

CBSE NCERT Solutions for Class 10 Mathematics Chapter 14

Back of Chapter Questions

1. A survey was conducted by a group of students as a part of their environment awareness program, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number of plants	0 – 2	2 – 4	4 – 6	6 – 8	8 – 10	10 – 12	12 – 14
Number of houses	1	2	1	5	6	2	3

Which method did you use for finding the mean, and why?

Solution:

First of all, we have to find class marks (x_i) for each interval.

We know that, class mark(x_i) = $\frac{\text{upper class limit} + \text{lower class limit}}{2}$

Now we may compute x_i and $f_i x_i$ as following

Number of plants	Number of houses (f_i)	x_i	$f_i x_i$
0 – 2	1	1	$1 \times 1 = 1$
2 – 4	2	3	$2 \times 3 = 6$
4 – 6	1	5	$1 \times 5 = 5$
6 – 8	5	7	$5 \times 7 = 35$
8 – 10	6	9	$6 \times 9 = 54$
10 – 12	2	11	$2 \times 11 = 22$
12 – 14	3	13	$3 \times 13 = 39$
Total	20		162

From the table we can see that

$$\Sigma f_i = 20$$

$$\Sigma f_i x_i = 162$$

$$\text{Mean } \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$= \frac{162}{20} = 8.1$$

So, mean number of plants per house is 8.1.

Direct method is used here as values of class marks (x_i) and f_i are small.

2. Consider the following distribution of daily wages of 50 worker of a factory.

Daily wages (in ₹)	500 – 520	520 – 540	540 – 560	560 – 580	580 – 600
Number of workers	12	14	8	6	10

Find the mean daily wages of the workers of the factory by using an appropriate method.

Solution:

First of all, we have to find class mark for each interval.

We know that,

$$x_i = \frac{\text{upper class limit} + \text{lower class limit}}{2}$$

Class size (h) of this data = 20

let us take 550 as assured mean (a). Now, we can calculate d_i, u_i and $f_i u_i$ as following.

Daily wages (in ₹)	Number of workers (f_i)	x_i	$d_i = x_i - 550$	$u_i = \frac{x_i - 550}{h}$	$f_i u_i$
500-520	12	510	-40	-2	-24
520 – 540	14	530	-20	-1	-14
540 – 560	8	550	0	0	0
560 – 580	6	570	20	1	6
580 – 600	10	590	40	2	20
Total	50				-12

From the table we can see that

$$\Sigma f_i = 50$$

$$\Sigma f_i x_i = -12$$

$$\text{Mean } \bar{x} = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i} \right) h$$

$$= 550 + \left(\frac{-12}{50} \right) 20$$

$$= 550 - \frac{24}{5}$$

$$= 550 - 4.8$$

$$= 545.2$$

So, mean daily wages of the workers of the factory is ₹ 545.20

3. The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is ₹ 18. Find the missing frequency f .

Daily pocket allowance (in ₹)	11 – 13	13 – 15	15 – 17	17 – 19	19 – 21	21 – 23	23 – 25
Number of workers	7	6	9	13	f	5	4

Solution:

First of all, we have to find class mark for each interval.

We know that,

$$\text{Class mark}(x_i) = \frac{\text{upper class limit} + \text{lower class limit}}{2}$$

Mean pocket allowance $\bar{x} = ₹ 18$ (Given)

Let us take 18 as assured mean (a). Now we can calculate d_i , u_i and $f_i u_i$ as following.

Daily pocket allowance (in ₹)	Number of children f_i	Class mark x_i	$d_i = x_i - 18$	$f_i d_i$
11 – 13	7	12	- 6	- 42
13 – 15	6	14	- 4	- 24
15 – 17	9	16	- 2	- 18
17 – 19	13	18	0	0
19 – 21	f	20	2	$2f$
21 – 23	5	22	4	20
23 – 25	4	24	6	24

Total	$\Sigma f_i = 44 + f$			$2f - 40$
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From the table we can see that

$$\Sigma f_i = 44 + f \text{ -----(1)}$$

$$\Sigma f_i d_i = 2f - 40 \text{ -----(2)}$$

$$\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} \text{ -----(3)}$$

Now put the values of Σf_i and $\Sigma f_i d_i$ from equations (1) and (2) to equation (3)

$$18 = 18 + \left(\frac{2f - 40}{44 + f} \right)$$

On solving, we get

$$0 = \left(\frac{2f - 40}{44 + f} \right)$$

$$2f - 40 = 0$$

$$2f = 40$$

$$f = 20$$

Hence the missing frequency f is 20.

