

CBSE NCERT Solutions for Class 10 Mathematics Chapter 4

Back of Chapter Questions

1. Check whether the following are quadratic equations:

- (i) $(x + 1)^2 = 2(x - 3)$
- (ii) $x^2 - 2x = (-2)(3 - x)$
- (iii) $(x - 2)(x + 1) = (x - 1)(x + 3)$
- (iv) $(x - 3)(2x + 1) = x(x + 5)$
- (v) $(2x - 1)(x - 3) = (x + 5)(x - 1)$
- (vi) $x^2 + 3x + 1 = (x - 2)^2$
- (vii) $(x + 2)^3 = 2x(x^2 - 1)$
- (viii) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

Solution:

- (i) We know that any equation of the form $ax^2 + bx + c = 0$ is called a quadratic equation, where a, b, c are real numbers and $a \neq 0$.

Given equation: $(x + 1)^2 = 2(x - 3)$

Using the formula $(a + b)^2 = a^2 + 2ab + b^2$

$$\Rightarrow x^2 + 2x + 1 = 2x - 6$$

$$\Rightarrow x^2 + 7 = 0$$

Here, $a = 1, b = 0$ and $c = 7$.

Thus, the given equation is a quadratic equation as $a \neq 0$.

- (ii) We know that any equation of the form $ax^2 + bx + c = 0$ is called a quadratic equation, where a, b, c are real numbers and $a \neq 0$.

Given equation: $x^2 - 2x = (-2)(3 - x)$

$$\Rightarrow x^2 - 2x = -6 + 2x$$

$$\Rightarrow x^2 - 4x + 6 = 0$$

Here, $a = 1, b = -4$ and $c = 6$.

Thus, the given equation is a quadratic equation as $a \neq 0$.

- (iii) We know that any equation of the form $ax^2 + bx + c = 0$ is called a quadratic equation, where a, b, c are real numbers and $a \neq 0$.

$$\text{Given equation: } (x - 2)(x + 1) = (x - 1)(x + 3)$$

$$\Rightarrow x^2 - x - 2 = x^2 + 2x - 3$$

$$\Rightarrow 3x - 1 = 0$$

But, here $a = 0$.

So, the given equation is not a quadratic equation.

- (iv) We know that any equation of the form $ax^2 + bx + c = 0$ is called a quadratic equation, where a, b, c are real numbers and $a \neq 0$.

$$\text{Given equation: } (x - 3)(2x + 1) = x(x + 5)$$

$$\Rightarrow 2x^2 - 5x - 3 = x^2 + 5x$$

$$\Rightarrow x^2 - 10x - 3 = 0$$

Here, $a = 1, b = -10$ and $c = -3$.

Thus, the given equation is a quadratic equation as $a \neq 0$.

- (v) We know that any equation of the form $ax^2 + bx + c = 0$ is called a quadratic equation, where a, b, c are real numbers and $a \neq 0$.

$$\text{Given equation: } (2x - 1)(x - 3) = (x + 5)(x - 1)$$

$$\Rightarrow 2x^2 - 7x + 3 = x^2 + 4x - 5$$

$$\Rightarrow x^2 - 11x + 8 = 0$$

Here, $a = 1, b = -11$ and $c = 8$.

Thus, the given equation is a quadratic equation as $a \neq 0$.

- (vi) We know that any equation of the form $ax^2 + bx + c = 0$ is called a quadratic equation, where a, b, c are real numbers and $a \neq 0$.

$$\text{Given equation: } x^2 + 3x + 1 = (x - 2)^2$$

$$\text{Using the formula } (a - b)^2 = a^2 - 2ab + b^2$$

$$\Rightarrow x^2 + 3x + 1 = x^2 - 4x + 4$$

$$\Rightarrow 7x - 3 = 0$$

But, here $a = 0$.

So, the given equation is not a quadratic equation.

- (vii) We know that any equation of the form $ax^2 + bx + c = 0$ is called a quadratic equation, where a, b, c are real numbers and $a \neq 0$.

$$\text{Given equation: } (x + 2)^3 = 2x(x^2 - 1)$$

$$\text{Using the formula } (a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$$

$$\Rightarrow x^3 + 8 + 6x^2 + 12x = 2x^3 - 2x$$

$$\Rightarrow x^3 - 14x - 6x^2 - 8 = 0$$

This equation is not of the form $ax^2 + bx + c = 0$

So, the given equation is not a quadratic equation.

- (viii) We know that any equation of the form $ax^2 + bx + c = 0$ is called a quadratic equation, where a, b, c are real numbers and $a \neq 0$.

Given equation: $x^3 - 4x^2 - x + 1 = (x - 2)^3$

Using the formula $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 8 - 6x^2 + 12x$$

$$\Rightarrow 2x^2 - 13x + 9 = 0$$

Here, $a = 2, b = -13$ and $c = 9$.

Thus, the given equation is a quadratic equation as $a \neq 0$.

2. Represent the following situations in the form of quadratic equations:

- (i) The area of a rectangular plot is 528 m^2 . The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.
- (ii) The product of two consecutive positive integers is 306. We need to find the integers.
- (iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.
- (iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Solution:

- (i) Let the breadth of the plot be x m.

