CBSE NCERT Solutions for Class 10 Mathematics Chapter 1

Back of Chapter Questions

1. Use Euclid's division algorithm to find the HCF of:

(i) 135 and 225

(ii) 196 and 38220

(iii) 867 and 255

Solution:

(i) 135 and 225

Step 1:

Since 225 is greater than 135, we can apply Euclid's division lemma to $a = 225$ and $b = 135$ to find q and r such that

$225 = 135q + r, 0 \leq r < 135$

So, dividing 225 by 135 we get 1 as the quotient and 90 as remainder.

i.e $225 = (135 \times 1) + 90$

Step 2:

Remainder r is 90 and is not equal to 0, we apply Euclid's division lemma to $b = 135$ and $r = 90$ to find whole numbers q and r such that

$135 = 90 \times q + r, 0 \leq r < 90$

So, dividing 135 by 90 we get 1 as the quotient and 45 as remainder.

$i.e\ 135 = (90 \times 1) + 45$

Step 3:

Again, remainder r is 45 and is not equal to 0, so we apply Euclid's division lemma to $b = 90$ and $r = 45$ to find q and r such that

$90 = 45 \times q + r, \quad 0 \leq r < 45$

So, dividing 90 by 45 we get 2 as the quotientand 0 as remainder.

$i.e. 90 = (2 \times 45) + 0$

Step 4:

Since the remainder is zero, the divisor at this stage will be HCF of $(135, 225)$

Since the divisor at this stage is 45, therefore, the HCF of 135 and 225 is 45.

(ii) 196 and 38220

Step 1:

Since 38220 is greater than 196, we can apply Euclid's division lemma to $a = 38220$ and $b = 196$ to find whole numbers q and r such that

$$38220 = 196\,q + r, 0 \leq r < 196$$

So dividing 38220 by 196, we get 195 as the quotient and 0 as remainder r

$$i.e\ 38220 = (196 \times 195) + 0$$

Because the remainder is zero, divisor at this stage will be HCF

Since divisor at this stage is 196, therefore, HCF of 196 and 38220 is 196.

(iii) 867 and 255

Step 1:

Since 867 is greater than 255, we can apply Euclid's division lemma, to $a = 867$ and $b = 255$ to find q and r such that $867 = 255q + r, 0 \leq r < 255$. So, dividing 867 by 255 we get 3 as the quotient and 102 as remainder. $i.e\ 867 = 255 \times 3 + 102$

Step 2:

Since remainder is 102 and is not equal to 0, we can apply the division lemma to $a = 255$ and $b = 102$ to find whole numbers q and r such that $255 = 102q + r$ where $0 \leq r < 102$. So, dividing 255 by 102 we get 2 as the quotient and 51 as remainder. $i.e\ 255 = 102 \times 2 + 51$

Step 3:

Again remainder 51 is not equal to zero, so we apply the division lemma to $a = 102$ and $b = 51$ to find whole numbers q and r such that $102 = 51\,q + r$ where $0 \leq r < 51$. So, dividing 102 by 51 we get 2 as the quotient and 0 as remainder. $i.e\ 102 = 51 \times 2 + 0$. Since, the remainder is zero, the divisor at this stage is the HCF. Since the divisor at this stage is 51, therefore, HCF of 867 and 255 is 51

2. Show that any positive odd integer is of the form $6q + 1$, or $6q + 3$, or $6q + 5$, where q is some integer.

Solution:

Let a be any odd positive integer, we need to prove that a is in the form of $6q + 1$, or $6q + 3$, or $6q + 5$, where q is some integer.

Because a is an integer, we can consider b to be 6 as another integer. Applying Euclid's division lemma, we get

$a = 6q + r$ for some integer $q \geq 0$, and $r = 0, 1, 2, 3, 4, 5$ Where $0 \leq r < 6$.

