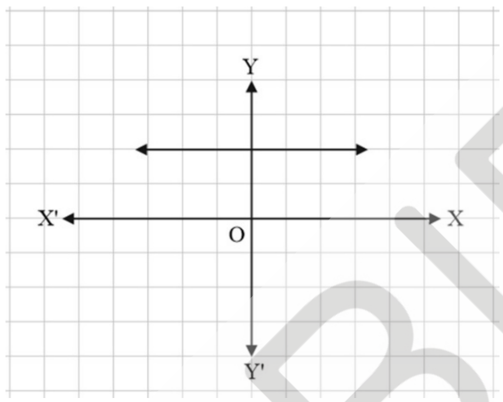
CBSE NCERT Solutions for Class 10 Mathematics Chapter 2

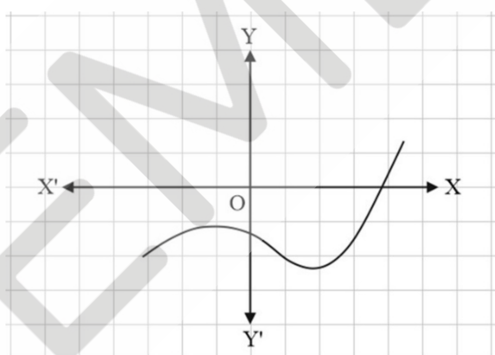
Back of Chapter Questions

1. The graphs of $y = p(x)$ are given in following figure, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.

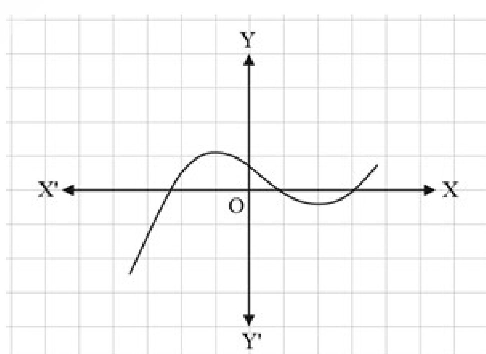
(i)



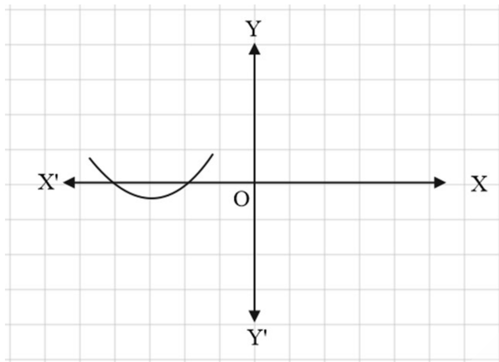
(ii)



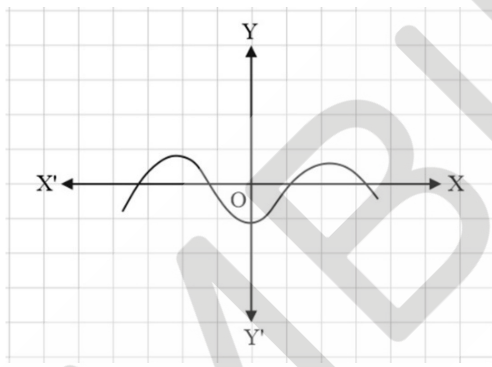
(iii)



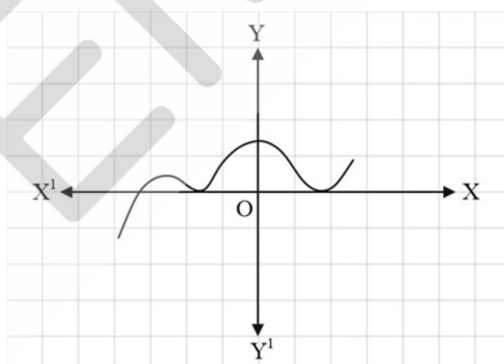
(iv)



(v)



(vi)



Solution:

(i) Since the graph of $\boldsymbol{p}(x)$ does not cut the X-axis at all. Therefore, the number of zeroes is **0.**

(ii) As the graph of $\boldsymbol{p}(x)$ intersects the X-axis at only **1** point. Therefore, the number of zeroes is **1.**

(iii) Since the graph of $\boldsymbol{p}(x)$ intersects the X-axis at **3** points. Hence, the number of zeroes is 3.