Hence, the length of the plot is $(2x + 1)$ m. (Since, given that length is one more than twice its breadth)

Therefore, area of a rectangle = length \times breadth

Given: area of rectangle = 528 m^2

$$\therefore 528 = x(2x + 1)$$

$$\Rightarrow 2x^2 + x - 528 = 0 \dots\dots\dots(i), \text{ which is of the form } ax^2 + bx + c = 0$$

Here $a = 2(\neq 0)$, $b = 1$ and $c = -528$

Thus, quadratic equation (i) represents the situation given in the question and roots of this equation will represent the breadth of the plot.

- (ii) We know that the difference between two consecutive positive integers is 1.

So, let the consecutive positive integers be x and $x + 1$.

Given that their product is 306.

$$\therefore x(x + 1) = 306$$

$$\Rightarrow x^2 + x - 306 = 0 \dots\dots\dots (i), \text{ which is of the form } ax^2 + bx + c = 0$$

Here $a = 1(\neq 0)$, $b = 1$ and $c = -306$

Thus, quadratic equation (i) represents the situation given in the question and roots of this equation will represent the smaller positive integer.

- (iii) Let Rohan's age be x ,

His mother's age = $x + 26$ (given that Rohan's mother is 26 years older than him)

3 years from now:

Rohan's age will be = $x + 3$

Mother's age will be = $x + 26 + 3 = x + 29$

Also given that the product of their ages after 3 years is 360.

$$\therefore (x + 3)(x + 29) = 360$$

On simplification, we get

$$x^2 + 32x - 273 = 0 \dots\dots\dots (i), \text{ which is of the form } ax^2 + bx + c = 0$$

Here $a = 1(\neq 0)$, $b = 32$ and $c = -273$

Thus, quadratic equation (i) represents the situation given in the question and positive root of this equation will represent the Rohan's present age.

- (iv) In first case,

Let the speed of train be x km/h.

$$\text{Total time taken to travel 480 km} = \frac{480}{x} \text{ hrs}$$

In second case,

Given: speed became 8 km/h less

So, the speed of train = $(x - 8)$ km/h

Also given that the train will take 3 more hours to cover the same distance.

Therefore, time take to travel 480 km = $\left(\frac{480}{x} + 3\right)$ hrs

Speed \times Time = Distance

$$(x - 8) \left(\frac{480}{x} + 3 \right) = 480$$

$$\Rightarrow 480 + 3x - \frac{3840}{x} - 24 = 480$$

$$\Rightarrow 3x - \frac{3840}{x} = 24$$

$$\Rightarrow 3x^2 - 24x - 3840 = 0$$

$$\Rightarrow x^2 - 8x - 1280 = 0 \dots\dots\dots (i), \text{ which is of the form } ax^2 + bx + c = 0$$

Here $a = 1 (\neq 0)$, $b = -8$ and $c = -1280$

Thus, quadratic equation (i) represents the situation given in the question and positive root of this equation will represent the speed of train.

EXERCISE 4.2

1. Find the roots of the following quadratic equations by factorisation:

(i) $x^2 - 3x - 10 = 0$

(ii) $2x^2 + x - 6 = 0$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

(iv) $2x^2 - x + \frac{1}{8} = 0$

(v) $100x^2 - 20x + 1 = 0$

Solution:

(i) To find the roots of given quadratic equation, let's first factorise the given quadratic expression $x^2 - 3x - 10$. The given quadratic expression can be written as follows:

$$x^2 - 3x - 10$$

$$= x^2 - 5x + 2x - 10 \text{ (we factorise by method of splitting the middle term)}$$

$$= x(x - 5) + 2(x - 5)$$

$$= (x - 5)(x + 2)$$

Now, the roots of this quadratic equation are the values of x for which $(x - 5)(x + 2) = 0$

$$\therefore x - 5 = 0 \text{ or } x + 2 = 0$$

$$\text{i. e., } x = 5 \text{ or } x = -2$$

Hence, the roots of this quadratic equation are 5 and -2 .

- (ii) To find the roots of given quadratic equation, let's first factorise the given quadratic expression $2x^2 + x - 6$. The given quadratic expression can be written as follows:

$$2x^2 + x - 6$$

$$= 2x^2 + 4x - 3x - 6 \text{ (we factorise by method of splitting the middle term)}$$

$$= 2x(x + 2) - 3(x + 2)$$

$$= (x + 2)(2x - 3)$$

Now, the roots of this quadratic equation are the values of x for which $(x + 2)(2x - 3) = 0$

$$\therefore x + 2 = 0 \text{ or } 2x - 3 = 0$$

$$\text{i. e., } x = -2 \text{ or } x = \frac{3}{2}$$

Hence, the roots of this quadratic equation are -2 and $\frac{3}{2}$.

- (iii) To find the roots of given quadratic equation, let's first factorise the given quadratic expression $\sqrt{2}x^2 + 7x + 5\sqrt{2}$. The given quadratic expression can be written as follows:

$$\sqrt{2}x^2 + 7x + 5\sqrt{2}$$

$$= \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} \text{ (we factorise by method of splitting the middle term)}$$

$$= x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5)$$

$$= (\sqrt{2}x + 5)(x + \sqrt{2})$$

Now, the roots of this quadratic equation are the values of x for which

$$(\sqrt{2}x + 5)(x + \sqrt{2}) = 0$$

$$\therefore \sqrt{2}x + 5 = 0 \text{ or } x + \sqrt{2} = 0$$

$$\text{i. e., } x = -\frac{5}{\sqrt{2}} \text{ or } x = -\sqrt{2}$$

Hence, the roots of this quadratic equation are $-\frac{5}{\sqrt{2}}$ and $-\sqrt{2}$.

