

CBSE NCERT Solutions for Class 10 Mathematics Chapter 8**Back of Chapter Questions**

1. In ΔABC , right-angled at B, $AB = 24$ cm, $BC = 7$ cm. Determine:

(i) $\sin A, \cos A$

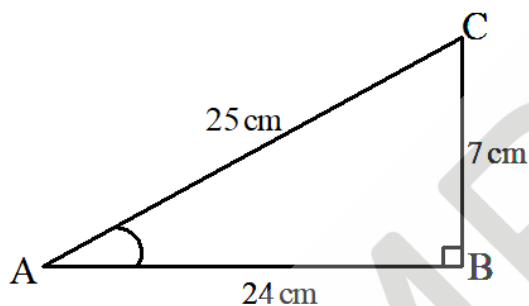
(ii) $\sin C, \cos C$

Solution:

In ΔABC , apply Pythagoras theorem

$$AC^2 = AB^2 + BC^2 = (24)^2 + (7)^2 = 625$$

$$AC = \sqrt{625} = 25 \text{ cm}$$



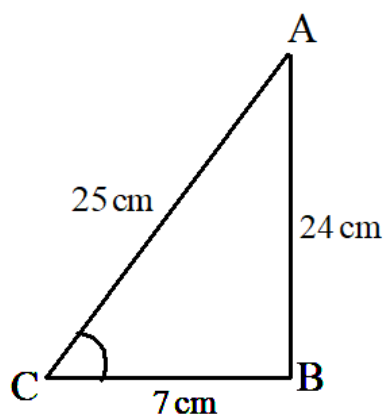
(i) $\sin A = \frac{\text{Side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC}$

$$= \frac{7}{25}$$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC}$$

$$= \frac{24}{25}$$

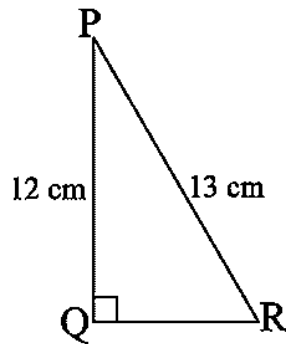
(ii)



$$\sin C = \frac{\text{Side opposite to } \angle C}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{24}{25}$$

$$\cos C = \frac{\text{Side adjacent to } \angle C}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{7}{25}$$

2. In Fig., find $\tan P - \cot R$.



Solution:

Apply Pythagoras theorem in ΔPQR

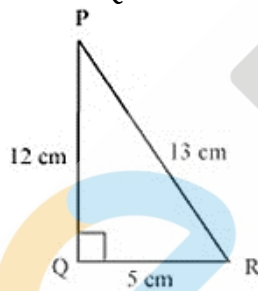
$$PR^2 = PQ^2 + QR^2$$

$$(13)^2 = (12)^2 + QR^2$$

$$169 = 144 + QR^2$$

$$25 = QR^2$$

$$QR = 5$$



$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ}$$

$$= \frac{5}{12}$$

$$\cot R = \frac{\text{Side adjacent to } \angle R}{\text{Side opposite to } \angle R} = \frac{QR}{PQ}$$

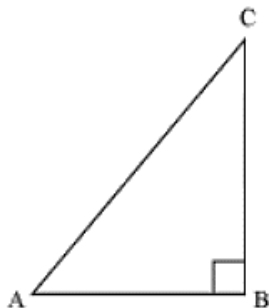
$$= \frac{5}{12}$$

$$\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$$

3. If $\sin A = \frac{3}{4}$, calculate $\cos A$ and $\tan A$.

Solution:

Let us assume that ΔABC is a right triangle, right angled at vertex B.



Given that

$$\sin A = \frac{3}{4}$$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{hypotenuse}}$$

$$\Rightarrow \frac{BC}{AC} = \frac{3}{4}$$

Let BC and AC be $3R$ and $4R$ respectively, where R is any positive number.

In ΔABC , apply Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow (4R)^2 = AB^2 + (3R)^2$$

$$\Rightarrow 16R^2 - 9R^2 = AB^2$$

$$\Rightarrow 7R^2 = AB^2$$

$$\Rightarrow AB = \sqrt{7}R$$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{hypotenuse}}$$

$$= \frac{AB}{AC} = \frac{\sqrt{7}R}{4R} = \frac{\sqrt{7}}{4}$$

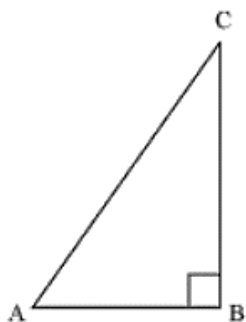
$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A}$$

$$= \frac{BC}{AB} = \frac{3R}{\sqrt{7}R} = \frac{3}{\sqrt{7}}$$

4. Given $15 \cot A = 8$, find $\sin A$ and $\sec A$.

Solution:

Let us assume that ΔABC is a right triangle, right angled at vertex B.



$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A}$$

$$= \frac{AB}{BC}$$

$$\cot A = \frac{8}{15} \quad (\text{given})$$

$$\Rightarrow \frac{AB}{BC} = \frac{8}{15}$$

Let AB and BC be 8R and 15R respectively, where R is a positive number.

Now applying Pythagoras theorem in $\triangle ABC$

$$AC^2 = AB^2 + BC^2$$

$$= (8R)^2 + (15R)^2$$

$$= 64R^2 + 225R^2$$

$$= 289R^2$$

$$\Rightarrow AC = 17R$$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{15R}{17R} = \frac{15}{17}$$

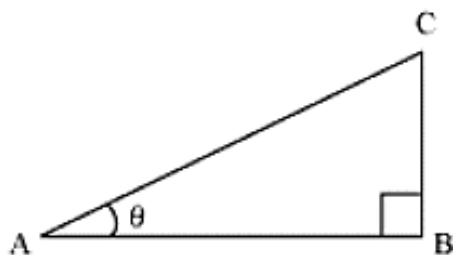
$$\sec A = \frac{\text{hypotenuse}}{\text{Side adjacent to } \angle A}$$

$$= \frac{AC}{AB} = \frac{17}{8}$$

5. Given $\sec \theta = \frac{13}{12}$, calculate all other trigonometric ratios.

Solution:

Let us assume that $\triangle ABC$ is a right angled triangle, right angled at vertex B.



$$\sec \theta = \frac{\text{hypotenuse}}{\text{Side adjacent to } \angle B}$$

$$\Rightarrow \frac{13}{12} = \frac{AC}{AB}$$

Let AC and AB be 13R and 12R, where R is a positive number.