4. Thirty women were examined in a hospital by a doctor and the number of heart beats per minute were recorded and summarized as follows. Find the mean heart beats per minute for these women, choosing a suitable method.

Number of heart beats per minute	65 – 68	68 – 71	71 – 74	74 – 77	77 – 80	80 – 83	83 – 86
Number of women	2	4	3	8	7	4	2

Solution:

First of all, we have to find class mark for each interval.

We know that,

$$\text{Class mark}(x_i) = \frac{\text{upperclasslimit} + \text{lowerclasslimit}}{2}$$

Class size h of this data = 3

Let us take 75.5 as assured mean (a). Now we can calculate d_i , u_i and $f_i u_i$ as following.

Number of heart beats per minute	Number of women f_i	x_i	$d_i = x_i - 75.5$	$u_i = \frac{x_i - 75.5}{h}$	$f_i u_i$
65 – 68	2	66.5	– 9	– 3	– 6
68 – 71	4	69.5	– 6	– 2	– 8
71 – 74	3	72.5	– 3	– 1	– 3
74 – 77	8	75.5	0	0	0
77 – 80	7	78.5	3	1	7
80 – 83	4	81.5	6	2	8
83 – 86	2	84.5	9	3	6
Total	30				4

Now we can see from table that

$$\Sigma f_i = 30$$

$$\Sigma f_i u_i = 4$$

$$\text{Mean } \bar{x} = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i} \right) h$$

$$= 75.5 + \left(\frac{4}{30} \right) \times 3$$

$$= 75.5 + 0.4 = 75.9$$

So mean heart beats per minute for these women are 75.9 beats per minute.

5. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes.

Number of mangoes	50 – 52	53 – 55	56 – 58	59 – 61	62 – 64
Number of boxes	15	110	135	115	25

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

Solution:

Number of mangoes	Number of boxes f_i
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50 – 52	15
53 – 55	110
56 – 58	135
59 – 61	115
62 – 64	25

There is a gap of 1 between any two consecutive class intervals. So, we can say that class intervals are not continuous. In order to remove this problem, we have to add $\frac{1}{2}$ to upper class limit and subtract $\frac{1}{2}$ from lower class limit of each interval.

And class mark (x_i) may be obtained by using the relation

$$\text{Class mark } (x_i) = \frac{\text{upper class limit} + \text{lower class limit}}{2}$$

Class size (h) of this data = 3

Let us take 57 as assured mean (a). Now we can calculate d_i , u_i and $f_i u_i$ as following.

Class interval	f_i	x_i	$d_i = x_i - 57$	$u_i = \frac{x_i - 57}{h}$	$f_i u_i$
49.5 – 52.5	15	51	-6	-2	-30
52.5 – 55.5	110	54	-3	-1	-110
55.5 – 58.5	135	57	0	0	0
58.5 – 61.5	115	60	3	1	115
61.5 – 64.5	25	63	6	2	50
Total	400				25

Now we can see that

$$\Sigma f_i = 400$$

$$\Sigma f_i u_i = 25$$

$$\begin{aligned}
 \text{Mean } \bar{x} &= a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \\
 &= 57 + \left(\frac{25}{400} \right) \times 3 \\
 &= 57 + \frac{3}{16} = 57 + 0.1875 \\
 &= 57.1875 \\
 &= 57.19
 \end{aligned}$$

So, mean number of mangoes kept in a packing box is 57.19.

Here, we chose step deviation method as values of f_i, d_i are big and also there is a common multiple (3) between all d_i .

- 6 The table below shows the daily expenditure on food of 25 households in a locality.

Daily expenditure (in ₹)	100 – 150	150 – 200	200 – 250	250 – 300	300 – 350
Number of households	4	5	12	2	2

Find the mean daily expenditure on food by a suitable method.

Solution:

First of all, we have to find class mark for each interval.

We know that,

$$\text{Class mark}(x_i) = \frac{\text{upper class limit} + \text{lower class limit}}{2}$$

Class size = 50

Let us take 225 as assured mean (a). Now we can calculate d_i, u_i and $f_i u_i$ as following.

