In each of the following, give the justification of the construction also:

1. Draw a line segment of length 7.6 cm and divide it in the ratio 5 : 8. Measure the two parts.

**Solution:**

Steps of Construction:

1. Draw line segment AB of 7.6 cm and draw any ray AX making an acute angle with AB.
2. Locate 13 (= 5 + 8) points A₁, A₂, A₃, A₄, . . . , A₁₃ on AX so that

   \[ AA₁ = A₁A₂ = A₂A₃ = . . . = A₁₂A₁₃ \]

4. Through the point A₅, draw a line parallel to A₁₃B (by making an angle equal to \( ∠AA₁₃B \)) at A₅ intersecting AB at the point C.

Now C is the point dividing line segment AB of 7.6 cm in the required ratio of 5 : 8.

We can measure the approximate lengths of AC and CB. The length of AC and CB comes to 2.9 cm and 4.7 cm respectively.

2. Construct a triangle of sides 4 cm, 5 cm and 6 cm and then a triangle similar to it whose sides are \( \frac{2}{3} \) of the corresponding sides of the first triangle.

**Solution:**

The steps of construction are as follows:
1. Draw a line segment $AB = 4$ cm. Taking point $A$ as centre draw an arc of $5$ cm radius. Similarly, taking point $B$ as its centre, draw an arc of $6$ cm radius. These arcs will intersect each other at point $C$. Now $AC = 5$ cm and $BC = 6$ cm and $\triangle ABC$ is the required triangle.

2. Draw any ray $AX$ making an acute angle with line $AB$ on opposite side of vertex $C$.

3. Locate 3 points $A_1, A_2, A_3$ (as 3 is greater between 2 and 3) on $AX$ such that $AA_1 = A_1A_2 = A_2A_3$.

4. Join $BA_3$ and draw a line through $A_2$ parallel to $BA_3$ to intersect $AB$ at point $B'$.

5. Draw a line through $B'$ parallel to the line $BC$ to intersect $AC$ at $C'$. $\triangle AB'C'$ is the required triangle.

Diagram:

3. Construct a triangle with sides $5$ cm, $6$ cm and $7$ cm and then another triangle whose sides are $\frac{2}{5}$ of the corresponding sides of the first triangle.

Solution:

The steps of construction are as follows:

1. Draw a line segment $AB$ of $5$ cm. Taking $A$ and $B$ as centre, draw arcs of $6$ cm and $7$ cm radius respectively. Let these arcs intersect each other at point $C$. $\triangle ABC$ is the required triangle having length of sides as $5$ cm, $6$ cm and $7$ cm respectively.

2. Draw any ray $AX$ making an acute angle with line $AB$ on opposite side of vertex $C$.

3. Locate 7 points $A_1, A_2, A_3, A_4, A_5, A_6, A_7$ (as 7 is greater between 5 and 7) on $AX$ such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7$.

4. Join $BA_5$ and draw a line through $A_7$ parallel to $BA_5$ to intersect extended line segment $AB$ at point $B'$. 
(5) Draw a line through $B'$ parallel to $BC$ intersecting the extended line segment $AC$ at $C'$. $\triangle AB'C'$ is required triangle.

![Diagram](https://via.placeholder.com/150)

4. Construct an isosceles triangle whose base is 8 cm and altitude 4 cm and then another triangle whose sides are $\frac{3}{2}$ times the corresponding sides of the isosceles triangle.

**Solution:**

Let $\triangle ABC$ be an isosceles triangle having $CA$ and $CB$ of equal lengths, base $AB$ is 8 cm and $AD$ is the altitude of length 4 cm.

Now, the steps of construction are as follows:

1. Draw a line segment $AB$ of 8 cm. Draw arcs of same radius on both sides of line segment while taking point $A$ and $B$ as its centre. Let these arcs intersect each other at $O$ and $O'$. Join $OO'$. Let $OO'$ intersect $AB$ at $D$.

2. Take $D$ as centre and draw an arc of 4 cm radius which cuts the extended line segment $OO'$ at point $C$. Now an isosceles $\triangle ABC$ is formed, having $CD$ (altitude) as 4 cm and $AB$ (base) as 8 cm.

3. Draw any ray $AX$ making an acute angle with line segment $AB$ on opposite side of vertex $C$.

4. Locate 3 points (as 3 is greater between 3 and 2) on $AX$ such that $AA_1 = A_1A_2 = A_2A_3$.

5. Join $BA_2$ and draw a line through $A_3$ parallel to $BA_2$ to intersect extended line segment $AB$ at point $B'$.

6. Draw a line through $B'$ parallel to $BC$ intersecting the extended line segment $AC$ at $C'$. $\triangle AB'C'$ is the required triangle.
5. Draw a triangle ABC with side BC = 6 cm, AB = 5 cm and ∠ABC = 60°. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC.