(iv) As the graph of $\boldsymbol{p}(x)$ intersects the X-axis at **2** points. So, the number of zeroes is **2.**

(v) Since the graph of $\boldsymbol{p}(x)$ intersects the X-axis at **4** points. Therefore, the number of zeroes is **4.**

(vi) As the graph of $\boldsymbol{p}(x)$ intersects the X-axis at **3** points. So, the number of zeroes is **3.**

♦ ♦ ♦

EXERCISE 2.2

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) $x^2 - 2x - 8$

(ii) $4s^2 - 4s + 1$

(iii) $6x^2 - 3 - 7x$

(iv) $4u^2 + 8u$

(v) $t^2 - 15$

(vi) $3x^2 - x - 4$

Solution:

(i) $x^2 - 2x - 8$

$= x^2 - 4x + 2x - 8$ [Factorisation by splitting the middle term]

$= x(x - 4) + 2(x - 4)$

$= (x - 4)(x + 2)$

We know that the zeroes of the quadratic polynomial $ax^2 + bx + c$ are the same as the roots of the quadratic equation $ax^2 + bx + c = 0$.

Therefore, by equating the given polynomial to zero. We get,

$x^2 - 2x - 8 = 0$

$\Rightarrow (x - 4)(x + 2) = 0$

$\Rightarrow x - 4 = 0$ or $x + 2 = 0$

$\Rightarrow x = 4$ or $x = -2$

Therefore, the zeroes of $x^2 - 2x - 8$ are 4 and -2.

Sum of zeroes $= 4 - 2 = 2 = \frac{-(-2)}{1} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $= 4 \times (-2) = -8 = \frac{(-8)}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Hence, the relationship between the zeroes and the coefficients is verified.

(ii) $4s^2 - 4s + 1 = (2s - 1)^2$ [Since, $a^2 - 2ab + b^2 = (a - b)^2$]

We know that the zeroes of the quadratic polynomial $ax^2 + bx + c$ are the same as the roots of the quadratic equation $ax^2 + bx + c = 0$.

Therefore, by equating the given polynomial to zero. We get,

$4s^2 - 4s + 1 = 0$

$\Rightarrow (2s - 1)^2 = 0$

Cancelling square on both the sides,

$\Rightarrow 2s - 1 = 0$

$\Rightarrow s = \frac{1}{2}$

Therefore, the zeroes of $4s^2 - 4s + 1$ are $\frac{1}{2}$ and $\frac{1}{2}$.

Sum of zeroes $= \frac{1}{2} + \frac{1}{2} = 1 = \frac{-(-4)}{4} = \frac{-(\text{Coefficient of } s)}{(\text{Coefficient of } s^2)}$

Product of zeroes $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{\text{Constant term}}{\text{Coefficient of } s^2}$

Hence, the relationship between the zeroes and the coefficients is verified.

(iii) $6x^2 - 3 - 7x = 6x^2 - 7x - 3$

$= 6x^2 - 9x + 2x - 3$ [Factorisation by splitting the middle term]

$= 3x(2x - 3) + (2x - 3)$

$= (3x + 1)(2x - 3)$

We know that the zeroes of the quadratic polynomial $ax^2 + bx + c$ are the same as the roots of the quadratic equation $ax^2 + bx + c = 0$.

Therefore, by equating the given polynomial to zero. We get,

$6x^2 - 3 - 7x = 0$

$\Rightarrow 3x + 1 = 0$ or $2x - 3 = 0$

$\Rightarrow x = \frac{-1}{3}$ or $x = \frac{3}{2}$

Therefore, the zeroes of $6x^2 - 3 - 7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$

Sum of zeroes $= \frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $= \frac{-1}{3} \times \frac{3}{2} = \frac{-1}{2} = \frac{-3}{6} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Hence, the relationship between the zeroes and the coefficients is verified.