Now applying Pythagoras theorem in ΔABC

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$\Rightarrow (13R)^2 = (12R)^2 + BC^2$$

$$\Rightarrow 169R^2 = 144R^2 + BC^2$$

$$\Rightarrow 25R^2 = BC^2$$

$$\Rightarrow BC = 5R$$

$$\sin \theta = \frac{\text{Side opposite to } \angle \theta}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{5R}{13R} = \frac{5}{13}$$

$$\cos \theta = \frac{\text{Side adjacent to } \angle \theta}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{12R}{13R} = \frac{12}{13}$$

$$\tan \theta = \frac{\text{Side opposite to } \angle \theta}{\text{Side adjacent to } \angle \theta} = \frac{BC}{AB} = \frac{5R}{12R} = \frac{5}{12}$$

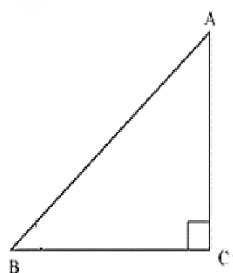
$$\cot \theta = \frac{\text{Side adjacent to } \angle \theta}{\text{Side opposite to } \angle \theta} = \frac{AB}{BC} = \frac{12R}{5R} = \frac{12}{5}$$

$$\operatorname{cosec} \theta = \frac{\text{hypotenuse}}{\text{Side opposite to } \angle \theta} = \frac{AC}{BC} = \frac{13R}{5R} = \frac{13}{5}$$

6. If $\angle A$ and $\angle B$ are acute angles such that $\cos A = \cos B$, then show that $\angle A = \angle B$.

Solution:

Let us assume a right triangle, right angled at vertex C.



$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{hypotenuse}}$$

$$= \frac{AC}{AB}$$

$$\cos B = \frac{\text{Side adjacent to } \angle B}{\text{hypotenuse}}$$

$$= \frac{BC}{AB}$$

Since $\cos A = \cos B$

$$\text{So } \frac{AC}{AB} = \frac{BC}{AB}$$

$$\Rightarrow AC = BC$$

So $\angle A = \angle B$ (Angle opposite to equal sides are equal in length)

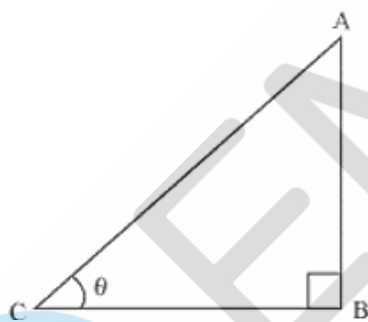
7. If $\cot \theta = \frac{7}{8}$, evaluate:

(i) $\frac{(1+\sin \theta)(1-\sin \theta)}{(1+\cos \theta)(1-\cos \theta)}$

(ii) $\cot^2 \theta$

Solution:

Let us assume that ΔABC is a right triangle, right angled at vertex B.



$$\cot \theta = \frac{\text{Side adjacent to } \angle \theta}{\text{Side opposite to } \angle \theta} = \frac{BC}{AB}$$

$$= \frac{7}{8}$$

Let BC and AB be 7R and 8R respectively, where R is a positive number.

In ΔABC , apply Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

$$= (8R)^2 + (7R)^2$$

$$= 64R^2 + 49R^2$$

$$= 113R^2$$

$$AC = \sqrt{113}R$$

$$\sin \theta = \frac{\text{Side opposite to } \angle \theta}{\text{hypotenuse}} = \frac{AB}{AC}$$

$$= \frac{8R}{\sqrt{113}R} = \frac{8}{\sqrt{113}}$$

$$\cos \theta = \frac{\text{Side adjacent to } \angle \theta}{\text{hypotenuse}} = \frac{BC}{AC}$$

$$\cos \theta = \frac{7R}{\sqrt{113}R} = \frac{7}{\sqrt{113}}$$

$$(i) \quad \frac{(1+\sin \theta)(1-\sin \theta)}{(1+\cos \theta)(1-\cos \theta)} = \frac{(1-\sin^2 \theta)}{(1-\cos^2 \theta)}$$

$$= \frac{1 - \left(\frac{8}{\sqrt{113}}\right)^2}{1 - \left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1 - \frac{64}{113}}{1 - \frac{49}{113}}$$

$$= \frac{\frac{113 - 64}{113}}{\frac{113 - 49}{113}} = \frac{49}{64}$$

$$(ii) \quad \cot^2 \theta$$

$$= (\cot \theta)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

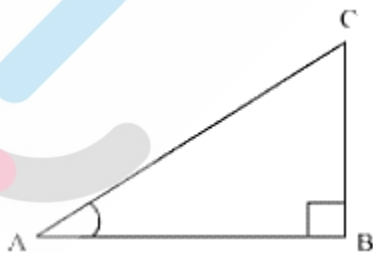
8. If $3 \cot A = 4$, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Solution:

Given that $3 \cot A = 4$

$$\text{or } \cot A = \frac{4}{3}$$

Let us assume that ΔABC is a right triangle, right angled at vertex B.



$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A}$$

$$\Rightarrow \frac{AB}{BC} = \frac{4}{3}$$

Let AB and BC be $4R$ and $3R$ respectively, where R is a positive number.