Therefore, a can be any of the form $6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$

However, since a is odd, a cannot take the values $6q$, $6q + 2$ and $6q + 4$

(since all these are divisible by 2)

Also, $6q + 1 = 2 \times 3q + 1 = 2k_1 + 1$, where k_1 is a positive integer

$6q + 3 = (6q + 2) + 1 = 2(3q + 1) + 1 = 2k_2 + 1$, where k_2 is an integer

$6q + 5 = (6q + 4) + 1 = 2(3q + 2) + 1 = 2k_3 + 1$, where k_3 is an integer

Clearly, $6q + 1, 6q + 3, 6q + 5$ are in the form of $2k + 1$, where k is an integer.

Therefore, $6q + 1, 6q + 3$ and $6q + 5$ are odd numbers.

Therefore, any odd integer can be expressed in the form of

$6q + 1$, or $6q + 3$, or $6q + 5$ where q is some integer

3. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Solution:

Maximum number of columns in which the Army contingent and the band can march is equal to the HCF of 616 and 32. Euclid's algorithm can be used here to find the HCF.

Step 1:

Since 616 is greater than 32, so by applying Euclid's division lemma to $a = 616$ and $b = 32$ we get integers q and r as 19 and 8

$i.e\ 616 = 32 \times 19 + 8$

Step 2:

Since remainder r is 8 and is not equal to 0, we can again apply Euclid's lemma to 32 and 8 to get integers 4 and 0 as the quotient and remainder respectively.

$i.e\ 32 = 8 \times 4 + 0$

Step 3:

Since remainder is zero so divisor at this stage will be the HCF. The HCF$(616, 32)$ is 8. Therefore, they can march in 8 columns each.

4. Use Euclid's division lemma to show that the square of any positive integer is either of the form $3m$ or $3m + 1$ for some integer m.

[**Hint:** Let x be any positive integer then it is of the form $3q, 3q + 1 or 3q + 2$. Now square each of these and show that they can be rewritten in the form $3m$ or $3m + 1$.]

Solution:

Let x be any positive integer we need to prove that x^2 is in the form of $3m$ or $3m + 1$ where m is an integer.

Let $b = 3$ be the other integer, so by applying Euclid's division lemma to x and $b = 3$

Then $x = 3q + r$ for another integer $q \geq 0$ and $r = 0, 1, 2$ because $0 \leq r < 3$

Therefore, $x = 3q$, for r = 0 or $3q + 1$, for r=1 or $3q + 2$, for r=2

Now Consider x^2

$x^2 = (3q)^2$ or $(3q + 1)^2$ or $(3q + 2)^2$

$x^2 = (9q^2)$ or $9q^2 + 6q + 1$ or $9q^2 + 12q + 4$

$x^2 = 3 \times (3q^2)$ or $3(3q^2 + 2q) + 1$ or $3(3q^2 + 4q + 1) + 1$

$x^2 = 3k_1$ or $3k_2 + 1$ or $3k_3 + 1$

Where $k_1 = 3q^2, k_2 = 3q^2 + 2q$ and $k_3 = 3q^2 + 4q + 1$ since $q, 2, 3, 1$ etc are all integers, so, their sum and product will be integer

So k_1, k_2, k_3 are all integers.

Hence, it can be said that the square of any positive integer is either in the form of $3m$ or $3m + 1$ for any integer m.

5. Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m, 9m + 1$ or $9m + 8$.

Solution:

Let a be any positive integer and b equals to 3, then $a = 3q + r$, where $q \geq 0$ and $0 \leq r < 3$, $\therefore a = 3q$ or $3q + 1$ or $3q + 2$

Therefore, every number can be represented in these three forms. There are three cases.