(iv) $4u^2 + 8u = 4u^2 + 8u + 0 = 4u(u + 2)$

We know that the zeroes of the quadratic polynomial $ax^2 + bx + c$ are the same as the roots of the quadratic equation $ax^2 + bx + c = 0$.

Therefore, by equating the given polynomial to zero. We get,

$4u^2 + 8u = 0$

$\Rightarrow 4u = 0$ or $u + 2 = 0$

$\Rightarrow u = 0$ or $u = -2$

So, the zeroes of $4u^2 + 8u$ are 0 and -2.

Sum of zeroes $= 0 + (-2) = -2 = \frac{-8}{4} = \frac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2}$

Product of zeroes $= 0 \times (-2) = 0 = \frac{0}{4} = \frac{\text{Constant term}}{\text{Coefficient of } u^2}$

Hence, the relationship between the zeroes and the coefficients is verified.

(v) $t^2 - 15 = t^2 - 0.t - 15 = (t - \sqrt{15})(t + \sqrt{15})$ [Since, $a^2 - b^2 = (a+b)(a-b)$]

We know that the zeroes of the quadratic polynomial $ax^2 + bx + c$ are the same as the roots of the quadratic equation $ax^2 + bx + c = 0$.

Therefore, by equating the given polynomial to zero. We get,

$t^2 - 15 = 0$

$\Rightarrow t - \sqrt{15} = 0$ or $t + \sqrt{15} = 0$

$\Rightarrow t = \sqrt{15}$ or $t = -\sqrt{15}$

Therefore, the zeroes of $t^2 - 15$ are $\sqrt{15}$ and $-\sqrt{15}$

Sum of zeroes $= \sqrt{15} + (-\sqrt{15}) = 0 = \frac{-0}{1} = \frac{-(\text{Coefficient of } t)}{(\text{Coefficient of } t^2)}$

Product of zeroes $= (\sqrt{15})(-\sqrt{15}) = -15 = \frac{-15}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Hence, the relationship between the zeroes and the coefficients is verified.

(vi) $3x^2 - x - 4$

$= 3x^2 - 4x + 3x - 4$ [Factorisation by splitting the middle term]

$= x(3x - 4) + (3x - 4)$

$= (3x - 4)(x + 1)$

We know that the zeroes of the quadratic polynomial $ax^2 + bx + c$ are the same as the roots of the quadratic equation $ax^2 + bx + c = 0$.

Therefore, by equating the given polynomial to zero. We get,

$3x^2 - x - 4 = 0$

$\Rightarrow 3x - 4 = 0$ or $x + 1 = 0$

$\Rightarrow x = \frac{4}{3}$ or $x = -1$

Hence, the zeroes of $3x^2 - x - 4$ are $\frac{4}{3}$ and -1.

Sum of zeroes $= \frac{4}{3} + (-1) = \frac{1}{3} = \frac{-(-1)}{3} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $= \frac{4}{3}(-1) = \frac{-4}{3} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Hence, the relationship between the zeroes and the coefficients is verified.

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\mathbf{\frac{1}{4}, -1}$

(ii) $\mathbf{\sqrt{2}, \frac{1}{3}}$

(iii) $\mathbf{0, \sqrt{5}}$

(iv) $\mathbf{1, 1}$

(v) $\mathbf{-\frac{1}{4}, \frac{1}{4}}$

(vi) $\mathbf{4, 1}$

Solution:

(i) We know that if α *and* β are the zeroes of a quadratic polynomial $p(x)$, then, the polynomial $p(x)$ can be written as $p(x) = a\{x^2 - (\alpha + \beta)x + \alpha\beta\}$ or $p(x) = a\{x^2 - (\text{Sum of the zeroes})x + \text{Product of the zeroes}\}$, where a is a non-zero real number.

