Sunday, July 29, 2007

1.00 pm - 4.00 pm

Question Booklet (A)

(This Question Booklet contains pages 1 to 32)

Note: Please return the Question Booklet to the room supervisor before leaving the examination hall.
Question Booklet (A)

Total Marks: 200

Time: 3 hours

Notes and Instructions

1. This question booklet contains 5 parts, with mark distribution as follows.

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Total = 20 + 3 x 60 = 200 marks

2. Part 1 (General) is compulsory. From parts 2 to 5, please answer any three parts of your choice, that is, please omit any one part from the 4 subjects: mathematics, physics, chemistry and biology.

3. Pocket calculators, log tables, cell phones, etc. are not permitted in the examination hall.

4. An Answer Folder is provided to you separately. Follow the instructions on the Answer Folder before filling in your answers. Remember to write all the particulars asked for in the Answer Folder.

5. Rough work should be done on the rough sheets provided separately.

6. Please return the Answer Folder and also the Question Booklet and the rough sheets to the room supervisor at the end of the examination. Do not staple any paper with the Answer Folder.

7. Read carefully the instructions given under each part of this Question Booklet.

8. For Question Booklet (A) you must use Answer Folder (A) only.
Part 1   General

Total marks for Part 1: 20

Note: This part contains 10 questions (1.1 to 1.10). For each question, only one of the 4 options is the correct answer. A correct answer will earn 2 marks, a wrong answer or an unattempted question will earn zero mark.

Passage for Questions 1.1 to 1.3

Read the passage below and answer the questions that follow:

There are four processes that the theory of evolution by natural selection explains satisfactorily:

Conservation refers to the fact that some species show no change over a long period in evolutionary history.

Modification is the process in which species undergo changes which are then passed on genetically to successive generations.

Branching is the appearance of two distinct and diverging life forms from a common ancestor.

Extinction is the cessation of offspring in a dwindling population.

As a simple model, suppose we consider a characteristic like ‘body size’ and assume that the population of a species is initially distributed about the mean value of the ‘body size’ according to the bell-shaped curve as shown below. In the graph, the x-coordinate represents the ‘body size’ and the y co-ordinate represents the population (per unit body size interval).
1.1 Suppose the temperature of the environment rises gradually over a long period and this puts those with mean size ($\mu$) at a disadvantage compared to both the lower and higher body sizes, of course only up to a limit on either side of $\mu$. A likely consequence of this is that

A) There will be gradual genetic mutation caused by the temperature rise which will eventually result in the extinction of the population.

B) The original species will split into two branches, one with a bell-shaped distribution about a lower size and the other with a bell-shaped distribution about a higher size.

C) The original bell-shaped distribution will become a flat distribution with population of all sizes (up to a limit on either side) being equally likely.

D) Because of interbreeding between higher and lower size members of the population, the original bell-shaped distribution will be preserved – an example of conservation.

1.2 Suppose the initial population of the species is randomly split into two groups and the two groups are separated geographically and isolated. Over many generations, it is found that the mean body size ($\mu$) and the shape of the distribution has not changed in spite of climatic changes. A likely reason for this is

A) The changed environments for the two groups were similar.

B) Body size does not affect the reproductive success of individuals in the changed environment.

C) The mean body size ($\mu$) is better adapted to the changed environments than both lower and higher sizes.

D) The changed environments reduce the width of the bell shaped distribution without altering its mean.

1.3 According to the theory of evolution by natural selection

A) Modification of species arises due to a natural instinct of survival of organisms.

B) Organisms change because of a natural desire to achieve a perfect fit with the environment.

C) Organisms modify themselves because the strong members naturally eliminate the weak in the competition for food.
D) Modification of species arises because the environmental changes put some members at a reproductive advantage over other members.

Passage for Questions 1.4 to 1.6

Read the passage below and answer the questions that follow:

In one method of cooling gases a constant high pressure is maintained on one side of a porous plug and a constant low pressure is maintained on its other side. As the gas goes from one side to the other there is a change in temperature. In this process (called throttling), a quantity called enthalpy remains unchanged. Figure below shows temperature-pressure (T-P) curves obtained for nitrogen gas, that are relevant to throttling. Each curve corresponds to a certain constant value of enthalpy, which differs from one curve to another. The slope of the curve at any point is called the Joule-Thomson coefficient and is denoted by μ. The solid dark curve in the figure (called inversion curve) is the locus of the maxima of the constant enthalpy T-P curves.
1.4 Choose the correct statement:

A) The inversion curve has \( \mu = 0 \) at every point on it.

B) \( \mu \) is maximum at every point of the inversion curve.

C) On the inversion curve, \( \mu \) increases to a maximum and then decreases to zero.

D) \( \mu \) is a non-zero constant at every point of the inversion curve.

1.5 Choose the correct statement:

A) Throttling of nitrozen gas always results in cooling whatever the initial temperature and pressure.

B) Throttling produces cooling of nitrogen only if the initial temperature is lower than 100 K, i.e. the lowest point of the inversion curve.

C) The magnitude of temperature drop from throttling is constant, independent of initial temperature and pressure.

D) For any given initial temperature between about 100 K to 620 K, the initial pressure that results in the greatest temperature drop is a point on the inversion curve.

1.6 Choose the correct statement:

A) All \((T,P)\) points inside the inversion curve have the same value of enthalpy.

