at \(M\). When the circles vary, the locus of \(M\) forms

(A) a circle with diameter \(AB\) minus points \(A\) and \(B\). 
(B) an ellipse with major axis \(AB\) minus points \(A\) and \(B\). 
(C) an ellipse with minor axis \(AB\) minus points \(A\) and \(B\). 
(D) a square with diagonal \(AB\) minus points \(A\) and \(B\).

For questions 4.11 to 4.14 one or more than one of the 4 options may be correct. Your answer is regarded correct only if you choose all the correct option(s) and no incorrect option(s). A correct answer will earn 5 marks, a wrong answer or an unattempted question will earn 0 marks.

4.11 Let \(f(x) = \left(\ln \left(\frac{7x-x^3}{12}\right)\right)^{3/2}\). Then

(A) \(f\) is defined only on \(\mathbb{R}^+\) and is strictly increasing.
(B) $f$ is defined on an interval of finite length and is strictly increasing.
(C) $f$ is defined on an interval of finite length and is bounded.
(D) range of $f$ contains 1.

4.12 Let $f(x) = e^{-\frac{1}{x^2}} + \int_0^{\pi x/2} \sqrt{1 + \sin t} \, dt$, $x \in (0, \infty)$.

Then,
(A) $f'$ exists and is continuous.
(B) $f''$ exists for all $x$.
(C) $f'$ is bounded.
(D) there exists $\alpha > 0$ such that $|f(x)| > |f'(x)|$ for every $x$ in $(\alpha, \infty)$.

4.13 Let $A, B, C, D, E$ be five real square matrices of the same order such that $ABCDE = I$, where $I$ is the unit matrix. Then,
(A) $B^{-1}A^{-1} = EDC$
(B) $BA$ is a nonsingular matrix
(C) $ABC$ commutes with $DE$
(D) $ABCD = \frac{1}{\det E} \text{Adj } E$.

4.14 Let $XY$ be a diameter of a circular pond of radius $R$. A vertical pole of height $H$ ($H < 2R$) is erected at $Y$. An observer at $X$ finds that the angle of elevation of the top of the pole is $\alpha$. He then walks along the circumference to a point $Z$ and finds that the angle of elevation of the top of the pole is $2\alpha$. If $L$ is the length of the minor arc $XZ$ then,
(A) $L < \frac{\pi R}{3}$
(B) $L > \frac{2\pi R}{3}$
(C) $L = 2R \cos^{-1} \left( \frac{4R^2 - H^2}{8R^2} \right)$
(D) $L = 2R \sin^{-1} \left( \frac{4R^2 - H^2}{8R^2} \right)$
Section 5: Physics

This section contains 14 questions.
For questions 5.1 to 5.10 only one of the 4 options is correct. A correct answer will earn 3 marks, a wrong answer will earn \((-1)\) mark, and an unattempted question will earn 0 marks.

5.1 Two identical particles are projected horizontally in opposite directions with a speed of 5 ms\(^{-1}\) each from the top of a tall tower as shown. Assuming \(g = 10\) ms\(^{-2}\), the distance between them at the moment when their velocity vectors become mutually perpendicular is

(A) 2.5 m  (B) 5 m  (C) 10 m  (D) 20 m

5.2 A particle of mass \(m\) goes counterclockwise around a square path ABCDA with vertices at \(A(-1, -1), B(1, -1), C(1, 1)\) and \(D(-1, 1)\) in a force, \(\vec{F} = \alpha x \sin (kx) \hat{x}\). Here, \(\alpha\) and \(k\) are positive constants. Consider the work done along the following paths:

(i) \(A \to B\)
(ii) \(B \to C\)
(iii) \(A \to B \to C \to D\)
(iv) \(A \to B \to C \to D \to A\)

The work done is zero for

(A) paths (i) and (ii) only.  (B) paths (i), (iii) and (iv) only.
(C) paths (ii), (iii) and (iv) only.  (D) paths (ii) and (iv) only.

5.3 A cylindrical steel rod of length 0.1 m and thermal conductivity 50 Wm\(^{-1}\)K\(^{-1}\) is welded end to end to a copper rod of thermal conductivity 400 Wm\(^{-1}\)K\(^{-1}\) and of the same area of cross section but 0.2 m long. One end of the copper rod is maintained at 0°C. The free end of the steel rod is maintained at a temperature of 200°C. Assuming that the rods are perfectly insulated, the temperature of the junction is

(A) 40°C  (B) 80°C  (C) 100°C  (D) 160°C

5.4 A standing wave pattern is formed when a plane electromagnetic wave from a transmitter falls on a perfect reflector at normal incidence. A receiver kept at a distance \(d\) from the reflector and along the line of normal incidence detects a minimum for a signal of frequency, \(\nu = 100\) MHz. When the frequency is decreased to \(\nu = 99.9\) MHz, the receiver records a maximum. Assuming \(c = 3 \times 10^8\) ms\(^{-1}\), the distance \(d\) is

(A) 75 m  (B) 1500 m  (C) 3000 m  (D) 750 m
5.5 An engine absorbs heat at a temperature of 727°C and exhausts heat at a temperature of 527°C. The engine operates at the maximum possible efficiency. The amount of work the engine performs for 2000 J of heat input can be