**Solution:**

The steps of construction are as follows:

1. Draw a line segment BC of length 6 cm. Draw an arc of any radius while taking B as centre. Let it intersect line BC at point O. Now taking O as centre draw another arc to cut the previous arc at point O'. Join BO' which is the ray making 60° with line BC.

2. Now draw an arc of 5 cm radius while taking B as centre, intersecting extended line segment BO' at point A. Join AC. ΔABC is having AB = 5 cm, BC = 6 cm and ∠ABC = 60°.

3. Draw any ray BX making an acute angle with BC on opposite side of vertex A.

4. Locate 4 points (as 4 is greater in 3 and 4). B₁, B₂, B₃, B₄ on line segment BX.

5. Join B₄C and draw a line through B₃, parallel to B₄C intersecting BC at C'.

6. Draw a line through C' parallel to AC intersecting AB at A'. ΔA'BC' is the required triangle.
6. Draw a triangle \( \triangle ABC \) with side \( BC = 7 \text{ cm} \), \( \angle B = 45^\circ \), \( \angle A = 105^\circ \). Then, construct a triangle whose sides are \( \frac{4}{3} \) times the corresponding sides of \( \triangle ABC \).

**Solution:**

\[ \angle B = 45^\circ, \angle A = 105^\circ \]

It is known that the sum of all interior angles in a triangle is \( 180^\circ \)

\[ \angle A + \angle B + \angle C = 180^\circ \]

\[ \Rightarrow 105^\circ + 45^\circ + \angle C = 180^\circ \]

\[ \Rightarrow \angle C = 180^\circ - 150^\circ = 30^\circ \]

Now, the steps of construction are as follows:

(1) Draw a line segment \( BC = 7 \text{ cm} \). Draw an arc of any radius while taking \( B \) as centre. Let it intersects \( BC \) at \( P \). Draw an arc from \( P \), of same radius as before, to intersect this arc at \( Q \). From \( Q \), again draw an arc, of same radius as before, to cut the arc at \( R \). Now from points \( Q \) and \( R \) draw arcs of same radius as before, to intersect each other at \( S \). Join \( BS \).

Let \( BS \) intersect the arc at \( T \). From \( T \) and \( P \) draw arcs of same radius as before to intersect each other at \( U \). Join \( BU \) which is making \( 45^\circ \) with \( BC \).

(2) Draw an arc of any radius taking \( C \) as its centre. Let it intersects \( BC \) at \( O \). Taking \( O \) as centre, draw an arc of same radius intersecting the previous arc at \( O' \). Now taking \( O \) and \( O' \) 's centre, draw arcs of same radius as before, to intersect each at \( Y \). Join \( CY \) which is making \( 30^\circ \) to \( BC \).

(3) Extend line segment \( CY \) and \( BU \). Let they intersect each other at \( A \). \( \triangle ABC \) is the triangle having \( \angle A = 105^\circ, \angle B = 45^\circ \) and \( BC = 7 \text{ cm} \).

(4) Draw any ray \( BX \) making an acute angle with \( BC \) on opposite side of vertex \( A \).

(5) Locate 4 points (as 4 is greater in 4 and 3) \( B_1, B_2, B_3 \) and \( B_4 \) on \( BX \).
(6) Join $B_3C$. Draw a line through $B_4$ parallel to $B_3C$ intersecting extended $BC$ at $C'$. 

(7) Through $C'$ draw a line parallel to $AC$ intersecting extended line segment $BA$ at $A'$. $\triangle A'BC'$ is required triangle.

7. Draw a right triangle in which the sides (other than hypotenuse) are of lengths 4 cm and 3 cm. Then construct another triangle whose sides are $\frac{5}{3}$ times the corresponding sides of the given triangle.

**Solution:**

The steps of construction are as follows:

(1) Draw a line segment $AB = 4\, \text{cm}$ and draw a ray $SA$ making $90^\circ$ with it.

(2) Draw an arc of $3\, \text{cm}$ radius while taking $A$ as its centre to intersect $SA$ at $C$. Join $BC$. $\triangle ABC$ is required triangle.

(3) Draw any ray $AX$ making an acute angle with $AB$ on the side opposite to vertex $C$.

(4) Locate 5 points (as 5 is greater in 5 and 3) $A_1, A_2, A_3, A_4, A_5$ on line segment $AX$.

(5) Join $A_3B$. Draw a line through $A_5$ parallel to $A_3B$ intersecting extended line segment $AB$ at $B'$.

(6) Through $B'$, draw a line parallel to $BC$ intersecting extended line segment $AC$ at $C'$. $\triangle AB'C'$ is required triangle.
EXERCISE 11.2

In each of the following, give also the justification of the construction:

1. Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths.

Solution:

The steps of construction are as follows:

(1) Taking any point O of the given plane as centre. Draw a circle of 6 cm radius. Locate a point P, 10 cm away from O. Join OP.

(2) Bisect OP. Let M be the midpoint of PO.