Case 1:

When $a = 3q$ (r=0)

$a^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9$ m

Where m is an integer such that $m = 3q^3$

Case 2:

When $a = 3q + 1$ (r=1), $a^3 = (3q + 1)^3$

$a^3 = 27q^3 + 27q^2 + 9q + 1$

$a^3 = 9(3q^3 + 3q^2 + q) + 1$

$a^3 = 9m + 1$

Where m is an integer such that $m = (3q^3 + 3q^2 + q)$

Case 3:

When $a = 3q + 2$ (r=2)

$a^3 = (3q + 2)^3$

$a^3 = 27q^3 + 54q^2 + 36q + 8$

$a^3 = 9(3q^3 + 6q^2 + 4q) + 8$

$a^3 = 9m + 8$

Where m is an integer such that $m = (3q^3 + 6q^2 + 4q)$

Therefore, the cube of any positive integer is of the form $9m, 9m + 1$ or $9m + 8$.

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EXERCISE 1.2

1. Express each number as a product of its prime factors:

(i) 140

(ii) 156

(iii) 3825

(iv) 5005

(v) 7429

Solution:

(i) $140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$

(ii) $156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$

(iii) $3825 = 3 \times 3 \times 5 \times 5 \times 17 = 3^2 \times 5^2 \times 17$

(iv) $5005 = 5 \times 7 \times 11 \times 13$

(v) $7429 = 17 \times 19 \times 23$

2. Find the LCM and HCF of the following pairs of integers and verify that LCM $\times$ HCF $=$ product of the two numbers.

(i) 26 and 91

(ii) 510 and 92

(iii) 336 and 54

Solution:

(i) 26 and 91

$26 = 2 \times 13$

$91 = 7 \times 13$

13 is the largest number which divides both 26 and 91. So, HCF = 13.

$\text{LCM} = 2 \times 7 \times 13 = 182$

Product of the two numbers $= 26 \times 91 = 2366$

$\text{HCF} \times \text{LCM} = 13 \times 182 = 2366$

Hence, product of two numbers $= \text{HCF} \times \text{LCM}$

(ii) 510 and 92

$510 = 2 \times 3 \times 5 \times 17$

$92 = 2 \times 2 \times 23$

2 is the largest number which divides both 510 and 92. So, HCF = 2.

$\text{LCM} = 2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$

Product of the two numbers$= 510 \times 92 = 46920$

$HCF \times LCM = 2 \times 23460 = 46920$

Hence, product of two numbers $=$ HCF $\times$ LCM

(iii) 336 and 54

$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7 = 2^4 \times 3 \times 7$

$54 = 2 \times 3 \times 3 \times 3 = 2 \times 3^3$

6 is the largest number which divides both 336 and 54. So, HCF = 6.

$LCM = 2^4 \times 3^3 \times 7 = 3024$

Product of the numbers $= 336 \times 54 = 18144$

$HCF \times LCM = 6 \times 3024 = 18144$

Hence, product of two numbers $=$ HCF $\times$ LCM

3. Find the LCM and HCF of the following integers by applying the prime factorization method.

(i) $12, 15$ and 21

(ii) $17, 23$ and 29

(iii) $8, 9$ and 25

Solution:

(i) $12, 15$ and 21

$12 = 2^2 \times 3$

$15 = 3 \times 5$

$21 = 3 \times 7$

3 is the largest number which divides 12, 15 and 21. So, HCF = 3.

$LCM = 2^2 \times 3 \times 5 \times 7 = 420$

(ii) $17, 23$ and 29

$17 = 1 \times 17$

$23 = 1 \times 23$

$29 = 1 \times 29$

1 is the largest number which divides 17, 23 and 29. So, HCF = 1.

$\text{LCM} = 17 \times 23 \times 29 = 11339$

(iii) $8, 9$ and 25

$8 = 2 \times 2 \times 2$

$9 = 3 \times 3$

$25 = 5 \times 5$

1 is the largest number which divides 8, 9 and 25. So, HCF = 1.

$\text{LCM} = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 1800$

4. Given that $\text{HCF}\,(306, 657) = 9$, find $\text{LCM}\,(306, 657)$.

Solution:

$\text{HCF}\,(306, 657) = 9$

We know that, Product of two numbers is equal to product of their LCM and HCF.