- (iv) To find the roots of given quadratic equation, let's first factorise the given quadratic expression $2x^2 - x + \frac{1}{8}$. The given quadratic expression can be written as follows:

$$\begin{aligned} & 2x^2 - x + \frac{1}{8} \\ &= \frac{1}{8}(16x^2 - 8x + 1) \\ &= \frac{1}{8}(16x^2 - 4x - 4x + 1) \quad (\text{we factorise by method of splitting the middle term}) \\ &= \frac{1}{8}(4x(4x - 1) - 1(4x - 1)) \\ &= \frac{1}{8}(4x - 1)^2 \end{aligned}$$

Now, the roots of this quadratic equation are the values of x for which $(4x - 1)^2 = 0$

Thus, $(4x - 1) = 0$ or $(4x - 1) = 0$

i. e., $x = \frac{1}{4}$ or $x = \frac{1}{4}$

Hence, the roots of this quadratic equation are $\frac{1}{4}$ and $\frac{1}{4}$.

- (v) To find the roots of given quadratic equation, let's first factorise the given quadratic expression $100x^2 - 20x + 1$. The given quadratic expression can be written as follows:

$$\begin{aligned} & 100x^2 - 20x + 1 \\ &= 100x^2 - 10x - 10x + 1 \quad (\text{we factorise by method of splitting the middle term}) \\ &= 10x(10x - 1) - 1(10x - 1) \\ &= (10x - 1)^2 \end{aligned}$$

Now, the roots of this quadratic equation are the values of x for which $(10x - 1)^2 = 0$

Thus, $(10x - 1) = 0$ or $(10x - 1) = 0$

i. e., $x = \frac{1}{10}$ or $x = \frac{1}{10}$

Hence, the roots of this quadratic equation are $\frac{1}{10}$ and $\frac{1}{10}$.

2. Solve the problems given below.

Represent the following situations mathematically:

- (i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.
- (ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ₹ 750. We would like to find out the number of toys produced on that day.

Solution:

- (i) Let the number of John's marbles be x .

So, the number of Jivanti's marbles = $45 - x$

If both lost 5 marbles each,

Then number of marbles left with John = $x - 5$

Then number of marbles left with Jivanti = $45 - x - 5 = 40 - x$

Given that the product of their marbles is 124.

$$\therefore (x - 5)(40 - x) = 124$$

$$\Rightarrow x^2 - 45x + 324 = 0$$

$$\Rightarrow x^2 - 36x - 9x + 324 = 0$$

$$\Rightarrow x(x - 36) - 9(x - 36) = 0$$

$$\Rightarrow (x - 36)(x - 9) = 0$$

Either $x - 36 = 0$ or $x - 9 = 0$

i.e., $x = 36$ or $x = 9$

If the number of John's marbles = 36

Then, the number of Jivanti's marbles = $45 - 36 = 9$

If the number of John's marbles = 9

Then, the number of Jivanti's marbles = $45 - 9 = 36$.

- (ii) Let the number of toys produced on that day be x .

\therefore The cost of production of each toy that day = ₹ $(55 - x)$

So, the total cost of production that day = $x(55 - x)$

As per the question, the total cost of production of the toys = ₹ 750

$$\begin{aligned}\therefore (55 - x)x &= 750 \\ \Rightarrow x^2 - 55x + 750 &= 0 \\ \Rightarrow x^2 - 25x - 30x + 750 &= 0 \\ \Rightarrow x(x - 25) - 30(x - 25) &= 0 \\ \Rightarrow (x - 25)(x - 30) &= 0 \\ \text{Either } x - 25 = 0 \text{ or } x - 30 &= 0 \\ \text{i. e., } x = 25 \text{ or } x = 30\end{aligned}$$

Thus, the number of toys produced that day will be either 25 or 30.

3. Find two numbers whose sum is 27 and product is 182.

Solution:

Let the first number be x .

Then the second number is $27 - x$. (Given sum of two numbers = 27)

Thus, their product = $x(27 - x)$

According to the question, the product of these numbers is 182.

Therefore, $x(27 - x) = 182$

$$\begin{aligned}\Rightarrow x^2 - 27x + 182 &= 0 \\ \Rightarrow x^2 - 13x - 14x + 182 &= 0 \\ \Rightarrow x(x - 13) - 14(x - 13) &= 0 \\ \Rightarrow (x - 13)(x - 14) &= 0\end{aligned}$$

Either $x - 13 = 0$ or $x - 14 = 0$

i. e., $x = 13$ or $x = 14$

If first number = 13, then

Second number = $27 - 13 = 14$

If first number = 14, then

Second number = $27 - 14 = 13$

Hence, the numbers are 13 and 14.

4. Find two consecutive positive integers, sum of whose squares is 365.

Solution:

We know that the difference between two consecutive positive integers is 1.

So, let the consecutive positive integers be x and $x + 1$.