B) For \((T,P)\) points outside the inversion curve, the enthalpy is always greater than that for the \((T,P)\) points inside the inversion curve.

C) Enthalpy is constant if either temperature or pressure of a gas is fixed.

D) A given value of enthalpy corresponds to many different values of \((T,P)\).

1.7 The ‘parallel postulate’ of Euclidean geometry states that through a point not on a line in a plane, there exists a single line parallel to the given line in the plane. Choose the correct statement concerning this postulate:

A) The ‘parallel postulate’ can be derived from the other postulates of Euclidean geometry and hence is not an independent postulate.
B) There were several failed attempts in the history of mathematics to prove the ‘parallel postulate’ from other Euclidean geometry postulates, but finally Bernhard Riemann succeeded in proving it.

C) The ‘parallel postulate’ is independent of the other Euclidean geometry postulates, giving the possibility of non-Euclidean geometries that do not satisfy this postulate.

D) The ‘parallel postulate’ is independent of the other Euclidean geometry postulates, but is satisfied by all Euclidean and non-Euclidean geometries.

1.8 Copernicus and later Kepler both favoured the heliocentric (sun at the centre and planets orbiting around it) view of the solar system. One important difference between the views of Copernicus and Kepler is:

A) The planetary orbits are circular according to Copernicus and elliptical according to Kepler.

B) The orbits are circular according to both, but Copernicus believed they are all coplanar while Kepler thought they are in different planes.

C) The orbits are elliptical according to both, but Keplerian orbits are supposed to precess slowly, i.e. the orbits are not fixed.

D) The orbits are circular according to both, but Copernicus thought circular motion is due to symmetry while Kepler regarded it as a consequence of the inverse-square law of force.

1.9 The scientific evidence for the existence of atoms/molecules got gradually built up in the following chronological order (from earlier to later times):


B) Dalton’s Laws of chemical combination, Kinetic theory of gases, Einstein’s explanation of Brownian motion.

C) Atomic hypothesis of Democritus, Thomson’s discovery of electron, Dalton’s Laws of chemical combination.


1.10 For a long time, proteins were believed to be the genetic material of a cell. The following experiments/discovery, however, established that only DNA could be the genetic material:
a) Fred Griffith’s discovery of transformation using virulent and non-virulent strains of *Streptococcus pneumoniae*.

b) Oswald Avery, Colin Macleod and Merclyn McCarty’s experiment on bacterial transformation using purified DNA.

c) Alfred D Hershey and Martha Chase’s experiment using T₂ bacteriophage.

d) James Watson and Francis Crick’s discovery of DNA double helical structure.

The correct chronological order of these experiments/discovery (from earlier to later times) is:

A) a, b, c, d

B) b, a, c, d

C) a, c, d, b

D) b, d, a, c

**Part 2  Mathematics**

*Total Marks for Part 2: 60*

*This part contains 16 questions.*

*For Questions 2.1 to 2.10, only one of the 4 options is the correct answer. A correct answer will earn 3 marks, a wrong answer will earn (-1) mark, and an unattempted question will earn zero mark.*

*For Questions 2.11 to 2.16, more than one of the 4 options are correct. Your answer is regarded correct only if you choose all the correct options and no incorrect options. A correct answer will earn 5 marks, and a wrong answer or an unattempted question will earn zero mark.*

2.1 The perimeter and area of a rectangle are respectively equal to the perimeter and area of a rhombus one of whose angles is 30°. The ratio of the longer to shorter side of the rectangle is:

A) \( \sqrt{3} + 1 \)  
B) \( 3 - \sqrt{2} \)  
C) \( \sqrt{5} - 1 \)  
D) \( 3 + 2\sqrt{2} \)

2.2 The number of diagonals of a convex polygon of 8 sides (octagon) is

A) 18  
B) 20  
C) 24  
D) 16
2.3 The co-efficient of \( t^2 \) in the expansion of

\[
(1 - 2tw + t^2)^{\frac{1}{2}} \quad (t \ll 1)
\]

is:

A) \( w \)  
B) \( w^2 \)  
C) \( \frac{1}{2} (3w^2 - 1) \)  
D) 1

2.4 The sum of all 3-digit numbers whose digits are all odd is:

A) 4,944,550  
B) 4,04,595  
C) 69,375  
D) 62,581

2.5 Let \( y^2 = 4ax \) be a parabola whose latus rectum is \( BC \). The line from \( B \) passing through the mid-point of the line joining the focus \( F \) and the origin \( O \) cuts the parabola again in \( A \). Then \( \tan \angle ABC + \tan \angle ACB \) equals

A) \( \sqrt{3} \)  
B) \( \frac{1}{2} \)  
C) \( \sqrt{3}/2 \)  
D) 1

2.6 If \( x \) is a positive real number, then the least possible value of \( (x + \frac{1}{9x}) \) is

A) \( \frac{1}{9} \)  
B) \( \frac{2}{3} \)  
C) \( \frac{1}{3} \)  
D) \( \frac{13}{18} \)

2.7 Mr and Mrs. X have two children. If one of them is known to be a girl, the probability that the other child is a girl is

A) \( \frac{1}{4} \)  
B) \( \frac{1}{2} \)  
C) \( \frac{3}{4} \)  
D) \( \frac{1}{3} \)

2.8 The value of \( S = \cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7} \) is

A) \( -\frac{1}{2} \)  
B) \( -\frac{3}{7} \)  
C) \( -\frac{4}{7} \)  
D) 0

[Hint: Multiply \( S \) by a suitable sine function.]