Daily expenditure (in ₹)	f_i	x_i	$d_i = x_i - 225$	$u_i = \frac{x_i - 225}{h}$	$f_i u_i$
100 – 150	4	125	-100	-2	-8
150 – 200	5	175	-50	-1	-5
200 – 250	12	225	0	0	0
250 – 300	2	275	50	1	2

300 – 350	2	325	100	2	4
Total	25				-7

Now we can see that -

$$\Sigma f_i = 25$$

$$\Sigma f_i u_i = -7$$

$$\text{Mean } \bar{x} = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i} \right) h$$

$$= 225 + \left(-\frac{7}{25} \right) \times (50)$$

$$= 225 - 14$$

$$= 211$$

So, mean daily expenditure on food is ₹ 211.

7. To find out the concentration of SO_2 in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below:

Concentration of SO_2 (in pmm)	Frequency
0.00 – 0.04	4
0.04 – 0.08	9
0.08 – 0.12	9
0.12 – 0.16	2
0.16 – 0.20	4
0.20 – 0.24	2

Find the mean concentration of SO_2 in the air.

Solution:

First of all, we have to find class mark for each interval.

We know that,

$$\text{Class mark}(x_i) = \frac{\text{upper class limit} + \text{lower class limit}}{2}$$

Let us take 0.14 as assured mean (a). Now we can calculate d_i, u_i and $f_i u_i$ as following.

Concentration of SO_2 (in ppm)	Frequency f_i	Class mark x_i	$d_i = x_i - 0.14$	$u_i = \frac{x_i - 0.14}{h}$	$f_i u_i$
0.00 – 0.04	4	0.02	-0.12	-3	-12
0.04 – 0.08	9	0.06	-0.08	-2	-18
0.08 – 0.12	9	0.10	-0.04	-1	-9
0.12 – 0.16	2	0.14	0	0	0
0.16 – 0.20	4	0.18	0.04	1	4
0.20 – 0.24	2	0.22	0.08	2	4
Total	30				-31

For the table we can see that

$$\Sigma f_i = 30$$

$$\Sigma f_i u_i = -31$$

$$\text{Mean, } \bar{x} = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i} \right) h$$

$$= 0.14 + \left(-\frac{31}{30} \right) \times (0.04)$$

$$= 0.14 - 0.04133$$

$$= 0.09867$$

$$= 0.099 \text{ ppm}$$

Hence, mean concentration of SO_2 in the air is 0.099 ppm.

8. A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

Number of days	0 – 6	6 – 10	10 – 14	14 – 20	20 – 28	28 – 38	38 – 40
Number of students	11	10	7	4	4	3	1

Solution:

First of all, we have to find class mark for each interval.

We know that,

$$\text{Class mark}(x_i) = \frac{\text{upper class limit} + \text{lower class limit}}{2}$$

Let us take 17 as assumed mean (a), we can calculate d_i and $f_i d_i$ as following

Number of days	Number of students f_i	x_i	$d_i = x_i - 17$	$f_i d_i$
0 – 6	11	3	-14	-154
6 – 10	10	8	-9	-90
10 – 14	7	12	-5	-35
14 – 20	4	17	0	0
20 – 28	4	24	7	28
28 – 38	3	33	16	48
38 – 40	1	39	22	22
Total	40			-181

Now we can see that

$$\Sigma f_i = 40$$

$$\Sigma f_i d_i = -181$$

$$\text{Mean } \bar{x} = a + \left(\frac{\Sigma f_i d_i}{\Sigma f_i} \right)$$

$$= 17 + \left(-\frac{181}{40} \right)$$

$$= 17 - 4.525$$

$$= 12.475$$

$$= 12.48$$

Hence, mean number of days for which a student was absent is 12.48 days.

9. The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

Literacy rate (in %)	45 – 55	55 – 65	65 – 75	75 – 85	85 – 95
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Number of cities	3	10	11	8	3
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Solution:

First of all, we have to find class mark for each interval.

We know that,

$$\text{Class mark}(x_i) = \frac{\text{upper class limit} + \text{lower class limit}}{2}$$

Class size (h) for this data = 10

Let us take 70 as assumed mean (a), we can calculate d_i , u_i , and $f_i u_i$ as following

Literacy rate (in %)	Number of cities f_i	x_i	$d_i = x_i - 70$	$u_i = \frac{d_i}{h}$	$f_i u_i$
45 – 55	3	50	-20	-2	-6
55 – 65	10	60	-10	-1	-10
65 – 75	11	70	0	0	0
75 – 85	8	80	10	1	8
85 – 95	3	90	20	2	6
Total	35				-2

Now we can see that

$$\Sigma f_i = 35$$

$$\Sigma f_i u_i = -2$$

$$\text{Mean } \bar{x} = a + \left(\frac{\Sigma f_i u_i}{\Sigma u_i} \right) h$$

$$= 70 + \left(-\frac{2}{35} \right) \times (10)$$

$$= 70 - \frac{20}{35}$$

$$= 70 - 0.57$$

$$= 69.43$$

So, mean literacy rate is 69.43%.