Given: sum of the roots = $\alpha + \beta = \frac{1}{4}$ and product of the roots = $\alpha\beta = -1$

Hence, the quadratic polynomial $p(x)$ can be written as:

$$p(x) = a\{x^2 - \frac{1}{4}x - 1\}$$

$$= a\left\{\frac{4x^2 - x - 4}{4}\right\}$$

By taking $a = 4$, we get one of the quadratic polynomials which satisfy the given conditions.

Therefore, the quadratic polynomial is $(4x^2 - x - 4)$.

(ii) We know that if α *and* β are the zeroes of a quadratic polynomial $p(x)$, then, the polynomial $p(x)$ can be written as $p(x) = a\{x^2 -$

$(\alpha+\beta)x+\alpha\beta\}$ or $p(x)=a\{x^2-(\text{Sum of the zeroes})x+\text{Product of the zeroes}\}$, where a is a non-zero real number.

Given: sum of the roots = $\alpha+\beta=\sqrt{2}$ and product of the roots = $\alpha\beta=\frac{1}{3}$

Hence, the quadratic polynomial $p(x)$ can be written as:

$$p(x)=a\{x^2-\sqrt{2}x+\frac{1}{3}\}$$

$$=a\left\{\frac{3x^2-3\sqrt{2}x+1}{3}\right\}$$

By taking $a=3$, we get one of the quadratic polynomials which satisfy the given conditions.

Therefore, the quadratic polynomial is $(3x^2-3\sqrt{2}x+1)$.

(iii) We know that if α *and* β are the zeroes of a quadratic polynomial $p(x)$, then, the polynomial $p(x)$ can be written as $p(x)=a\{x^2-(\alpha+\beta)x+\alpha\beta\}$ or $p(x)=a\{x^2-(\text{Sum of the zeroes})x+\text{Product of the zeroes}\}$, where a is a non-zero real number

Given: sum of the roots = $\alpha+\beta=0$ and product of the roots = $\alpha\beta=\sqrt{5}$

Hence, the quadratic polynomial $p(x)$ can be written as:

$$p(x)=a\{x^2-0.x+\sqrt{5}\}$$

$$=a\{x^2+\sqrt{5}\}$$

By taking $a=1$, we get one of the quadratic polynomials which satisfy the given conditions.

Therefore, the quadratic polynomial is $(x^2+\sqrt{5})$.

(iv) We know that if α *and* β are the zeroes of a quadratic polynomial $p(x)$, then, the polynomial $p(x)$ can be written as $p(x)=a\{x^2-(\alpha+\beta)x+\alpha\beta\}$ or $p(x)=a\{x^2-(\text{Sum of the zeroes})x+\text{Product of the zeroes}\}$, where a is a non-zero real number.

Given: sum of the roots = $\alpha+\beta=1$ and product of the roots = $\alpha\beta=1$

Hence, the quadratic polynomial $p(x)$ can be written as:

$p(x) = a\{x^2 - 1.x + 1\}$

$= a\{x^2 - x + 1\}$

By taking $a = 1$, we get one of the quadratic polynomials which satisfy the given conditions.

Therefore, the quadratic polynomial is $(x^2 - x + 1)$.

(v) We know that if α *and* β are the zeroes of a quadratic polynomial $p(x)$, then, the polynomial $p(x)$ can be written as $p(x) = a\{x^2 - (\alpha + \beta)x + \alpha\beta\}$ or $p(x) = a\{x^2 - (\text{Sum of the zeroes})x + \text{Product of the zeroes}\}$, where a is a non-zero real number.

Given: sum of the roots = $\alpha + \beta = -\frac{1}{4}$ and product of the roots = $\alpha\beta = \frac{1}{4}$

Hence, the quadratic polynomial $p(x)$ can be written as:

$$p(x) = a\{x^2 + \frac{1}{4}x + \frac{1}{4}\}$$

$$= a\left\{\frac{4x^2 + x + 1}{4}\right\}$$

By taking $a = 4$, we get one of the quadratic polynomials which satisfy the given conditions.