2.9 The number of positive real solutions of the equation

\[
x^2 - x - 1 = 2^x - \log_2(x^2 + 2^x)
\]

is

A) 2  
B) 1  
C) 0  
D) infinite
2.10 The limit of \( \frac{\sin^{-1} x - \sin x}{x^3} \) as \( x \to 0 \) is:

A) 0  
B) \( \frac{1}{3} \)  
C) \( \frac{1}{2} \)  
D) 1

2.11 The set of complex numbers \( z \neq 0 \) such that \( z + \frac{1}{z} \) is either real or purely imaginary, when represented in the \( xy \)-plane (Argand diagram) include all the points of

A) the union of the \( x \)--axis and the \( y \)--axis excluding the origin.
B) the curve \( y = x + \frac{1}{x} \), \( x \neq 0 \).
C) the unit circle \( |z| = 1 \)
D) the rectangular hyperbola \( xy = 1 \)

2.12 If \( \overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OC} \) are three distinct vectors lying in the same plane in space, then

A) \( \overrightarrow{OA} \cdot (\overrightarrow{OB} \times \overrightarrow{OC}) = 0 \)
B) \( \overrightarrow{OA} \times (\overrightarrow{OB} \times \overrightarrow{OC}) \) is a vector normal to the plane.
C) There exist three real numbers \( \alpha, \beta, \gamma \) (not all zero) such that
\[
\alpha \overrightarrow{OA} + \beta \overrightarrow{OB} + \gamma \overrightarrow{OC} = 0
\]
D) The determinant
\[
\begin{vmatrix}
\overrightarrow{OA} \cdot \overrightarrow{OA} & \overrightarrow{OA} \cdot \overrightarrow{OB} & \overrightarrow{OA} \cdot \overrightarrow{OC} \\
\overrightarrow{OB} \cdot \overrightarrow{OA} & \overrightarrow{OB} \cdot \overrightarrow{OB} & \overrightarrow{OB} \cdot \overrightarrow{OC} \\
\overrightarrow{OC} \cdot \overrightarrow{OA} & \overrightarrow{OC} \cdot \overrightarrow{OB} & \overrightarrow{OC} \cdot \overrightarrow{OC}
\end{vmatrix} = 0
\]

2.13 Consider the polynomial function
\[
f(x) = x^3 - 9x^2 + 23x - 15 \quad x \in (-\infty, \infty)
\]
A) \( f(x) \) assumes every real value at least once.
B) \( |f(x)| \) is differentiable everywhere.
C) \( f(x) \) has a local minimum in the interval \( 3 \leq x \leq 5 \)
D) for \( x < 0 \), the curve \( y = f(x) \) lies entirely in the third quadrant of the \( xy \)-plane.

2.14 Consider the integral

\[
I_n = \int_0^1 x^n \, dx
\]

A) For \( n \geq -1 \), the integral exists.

B) For \( n < 0 \) the integrand (i.e. \( x^n \)) is unbounded but finite over the open interval \((0, 1)\).

C) For any positive integer \( m \), the product

\[
\frac{1}{I_1 \cdots I_2 \cdots I_m} = \frac{1}{(m+1)!}
\]

D) For any positive integer \( m \)

\[
I_1 < I_2 < I_3 \ldots < I_m
\]

2.15 Consider the differential equation

\[
a \frac{dx}{dt} + bx = c
\]

where \( a, b, c \) are positive constants.

A) If \( x = f(t) \) is a solution of the equation, \( \alpha f(t) \) is also a solution of the equation, if \( \alpha \) is a constant.

B) Given that at \( t = 0, x = x_0, \) \( (x_0 \neq 0) \)

\[
x \to \frac{c}{b} \text{ as } t \to \infty
\]

C) Given that at \( t = 0, x = 0, \)

\[
\frac{dx}{dt} = 0 \text{ at } t = 0
\]

D) For the equation \( g \frac{dx}{dt} + hx = 0, \quad \frac{g}{h} > 0 \) \( (g, h \text{ are constants}) \)

\[
x \to 0 \text{ as } t \to \infty
\]
2.16 The system of equations

\[\begin{align*}
2x + 3y - z &= 4 \\
x + y + 3z &= 5 \\
ax + by + cz &= a + b + c
\end{align*}\]

has a unique solution \((1, 1, 1)\).

A) \(\begin{vmatrix}
2 & 3 & -1 \\
1 & 1 & 3 \\
a & b & c
\end{vmatrix} \neq 0\)

B) The equations represent three concurrent lines in space.

C) The equations represent three non-parallel planes in space, and the lines of intersection of the planes taken pair wise are concurrent.

D) Eliminating \(x\) in the first two of the equations, gives \(y - 7z = -6\), which represents a line in the \(yz\) plane in space.

Part 3 Physics

This part contains 16 questions.

For Questions 3.1 to 3.10, only one of the 4 options is the correct answer. A correct answer will earn 3 marks, a wrong answer will earn \((-1)\) mark, and an unattempted question will earn zero mark.

For Questions 3.11 to 3.16, more than one of the 4 options are correct. Your answer is regarded correct only if you choose all the correct options and no incorrect options. A correct answer will earn 5 marks, and a wrong answer or an unattempted question will earn zero mark.