Now in ΔABC

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$= (4R)^2 + (3R)^2$$

$$= 16R^2 + 9R^2$$

$$= 25R^2$$

$$AC = 5R$$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC}$$

$$= \frac{4R}{5R} = \frac{4}{5}$$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{3R}{5R} = \frac{3}{5}$$

$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AB}$$

$$= \frac{3R}{4R} = \frac{3}{4}$$

$$LHS = \frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2} = \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}}$$

$$= \frac{\frac{7}{16}}{\frac{25}{16}} = \frac{7}{25}$$

$$RHS = \cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

Since, LHS = RHS

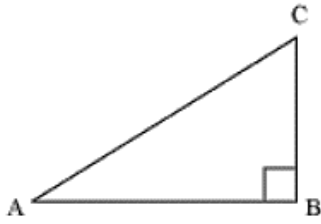
Hence proved.

9. In triangle ABC, right-angled at B, if $\tan A = \frac{1}{\sqrt{3}}$, find the value of:

(i) $\sin A \cos C + \cos A \sin C$

(ii) $\cos A \cos C - \sin A \sin C$

Solution:



$$\tan A = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{BC}{AB} = \frac{1}{\sqrt{3}}$$

Let BC and AB be R and $\sqrt{3}R$, Where R is a positive number.

In $\triangle ABC$, apply Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

$$= (\sqrt{3}R)^2 + (R)^2$$

$$= 3R^2 + R^2 = 4R^2$$

$$\Rightarrow AC = 2R$$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{R}{2R} = \frac{1}{2}$$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}R}{2R} = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{\text{Side opposite to } \angle C}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}R}{2R} = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{\text{Side adjacent to } \angle C}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{R}{2R} = \frac{1}{2}$$

$$(i) \sin A \cos C + \cos A \sin C$$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{4} + \frac{3}{4} = 1$$

$$(ii) \cos A \cos C - \sin A \sin C$$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

10. In $\triangle PQR$, right-angled at Q, $PR + QR = 25$ cm and $PQ = 5$ cm. Determine the values of $\sin P$, $\cos P$ and $\tan P$.

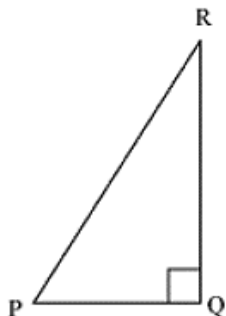
Solution:

Given that $PR + QR = 25$

$PQ = 5$

Let PR be a

So, QR = $25 - a$



In ΔPQR , apply Pythagoras theorem

$$PR^2 = PQ^2 + QR^2$$

$$a^2 = (5)^2 + (25 - a)^2$$

$$a^2 = 25 + 625 + a^2 - 50a$$

$$50a = 650$$

$$a = 13$$

So, PR = 13 cm

$$QR = 25 - 13 = 12 \text{ cm}$$

$$\sin P = \frac{\text{Side opposite to } \angle P}{\text{hypotenuse}} = \frac{QR}{PR} = \frac{12}{13}$$

$$\cos P = \frac{\text{Side adjacent to } \angle P}{\text{hypotenuse}} = \frac{PQ}{PR} = \frac{5}{13}$$

$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ} = \frac{12}{5}$$

11. State whether the following are true or false. Justify your answer.

- (i) The value of $\tan A$ is always less than 1.
- (ii) $\sec A = \frac{12}{5}$ for some value of angle A .
- (iii) $\cos A$ is the abbreviation used for the cosecant of angle A .
- (iv) $\cot A$ is the product of \cot and A .
- (iv) $\sin \theta = \frac{4}{3}$ for some angle θ .

Solution:

- (i) If $0^\circ \leq A \leq 45^\circ$
 $0 \leq \tan A \leq 1$
 If $45^\circ \leq A \leq 90^\circ$

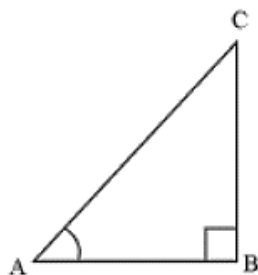
$$1 \leq \tan A < \infty$$

Clearly value of $\tan A$ is from 0 to ∞ and not always less than 1.

Hence, the given statement is false.

- (ii) Let us assume that ΔABC is a right-angled triangle, right angled at vertex B.

$$\sec A = \frac{12}{5}$$



$$\frac{\text{hypotenuse}}{\text{Side adjacent to } \angle A} = \frac{12}{5}$$

Let AC and AB be $12R$ and $5R$ respectively, where R is a positive number.

In ΔABC , apply Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

$$(12R)^2 = (5R)^2 + BC^2$$

$$144R^2 = 25R^2 + BC^2$$

$$BC^2 = 119R^2$$

$$BC = 10.9R$$

We may observe that for given two sides $AC = 12R$ and $AB = 5R$

BC should be such that –

$AC - AB < BC < AC + AB$ (sum of any two sides of a triangle is greater than third side and difference of any two sides of a triangle is smaller than the third side)

$$\text{so, } 12R - 5R < BC < 12R + 5R$$

$$7R < BC < 17R$$

But $BC = 10.9R$. Clearly such a triangle is possible and hence such value of $\sec A$ is possible. Hence, the given statement is true.

- (iii) $\operatorname{cosec} A$ is the abbreviation used for cosecant of angle A , and $\cos A$ is the abbreviation used for cosine of angle A . Hence, the given statement is false.
- (iv) $\cot A$ is the abbreviation used for cotangent of angle A and not the product of \cot and A . So, given statement is false.

$$(v) \quad \sin \theta = \frac{4}{3}$$

In a right-angle triangle, we know that

$$\sin \theta = \frac{\text{Side opposite to } \angle \theta}{\text{hypotenuse}} = \frac{4}{3}$$

So, side opposite to $\angle \theta = \frac{4}{3} \text{hypotenuse}$

But in a right-angle triangle hypotenuse is always greater than the remaining two sides.