EXERCISE 14.2

1. The following table shows the ages of the patients admitted in a hospital during a year:

Age (in years)	5 – 15	15 – 25	25 – 35	35 – 45	45 – 55	55 – 65
Number of patients	6	11	21	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

Solution:

First of all, we have to find class mark for each interval.

We know that,

$$x_i = \frac{\text{upper class limit} + \text{lower class limit}}{2}$$

Now, let us take 30 as assumed mean (a), we can calculate d_i and $f_i d_i$ as following.

Age (in years)	Number of patients f_i	Class mark x_i	$d_i = x_i - 30$	$f_i d_i$
5 – 15	6	10	-20	-120
15 – 25	11	20	-10	-110
25 – 35	21	30	0	0
35 – 45	23	40	10	230
45 – 55	14	50	20	280
55 – 65	5	60	30	150

From the table we may observe that

$$\Sigma f_i = 80$$

$$\Sigma f_i d_i = 430$$

$$\text{Mean } \bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

$$= 30 + \left(\frac{430}{80} \right)$$

$$= 30 + 5.375$$

$$= 35.375$$

$$\approx 35.38$$

Clearly, mean of this data is 35.38 which means that the average age of a patient admitted to hospital was 35.38 years.

As we can see that maximum class frequency is 23 which belongs to class interval 35 – 45.

So, modal class = 35 – 45

Lower limit (l) of modal class = 35

Frequency (f_1) of modal class = 23

Class size (h) = 10

Frequency (f_0) of class preceding the modal class = 21

Frequency (f_2) of class succeeding the modal class = 14

$$\text{Now mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

$$= 35 + \left(\frac{23 - 21}{2(23) - 21 - 14} \right) \times 10$$

$$= 35 + \frac{20}{11}$$

$$= 35 + 1.81$$

$$= 36.8$$

Clearly, mode is 36.8. It represents that maximum number of patients admitted in hospital were of 36.8 years.

2. The following data gives the information on the observed lifetimes (in hours) of 225 electrical components:

Lifetimes (in hours)	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120
Frequency	10	35	52	61	38	29

Determine the modal lifetimes of the components.

Solution:

From the given data, it is clear that maximum class frequency is 61 which belongs to class interval 60 – 80.

So, modal class = 60 – 80

Lower class limit (l) of modal class = 60

Frequency (f_1) of modal class = 61

Frequency (f_0) of class preceding the modal class = 52

Frequency (f_2) of class succeeding the modal class = 38

Class size (h) = 20

$$\begin{aligned} \text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\ &= 60 + \left(\frac{61 - 52}{2(61) - 52 - 38} \right) \times (20) \\ &= 60 + \left(\frac{9}{122 - 90} \right) (20) \\ &= 60 + \left(\frac{9}{32} \right) (20) \\ &= 60 + \frac{90}{16} = 60 + 5.625 \\ &= 65.625 \end{aligned}$$

So, modal lifetime of electrical components is 65.625 hours.

3. The following data gives the distribution of total monthly household expenditure of **200** families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure.

Expenditure (in ₹)	Number of families
1000 – 1500	24
1500 – 2000	40
2000 – 2500	33
2500 – 3000	28
3000 – 3500	30
3500 – 4000	22
4000 – 4500	16
4500 – 5000	7

Solution:

From the given data, it is clear that maximum class frequency is 40 which belongs to 1500 – 2000 intervals.