3.1 The scattering of an electromagnetic wave by a charged particle of charge \(e\) and mass \(m\) is described in terms of the 'Thomson Cross Section' \(\sigma_T\), which has the dimension of area. On the basis of dimensional analysis, the possible formula for \(\sigma_T\) is

A) \(\sigma_T = \frac{8\pi}{3} \left( \frac{e^2}{4\pi \varepsilon_0 mc^2} \right)^2\)

B) \(\sigma_T = \frac{8\pi}{3} \left( \frac{e^2}{4\pi \varepsilon_0 mc^3} \right)\)

C) \(\sigma_T = \frac{8\pi}{3} \left( \frac{\mu_e e^2}{4\pi \varepsilon_0 m} \right)\)

D) \(\sigma_T = \frac{8\pi}{3} \left( \frac{e^2}{4\pi \varepsilon_0 m c} \right)\)

[The symbols have the usual meanings.]
3.2 A cubical container is placed on a frictionless horizontal surface. A molecule of a
gas in the container hits a vertical wall of the container normally and rebounds
with the same speed. In this elastic collision the container picks up

A) finite momentum, but negligible kinetic energy and velocity.
B) finite kinetic energy, but negligible momentum and velocity.
C) negligible momentum, kinetic energy and velocity.
D) finite momentum and finite kinetic energy but negligible velocity.

[Here ‘finite’ means ‘comparable to the corresponding molecular quantity’.]

3.3 A flexible conducting wire of rectangular shape (P) of sides $a$ and $b$ lies in the $xy$
plane. There is a uniform magnetic field along the +ve $z$-axis. The wire is
deformed continuously in the same plane from shape (P) to another rectangular
shape (Q) of sides $b$ and $a$. Let $\phi$ denote the magnetic flux through the loop.
Then, independent of how the loop is deformed,

A) $\phi$ is constant throughout the process.
B) Since $\phi_P = \phi_Q$, there is no induced emf in the wire at any time.
C) The net charge flowing through the wire in the entire process is zero.
D) There is no heat generated in the entire process.

3.4 In a double slit experiment, two identical point sources $S_1$ and $S_2$ are placed on the
line perpendicular to the two slits P and Q as shown. (The figure is not to scale.
The screen is placed far away compared to the distance between the slits.)
When only $S_1$ is switched on, there is an interference pattern with a peak intensity $I_1$. When only $S_2$ is switched on, there is an interference pattern with peak intensity $I_2$ ($I_2 < I_1$). When both $S_1$ and $S_2$ are switched on,

A) there will be a uniform intensity on the screen.

B) the intensity on the screen will vary from $I_1 - I_2$ to $I_1 + I_2$.

C) the intensity on the screen will vary from 0 to $(\sqrt{I_1} + \sqrt{I_2})^2$.

D) the intensity on the screen will vary from 0 to $I_1 + I_2$.

3.5 An infinite V-shaped wire carrying a steady current $I$ is shown in the figure. The magnitude of the magnetic field at a point $P$ ($AP = r$) due to the wire is:

\[
\begin{align*}
A) & \quad \frac{\mu_0 I}{2\pi r} \tan \theta \\
B) & \quad \frac{\mu_0 I}{2\pi r} \tan \frac{\theta}{2} \\
C) & \quad \frac{\mu_0 I}{2\pi r} \cot \frac{\theta}{2} \\
D) & \quad \frac{\mu_0 I}{2\pi r} \sin \theta
\end{align*}
\]
3.6 A black hole is an object from which even light cannot escape. A star of mass $M$, collapsing due to gravity, becomes a black hole when its radius is $R$. At this stage, the density $\rho$ of the black hole depends on $M$ as:

A) $\rho$ is proportional to $M$.
B) $\rho$ is independent of $M$.
C) $\rho$ is proportional to $M^{-1}$.
D) $\rho$ is proportional to $M^{-2}$.

3.7 Figure below shows the graph of potential energy $V(x)$ of a particle as a function of $x$. The positions 'b' and 'd' are local minima, 'c' is a local maximum, 'e–f' is a flat region of constant potential. The particle is released from rest at $x = a$. Subsequently the particle will

A) oscillate back and forth between $x = a$ and $x = g$.
B) oscillate back and forth between $x = a$ and $x = e$.
C) move to $x = g$ where it remains at rest.
D) move to $x = d$ where it remains at rest.

3.8 Electrons cannot remain confined in a nucleus. This is because

A) The size of an electron is about $10^{-10}$ m, which is much bigger than the typical size of a nucleus ($10^{-14}$ m to $10^{-15}$ m).

B) Electrons confined in a nucleus will, according to the Uncertainty Principle, have so much kinetic energy that even electron-proton attraction will not be able to hold them bound.
C) Electron–electron repulsion will dominate electron–proton attraction inside the nucleus.

D) Pauli’s exclusion principle forbids electrons to be bound together in the same nucleus.