Hence such value of $\sin \theta$ is not possible.

Hence the given statement is false.

EXERCISE 8.2

1. Evaluate the following:

$$(i) \quad \sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$$

$$(ii) \quad 2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$$

$$(iii) \quad \frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$$

$$(iv) \quad \frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$$

$$(v) \quad \frac{5 \cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$

Solution:

$$(i) \quad \sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$$

$$= \left(\frac{\sqrt{3}}{2}\right) \left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right) \left(\frac{1}{2}\right)$$

$$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

$$(ii) \quad 2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$$

$$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2 + \frac{3}{4} - \frac{3}{4} = 2$$

$$(iii) \quad \frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$$

$$\begin{aligned}
 &= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2 + 2\sqrt{3}}{\sqrt{3}}} \\
 &= \frac{\sqrt{3}}{\sqrt{2}(2 + 2\sqrt{3})} = \frac{\sqrt{3}}{2\sqrt{2} + 2\sqrt{6}} \\
 &= \frac{\sqrt{3}(2\sqrt{6} - 2\sqrt{2})}{(2\sqrt{6} + 2\sqrt{2})(2\sqrt{6} - 2\sqrt{2})} \\
 &= \frac{2\sqrt{3}(\sqrt{6} - \sqrt{2})}{(2\sqrt{6})^2 - (2\sqrt{2})^2} = \frac{2\sqrt{3}(\sqrt{6} - \sqrt{2})}{24 - 8} = \frac{2\sqrt{3}(\sqrt{6} - \sqrt{2})}{16} \\
 &= \frac{\sqrt{18} - \sqrt{6}}{8} = \frac{3\sqrt{2} - \sqrt{6}}{8}
 \end{aligned}$$

$$(iv) \frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$$

$$\begin{aligned}
 &= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \frac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{3}{2} + \frac{2}{\sqrt{3}}} \\
 &= \frac{\frac{3\sqrt{3}-4}{2\sqrt{3}}}{\frac{3\sqrt{3}+4}{2\sqrt{3}}} = \frac{(3\sqrt{3}-4)}{(3\sqrt{3}+4)}
 \end{aligned}$$

On multiplying and dividing by $3\sqrt{3} - 4$, we get

$$\begin{aligned}
 &= \frac{(3\sqrt{3} - 4)^2}{(3\sqrt{3} + 4)(3\sqrt{3} - 4)} \\
 &= \frac{27 + 16 - 24\sqrt{3}}{27 - 16} = \frac{43 - 24\sqrt{3}}{11}
 \end{aligned}$$

$$(v) \frac{5 \cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$

$$\begin{aligned}
 &= \frac{5 \left(\frac{1}{2}\right)^2 + 4 \left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \\
 &= \frac{5 \left(\frac{1}{4}\right) + \left(\frac{16}{3}\right) - 1}{\frac{1}{4} + \frac{3}{4}} \\
 &= \frac{\frac{15 + 64 - 12}{12}}{\frac{4}{4}} = \frac{67}{12}
 \end{aligned}$$

2. Choose the correct option and justify your choice:

(i) $\frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} =$

(A) $\sin 60^\circ$

(B) $\cos 60^\circ$

(C) $\tan 60^\circ$

(D) $\sin 30^\circ$

(ii) $\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ}$

(A) $\tan 90^\circ$

(B) 1

(C) $\sin 45^\circ$

(D) 0

(iii) $\sin 2A = 2 \sin A$ is true when $A =$

(A) 0°

(B) 30°

(C) 45°

(D) 60°

(iv) $\frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} =$

(A) $\cos 60^\circ$

(B) $\sin 60^\circ$

(C) $\tan 60^\circ$

(D) $\sin 30^\circ$

Solution:

(i) $\frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ}$

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$

$$= \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$\text{Also, } \sin 60^\circ = \frac{\sqrt{3}}{2}$$

So option (A) is correct.

$$(ii) \quad \frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ}$$

$$= \frac{1 - (1)^2}{1 + (1)^2} = \frac{1 - 1}{1 + 1} = \frac{0}{2} = 0$$

Hence (D) is correct.

(iii) Out of given options only option (A) is correct.

$$\text{As } \sin 2A = \sin 0^\circ = 0$$

$$2 \sin A = 2 \sin 0^\circ = 2(0) = 0$$

$$(iv) \quad \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} =$$

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$$

$$= \sqrt{3}$$

$$\text{Also, } \tan 60^\circ = \sqrt{3}$$

So option (C) is correct.

3. If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$; $0^\circ < A + B \leq 90^\circ$; $A > B$, find A and B.

Solution:

$$\tan(A + B) = \sqrt{3}$$

$$\Rightarrow \tan(A + B) = \tan 60^\circ$$

$$\Rightarrow A + B = 60^\circ \quad \dots(i)$$

$$\tan(A - B) = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan(A - B) = \tan 30^\circ$$

$$\Rightarrow A - B = 30^\circ \quad \dots(ii)$$

Adding both equations

$$2A = 90^\circ$$

$$\Rightarrow A = 45^\circ$$

From equation (i)

$$45^\circ + B = 60^\circ$$

$$B = 15^\circ$$

So, $\angle A = 45^\circ$ and $\angle B = 15^\circ$

4. State whether the following are true or false. Justify your answer.

- (i) $\sin(A + B) = \sin A + \sin B$
- (ii) The value of $\sin \theta$ increases as θ increases.
- (iii) The value of $\cos \theta$ increases as θ increases.
- (iv) $\sin \theta = \cos \theta$ for all values of θ .
- (v) $\cot A$ is not defined for $A = 0^\circ$.