Therefore, the quadratic polynomial is $(4x^2 + x + 1)$.

(vi) We know that if α *and* β are the zeroes of a quadratic polynomial $p(x)$, then, the polynomial $p(x)$ can be written as $p(x) = a\{x^2 - (\alpha + \beta)x + \alpha\beta\}$ or $p(x) = a\{x^2 - (\text{Sum of the zeroes})x + \text{Product of the zeroes}\}$, where a is a non-zero real number.

Given: sum of the roots = $\alpha + \beta = 4$ and product of the roots = $\alpha\beta = 1$

Hence, the quadratic polynomial $p(x)$ can be written as:

$$p(x) = a\{x^2 - 4x + 1\}$$

By taking $a = 1$, we get one of the quadratic polynomials which satisfy the given conditions.

Therefore, the quadratic polynomial is $(x^2 - 4x + 1)$.

♦ ♦ ♦

EXERCISE 2.3

1. Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following:

    (i) $p(x) = x^3 - 3x^2 + 5x - 3,\ g(x) = x^2 - 2$

    (ii) $p(x) = x^4 - 3x^2 + 4x + 5,\ g(x) = x^2 + 1 - x$

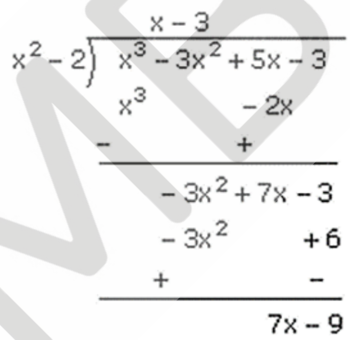
    (iii) $p(x) = x^4 - 5x + 6,\ g(x) = 2 - x^2$

Solution:

(i) $p(x) = x^3 - 3x^2 + 5x - 3,\ g(x) = x^2 - 2$

Here, both the polynomials are already arranged in the descending powers of variable.

The polynomial $p(x)$ can be divided by the polynomial $g(x)$ as follows:



$Quotient = x - 3$

$Remainder = 7x - 9$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5 = x^4 + 0.x^3 - 3x^2 + 4x + 5,$

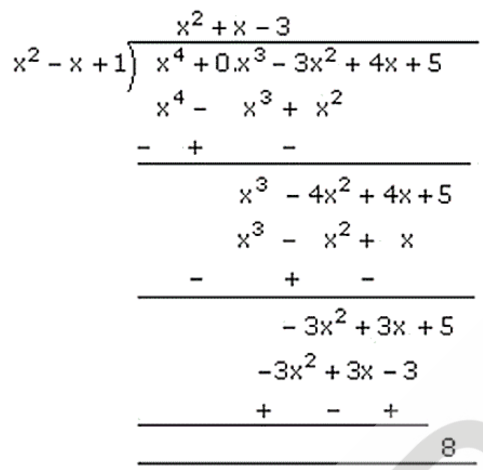
Here, the polynomial $p(x)$ is already arranged in the descending powers of variable.

$g(x) = x^2 + 1 - x$

Here, the polynomial $g(x)$ is not arranged in the descending powers of variable.

Now, $g(x) = x^2 - x + 1$

The polynomial $p(x)$ can be divided by the polynomial $g(x)$ as follows:



$Quotient = x^2 + x - 3$

$Remainder = 8$

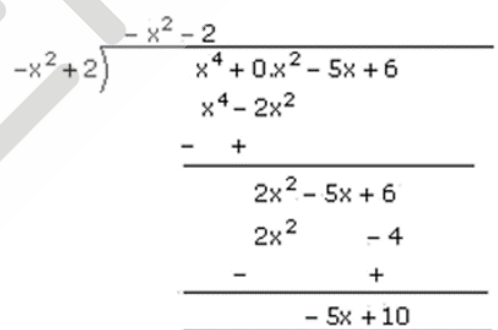
(iii) $p(x) = x^4 - 5x + 6 = x^4 + 0.x^2 - 5x + 6$

$g(x) = 2 - x^2$

Here, the polynomial $g(x)$ is not arranged in the descending powers of variable.