3.9 2 moles of hydrogen gas at standard temperature ($T_0$) and pressure ($P_0$) are contained in a cylinder with a movable piston. The walls of the cylinder and also the piston are insulated. If the gas is compressed to one third of its original volume, the pressure of the gas will be

A) $[2]^{5/3} P_0$

B) $[2]^{7/5} P_0$

C) $[3]^{7/5} P_0$

D) $[3]^{-5/3} P_0$

3.10 An illuminated object is fixed at one end of an optical bench of length $L$ and a screen is fixed at the other end of the bench. A convex lens is to be used to obtain the image of the object on the screen. The maximum value of the focal length of the lens that can be used (without changing the object and screen positions) for the purpose is:

A) $\frac{L}{2}$

B) $L$

C) $\frac{L}{3}$

D) $\frac{L}{4}$

3.11 There is a spherically symmetric radial electric field in space as shown below. For $r > r_2$, $E(r)$ varies as $\frac{1}{r^2}$. The graph implies
A) $T$ (in Kelvin) $>> 273$ K.

B) Entropy of the system (water) decreases on freezing.

C) If the phase transition is considered reversible the decrease in entropy of the system is exactly compensated by the increase in entropy of the environment.

D) If water was supercooled and it then suddenly freezes to ice, the total entropy of the universe increases.

3.14 The displacement ($y$) of a wave disturbance in a medium as a function of $x$ – coordinate and time $t$ is given by

$$ y = \frac{2}{3 + (x - 4t)^2} $$

where $x$ and $y$ are in meters, and $t$ is in seconds.

A) The numerical constants 2, 3, 4 appearing on the right side are not dimensionless.

B) The wave is moving in the positive $x$ direction with a speed of 4 ms$^{-1}$, with its shape unchanged during propagation.

C) The wave has a definite wavelength of 4m, frequency 1ms$^{-1}$ and speed 4ms$^{-1}$.

D) At any instant the peak value of displacement is $\frac{2}{3}$ m.

3.15 Given below are statements on the rough magnitudes of some important physical quantities in different situations. Choose the approximately true statements as the correct options:

A) The wavelength of visible light is approximately $10^7$ to $10^8$ times the typical atomic sizes.

B) The binding energy per nucleon in a nucleus of intermediate mass number is approximately equal to the electron proton binding energy in the ground state of hydrogen atom.

C) A 10m x 10m room with the ceiling at a height of 4m has at standard temperature and pressure approximately $10^{28}$ molecules of air.

D) If sound takes 1 s to cover a certain distance in air, light will take approximately $10^{-6}$ s.
A) Both positive and negative charges exist in space.
B) Total charge inside a sphere of radius \( r = r_i \) is zero.
C) The total charge (in all space) is positive.
D) There is no charge distribution beyond \( r = r_z \).

3.12 Figure shows an open L-shaped glass tube of uniform cross section. One arm is immersed in a tub of water and the other (horizontal) arm remains in air. When the tube is rotated about the vertical arm with a constant angular velocity \( \omega \), water rises in the vertical arm up a height \( h \). \( P_0 \) is the atmosphere pressure.

\[ B \text{ is the mid-point of the arm } AC \text{ of length } L. \quad \rho_w \text{ is density of water and } g \text{ is acceleration due to gravity. Assuming air to be an ideal gas, pressures at the points } A, B, C \text{ and } D \text{ in the rotating tube satisfy the relations:} \]

A) \[ P_C = P_0 - h \rho_w g \quad ; \quad P_B = \frac{P_A + P_C}{2} \]

B) \[ P_C < P_B < P_0 \quad ; \quad P_D \text{ is nearly equal to } P_C \]

C) Along the arm \( CA \), pressure increases logarithmically with distance from \( C \), i.e. \( P(x) = P_C + \log \frac{x}{\alpha} \), where \( x \) is the distance from \( C \) and \( \alpha \) is a constant at the given temperature.

D) \[ P_C = P_0 \ e^{-\beta \omega^2 L^2} \], where \( \beta \) is a constant at the given temperature.

3.13 Water in a container is cooled slowly at constant pressure \( P \) which is much greater than standard atmosphere pressure. At a certain temperature \( T \) it freezes to ice.
3.16 A capacitor of capacitance \( C \) discharges through an inductor of inductance \( L \) in a circuit of low resistance \( R \).

A) The potential difference \( (V) \) across the capacitor will show damped oscillations as in the figure below:

![Graph showing damped oscillations](image)

B) The current \( (I) \) in the circuit will show an exponential decay as in the figure below:

![Graph showing exponential decay](image)

C) The potential difference across the resistance \( R \) will be rectified as

D) If \( C = 0 \) and \( L = 0 \) and the resistance is connected to an a.c source of frequency \( f \), power varies at twice the frequency (about a constant value) of either \( V \) or \( I \).
Part 4  Chemistry

This part contains 16 questions.

For Questions 4.1 to 4.10, only one of the 4 options is the correct answer. A correct answer will earn 3 marks, a wrong answer will earn (−1) mark, and an unattempted question will earn zero mark.

For Questions 4.11 to 4.16, more than one of the 4 options are correct. Your answer is regarded correct only if you choose all the correct options and no incorrect options. A correct answer will earn 5 marks, and a wrong answer or an unattempted question will earn zero mark.