Solution:

(i) $\sin(A + B) = \sin A + \sin B$

Let $A = 45^\circ$ and $B = 45^\circ$

$$LHS = \sin(A + B) = \sin(45^\circ + 45^\circ)$$

$$= \sin 90^\circ$$

$$= 1$$

$$RHS = \sin A + \sin B = \sin 45^\circ + \sin 45^\circ$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}}$$

$$= \sqrt{2}$$

Clearly $LHS \neq RHS$.

Hence the given statement is false.

(ii) We know that,

$$\sin 0^\circ = 0$$

$$\sin 30^\circ = \frac{1}{2} = 0.5$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}} = 0.707$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} = 0.866$$

$$\sin 90^\circ = 1$$

Clearly, we can see that value of $\sin \theta$ increases as θ increases in the interval of $0^\circ < \theta < 90^\circ$

Hence the given statement is true.

(iii) $\cos 0^\circ = 1$

$$\cos 30^\circ = \frac{\sqrt{3}}{2} = 0.866$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} = 0.707$$

$$\cos 60^\circ = \frac{1}{2} = 0.5$$

$$\cos 90^\circ = 0$$

Clearly, we can see that value of $\cos \theta$ decreases as θ increases from 0° to 90° .

Hence the given statement is false.

(iv) $\sin \theta = \cos \theta$ for all values of θ .

when $\theta = 45^\circ$

$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$

This is true for $\theta = 45^\circ$. But not true for all other values of θ .

As $\sin 30^\circ = \frac{1}{2}$ and $\cos 30^\circ = \frac{\sqrt{3}}{2}$

Hence the given statement is false.

(v) $\cot A$ is not defined for $A = 0^\circ$

We know that,

$$\cot A = \frac{\cos A}{\sin A}$$

$$\cot 0^\circ = \frac{\cos 0^\circ}{\sin 0^\circ} = \frac{1}{0} = \text{undefined.}$$

Hence the given statement is true.

EXERCISE 8.3

1. Evaluate:

(i) $\frac{\sin 18^\circ}{\cos 72^\circ}$

(ii) $\frac{\tan 26^\circ}{\cot 64^\circ}$

(iii) $\cos 48^\circ - \sin 42^\circ$

(iv) $\operatorname{cosec} 31^\circ - \sec 59^\circ$

Solution:

(i) $\frac{\sin 18^\circ}{\cos 72^\circ} = \frac{\sin(90^\circ - 72^\circ)}{\cos 72^\circ}$

$$= \frac{\cos 72^\circ}{\cos 72^\circ} = 1$$

$$(ii) \quad \frac{\tan 26^\circ}{\cot 64^\circ} = \frac{\tan(90^\circ - 64^\circ)}{\cot 64^\circ}$$

$$= \frac{\cot 64^\circ}{\cot 64^\circ} = 1$$

$$(iii) \quad \cos 48^\circ - \sin 42^\circ = \cos(90^\circ - 42^\circ) - \sin 42^\circ$$

$$= \sin 42^\circ - \sin 42^\circ$$

$$= 0$$

$$(v) \quad \operatorname{cosec} 31^\circ - \sec 59^\circ = \operatorname{cosec}(90^\circ - 59^\circ) - \sec 59^\circ$$

$$= \sec 59^\circ - \sec 59^\circ$$

$$= 0$$

2. Show that:

$$(i) \quad \tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ = 1$$

$$(ii) \quad \cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ = 0$$

Solution:

$$(i) \quad \text{LHS} = \tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ$$

$$= \tan(90^\circ - 42^\circ) \tan(90^\circ - 67^\circ) \tan 42^\circ \tan 67^\circ$$

$$= \cot 42^\circ \cot 67^\circ \tan 42^\circ \tan 67^\circ$$

$$= (\cot 42^\circ \tan 42^\circ)(\cot 67^\circ \tan 67^\circ)$$

$$= (1)(1)$$

$$= 1 = \text{RHS}$$

$$(ii) \quad \text{LHS} = \cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ$$

$$= \cos(90^\circ - 52^\circ) \cos(90^\circ - 38^\circ) - \sin 38^\circ \sin 52^\circ$$

$$= \sin 52^\circ \sin 38^\circ - \sin 38^\circ \sin 52^\circ = 0 = \text{RHS}$$

3. If $\tan 2A = \cot(A - 18^\circ)$, where $2A$ is an acute angle, find the value of A .

Solution:

Given that

$$\tan 2A = \cot(A - 18^\circ)$$

$$\Rightarrow \cot(90^\circ - 2A) = \cot(A - 18^\circ)$$

$$\Rightarrow 90^\circ - 2A = A - 18^\circ$$

$$\Rightarrow 108^\circ = 3A$$

$$\Rightarrow A = 36^\circ$$

4. If $\tan A = \cot B$, prove that $A + B = 90^\circ$.

Solution:

Given that

$$\tan A = \cot B$$

$$\Rightarrow \tan A = \tan(90^\circ - B)$$

$$\Rightarrow A = 90^\circ - B$$

$$\Rightarrow A + B = 90^\circ$$

Hence Proved

5. If $\sec 4A = \operatorname{cosec}(A - 20^\circ)$, where $4A$ is an acute angle, find the value of A .

Solution:

Given that

$$\sec 4A = \operatorname{cosec}(A - 20^\circ)$$

$$\Rightarrow \operatorname{cosec}(90^\circ - 4A) = \operatorname{cosec}(A - 20^\circ)$$

$$\Rightarrow 90^\circ - 4A = A - 20^\circ$$

$$\Rightarrow 110^\circ = 5A$$

$$\Rightarrow A = 22^\circ$$

6. If A, B and C are interior angles of a triangle ABC , then show that

$$\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$$

Solution:

We know that, sum of all angles of a triangle is 180°

So, for a triangle ΔABC

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle B + \angle C = 180^\circ - \angle A$$

$$\Rightarrow \frac{\angle B + \angle C}{2} = 90^\circ - \frac{\angle A}{2}$$

$$\Rightarrow \sin\left(\frac{B+C}{2}\right) = \sin\left(90^\circ - \frac{A}{2}\right)$$

$$\Rightarrow \sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$$

7. Express $\sin 67^\circ + \cos 75^\circ$ in terms of trigonometric ratios of angles between 0° and 45° .

Solution:

$$\sin 67^\circ + \cos 75^\circ$$

$$= \sin(90^\circ - 23^\circ) + \cos(90^\circ - 15^\circ)$$

$$= \cos 23^\circ + \sin 15^\circ$$

EXERCISE 8.4

1. Express the trigonometric ratios $\sin A$, $\sec A$ and $\tan A$ in terms of $\cot A$.

Solution:

We know that

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\frac{1}{\operatorname{cosec}^2 A} = \frac{1}{1 + \cot^2 A}$$

$$\sin^2 A = \frac{1}{1 + \cot^2 A} \quad \left(\text{Since, } \operatorname{cosec} A = \frac{1}{\sin A}\right)$$

$$\text{So, } \sin A = \frac{1}{\sqrt{1 + \cot^2 A}}$$

$$\text{As we know, } \tan A = \frac{\sin A}{\cos A}$$

$$\text{Also, } \cot A = \frac{\cos A}{\sin A}$$

$$\text{So, we can easily see that } \tan A = \frac{1}{\cot A}$$

$$\text{Also, } \sec^2 A = 1 + \tan^2 A$$

$$= 1 + \frac{1}{\cot^2 A} \quad \left(\tan A = \frac{1}{\cot A}\right)$$

$$= \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\Rightarrow \sec A = \frac{\sqrt{\cot^2 A + 1}}{\cot A}$$

2. Write all the other trigonometric ratios of $\angle A$ in terms of $\sec A$.

Solution:

$$\text{As we know, } \cos A = \frac{1}{\sec A}$$

$$\text{Also, } \sin^2 A + \cos^2 A = 1$$

$$\sin^2 A = 1 - \cos^2 A$$

$$\Rightarrow \sin A = \sqrt{1 - \left(\frac{1}{\sec A}\right)^2}$$

$$= \frac{\sqrt{\sec^2 A - 1}}{\sec A} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

$$\tan^2 A + 1 = \sec^2 A$$

$$\Rightarrow \tan^2 A = \sec^2 A - 1$$

$$\Rightarrow \tan A = \sqrt{\sec^2 A - 1}$$

$$\cot A = \frac{\cos A}{\sin A} = \frac{\frac{1}{\sec A}}{\sqrt{\sec^2 A - 1}}$$

$$= \frac{1}{\sqrt{\sec^2 A - 1}}$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

3. Evaluate:

(i) $\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

(ii) $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

Solution:

(i)
$$\begin{aligned} & \frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ} \\ &= \frac{[\sin(90^\circ - 27^\circ)]^2 + \sin^2 27^\circ}{[\cos(90^\circ - 73^\circ)]^2 + \cos^2 73^\circ} \\ &= \frac{[\cos 27^\circ]^2 + \sin^2 27^\circ}{[\sin 73^\circ]^2 + \cos^2 73^\circ} \\ &= \frac{\cos^2 27^\circ + \sin^2 27^\circ}{\sin^2 73^\circ + \cos^2 73^\circ} \\ &= \frac{1}{1} \quad (\text{As } \sin^2 A + \cos^2 A = 1) \\ &= 1 \end{aligned}$$

(ii)
$$\begin{aligned} & \sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ \\ &= (\sin 25^\circ)\{\cos(90^\circ - 25^\circ)\} + \cos 25^\circ\{\sin(90^\circ - 25^\circ)\} \\ &= (\sin 25^\circ)(\sin 25^\circ) + (\cos 25^\circ)(\cos 25^\circ) \\ &= \sin^2 25^\circ + \cos^2 25^\circ \\ &= 1 \quad (\text{As, } \sin^2 A + \cos^2 A = 1) \end{aligned}$$

4. Choose the correct option. Justify your choice.

(i) $9 \sec^2 A - 9 \tan^2 A =$

(A) 1

- (B) 9
 (C) 8
 (D) 0
- (ii) $(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta) =$
 (A) 0
 (B) 1
 (C) 2
 (D) -1
- (iii) $(\sec A + \tan A)(1 - \sin A) =$
 (A) $\sec A$
 (B) $\sin A$
 (C) $\operatorname{cosec} A$
 (D) $\cos A$
- (iv) $\frac{1 + \tan^2 A}{1 + \cot^2 A} =$
 (A) $\sec^2 A$
 (B) -1
 (C) $\cot^2 A$
 (D) $\tan^2 A$

Solution:

$$\begin{aligned}
 \text{(i)} \quad & 9 \sec^2 A - 9 \tan^2 A \\
 &= 9(\sec^2 A - \tan^2 A) \\
 &= 9(1) \quad [\text{as, } \sec^2 A = \tan^2 A + 1] \\
 &= 9
 \end{aligned}$$

Hence option (B) is correct.

$$\begin{aligned}
 \text{(ii)} \quad & (1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta) \\
 &= \left(1 + \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta}\right) \left(1 + \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta}\right) \\
 &= \left(\frac{\cos \theta + \sin \theta + 1}{\cos \theta}\right) \left(\frac{\sin \theta + \cos \theta - 1}{\sin \theta}\right) \\
 &= \frac{(\sin \theta + \cos \theta)^2 - (1)^2}{\sin \theta \cos \theta} \\
 &= \frac{\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta}
 \end{aligned}$$

$$= \frac{1 + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta}$$

$$= \frac{2 \sin \theta \cos \theta}{\sin \theta \cos \theta} = 2$$

Hence option (C) is correct.

$$(iii) \quad (\sec A + \tan A)(1 - \sin A)$$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A} \right) (1 - \sin A)$$

$$= \left(\frac{1 + \sin A}{\cos A} \right) (1 - \sin A)$$

$$= \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A}$$

$$= \cos A$$

So, option (D) is correct.

$$(iv) \quad \frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{1 + \frac{\sin^2 A}{\cos^2 A}}{1 + \frac{\cos^2 A}{\sin^2 A}}$$

$$= \frac{\frac{\cos^2 A + \sin^2 A}{\cos^2 A}}{\frac{\sin^2 A + \cos^2 A}{\sin^2 A}} = \frac{1}{\frac{\cos^2 A}{\sin^2 A}}$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

Hence option (D) is correct.

5. Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

$$(i) \quad (\operatorname{cosec} \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$$

$$(ii) \quad \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$$

$$(iii) \quad \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$$

[Hint: Write the expression in terms of $\sin \theta$ and $\cos \theta$]

$$(iv) \quad \frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$$

[Hint: Simplify LHS and RHS separately]

$$(v) \quad \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A, \text{ using the identity}$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

$$(vi) \quad \sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$$

$$(vii) \quad \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} = \tan \theta$$

$$(viii) \quad (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$$

$$(ix) \quad (\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$$

[**Hint:** Simplify LHS and RHS separately]

$$(x) \quad \left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$$

Solution:

$$(i) \quad (\operatorname{cosec} \theta - \cot \theta)^2 = \frac{1-\cos \theta}{1+\cos \theta}$$

$$\text{LHS} = (\operatorname{cosec} \theta - \cot \theta)^2$$

$$= \left(\frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta}\right)^2$$

$$= \frac{(1 - \cos \theta)^2}{(\sin \theta)^2} = \frac{(1 - \cos \theta)^2}{\sin^2 \theta}$$

$$= \frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta} = \frac{(1 - \cos \theta)^2}{(1 - \cos \theta)(1 + \cos \theta)} = \frac{1 - \cos \theta}{1 + \cos \theta}$$

$$\text{RHS} = \frac{1 - \cos \theta}{1 + \cos \theta}$$

So, LHS = RHS

Hence Proved.

$$(ii) \quad \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2 \sec A$$

$$\text{LHS} = \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A}$$

$$= \frac{\cos^2 A + (1 + \sin A)^2}{(1 + \sin A)(\cos A)}$$

$$= \frac{\cos^2 A + 1 + \sin^2 A + 2 \sin A}{(1 + \sin A)(\cos A)}$$

$$= \frac{\sin^2 A + \cos^2 A + 1 + 2 \sin A}{(1 + \sin A)(\cos A)}$$

$$= \frac{1 + 1 + 2 \sin A}{(1 + \sin A)(\cos A)} = \frac{2 + 2 \sin A}{(1 + \sin A)(\cos A)}$$

$$= \frac{2(1 + \sin A)}{(1 + \sin A)(\cos A)} = \frac{2}{\cos A} = 2 \sec A$$

$$\text{RHS} = 2 \sec A$$

Clearly, we can see LHS = RHS

Hence proved.

$$(iii) \quad \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$$

$$\begin{aligned} \text{LHS} &= \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} \\ &= \frac{\frac{\sin \theta}{\cos \theta}}{1 - \frac{\cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{1 - \frac{\sin \theta}{\cos \theta}} \\ &= \frac{\frac{\sin \theta}{\cos \theta}}{\frac{\sin \theta - \cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{\frac{\cos \theta - \sin \theta}{\cos \theta}} \\ &= \frac{\sin^2 \theta}{\cos \theta (\sin \theta - \cos \theta)} - \frac{\cos^2 \theta}{\sin \theta (\sin \theta - \cos \theta)} \\ &= \frac{1}{(\sin \theta - \cos \theta)} \left[\frac{\sin^2 \theta}{\cos \theta} - \frac{\cos^2 \theta}{\sin \theta} \right] \\ &= \frac{1}{(\sin \theta - \cos \theta)} \left[\frac{\sin^3 \theta - \cos^3 \theta}{\sin \theta \cos \theta} \right] \\ &= \frac{1}{(\sin \theta - \cos \theta)} \left[\frac{(\sin \theta - \cos \theta)(\sin^2 \theta + \cos^2 \theta + \sin \theta \cos \theta)}{\sin \theta \cos \theta} \right] \\ &= \frac{(1 + \sin \theta \cos \theta)}{\sin \theta \cos \theta} \\ &= \frac{(1)}{(\sin \theta \cos \theta)} + \frac{(\sin \theta \cos \theta)}{(\sin \theta \cos \theta)} \\ &= \sec \theta \operatorname{cosec} \theta + 1 \end{aligned}$$

$$\text{RHS} = \sec \theta \operatorname{cosec} \theta + 1$$

Clearly, we can see that LHS = RHS.

Hence Proved

$$(iv) \quad \frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$$

$$\text{LHS} = \frac{1 + \sec A}{\sec A} = \frac{1 + \frac{1}{\cos A}}{\frac{1}{\cos A}}$$

$$\begin{aligned} & \frac{\cos A + 1}{\frac{\cos A}{\cos A}} = (\cos A + 1) \\ & = \frac{(1 - \cos A)(1 + \cos A)}{(1 - \cos A)} \\ & = \frac{1 - \cos^2 A}{1 - \cos A} = \frac{\sin^2 A}{1 - \cos A} \end{aligned}$$