$\therefore \text{LCM} \times \text{HCF} = 306 \times 657$

$$\text{LCM} = \frac{306 \times 657}{\text{HCF}} = \frac{306 \times 657}{9}$$

$\text{LCM} = 22338$

5. Check whether 6^n can end with the digit 0 for any natural number n.

Solution:

If any number ends with the digit 0, it should be divisible by 10 or in other words the prime factorization of the number must include 2 and 5 both.

Prime factorization of $6^n = (2 \times 3)^n$

We can see that 5 is not a prime factor of 6^n.

Hence, for any value of n, 6^n will not be divisible by 5.

Therefore, 6^n cannot end with the digit 0 for any natural number n.

6. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Solution:

There are two types of numbers, namely – prime and composite. Prime numbers have only two factors namely 1 and the number itself whereas composite numbers have factors other than 1 and itself.

It can be observed that

$7 \times 11 \times 13 + 13 = 13 \times (7 \times 11 + 1) = 13 \times (77 + 1)$

$= 13 \times 78$

$= 13 \times 13 \times 6$

The given expression has 6 and 13 as its factors. Therefore, it is a composite number.

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$

$= 5 \times (1008 + 1)$

$= 5 \times 1009$

1009 cannot be factorised further. Therefore, the given expression has 5 and 1009 as its factors. Hence, it is a composite number.

7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time and go in the same direction. After how many minutes will they meet again at the starting point?

Solution:

It is given that Ravi and Sonia do not take same of time. Ravi takes lesser time than Sonia for completing 1 round of the circular path.

As they are going in the same direction, they will meet again when Ravi will complete 1 round of that circular path with respect to Sonia.

i.e., When Sonia completes one round then Ravi completes 1.5 rounds. So they will meet first time at the time which is a common multiple of the time taken by them to complete 1 round

i.e.,LCM of 18 minutes and 12 minutes.

Now

$18 = 2 \times 3 \times 3 = 2 \times 3^2$

and, $12 = 2 \times 2 \times 3 = 2^2 \times 3$

LCM of 12 and 18 = product of factors raised to highest exponent $= 2^2 \times 3^2 = 36$

Therefore, Ravi and Sonia will meet together at the starting point after 36 minutes.

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EXERCISE 1.3

1. Prove that $\sqrt{5}$ is irrational.

Solution:

Let us assume, on the contrary, that $\sqrt{5}$ is a rational number.

Therefore, we can find two co-prime integers a, b $(\mathrm{b} \neq 0)$ such that $\sqrt{5} = \frac{a}{\mathrm{b}}$

$$\sqrt{5} = \frac{a}{\mathrm{b}}$$

$$\Rightarrow a = \sqrt{5}\,\mathrm{b}$$

$$\Rightarrow a^2 = 5\,\mathrm{b}^2$$

Therefore, a^2 is divisible by 5, then a is also divisible by 5.

So, $a^2 = (5\mathrm{k})^2 = 5(5\mathrm{k}^2) = 5\mathrm{b}^2$

$$\Rightarrow \mathrm{b}^2 = 5\mathrm{k}^2$$

This means that b^2 is divisible by 5 and hence, b is divisible by 5.

This implies that a and b have 5 as a common factor.

And this is a contradiction to the fact that a and b are co-prime.

So our assumption that $\sqrt{5}$ is rational is wrong.

Hence, $\sqrt{5}$ cannot be a rational number. Therefore, $\sqrt{5}$ is irrational.

2. Prove that $3 + 2\sqrt{5}$ is irrational.

Solution:

Let us assume, on the contrary, that $3 + 2\sqrt{5}$ is rational.

Therefore, we can find two integers a, b $(\mathrm{b} \neq 0)$ such that

$$3 + 2\sqrt{5} = \frac{a}{\mathrm{b}}$$

$$2\sqrt{5} = \frac{a}{b} - 3$$

$$\sqrt{5} = \frac{1}{2}\left(\frac{a}{b} - 3\right)$$

Since a and b are integers, $\frac{1}{2}\left(\frac{a}{b} - 3\right)$ will also be rational and therefore, $\sqrt{5}$ is rational.