As per the question, $x^2 + (x + 1)^2 = 365$

$$\Rightarrow x^2 + x^2 + 1 + 2x = 365$$

$$\Rightarrow 2x^2 + 2x - 364 = 0$$

$$\Rightarrow x^2 + x - 182 = 0$$

$$\Rightarrow x^2 + 14x - 13x - 182 = 0$$

$$\Rightarrow x(x + 14) - 13(x + 14) = 0$$

$$\Rightarrow (x + 14)(x - 13) = 0$$

Either $x + 14 = 0$ or $x - 13 = 0$, i.e., $x = -14$ or $x = 13$

Since given that integers are positive, x can only be 13.

$$\therefore x + 1 = 13 + 1 = 14$$

Hence, the two consecutive positive integers are 13 and 14.

5. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Solution: As per the question, hypotenuse is 13 cm

Let the base of the right triangle be x cm.

Its altitude = $(x - 7)$ cm

From Pythagoras theorem,

$$\text{Base}^2 + \text{Altitude}^2 = \text{Hypotenuse}^2$$

$$\therefore x^2 + (x - 7)^2 = 13^2$$

$$\Rightarrow x^2 + x^2 + 49 - 14x = 169$$

$$\Rightarrow 2x^2 - 14x - 120 = 0$$

$$\Rightarrow x^2 - 7x - 60 = 0$$

$$\Rightarrow x^2 - 12x + 5x - 60 = 0$$

$$\Rightarrow x(x - 12) + 5(x - 12) = 0$$

$$\Rightarrow (x - 12)(x + 5) = 0$$

Either $x - 12 = 0$ or $x + 5 = 0$, i.e., $x = 12$ or $x = -5$

Since sides of a triangle are positive, x can only take 12.

Hence, the base of the right triangle is 12 cm and the altitude of this triangle is $(12 - 7)$ cm = 5 cm.

6. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was ₹. 90, find the number of articles produced and the cost of each article.

Solution:

Let the number of articles produced on that day be x .

So, the cost of production of each article = ₹ $(2x + 3)$

According to the question, the total cost of production on that day was ₹ 90.

We know that

Total cost of production = Cost of each article \times Number of articles produced

$$\therefore x(2x + 3) = 90$$

$$\Rightarrow 2x^2 + 3x - 90 = 0$$

$$\Rightarrow 2x^2 + 15x - 12x - 90 = 0$$

$$\Rightarrow x(2x + 15) - 6(2x + 15) = 0$$

$$\Rightarrow (2x + 15)(x - 6) = 0$$

Either $2x + 15 = 0$ or $x - 6 = 0$, i.e., $x = \frac{-15}{2}$ or $x = 6$

It's clear that number of articles produced can only be a positive integer, so, x can only be 6.

Therefore, number of articles produced on that day = 6

Cost of each article = $(2 \times 6) + 3 = ₹ 15$

EXERCISE 4.3

1. Find the roots of the following quadratic equations, if they exist, by the method of completing the square:

(i) $2x^2 - 7x + 3 = 0$

(ii) $2x^2 + x - 4 = 0$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

(iv) $2x^2 + x + 4 = 0$

Solution:

(i) Given quadratic equation: $2x^2 - 7x + 3 = 0$

$$\Rightarrow 2x^2 - 7x = -3$$

On dividing both sides of the equation by 2, we obtain

$$\Rightarrow x^2 - \frac{7}{2}x = -\frac{3}{2}$$

$$\Rightarrow x^2 - 2 \times x \times \frac{7}{4} = -\frac{3}{2}$$

On adding $\left(\frac{7}{4}\right)^2$ to both sides of equation by, we obtain

$$\Rightarrow (x)^2 - 2 \times x \times \frac{7}{4} + \left(\frac{7}{4}\right)^2 = \left(\frac{7}{4}\right)^2 - \frac{3}{2}$$

$$\Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{49}{16} - \frac{3}{2}$$

$$\Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{25}{16}$$

$$\Rightarrow \left(x - \frac{7}{4}\right) = \pm \frac{5}{4} \quad (\text{Cancelling square both the sides})$$

$$\Rightarrow x = \frac{7}{4} \pm \frac{5}{4}$$

$$\Rightarrow x = \frac{7}{4} + \frac{5}{4} \text{ or } x = \frac{7}{4} - \frac{5}{4}$$

$$\Rightarrow x = \frac{12}{4} \text{ or } x = \frac{2}{4}$$

$$\Rightarrow x = 3 \text{ or } \frac{1}{2}$$

Hence, the roots of this quadratic equation are 3 and $\frac{1}{2}$.

(ii) $2x^2 + x - 4 = 0$

$$\Rightarrow 2x^2 + x = 4$$

On dividing both sides of the equation by 2, we obtain

$$\Rightarrow x^2 + \frac{1}{2}x = 2$$

On adding $\left(\frac{1}{4}\right)^2$ to both sides of the equation, we obtain

$$\Rightarrow (x)^2 + 2 \times x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 = 2 + \left(\frac{1}{4}\right)^2$$

$$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{33}{16}$$

$$\Rightarrow x + \frac{1}{4} = \pm \frac{\sqrt{33}}{4} \quad (\text{Cancelling square both the sides})$$

$$\Rightarrow x = \pm \frac{\sqrt{33}}{4} - \frac{1}{4}$$

$$\Rightarrow x = \frac{\pm\sqrt{33} - 1}{4}$$

$$\Rightarrow x = \frac{\sqrt{33}-1}{4} \text{ or } \frac{-\sqrt{33}-1}{4}$$

Hence, the roots of this quadratic equation are $\frac{-1+\sqrt{33}}{4}$ and $\frac{-1-\sqrt{33}}{4}$.