$$\text{RHS} = \frac{\sin^2 A}{1 - \cos A}$$

Clearly, LHS = RHS

Hence Proved

$$(v) \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$$

Using the identity $\operatorname{cosec}^2 A = 1 + \cot^2 A$

$$\text{LHS} = \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$$

On dividing the numerator and denominator by $\sin A$, we get

$$\begin{aligned} & = \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}} \\ & = \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A} \\ & = \frac{\{(\cot A) - (1 - \operatorname{cosec} A)\}\{(\cot A) - (1 - \operatorname{cosec} A)\}}{\{(\cot A) + (1 - \operatorname{cosec} A)\}\{(\cot A) - (1 - \operatorname{cosec} A)\}} \\ & = \frac{(\cot A - 1 + \operatorname{cosec} A)^2}{(\cot A)^2 - (1 - \operatorname{cosec} A)^2} \\ & = \frac{\cot^2 A + 1 + \operatorname{cosec}^2 A - 2 \cot A - 2 \operatorname{cosec} A + 2 \cot A \operatorname{cosec} A}{\cot^2 A - (1 + \operatorname{cosec}^2 A - 2 \operatorname{cosec} A)} \\ & = \frac{2 \operatorname{cosec}^2 A + 2 \cot A \operatorname{cosec} A - 2 \cot A - 2 \operatorname{cosec} A}{\cot^2 A - 1 - \operatorname{cosec}^2 A + 2 \operatorname{cosec} A} \\ & = \frac{2 \operatorname{cosec} A(\operatorname{cosec} A + \cot A) - 2(\cot A + \operatorname{cosec} A)}{\cot^2 A - \operatorname{cosec}^2 A - 1 + 2 \operatorname{cosec} A} \\ & = \frac{(\operatorname{cosec} A + \cot A)(2 \operatorname{cosec} A - 2)}{(2 \operatorname{cosec} A - 2)} \end{aligned}$$

$$= \operatorname{cosec} A + \cot A$$

$$\text{RHS} = \operatorname{cosec} A + \cot A$$

Clearly, LHS = RHS

Hence Proved

$$(vi) \quad \sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$$

$$\text{LHS} = \sqrt{\frac{1 + \sin A}{1 - \sin A}}$$

$$= \sqrt{\frac{(1 + \sin A)(1 + \sin A)}{(1 - \sin A)(1 + \sin A)}}$$

$$= \frac{(1 + \sin A)}{\sqrt{1 - \sin^2 A}} = \frac{1 + \sin A}{\sqrt{\cos^2 A}}$$

$$= \frac{1 + \sin A}{\cos A} = \sec A + \tan A$$

RHS = $\sec A + \tan A$

Clearly, LHS = RHS

Hence Proved

$$(vii) \quad \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} = \tan \theta$$

$$\text{LHS} = \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta}$$

$$= \frac{\sin \theta (1 - 2 \sin^2 \theta)}{\cos \theta (2 \cos^2 \theta - 1)}$$

$$= \frac{\sin \theta \times (1 - 2 \sin^2 \theta)}{\cos \theta \times \{2(1 - \sin^2 \theta) - 1\}}$$

$$= \frac{\sin \theta \times (1 - 2 \sin^2 \theta)}{\cos \theta \times (1 - 2 \sin^2 \theta)}$$

= $\tan \theta$

RHS = $\tan \theta$

Clearly, LHS = RHS

Hence Proved

$$(viii) \quad (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$$

$$\text{LHS} = (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$$

$$= \sin^2 A + \operatorname{cosec}^2 A + 2 \sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2 \cos A \sec A$$

$$\begin{aligned}
 &= \sin^2 A + \cos^2 A + \operatorname{cosec}^2 A + \sec^2 A + 2 \sin A \left(\frac{1}{\sin A} \right) + \\
 &2 \cos A \left(\frac{1}{\cos A} \right) \quad \left(\text{Since, } \operatorname{cosec} A = \frac{1}{\sin A} \text{ and } \sec A = \frac{1}{\cos A} \right) \\
 &= 1 + 1 + \cot^2 A + 1 + \tan^2 A + 2 + 2 \\
 &= 7 + \tan^2 A + \cot^2 A \\
 &= \text{RHS}
 \end{aligned}$$

Hence Proved

$$(ix) \quad (\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$$

$$\text{LHS} = (\operatorname{cosec} A - \sin A)(\sec A - \cos A)$$

$$= \left(\frac{1}{\sin A} - \sin A \right) \left(\frac{1}{\cos A} - \cos A \right) \quad \left(\text{Since, } \operatorname{cosec} A = \frac{1}{\sin A} \text{ and } \sec A = \frac{1}{\cos A} \right)$$

$$= \left(\frac{1 - \sin^2 A}{\sin A} \right) \left(\frac{1 - \cos^2 A}{\cos A} \right)$$

$$= \frac{(\cos^2 A)(\sin^2 A)}{\sin A \cos A}$$

$$= \sin A \cos A$$

$$\text{RHS} = \frac{1}{\tan A + \cot A}$$

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}}$$

$$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A} = \sin A \cos A$$

Since, LHS = RHS

Hence Proved

$$(x) \quad \left(\frac{1 + \tan^2 A}{1 + \cot^2 A} \right) = \left(\frac{1 - \tan A}{1 - \cot A} \right)^2 = \tan^2 A$$

$$\frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{1 + \frac{\sin^2 A}{\cos^2 A}}{1 + \frac{\cos^2 A}{\sin^2 A}} = \frac{\frac{\cos^2 A + \sin^2 A}{\cos^2 A}}{\frac{\sin^2 A + \cos^2 A}{\sin^2 A}}$$

$$= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A}$$

$$= \tan^2 A$$

$$\begin{aligned}\left(\frac{1 - \tan A}{1 - \cot A}\right)^2 &= \frac{1 + \tan^2 A - 2 \tan A}{1 + \cot^2 A - 2 \cot A} \\ &= \frac{\sec^2 A - 2 \tan A}{\operatorname{cosec}^2 A - 2 \cot A} \\ &= \frac{\frac{1}{\cos^2 A} - \frac{2 \sin A}{\cos A}}{\frac{1}{\sin^2 A} - \frac{2 \cos A}{\sin A}} = \frac{\frac{1 - 2 \sin A \cos A}{\cos^2 A}}{\frac{1 - 2 \sin A \cos A}{\sin^2 A}} \\ &= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A\end{aligned}$$

Hence Proved

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