(A) 550 J  
(B) 1600 J  
(C) 400 J  
(D) 2000 J

5.6 A circular loop of radius \(a\), centered at origin has five positive charges equally spaced on it. All charges have a magnitude \(Q\), except the one at \((0,a)\) which has a magnitude \(q\), as shown in the figure. The electric field at the origin will be

\[
\begin{align*}
\text{(A)} & \quad \frac{1}{4\pi \varepsilon_0} \left( \frac{4Q + q}{a^2} \right) \hat{j} \\
\text{(B)} & \quad \frac{1}{4\pi \varepsilon_0} \left( \frac{Q - q}{a^2} \right) \hat{j} \\
\text{(C)} & \quad \frac{1}{4\pi \varepsilon_0} \left[ \frac{4Q a^2 \sin\left(\frac{2\pi}{5}\right)}{a} + \frac{q a^2}{a^2} \right] \hat{j} \\
\text{(D)} & \quad \frac{1}{4\pi \varepsilon_0} \left[ \frac{4Q a^2 \cos\left(\frac{2\pi}{5}\right)}{a} + \frac{q a^2}{a^2} \right] \hat{j}
\end{align*}
\]

5.7 An airplane of wingspan 50 m is traveling at 1800 kmph in a region where the earth’s magnetic field is \(4.0 \times 10^{-4}\) T. The maximum voltage that can possibly be developed between the ends of the wing is

(A) 10 V  
(B) 0 V  
(C) 2 V  
(D) 5 V

5.8 The mean momentum \(p\) of a particle has uncertainty \(\Delta p\). Taking Planck’s constant to be \(h\), the uncertainty in the de Broglie wavelength of the particle is

\[
\begin{align*}
\text{(A)} & \quad \frac{h}{\Delta p} \\
\text{(B)} & \quad \frac{h \Delta p}{p} \\
\text{(C)} & \quad 0 \\
\text{(D)} & \quad \frac{h \Delta p}{p^2}
\end{align*}
\]

5.9 The binding energy per nucleon of \(^{10}\text{X}\) is 8 MeV and that of \(^{11}\text{X}\) is 7.5 MeV where \(\text{X}\) represents an element. The minimum energy required to remove a neutron from \(^{11}\text{X}\) is

(A) 7.5 MeV  
(B) 2.5 MeV  
(C) 8 MeV  
(D) 0.5 MeV

5.10 Consider the following pairs of quantities:

(i) Coefficient of friction and relative permittivity
(ii) Breakdown voltage in lightning discharge and electric potential
(iii) Torque and kinetic energy
(iv) Light year and time period of simple harmonic motion

Which of the above pairs of quantities have the same dimensions?

(A) (i), (ii), and (iii) only  
(B) (ii) and (iii) only  
(C) (i), (ii), (iii) and (iv)  
(D) (i) and (iii) only
For questions 5.11 to 5.14 one or more than one of the 4 options may be correct. Your answer is regarded correct only if you choose all the correct option(s) and no incorrect option(s). A correct answer will earn 5 marks, a wrong answer or an unattempted question will earn 0 marks.

5.11 A particle with mass $m$ is at rest on top of a smooth massive infinitely long cylinder lying on a horizontal plane. The radius of the cylinder is $r$. A small push is applied to the particle to get it sliding down the cylindrical surface. The particle leaves the surface at a point $B$ on the cylinder which is at a height $h$ and reaches the ground at a distance $x$ from the cylinder.

A cross-sectional view is shown in the figure. The radius vector connecting the point $B$ makes an angle $\theta$ with the vertical. Then,

(A) $h$ is independent of $m$ and depends on $r$.
(B) $x$ is independent of $m$ and $r$.
(C) the speed of the particle at point $B$ is independent of $m$ and depends on $r$.
(D) the angle $\theta$ is greater than $45^0$.

5.12 Which statement(s) about ideal gases and their constituent molecules at a given temperature is/are correct?

(A) The mean free path of the molecules increases with decrease in pressure.
(B) The coefficient of isobaric volume expansion is the same for all ideal gases.
(C) The interaction between the molecules is negligible.
(D) The average translational kinetic energy of random motion of the molecules is different for different components in a gaseous mixture.

5.13 A siren produces a sound whose amplitude goes as $x = 10 \cos(3\omega t) \cos(2\omega t) + 5 \cos(5\omega t)$. Tuning forks with natural frequency $f$ will resonate with the siren for the value(s) of $f$ given by

(A) $f = \frac{5\omega}{2\pi}$
(B) $f = \frac{\omega}{2\pi}$
(C) $f = \frac{3\omega}{2\pi}$
(D) $f = \frac{\omega}{\pi}$

5.14 A loop of radius $r$ lies in the plane of the page as shown in the figure. The loop is shrunk radially at a constant rate $\alpha = |\frac{dr}{dt}|$ in a uniform magnetic field $\vec{B}$ that points into the plane of the paper. Then,

(A) the induced emf is proportional to the square of the radius of the loop.
(B) the induced emf depends on $\alpha$.
(C) the induced current will be clockwise in direction.
(D) the induced emf increases with time.