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

$$\Rightarrow (2x)^2 + 2 \times 2x \times \sqrt{3} + (\sqrt{3})^2 = 0$$

$$\Rightarrow (2x + \sqrt{3})^2 = 0$$

$$\Rightarrow (2x + \sqrt{3}) = 0 \text{ and } (2x + \sqrt{3}) = 0$$

$$\Rightarrow x = \frac{-\sqrt{3}}{2} \text{ and } x = \frac{-\sqrt{3}}{2}$$

Hence, the roots of this quadratic equation are $\frac{-\sqrt{3}}{2}$ and $\frac{-\sqrt{3}}{2}$.

(iv) $2x^2 + x + 4 = 0$

$$\Rightarrow 2x^2 + x = -4$$

On dividing both sides of the equation by 2, we obtain

$$\Rightarrow x^2 + \frac{1}{2}x = -2$$

$$\Rightarrow x^2 + 2 \times x \times \frac{1}{4} = -2$$

On adding $\left(\frac{1}{4}\right)^2$ to both sides of the equation, we obtain

$$\Rightarrow (x)^2 + 2 \times x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 = \left(\frac{1}{4}\right)^2 - 2$$

$$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{1}{16} - 2$$

$$\Rightarrow \left(x + \frac{1}{4}\right)^2 = -\frac{31}{16}$$

Since, the square of a number cannot be negative.

Thus, there is no real root for the given equation.

2. Find the roots of the quadratic equations given by applying the quadratic formula.

(i) $2x^2 - 7x + 3 = 0$

(ii) $2x^2 + x - 4 = 0$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

(iv) $2x^2 + x + 4 = 0$

Solution:

(i) $2x^2 - 7x + 3 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$$a = 2, b = -7, c = 3$$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{49 - 24}}{2a}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{25}}{4}$$

$$\Rightarrow x = \frac{7 \pm 5}{4}$$

$$\Rightarrow x = \frac{7+5}{4} \text{ or } \frac{7-5}{4}$$

$$\Rightarrow x = \frac{12}{4} \text{ or } \frac{2}{4}$$

$$\therefore x = 3 \text{ or } \frac{1}{2}$$

Hence, the roots of this quadratic equation are 3 and $\frac{1}{2}$.

(ii) $2x^2 + x - 4 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$$a = 2, b = 1, c = -4$$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1 + 32}}{4}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{33}}{4}$$

$$\therefore x = \frac{-1 + \sqrt{33}}{4} \text{ or } \frac{-1 - \sqrt{33}}{4}$$

Hence, the roots of this quadratic equation are $\frac{-1 + \sqrt{33}}{4}$ and $\frac{-1 - \sqrt{33}}{4}$.

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

On comparing this equation $ax^2 + bx + c = 0$, we obtain $a = 4, b = 4\sqrt{3}, c = 3$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-4\sqrt{3} \pm \sqrt{48 - 48}}{8}$$

$$\Rightarrow x = \frac{-4\sqrt{3} \pm 0}{8}$$

$$\therefore x = \frac{-\sqrt{3}}{2} \text{ or } \frac{-\sqrt{3}}{2}$$

Hence, the roots of this quadratic equation are $\frac{-\sqrt{3}}{2}$ and $\frac{-\sqrt{3}}{2}$.

(iv) $2x^2 + x + 4 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$$a = 2, b = 1, c = 4$$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1 - 32}}{4}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{-31}}{4}$$

Hence, roots do not exist for this quadratic equation as $D = b^2 - 4ac = -31 < 0$.

3. Find the roots of the following equations:

$$(i) \quad x - \frac{1}{x} = 3, \quad x \neq 0$$

$$(ii) \quad \frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, \quad x \neq -4, 7$$

Solution:

$$(i) \quad x - \frac{1}{x} = 3 \Rightarrow x^2 - 3x - 1 = 0$$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$$a = 1, b = -3, c = -1$$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{9 + 4}}{2}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{13}}{2}$$

$$\text{Therefore, } x = \frac{3 + \sqrt{13}}{2} \text{ or } \frac{3 - \sqrt{13}}{2}$$

$$(ii) \quad \frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$$

$$\Rightarrow \frac{x - 7 - x - 4}{(x + 4)(x - 7)} = \frac{11}{30}$$

$$\Rightarrow \frac{-11}{(x + 4)(x - 7)} = \frac{11}{30}$$

$$\Rightarrow (x + 4)(x - 7) = -30$$

$$\Rightarrow x^2 - 3x - 28 = -30$$

$$\Rightarrow x^2 - 3x + 2 = 0$$

$$\Rightarrow x^2 - 2x - x + 2 = 0$$

$$\Rightarrow x(x - 2) - 1(x - 2) = 0$$

$$\Rightarrow (x - 2)(x - 1) = 0$$

$$\Rightarrow x = 1 \text{ or } 2$$

4. The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Solution:

Let Rehman's present age = x years.

His age three years ago = $(x - 3)$ years.

His age five years from now = $(x + 5)$ years.