4.1  Sodium chloride crystallizes in a face centered cubic lattice. The number of Cl⁻ ions surrounding any single Na⁺ ion as nearest neighbour is

A) 4  
B) 12  
C) 8  
D) 6

4.2  Among the following compounds, the one that will exhibit aromaticity is:

\[ \text{(A) } \text{(B) } \text{(C) } \text{(D) } \]

\[ \text{(A) } \text{(B) } \text{(C) } \text{(D) } \]

4.3  The pair of species which will have identical bond order consists of

A) CN⁻ and O₂⁻  
B) O₂⁻ and NO⁺  
C) CN⁻ and NO⁺  
D) CN⁺ and O₂⁻

4.4  The solubility product of a salt having the same molecular formula as that of Ca₃(PO₄)₂ is $1.08 \times 10^{-28}$ at 25°C. The total number of ions present in 1 litre of saturated solution of Ca₃(PO₄)₂ is approximately

A) $3 \times 10^{18}$  
B) $6 \times 10^{17}$  
C) $6.2 \times 10^{9}$  
D) $7.65 \times 10^{18}$
4.5 For a chemical reaction,

A) If $\Delta H$ is negative but $\Delta S$ is positive, the reaction will never take place.
B) If both $\Delta H$ and $\Delta S$ are positive, the reaction will always be spontaneous.
C) If $\Delta G^\circ$ for a reaction is positive, the reaction can still take place under suitable conditions.
D) If $\Delta G^\circ$ for the forward reaction is negative, only the forward reaction will take place and such reactions can never be reversible.

4.6 The standard electrode potential $E^\circ (\text{Fe}^{2+}/\text{Fe}) = -0.41 \text{ V}$ and $E^\circ (\text{Fe}^{3+}/\text{Fe}^{2+}) = 0.77 \text{ V}$. The value for $E^\circ (\text{Fe}^{3+}/\text{Fe})$ in Volts will be equal to

A) 0.36
B) -0.017
C) 0.188
D) 1.18

4.7 An aqueous solution of glucose containing 18 g of glucose in $X$ g of water freezes at $-0.2$ °C. The amount of NaCl to be dissolved in $X$ g of water to have a solution with the same freezing point is

A) 5.85 g
B) 18.0 g
C) 11.70 g
D) 2.93 g

4.8 The compound X undergoes ozonolysis to yield 5-oxo-2-methylhexanal. The structure of X is

(A) ![Structure A](image)
(B) ![Structure B](image)
(C) ![Structure C](image)
(D) ![Structure D](image)

4.9 The conductivity of an aqueous solution of acetic acid changes with addition of sodium hydroxide. The graph that best represents this conductivity behavior is
4.10  0.10 mL of liquid water is introduced into a 1.0 L flask at 27°C and the flask is then closed. The vapour pressure of water at that temperature is 3.567 x 10³ Pa the gas constant R = 8.314 J mol⁻¹ K⁻¹. The number of moles of water in the vapour phase when equilibrium is established is

A) 1.43 x 10⁻³
B) 5.56 x 10⁻³
C) 2.54 x 10⁻²
D) 4.45 x 10⁻⁴

4.11  The initial state of an ideal gas is represented by the point X. On reversible isothermal expansion, the state X changes to Y. On reversible adiabatic expansion the state X changes to Z. The graphs which correctly represent the processes are:

(A) [Diagram of P vs V with X, Y, Z]
(B) [Diagram of T vs S with X, Y, Z]
(C) [Diagram of T vs V with X, Y, Z]
(D) [Diagram of U vs T with X, Y, Z]

The Diels-Alder reaction, a concerted [4+2]-cycloaddition between a diene and an olefin to yield a cyclohexene, was discovered in 1928. In the reaction of benzoquinone with an excess of cyclopentadiene, an adduct (M) was obtained.

Theoretically six stereoisomers are possible for compound M. The structures of these stereoisomers are given below.

However, in the original reaction (formation of M from cyclopentadiene and benzoquinone) Diels and Alder found that only one of the above six conceivable stereoisomers is obtained. After prolonged heating (15h, 120°C) of the originally isolated stereoisomer M, they obtained two new stereoisomers P (20%) and R (70%). Choose the correct statements:

A) Only the stereoisomer represented by structure 2 is obtained as H atoms and the CH₂-bridge present on one bicyclic ring are trans to H atoms and the CH₂-bridge present on another bicyclic ring.

B) Only stereoisomer represented by structure 5 will be obtained as H atoms and the CH₂-bridge present on any bicyclic ring are trans to each other.

C) Higher rate of formation of R does not necessarily imply that the free energy change for M → R is more negative than that for M → P.

D) Low yield of P arises only when the process M → P involves large activation energy.

Water at its freezing point $T_1$, at pressure $P_1$ completely fills a strong steel container. The temperature is then reduced to $T_2$ at constant volume, with the pressure rising to $P_2$. Let the molar volume of water be $V_1$ at $(T_1, P_1)$ and $V_2$ at $(T_2, P_2)$, while the molar volume of ice at $(T_2, P_2)$ be $V_3$. Choose the correct statements for this situation:
A) There will not be any freezing of water since the constant volume condition does not allow expansion in volume due to conversion of water to ice.

B) Part of water will freeze and the fraction $f$ of water that will freeze is given by $f = (V_1 - V_2) / (V_3 - V_2)$.

C) Part of water will freeze since the contraction in volume of water due to lowering of temperature can make room for the expansion due to conversion of water to ice.

D) Part of water will freeze and the fraction of water that freezes is given by $f = C_v(T_1 - T_2) / \Delta H$, where $\Delta H$ is the latent heat of fusion and $C_v$ is the heat capacity of water.