So, modal class = 1500 – 2000

Lower limit (l) of modal class = 1500

Frequency (f_1) of modal class = 40

Frequency (f_0) of class preceding modal class = 24

Frequency (f_2) of class succeeding modal class = 33

Class size (h) = 500

$$\begin{aligned} \text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\ &= 1500 + \left(\frac{40 - 24}{2(40) - 24 - 33} \right) \times (500) \\ &= 1500 + \left(\frac{16}{80 - 57} \right) (500) \\ &= 1500 + \frac{8000}{23} \\ &= 1500 + 347.826 \\ &= 1847.826 = 1847.83 \end{aligned}$$

So modal monthly expenditure was ₹ 1847.83

Now, we have to find class mark

$$\text{Class mark} = \frac{\text{upper class limit} + \text{lower class limit}}{2}$$

Class size (h) of give data = 500

Let us take 2750 as assumed mean (a), we can calculate $d_i u_i$ and $f_i u_i$ as following

Expenditure (in ₹)	Number of families f_i	x_i	$d_i = x_i - 2750$	$u_i = \frac{x_i - 2750}{h}$	$f_i u_i$
1000 – 1500	24	1250	–1500	–3	–72
1500 – 2000	40	1750	–1000	–2	–80
2000 – 2500	33	2250	–500	–1	–33
2500 – 3000	28	2750	0	0	0
3000 – 3500	30	3250	500	1	30
3500 – 4000	22	3750	1000	2	44

4000 – 4500	16	4250	1500	3	48
4500 – 5000	7	4750	2000	4	28
Total	200				-35

Now, from the table it is clear that

$$\Sigma f_i = 200$$

$$\Sigma f_i u_i = -35$$

$$\bar{x}(\text{mean}) = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i} \right) \times h$$

$$\bar{x} = 2750 + \left(\frac{-35}{200} \right) \times 500$$

$$= 2750 - 87.5$$

$$= 2662.5$$

So, mean monthly expenditure was ₹ 2662.50.

4. The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret the two measures.

Number of students per teacher	Number of states/U.T
15 – 20	3
20 – 25	8
25 – 30	9
30 – 35	10
35 – 40	3
40 – 45	0
45 – 50	0
50 – 55	2

Solution:

From the given data, it is clear that maximum class frequency is 10 which belongs to class interval 30 – 35.

So, modal class = 30 – 35

Class size (h) = 5

Lower limit (l) of modal class = 30

Frequency (f_1) of modal class = 10

Frequency (f_0) of class preceding modal class = 9

Frequency (f_2) of class succeeding modal class = 3

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

$$= 30 + \left(\frac{10 - 9}{2(10) - 9 - 3} \right) \times (5)$$

$$= 30 + \left(\frac{1}{20 - 12} \right) 5$$

$$= 30 + \frac{5}{8} = 30.625$$

Mode = 30.6

It represents that teacher student ratio of most of states/U.T is 30.6

Now, we have to find out class mark by using the relation

$$\text{Class mark} = \frac{\text{upper class limit} + \text{lower class limit}}{2}$$

Let us take 32.5 as assumed mean (a), we can calculate d_i , u_i and $f_i u_i$ as following.

Number of students per teacher	Number of states/U. T (f_i)	x_i	$d_i = x_i - 32.5$	$u_i = \frac{x_i - 32.5}{h}$	$f_i u_i$
15 - 20	3	17.5	-15	-3	-9
20 - 25	8	22.5	-10	-2	-16
25 - 30	9	27.5	-5	-1	-9
30 - 35	10	32.5	0	0	0
35 - 40	3	37.5	5	1	3
40 - 45	0	42.5	10	2	0
45 - 50	0	47.5	15	3	0
50 - 55	2	52.5	20	4	8
Total	35				-23

$$\text{Now mean } \bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) h$$

$$\begin{aligned}
 &= 32.5 + \left(-\frac{23}{35}\right) \times 5 \\
 &= 32.5 - \frac{23}{7} = 32.5 - 3.28 \\
 &= 29.22
 \end{aligned}$$

So mean of data is 29.2 which means that on an average teacher-student ratio was 29.2

5. The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches.

Number of students per teacher	Number of states/U.T
3000 – 4000	4
4000 – 5000	18
5000 – 6000	9
6000 – 7000	7
7000 – 8000	6
8000 – 9000	3
9000 – 10000	1
10000 – 11000	1

Find the mode of the data.

Solution:

It is clear from the given data that maximum class frequency is 18 which belongs to class interval 4000 – 5000.

So, modal class = 4000 – 5000

Lower limit (l) of modal class = 4000

Frequency (f_1) of modal class = 18

Frequency (f_0) of class preceding modal class = 4

Frequency (f_2) of class succeeding modal class = 9

Class size (h) = 1000

$$\text{Now Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times (h)$$

$$= 4000 + \left(\frac{18 - 4}{2(18) - 4 - 9} \right) \times 1000$$

$$= 4000 + \left(\frac{14000}{23} \right)$$

$$= 4000 + 608.695$$

$$= 4608.695$$

So mode of given data is 4608.7 runs.