This contradicts the fact that $\sqrt{5}$ is irrational.

Hence, our assumption that $3 + 2\sqrt{5}$ is rational is false.

Therefore, $3 + 2\sqrt{5}$ is irrational.

3. Prove that the following are irrationals:

(i) $\frac{1}{\sqrt{2}}$

(ii) $7\sqrt{5}$

(iii) $6 + \sqrt{2}$

Solution:

(i) $\frac{1}{\sqrt{2}}$

Let us assume that $\frac{1}{\sqrt{2}}$ is rational.

Therefore, we can find two integers a, b $(b \neq 0)$ such that

$$\frac{1}{\sqrt{2}} = \frac{a}{b}$$

$$\sqrt{2} = \frac{b}{a}$$

$\frac{b}{a}$ is rational as a and b are integers.

Therefore, $\sqrt{2}$ is rational.

This contradicts the fact that $\sqrt{2}$ is irrational.

Hence, our assumption is false and $\frac{1}{\sqrt{2}}$ is irrational.

(ii) $7\sqrt{5}$

Let us assume that $7\sqrt{5}$ is rational.

Therefore, we can find two integers a, b (b ≠ 0) such that $7\sqrt{5} = \frac{a}{b}$

$\therefore \sqrt{5} = \frac{a}{7b}$

$\frac{a}{7b}$ is rational as a and b are integers.

Therefore, $\sqrt{5}$ should be rational.

This contradicts the fact that $\sqrt{5}$ is irrational.

Therefore, our assumption that $7\sqrt{5}$ is rational is false.

Hence, $7\sqrt{5}$ is irrational.

(iii) $6 + \sqrt{2}$

Let $6 + \sqrt{2}$ be rational.

Therefore, we can find two integers a, b (b ≠ 0) such that$6 + \sqrt{2} = \frac{a}{b}$

$\sqrt{2} = \frac{a}{b} - 6$

Since a and b are integers, $\frac{a}{b} - 6$ is a rational number and hence, $\sqrt{2}$ should be rational.

This contradicts the fact that $\sqrt{2}$ is irrational.

Therefore, our assumption is false and hence, $6 + \sqrt{2}$ is irrational.

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EXERCISE 1.4

1. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(i) $\frac{13}{3125}$

(ii) $\frac{17}{8}$

(iii) $\frac{64}{455}$

(iv) $\frac{15}{1600}$

(v) $\frac{29}{343}$

(vi) $\frac{23}{2^3 5^2}$

(vii) $\frac{129}{2^2 5^7 7^5}$

(viii) $\frac{6}{15}$

(ix) $\frac{35}{50}$

(x) $\frac{77}{210}$

Solution:

(i) $\frac{13}{3125}$

$3125 = 5^5$

The denominator is of the form $2^m \times 5^n$.

Here, m=0 and n=5

Hence, the decimal expansion of $\frac{13}{3125}$ is terminating

(ii) $\frac{17}{8}$

$8 = 2^3$

The denominator is of the form $2^m \times 5^n$.

Here m=3 and n=0.

Hence, the decimal of expansion of $\frac{17}{8}$ is terminating.

(iii) $\frac{64}{455}$

$455 = 5 \times 7 \times 13$

Since the denominator is not in the form $2^m \times 5^n$, as it also contains 7 and 13 as its factors.

Hence, its decimal expansion will be non-terminating repeating.

(iv) $\frac{15}{1600}$

$1600 = 2^6 \times 5^2$

The denominator is of the form $2^m \times 5^n$.

Here m=6 and n=2.

Hence, the decimal expansion of $\frac{15}{1600}$ is terminating.

(v) $\frac{29}{343}$

$343 = 7^3$

Since the denominator is not in the form $2^m \times 5^n$, as it has 7 as its factor.

Hence, decimal expansion of $\frac{29}{343}$ is non-terminating repeating.

(vi) $\frac{23}{2^3 \times 5^2}$

The denominator is of the form $2^m \times 5^n$.