As per the question, the sum of the reciprocals of Rehman's ages 3 years ago and 5 years from now is $\frac{1}{3}$.

$$\therefore \frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$

$$\frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3}$$

$$\frac{2x+2}{(x-3)(x+5)} = \frac{1}{3}$$

$$\Rightarrow 3(2x+2) = (x-3)(x+5)$$

$$\Rightarrow 6x+6 = x^2+2x-15$$

$$\Rightarrow x^2-4x-21 = 0$$

$$\Rightarrow x^2-7x+3x-21 = 0$$

$$\Rightarrow x(x-7)+3(x-7) = 0$$

$$\Rightarrow (x-7)(x+3) = 0$$

$$\Rightarrow x = 7 \text{ or } x = -3$$

It's clear that age is always positive.

Thus, Rehman's present age is 7 years.

5. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Solution:

Let Shefali's marks in Mathematics = x .

Then, her marks in English = $30 - x$. (Given in question)

As per the question, we get

$$(x+2)(30-x-3) = 210$$

$$(x+2)(27-x) = 210$$

$$\Rightarrow -x^2 + 25x + 54 = 210$$

$$\Rightarrow x^2 - 25x + 156 = 0$$

$$\Rightarrow x^2 - 12x - 13x + 156 = 0$$

$$\Rightarrow x(x + 2) - 13(x - 12) = 0$$

$$\Rightarrow (x - 12)(x - 13) = 0$$

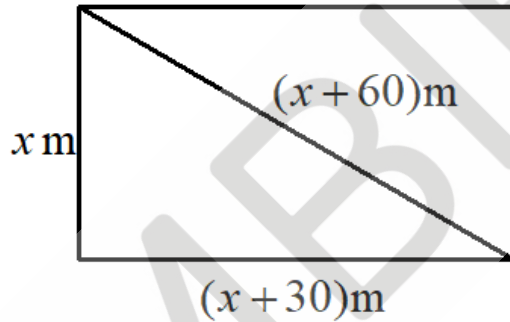
$$\Rightarrow x = 12 \text{ or } x = 13$$

If the marks in Mathematics is 12, then marks in English will be $30 - 12 = 18$

If the marks in Mathematics is 13, then marks in English will be $30 - 13 = 17$

6. The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 meters more than the shorter side, find the sides of the field.

Solution:



Let the shorter side of the rectangle be $x \text{ m}$.

Then, longer side of the rectangle = $(x + 30) \text{ m}$

Diagonal of the rectangle = $\sqrt{x^2 + (x + 30)^2}$ (By Pythagoras theorem)

But in question, it is given that the diagonal of the rectangular field is 60 m more than the shorter side.

$$\therefore \sqrt{x^2 + (x + 30)^2} = x + 60$$

$$\Rightarrow x^2 + (x + 30)^2 = (x + 60)^2 \quad (\text{By squaring on both the sides})$$

$$\Rightarrow x^2 + x^2 + 900 + 60x = x^2 + 3600 + 120x$$

$$\Rightarrow x^2 - 60x - 2700 = 0$$

$$\Rightarrow x^2 - 90x + 30x - 2700 = 0$$

$$\Rightarrow x(x - 90) + 30(x - 90)$$

$$\Rightarrow (x - 90)(x + 30) = 0$$

$$\Rightarrow x = 90 \text{ or } x = -30$$

But, side of a rectangle cannot be negative. So, the length of the shorter side is 90 m. Thus, the length of the longer side will be $(90 + 30) \text{ m} = 120 \text{ m}$

7. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Solution:

Let the larger and smaller number be x and y respectively.

It is given in the question that,

$$x^2 - y^2 = 180 \text{ and } y^2 = 8x$$

$$\Rightarrow x^2 - 8x = 180$$

$$\Rightarrow x^2 - 8x - 180 = 0$$

$$\Rightarrow x^2 - 18x + 10x - 180 = 0$$

$$\Rightarrow x(x - 18) + 10(x - 18) = 0$$

$$\Rightarrow (x - 18)(x + 10) = 0$$

$$\Rightarrow x = 18, -10$$

If larger number, $x = -10$

then smaller number, $y = \pm\sqrt{8x}$

$$= \pm\sqrt{8(-10)}$$

$$= \pm\sqrt{-80}$$

Since we cannot have negative number in roots

$x = -10$ is not possible

Therefore, the larger number will be 18 only.

$$x = 18$$

$$\therefore y^2 = 8x = 8 \times 18 = 144$$

$$\Rightarrow y = \pm\sqrt{144} = \pm 12$$

$$\therefore \text{Smaller number} = \pm 12$$

Therefore, the numbers are 18 and 12 or 18 and -12 .

8. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Solution:

Let the speed of the train be x km/h.

$$\text{Total time taken to cover 360 km} = \frac{360}{x} \text{ hr}$$

It is given in question,

$$(x + 5) \left(\frac{360}{x} - 1 \right) = 360 \quad (\text{Distance} = \text{Speed} \times \text{Time})$$

$$\Rightarrow 360 - x + \frac{1800}{x} - 5 = 360$$

$$\Rightarrow x^2 + 5x - 1800 = 0$$

$$\Rightarrow x^2 + 45x - 40x - 1800 = 0$$

$$\Rightarrow x(x + 45) - 40(x + 45) = 0$$

$$\Rightarrow (x + 45)(x - 40) = 0$$

$$\Rightarrow x = 40 \text{ or } x = -45$$

But, speed cannot be negative.

Thus, the speed of the train is 40 km/h

9. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Solution:

Let the time taken by the smaller diameter tap to fill the tank be x hr.