Now, $g(x) = -x^2 + 2$

The polynomial $p(x)$ can be divided by the polynomial $g(x)$ as follows:



$Quotient = -x^2 - 2$

$Remainder = -5x + 10$

2 Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) $t^2 - 3,\ 2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii) $x^2 + 3x + 1,\ 3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii) $x^3 - 3x + 1,\ x^5 - 4x^3 + x^2 + 3x + 1$

Solution:

(i) The polynomial $2t^4 + 3t^3 - 2t^2 - 9t - 12$ can be divided by the polynomial $t^2 - 3 = t^2 + 0.t - 3$ as follows:

$$
\begin{array}{r|l}
 & 2t^2 + 3t + 4 \\
t^2 + 0.t - 3 & 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\
 & 2t^4 + 0.t^3 - 6t^2 \\
 & - \quad - \quad + \\
\hline
 & 3t^3 + 4t^2 - 9t - 12 \\
 & 3t^3 + 0.t^2 - 9t \\
 & - \quad - \quad + \\
\hline
 & 4t^2 + 0.t - 12 \\
 & 4t^2 + 0.t - 12 \\
 & - \quad - \quad + \\
\hline
 & 0
\end{array}
$$

Since the remainder is 0, hence $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

(ii) The polynomial $3x^4 + 5x^3 - 7x^2 + 2x + 2$ can be divided by the polynomial $x^2 + 3x + 1$ as follows:

$$
\begin{array}{r|l}
 & 3x^2 - 4x + 2 \\
x^2 + 3x + 1 & 3x^4 + 5x^3 - 7x^2 + 2x + 2 \\
 & 3x^4 + 9x^3 + 3x^2 \\
 & - \quad - \quad - \\
\hline
 & -4x^3 - 10x^2 + 2x + 2 \\
 & -4x^3 - 12x^2 - 4x \\
 & + \quad + \quad + \\
\hline
 & 2x^2 + 6x + 2 \\
 & 2x^2 + 6x + 2 \\
\hline
 & 0
\end{array}
$$

Since the remainder is 0, hence $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii) The polynomial $x^5 - 4x^3 + x^2 + 3x + 1$ can be divided by the polynomial $x^3 - 3x + 1$ as follows:

$$\begin{array}{r|l} & x^2 - 1 \\ \hline x^3 - 3x + 1 & x^5 - 4x^3 + x^2 + 3x + 1 \\ & x^5 - 3x^3 + x^2 \\ & - \quad + \quad - \\ \hline & -x^3 \qquad\qquad + 3x + 1 \\ & -x^3 \qquad\qquad + 3x - 1 \\ & + \qquad\qquad - \quad + \\ \hline & \qquad\qquad\qquad\qquad 2 \\ \hline \end{array}$$

Since the remainder is not equal to 0, hence $x^3 - 3x + 1$ is not a factor of $x^5 - 4x^3 + x^2 + 3x + 1$.

3. Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

Solution:

Let $p(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$

It is given that the two zeroes of $p(x)$ are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

$\therefore \left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = \left(x^2 - \frac{5}{3}\right)$ is a factor of $p(x)$ $\{$Since, $(a - b)(a + b) = a^2 - b^2\}$

Therefore, on dividing the given polynomial by $x^2 - \frac{5}{3}$, we obtain remainder as 0.

$$
\begin{array}{r|l}
 & 3x^2 + 6x + 3 \\
x^2 + 0.x - \frac{5}{3} & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\
 & 3x^4 + 0x^3 - 5x^2 \\
 & - \quad - \quad + \\
\hline
 & 6x^3 + 3x^2 - 10x - 5 \\
 & 6x^3 + 0x^2 - 10x \\
 & - \quad - \quad + \\
\hline
 & 3x^2 + 0x - 5 \\
 & 3x^2 + 0x - 5 \\
 & - \quad - \quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

Hence, $3x^4 + 6x^3 - 2x^2 - 10x - 5 = \left(x^2 - \frac{5}{3}\right)(3x^2 + 6x + 3)$

$= 3\left(x^2 - \frac{5}{3}\right)(x^2 + 2x + 1)$

Now, $x^2 + 2x + 1 = (x + 1)^2$

Thus, the two zeroes of $x^2 + 2x + 1$ are -1 and -1

Therefore, the zeroes of the given polynomial are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, -1$ and -1.

4. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$.

Solution:

Dividend, $p(x) = x^3 - 3x^2 + x + 2$

$Quotient = (x - 2)$

$Remainder = (-2x + 4)$

$g(x)$ be the divisor.

According to the division algorithm,

Dividend $=$ Divisor $\times$ Quotient $+$ Remainder

$x^3 - 3x^2 + x + 2 = g(x) \times (x - 2) + (-2x + 4)$

$x^3 - 3x^2 + x + 2 + 2x - 4 = g(x)(x - 2)$

$x^3 - 3x^2 + 3x - 2 = g(x)(x - 2)$

Now, $g(x)$ is the quotient when $x^3 - 3x^2 + 3x - 2$ is divided by $x - 2$. (Since, Remainder = 0)

$$\begin{array}{r|l} & x^2 - x + 1 \\ x - 2 & x^3 - 3x^2 + 3x - 2 \\ & x^3 - 2x^2 \\ & - \quad + \\ \hline & -x^2 + 3x - 2 \\ & -x^2 + 2x \\ & + \quad - \\ \hline & x - 2 \\ & x - 2 \\ & - \quad + \\ \hline & 0 \end{array}$$

$\therefore g(x) = x^2 - x + 1$

6. Give examples of polynomials $p(x)$, $g(x)$, $q(x)$ and $r(x)$, which satisfy the division algorithm and

(i) $\deg p(x) = \deg q(x)$

(ii) $\deg q(x) = \deg r(x)$

(iii) $\deg r(x) = 0$

Solution:

According to the division algorithm, if $p(x)$ and $g(x)$ are two polynomials with $g(x) \neq 0$, then we can find polynomials q(x) and r(x) such that

$p(x) = g(x) \times q(x) + r(x)$, where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$.

(i) Degree of quotient will be equal to degree of dividend when divisor is constant.

Let us consider the division of $2x^2 + 2x - 16$ by 2.

Here, $p(x) = 2x^2 + 2x - 16$ and $g(x) = 2$

$q(x) = x^2 + x - 8$ and $r(x) = 0$

Clearly, the degree of $p(x)$ and $q(x)$ is the same which is 2.

Verification:

$p(x) = g(x) \times q(x) + r(x)$

$2x^2 + 2x - 16 = 2(x^2 + x - 8) + 0$

$= 2x^2 + 2x - 16$

Thus, the division algorithm is satisfied.

(ii) Let us consider the division of $4x + 3$ by $x + 2$.

Here, $p(x) = 4x + 3$ and $g(x) = x + 2$

$q(x) = 4$ and $r(x) = -5$

Here, degree of $q(x)$ and $r(x)$ is the same which is 0.

Verification:

$p(x) = g(x) \times q(x) + r(x)$

$4x + 3 = (x + 2) \times 4 + (-5)$

$4x + 3 = 4x + 3$

Thus, the division algorithm is satisfied.

(iii) Degree of remainder will be 0 when remainder obtained on division is a constant.

Let us consider the division of $4x + 3$ by $x + 2$.

Here, $p(x) = 4x + 3$ and $g(x) = x + 2$

$q(x) = 4$ and $r(x) = -5$

Here, we get remainder as a constant. Therefore, the degree of $r(x)$ is 0.

Verification:

$p(x) = g(x) \times q(x) + r(x)$

$4x + 3 = (x + 2) \times 4 + (-5)$

$4x + 3 = 4x + 3$

Thus, the division algorithm is satisfied.

♦ ♦ ♦