Here m=3 and n=2.

Hence, the decimal expansion of $\frac{23}{2^3 \times 5^2}$ is terminating.

(vii) $\frac{129}{2^2 \times 5^7 \times 7^5}$

Since the denominator is not of the form $2^m \times 5^n$ as it also has 7 as its factor.

The decimal expansion of $\frac{129}{2^2 \times 5^7 \times 7^5}$ is non-terminating repeating.

(viii) $\frac{6}{15} = \frac{2 \times 3}{3 \times 5} = \frac{2}{5}$

The denominator is of the form $2^m \times 5^n$.

Here m=0 and n=1.

Hence, the decimal expansion of $\frac{6}{15}$ is terminating.

(ix) $\frac{35}{50}$

The denominator is of the form $2^m \times 5^n$

Here m=1 and n=2

Hence, the decimal expansion of $\frac{35}{50}$ is terminating.

(x) $\frac{77}{210} = \frac{11 \times 7}{30 \times 7} = \frac{11}{30}$

$30 = 2 \times 3 \times 5$

Since the denominator is not in the form $2^m \times 5^n$, as it also has 3 as its factor. Hence, the decimal expansion of $\frac{77}{210}$ is non-terminating repeating.

2. Write down the decimal expansions of those rational numbers in Question1 above which have terminating decimal expansions.

Solution:

(i) $\frac{13}{3125} = 0.00416$

$$\begin{array}{r} 0.00416 \\ 3125\overline{)13.00000} \end{array}$$

0
130
0
1300
0
13000
12500
5000
3125
18750
18750
×

(ii) $\frac{17}{8} = 2.125$

$$\begin{array}{r} 2.125 \\ 8\overline{)\ 17} \end{array}$$

16
10
8
20
16
40
40
×

(iv) $\frac{15}{1600} = 0.009375$

$$\begin{array}{r} 0.009375 \\ 1600\overline{)15.000000} \end{array}$$

$$\begin{array}{r} 0 \\ \hline 150 \\ 0 \\ \hline 1500 \\ 0 \\ \hline 15000 \\ 14400 \\ \hline 6000 \\ 4800 \\ \hline 12000 \\ 11200 \\ \hline 8000 \\ 8000 \\ \hline \times \end{array}$$

(vi) $\frac{23}{2^3 \times 5^2} = \frac{23}{200} = 0.115$

$$200\overline{)23.00}\quad 0.115$$

$$\begin{array}{r} 0 \\ \hline 230 \\ 200 \\ \hline 300 \\ 200 \\ \hline 1000 \\ 1000 \\ \hline \times \end{array}$$

(viii) $\frac{6}{15} = \frac{2 \times 3}{3 \times 5} = \frac{2}{5} = 0.4$

$$5\overline{)2.0}\quad 0.4$$

$$\begin{array}{r} 0 \\ \hline 20 \\ 20 \\ \hline \times \end{array}$$

(ix) $\frac{35}{50} = 0.7$

$$\begin{array}{r} 0.7 \\ 50\overline{)35.0} \\ 0 \\ \hline 350 \\ 350 \\ \hline \times \end{array}$$

3. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form $\frac{p}{q}$, what can you say about the prime factors of q?

(i) 43.123456789

(ii) $0.120120012000120000\ldots$

(iii) $43.\overline{123456789}$

Solution:

(i) 43.123456789

Since this has a terminating decimal expansion, it is a rational number of the form $\frac{p}{q}$ and q is of the form $2^m \times 5^n$,

i.e., the prime factors of q will be either 2 or 5 or both.

(ii) 0.120120012000120000 ...

The decimal expansion is neither terminating nor recurring. Therefore, the given number is an irrational number.

(iii) $43.\overline{123456789}$

Since the decimal expansion is non-terminating recurring, the given number is a rational number of the form $\frac{p}{q}$ and q is not of the form $2^m \times 5^n$ i.e., the prime factors of q will also have a factors other than 2 or 5.

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