4.14. Consider a first order consecutive reaction, without a reverse reaction:

$$P \underset{k_1}{\rightarrow} M \underset{k_2}{\rightarrow} F$$

where $k_1$ is the rate constant for the first step and $k_2$ is the rate constant for the second step.

A) The concentration of $P$ decreases exponentially with time like $e^{-k_1 t}$

B) For $k_2$ much greater than $k_1$ (say $\frac{k_2}{k_1} = 5$) the intermediate $M$ is used up rapidly and its concentration remains small.

C) For $k_1$ much greater than $k_2$ (say $\frac{k_1}{k_2} = 5$), the intermediate $M$ rises to a large value in the intermediate stages of the reaction.

D) If reverse reactions are included in both the steps; with $k_1'$ and $k_2'$ denoting respective backward rate constants

$$P \underset{k_1'}{\leftrightarrow} M \underset{k_1'}{\leftrightarrow} F$$

$$M \underset{k_2'}{\leftrightarrow} F \underset{k_2'}{\leftrightarrow} F$$

the equilibrium constant $K$ for the overall reaction is given by

$$K = \frac{k_2 k_1}{k_1' k_2'}$$

4.15. Copper (II) content of a sample is generally estimated by using iodometric method. In this method, excess of KI, either solid or solution, is added to the sample solution containing Cu(II) ions. The liberated $I_2$ is then titrated with Na$_2$S$_2$O$_3$ solution using starch as an indicator.

Data given below are for standard reduction potentials at 25 °C:

24
\[
\begin{align*}
\text{Cu}^{2+} + e^- &= \text{Cu}^+ & E^0 &= +0.15 \text{ V} \\
\text{I}_2 + 2e^- &= 2\text{I}^- & E^0 &= +0.54 \text{ V} \\
2\text{Cu}^{2+} + 4\text{I}^- &= 2\text{CuI} + \text{I}_2 & E^0 &= +0.86 \text{ V}
\end{align*}
\] (1) (2) (3)

Choose the correct statements for the above reactions:

A) The standard free energy change for the reaction between I\(^-\) and Cu\(^{2+}\) as calculated from \(E^0\) values of half cell reactions (1) and (2) is positive.

B) The presence of excess I\(^-\) leads to formation of sparingly soluble CuI precipitate. Formation of CuI makes the free energy change for the reaction negative. Thus the reaction becomes spontaneous only with excess of I\(^-\).

C) The acidic medium is necessary for the reaction between Cu\(^{2+}\) and I\(^-\), as in basic medium copper will get precipitated as Cu(OH)\(_2\) and will not react with I\(^-\).

D) In basic medium, the liberated I\(_2\) will be partly converted to I\(^-\) and OI\(^-\). Hence the reaction can be studied quantitatively.

4.16. An alloy containing Sn (II) and Pb(II) is generally analysed by complexometric titration technique. In such titrations, the reagent used for titration forms stable complexes with the metal ions. The reagent used for estimation of Sn and Pb is disodium salt of ethylene diamine tetraacetic acid (abbreviation: Na\(_2\)EDTA or Na\(_2\)H\(_2\)Y). Regardless of the charge on the metal ion, this titrant forms stable 1:1 complex with the metal ion.

The reactions of (Na\(_2\)H\(_2\)Y) with Sn(II) and Pb(II) are given below

\[
\text{Sn}^{2+} + \text{H}_2\text{Y}^{2-} = \text{SnY}^{2-} + 2\text{H}^+
\]

\[
\text{Pb}^{2+} + \text{H}_2\text{Y}^{2-} = \text{PbY}^{2-} + 2\text{H}^+
\]

The equilibrium constants for the above reactions (formation constants) are indicated by the symbol \(K_{MY}\). In the following figure, four curves are given showing variation in log \(K_{MY}\) as function of pH. Curves (3) and (4) represent formation of EDTA complexes of Pb(II) and Sn(II) in presence of F\(^-\) ions.
Choose the correct statements:

A) Total amount of Sn(II) and Pb(II) can be estimated above pH 7.
B) Addition of fluoride ions helps in quantitative estimation of Pb(II) in presence Sn (II) at suitable pH between 4 to 7.
C) Sn(II) can be estimated by direct titration in presence of Pb(II) by adding F⁻ ions and adjusting the pH of solution equal to 12.
D) Both Sn(II) and Pb(II) can be estimated independently in presence of F⁻ at pH = 6.

Part 5  Biology  Total Marks for Part 5:  60

This part contains 16 questions.

For Questions 5.1 to 5.10, only one of the 4 options is the correct answer. A correct answer will earn 3 marks, a wrong answer will earn (-1) mark, and an unattempted question will earn zero mark.

For Questions 5.11 to 5.16, more than one of the 4 options are correct. Your answer is regarded correct only if you choose all the correct options and no incorrect options. A correct answer will earn 5 marks, and a wrong answer or an unattempted question will earn zero mark.

5.1 Anhidrotic dysplasia is a condition in which development of sweat glands is prevented. It is caused by a mutation present on X-chromosome. A heterozygous female for this allele will

A) show absence of sweat glands on the body.
B) have a normal phenotype.
C) show mosaic pattern of presence and absence of sweat glands.
D) show suppressed activity of sweat glands.