(3) Taking M as centre and MO as radius, draw a circle.

(4) Let this circle intersect our first circle at point Q and R.

(5) Join PQ and PR. PQ and PR are the required tangents. The length of tangents PQ and PR are 8 cm each.
2. Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also verify the measurement by actual calculation.

**Solution:**

The steps of construction are as follows:

1. Draw a circle of 4 cm radius with centre as O on the given plane.
2. Draw a circle of 6 cm radius taking O as its centre. Locate a point P on this circle and join OP.
3. Bisect OP. Let M be the midpoint of PO.
4. Taking M as its centre and MO as its radius draw a circle. Let it intersect the given circle at the points Q and R.
5. Join PQ and PR. PQ and PR are the required tangents.

Now, PQ and PR are of length 4.47 cm each.

In ΔPQO, since PQ is tangent, ∠PQO = 90°.

PO = 6 cm
QO = 4 cm

Applying Pythagoras theorem in ΔPQO,

\[ \text{PQ}^2 + \text{QO}^2 = \text{PO}^2 \]
\[ \Rightarrow \text{PQ}^2 + (4)^2 = (6)^2 \]
\[ \Rightarrow \text{PQ}^2 = 20 \]
\[ \therefore \text{PQ} = 2\sqrt{5} = 4.47 \text{ cm} \]
3. Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q.

**Solution:**

The steps of construction are as follows:

1. Taking any point O on given plane as centre, draw a circle of 3 cm radius.
2. Take one of its diameters, RS, extended it on both sides. Locate two points on this diameter such that OP = OQ = 7 cm.
3. Bisect OP and OQ. Let T and U be the midpoints of OP and OQ respectively.
4. Taking T and U as its centre, with TO and UO as radius, draw two circles. These two circles will intersect our circle at point V, W, X and Y respectively. Join PV, PW, QX and QY. These are required tangents.

![Diagram](image)

4. Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°.

Consider the above figure. PQ and PR are the tangents to the given circle.

If they are inclined at 60°, then \( \angle QPO = \angle OPR = 30° \)

Hence, \( \angle POQ = \angle POR = 60° \)

Consider \( \triangle QSO \),

\( \angle QOS = 60° \)

\( OQ = OS \) (radius)
So, \( \angle OQS = \angle OSQ = 60^\circ \)
\[ \therefore \triangle QSO \text{ is an equilateral triangle} \]
So, \( QS = SO = QO \text{ = radius} \)
\[ \angle PQS = 90^\circ - \angle OQS = 90^\circ - 60^\circ = 30^\circ \]
\[ \angle QPS = 30^\circ \]
\( PS = SQ \) (Isosceles triangle)
Hence, \( PS = SQ = OS \) (radius)

Now, the steps of construction are as follows:
1. Draw a circle of 5 cm radius and with centre O.
2. Take a point P on circumference of this circle. Extend OP to Q such that OP = PQ.
3. Midpoint of OQ is P. Draw a circle with radius OP with centre as P. Let it Intersect our circle at R and S. Join QR and QS. QR and QS are required tangents.

5. Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle.

**Solution:**

The steps of construction are as follows:
1. Draw a line segment AB of 8 cm. Taking A and B as centre, draw two circles of 4 cm and 3 cm radius.
2. Bisect the line AB. Let midpoint of AB is C. Taking C as centre draw a circle of radius AC which will intersect our circles at point P, Q, R and S. Join BP, BQ, AS and AR. These are our required tangents.
6. Let ABC be a right triangle in which AB = 6 cm, BC = 8 cm and \( \angle B = 90^\circ \). BD is the perpendicular from B on AC. The circle through B, C, D is drawn. Construct the tangents from A to this circle.

**Solution:**

In the following figure, it can be seen that if a circle is drawn through B, D and C, then BC will be its diameter as \( \angle BDC = 90^\circ \). The centre \( E \) of this circle will be the midpoint of BC.

The steps of construction are as follows:

1. Join AE and bisect it. Let \( F \) be the midpoint of AE.
2. Now with \( F \) as centre and radius FE, draw a circle intersecting the first circle at point B and G.
3. Join AG.

Thus, AB and AG are the required tangents.
7. Draw a circle with the help of a bangle. Take a point outside the circle. Construct the pair of tangents from this point to the circle.

**Solution:**

The steps of construction are as follows:

1. Draw a circle with bangle.
2. Take a point \( P \) outside this circle and take two non-parallel chords \( QR \) and \( ST \).
3. Draw perpendicular bisectors of these chords intersecting each other at point \( O \) which is centre of the given circle.
4. Join \( OP \) and bisect it. Let \( U \) be the midpoint of \( PO \). With \( U \) as centre and radius \( OU \), draw a circle, intersecting our first circle at \( V \) and \( W \). Join \( PV \) and \( PW \).

Thus, \( PV \) and \( PW \) are the required tangents.