Time taken by the larger diameter tap = $(x - 10)$ hr

Volume of tank filled by smaller tap in 1 hour = $\frac{1}{x}$

Volume of tank filled by larger tap in 1 hour = $\frac{1}{x-10}$

As per the question, the tank can be filled in $9\frac{3}{8} = \frac{75}{8}$ hours by both the taps together.

Hence,

$$\frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$$

$$\frac{x-10+x}{x(x-10)} = \frac{8}{75}$$

$$\Rightarrow \frac{2x-10}{x(x-10)} = \frac{8}{75}$$

$$\Rightarrow 75(2x-10) = 8x^2 - 80x$$

$$\Rightarrow 150x - 750 = 8x^2 - 80x$$

$$\Rightarrow 8x^2 - 230x + 750 = 0$$

$$\Rightarrow 8x^2 - 200x - 30x + 750 = 0$$

$$\Rightarrow 8x(x - 25) - 30(x - 25) = 0$$

$$\Rightarrow (x - 25)(8x - 30) = 0$$

$$i. e., x = 25 \text{ or } x = \frac{30}{8} = \frac{15}{4}$$

$$\text{Taking } x = \frac{15}{4}$$

Time taken by smaller tap

$$= x = \frac{15}{4} \text{ hrs}$$

Time taken by larger tap = $x - 10$

$$= \frac{15}{4} - 10 = \frac{15 - 40}{4} = -\frac{25}{4}$$

Since time is negative,

$$x = \frac{15}{4} \text{ is not the solution}$$

Thus, the time taken separately by the smaller diameter tap and the larger diameter tap will be 25 hours and $(25 - 10) = 15$ hours respectively.

10. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11 km/h more than that of the passenger train, find the average speed of the two trains.

Solution:

Let the average speed of passenger train be x km/h.

Average speed of express train = $(x + 11)$ km/h (Given in question)

According to the question, the time taken by the express train to cover 132 km is 1 hour less than a passenger train.

Therefore, time taken by passenger train – time taken by express train = 1 hour

$$\therefore \frac{132}{x} - \frac{132}{x+11} = 1 \quad \left(\text{Total time} = \frac{\text{Distance}}{\text{Average Speed}} \right)$$

$$\Rightarrow 132 \left[\frac{x + 11 - x}{x(x + 11)} \right] = 1$$

$$\begin{aligned} \Rightarrow \frac{132 \times 11}{x(x+11)} &= 1 \\ \Rightarrow 132 \times 11 &= x(x+11) \\ \Rightarrow x^2 + 11x - 1452 &= 0 \\ \Rightarrow x^2 + 44x - 33x - 1452 &= 0 \\ \Rightarrow x(x+44) - 33x + 44 &= 0 \\ \Rightarrow (x+44)(x-33) &= 0 \\ \Rightarrow x &= -44, 33 \end{aligned}$$

Average speed of passenger train cannot be negative. Hence, the speed of the passenger train is 33km/h and thus, the speed of the express train will be $33 + 11 = 44$ km/h.

11. Sum of the areas of two squares is 468 m^2 . If the difference of their perimeters is 24 m, find the sides of the two squares.

Solution:

Let the sides of the two squares be x m and y m.

So, their perimeter will be $4x$ and $4y$ respectively and their areas will be x^2 and y^2 respectively.

According to the question, $4x - 4y = 24$

$$\Rightarrow x - y = 6$$

$$\Rightarrow x = y + 6$$

Also, $x^2 + y^2 = 468$

$$\Rightarrow (6 + y)^2 + y^2 = 468$$

$$\Rightarrow 36 + y^2 + 12y + y^2 = 468$$

$$\Rightarrow 2y^2 + 12y - 432 = 0$$

$$\Rightarrow y^2 + 6y - 216 = 0$$

$$\Rightarrow y^2 + 18y - 12y - 216 = 0$$

$$\Rightarrow y(y + 18) - 12(y + 18) = 0$$

$$\Rightarrow (y + 18)(y - 12) = 0$$

$$\Rightarrow y = -18 \text{ or } 12.$$

But, side of a square cannot be negative.

Hence, $y = 12$ & $x = 12 + 6 = 18$

Therefore, the sides of the squares are 12 m and 18 m

EXERCISE 4.4

1. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:

(i) $2x^2 - 3x + 5 = 0$

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

(iii) $2x^2 - 6x + 3 = 0$

Solution:

We know that for a quadratic equation $ax^2 + bx + c = 0$, discriminant is $b^2 - 4ac$.

(A) If $b^2 - 4ac > 0$ implies two distinct real roots

(B) If $b^2 - 4ac = 0$ implies two equal real roots

(C) If $b^2 - 4ac < 0$ implies imaginary roots

(i) $2x^2 - 3x + 5 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain $a = 2, b = -3, c = 5$

$$\text{Discriminant} = b^2 - 4ac = (-3)^2 - 4(2)(5) = 9 - 40 = -31$$

As $b^2 - 4ac < 0$,

Hence, no real root is possible for the given equation.