5.2 The generalized life cycle of a moss is shown. The ploidy levels at stages A, B and C respectively are

[Diagram showing the life cycle of a moss with labels: Zygote, Female gametophyte, Male gametophyte, A sporophyte, B spores, C]
5.3 Allele ‘A’ is dominant over allele ‘a’ and results in black pigmentation of skin. In a mating of Aa x Aa, if 6 offspring are produced, the probability of all having black pigmentation is

A) 0.18  
B) 0.75  
C) 0.24  
D) 0.14

5.4 Utilization of fuel during migration and spawning of salmon fish is shown in the graph. Migration is a steady continuous upstream swimming while spawning is energetically expensive swimming behavior. Identify the types of biomolecules (P, Q and R) expended in the two activities:

![Graph showing energy expenditure]  

A) P: protein  
B) P: fat  
C) P: carbohydrate  
D) P protein  
Q: fat  
Q: protein  
Q carbohydrates  
R: carbohydrate  
R: carbohydrate  
R: fat  
R: fat

5.5 Mr. and Mrs. X have two children, a son and a daughter. The correct statement is:

A) The son and daughter received equal amount of chromosomal DNA from the father as well as the mother.

B) The daughter received more chromosomal DNA from the mother as compared to the father.
C) The son received more chromosomal DNA from the father compared to mother.

D) The son received more chromosomal DNA from the mother compared to father.

5.6 The pair in which the members would compete most with each other for food is
A) Lion and Elephant    B) Spider and Butterfly
C) Honey bee and Beetle  D) Horse and Zebra.

5.7 In a pond, population of two kinds of fish namely ‘Rohu’ and ‘Catla’ were estimated. 100 tagged Catla were released into the tank. In the fishing operation, 50 Catla and 30 Rohu were captured of which 5 of the Catla were found to be tagged. The population of the Rohu in the pond is
A) 500              B) 300
C) 1500             D) 30

5.8 A microscope has a 4x eye piece and 44x objective. The magnification one can obtain is
A) \( \frac{44}{4} = 11 \)    B) \( 44 \times 4 = 176 \)
C) \( 44 + 4 = 48 \)          D) \( (44) + (4)^2 = 60 \)

5.9 A mouse, a rabbit, a goat and a buffalo were given food proportionate to their body weight. Assuming that the food given to the goat was in the right amount, the correct statement is:
A) The rabbit and mouse are under-fed.
B) The rabbit and buffalo are over-fed.
C) The mouse and rabbit are over-fed.
D) The buffalo is under-fed.

5.10 Red blood cells were separated from blood. An excess quantity of a radioactive membrane impermeable compound capable of covalently linking up with free \( \text{NH}_2 \) group of a molecule was used to label the intact cells. On lysis of the labeled RBCs, radioactivity was found on 25% of the total proteins present in RBCs. This is because
A) RBCs have only 25% protein.
B) 25% of the RBC proteins have free NH₂ groups.
C) 25% proteins of the RBC are present on the surface.
D) the molecule did not react with 75% of the proteins present on the RBC surface.

5.11 A double stranded DNA was denatured in a mixture that included mRNA from a gene in the segment. The mixture was then cooled to promote RNA and DNA hybridization. The following figure reveals the results. Choose the correct interpretations:

A) Molecule I is DNA and II is RNA.
B) Source of DNA is bacterial cell.
C) Segment P is intron and Q is exon.
D) Region P is exon and Q is intron.

5.12 Part of the glycolytic pathway in a cell is a shown in the diagram.
Consider the following two situations:
(1) The cell has large supply of ATP.
(2) The cell has large supply of citrate.

Choose the correct interpretations:
A) Reaction A will be favoured in situation 1.
B) Reaction A will be favoured in situation 2.
C) Reaction B will be favoured in situation 1.
D) Reaction B will be favoured in situation 2.

5.13 Study the following metabolic pathways occurring in a cell.

If the cell receives excess of E, the consequences will be:
A) Flow of A in the pathway will decrease.
B) Concentration of D in the cell will increase.
C) Concentration of H in the cell will increase.
D) Flow of B and G in the pathway will remain unaffected.

5.14 Biological nitrogen fixation is given in the following equation:
$$N_2 + 8H^+ + 16ATP \rightarrow 2NH_3 + H_2 + 16Pi$$

Choose the correct statements:
A) The equation is incomplete because ADP molecules need to be included as products.
B) The equation is incorrect because nitrogen fixation needs O_2 and water is formed as a by-product.
C) The equation is incorrect because nitrogen fixation will yield ATP and not consume it.

D) The equation is incomplete because electrons need to be included as reactants.

5.15 For a mesophyte growing in a moist soil, water potential of its root sap is $-3$ atm, then water potential of moist soil and leaf may respectively be

A) $-1$ atm and $-5$ atm  
B) $+3$ atm and $-2$ atm  
C) $-2$ atm and $+5$ atm  
D) $-0.5$ atm and $-15$ atm

5.16 In drosophila, the following pattern of sex determination is observed in which $O$ represents autosomes:

- $OO$ XX: normal female
- $OO$ XY: normal male
- $OO$ XO: sterile male
- $OO$ XXY: normal female

Choose the correct statements:

A) The ratio of X chromosomes to autosomes determines the sex.
B) The presence of Y chromosome plays sex determining role.
C) The presence of X chromosomes decides the sex.
D) The Y chromosome is needed for fertility in males.

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