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain

$$a = 3, b = -4\sqrt{3}, c = 4$$

$$\text{Discriminant} = b^2 - 4ac = (-4\sqrt{3})^2 - 4(3)(4) = 48 - 48 = 0$$

As $b^2 - 4ac = 0$,

So, real roots exist for the given equation and they are equal to each other and the roots will be $-\frac{b}{2a}$ and $-\frac{b}{2a}$.

$$-\frac{b}{2a} = \frac{-(-4\sqrt{3})}{2 \times 3} = \frac{4\sqrt{3}}{6} = \frac{2\sqrt{3}}{3} = \frac{2}{\sqrt{3}}$$

Hence, the roots are $\frac{2}{\sqrt{3}}$ and $\frac{2}{\sqrt{3}}$.

(iii) $2x^2 - 6x + 3 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain $a = 2, b = -6, c = 3$

$$\text{Discriminant} = b^2 - 4ac = (-6)^2 - 4(2)(3) = 36 - 24 = 12$$

As $b^2 - 4ac > 0$,

So, two distinct real roots exist for this equation as follows:

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(3)}}{2(2)} \\ &= \frac{6 \pm \sqrt{12}}{4} = \frac{6 \pm 2\sqrt{3}}{4} \\ &= \frac{3 \pm \sqrt{3}}{2} \end{aligned}$$

Hence, the roots are $\frac{3+\sqrt{3}}{2}$ or $\frac{3-\sqrt{3}}{2}$.

2. Find the values of k for each of the following quadratic equations, so that they have two equal roots.

(i) $2x^2 + kx + 3 = 0$

(ii) $kx(x - 2) + 6 = 0$

Solution:

We know that if an equation $ax^2 + bx + c = 0$ has two equal roots, its discriminant ($b^2 - 4ac$) will be 0.

(i) $2x^2 + kx + 3 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we get $a = 2, b = k, c = 3$

$$\text{Discriminant} = b^2 - 4ac = (k)^2 - 4(2)(3) = k^2 - 24$$

For equal roots,

$$\text{Discriminant} = 0$$

$$k^2 - 24 = 0$$

$$\Rightarrow k^2 = 24$$

$$\Rightarrow k = \pm\sqrt{24} = \pm 2\sqrt{6}$$

(ii) $kx(x - 2) + 6 = 0$ or $kx^2 - 2kx + 6 = 0$

Comparing the equation with $ax^2 + bx + c = 0$, we get $a = k, b = -2k, c = 6$

$$\text{Discriminant} = b^2 - 4ac = (-2k)^2 - 4(k)(6) = 4k^2 - 24k$$

For equal roots,

$$D = b^2 - 4ac = 0$$

$$4k^2 - 24k = 0$$

$$4k(k - 6) = 0$$

$$\text{Either } 4k = 0 \text{ or } k - 6 = 0$$

$$\Rightarrow k = 0 \text{ or } k = 6$$

But, if $k = 0$, then the equation will not have the terms ' x^2 ' and ' x '.

Hence, if this quadratic equation has two equal roots, then k should be 6 only.

3. Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2 ? If so, find its length and breadth.

Solution:

Breadth = x



Length = $2x$

Let the breadth of mango grove be x .

Length of mango grove will be $2x$.

$$\text{Area of mango grove} = (2x)(x) = 2x^2$$

$$\text{Hence, } 2x^2 = 800$$

$$\Rightarrow x^2 = \frac{800}{2}$$

$$\Rightarrow x^2 = 400$$

Cancelling square on both the sides, we get $x = \pm 20$

But length cannot be negative.

So, breadth of mango grove = 20 m

Length of mango grove = $2 \times 20 = 40$ m

4. Is the following situation possible? If so, determine their present ages.

The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Solution:

Let the age of one friend be x years.

Age of the other friend will be = $(20 - x)$ years. (Given in question)

Four years ago, age of 1st friend = $(x - 4)$ years

and, age of 2nd friend = $((20 - x) - 4) = (16 - x)$ years

As per the question,

$$(x - 4)(16 - x) = 48$$

$$16x - 64 - x^2 + 4x = 48$$

$$-x^2 + 20x - 112 = 0$$

$$x^2 - 20x + 112 = 0$$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain

$$a = 1, b = -20, c = 112$$

$$\text{Discriminant} = b^2 - 4ac = (-20)^2 - 4(1)(112)$$

$$= 400 - 448 = -48$$

$$\text{As } b^2 - 4ac < 0,$$

Thus, no real roots are possible for this equation and hence, this situation is not possible.

5. Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2 ? If so, find its length and breadth.

Solution:

Let the length and breadth of the rectangular park be l and b .

$$\text{Perimeter} = 2(l + b) = 80$$

$$l + b = 40$$

$$\text{or, } b = 40 - l$$

$$\text{Area} = l \times b = l(40 - l) = 40l - l^2$$

$$40l - l^2 = 400$$

$$l^2 - 40l + 400 = 0$$

Comparing this equation with $al^2 + bl + c = 0$, we obtain

$$a = 1, b = -40, c = 400$$

$$\text{Discriminant} = b^2 - 4ac = (-40)^2 - 4(1)(400)$$

$$= 1600 - 1600 = 0$$

$$\text{As } b^2 - 4ac = 0,$$

Thus, this equation has equal real roots, and hence, this situation is possible.

Root of this equation,

$$l = -\frac{b}{2a}$$

$$l = -\frac{(-40)}{2(1)} = \frac{40}{2} = 20$$

So, length of park, $l = 20$ m

and breadth of park, $b = 40 - l = 40 - 20 = 20$ m