6. A student noted the number of cars passing through a spot on a road for **100** periods each of **3** minutes and summarised it in the table given below. Find the mode of the data:

Number of cars	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Frequency	7	14	13	12	20	11	15	8

Solution:

It is clear from the given data that maximum class frequency is 20 which belongs to 40 - 50 class intervals.

So, modal class = 40 – 50

Lower limit (l) of modal class = 40

Frequency (f_1) of modal class = 20

Frequency (f_0) of class preceding modal class = 12

Frequency (f_2) of class succeeding modal class = 11

Class size = 10

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times (h)$$

$$= 40 + \left(\frac{20 - 12}{2(20) - 12 - 11} \right) \times 10$$

$$= 40 + \left(\frac{80}{40 - 23} \right)$$

$$= 40 + \frac{80}{17}$$

$$= 40 + 4.7$$

So mode of this data is 44.7 cars.



EXERCISE 14.3

1. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.

Monthly consumption (in units)	Number of consumers
65 - 85	4
85 - 105	5
105 - 125	13
125 - 145	20
145 - 165	14
165 - 185	8
185 - 205	4

Solution:

First of all, we have to find class mark for each interval.

We know that,

$$\text{Class mark} = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Let us take 135 as assumed mean (a), according to step deviation method we may find $d_i, u_i, f_i \times u_i$, as following

Monthly consumption (in units)	Number of consumers (f_i)	x_i class mark	$d_i = x_i - 135$	$u_i = \frac{d_i}{20}$	$f_i u_i$
65 - 85	4	75	-60	-3	-12
85 - 105	5	95	-40	-2	-10
105 - 125	13	115	-20	-1	-13
125 - 145	20	135	0	0	0
145 - 165	14	155	20	1	14
165 - 185	8	175	40	2	16
185 - 205	4	195	60	3	12

Total	68				7
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We can observe from the table

$$\sum f_i u_i = 7$$

$$\sum f_i = 68$$

$$\text{Class size } (h) = 20$$

$$\text{Mean } \bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h$$

$$= 135 + \frac{7}{68} \times 20$$

$$= 135 + \frac{140}{68}$$

$$= 137.058$$

It is clear from the table that maximum class frequency is 20 which belongs to class interval 125 - 145.

$$\text{Modal class} = 125 - 145$$

$$\text{Lower limit } (l) \text{ of modal class} = 125$$

$$\text{Class size } (h) = 20$$

$$\text{Frequency } (f_1) \text{ of modal class} = 20$$

$$\text{Frequency } (f_0) \text{ of class preceding modal class} = 13$$

$$\text{Frequency } (f_2) \text{ of class succeeding the modal class} = 14$$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

$$= 125 + \left[\frac{20 - 13}{2(20) - 13 - 14} \right] \times 20$$

$$= 125 + \frac{7}{13} \times 20$$

$$= 125 + \frac{140}{13} = 135.76$$

We know that

$$3 \text{ median} = \text{mode} + 2 \text{ mean}$$

$$= 135.76 + 2 (137.058)$$

$$= 135.76 + 274.116$$

$$= 409.876$$

$$\Rightarrow \text{Median} = \frac{409.876}{3}$$

$$\text{Median} = 136.625$$

So median, mode, mean of given data is 136.625, 135.76, 137.05 respectively.

2. If the median of the distribution is given below is 28.5, find the values of x and y.

Class interval	Frequency
0 - 10	5
10 - 20	x
20 - 30	20
30 - 40	15
40 - 50	y
50 - 60	5
Total	60

Solution:

First of all we have to find cumulative frequency of the given data

Class interval	Frequency	Cumulative frequency
0 - 10	5	5
10 - 20	x	5 + x
20 - 30	20	25 + x
30 - 40	15	40 + x
40 - 50	y	40 + x + y
50 - 60	5	45 + x + y
Total (n)	60	

It is clear that $n = 60$

$$45 + x + y = 60$$

$$x + y = 15 \dots\dots\dots (1)$$

Median of data = 28.5 (given)

Median of data lies in interval 20 – 30.

So, median class = 20 – 30

Lower limit (l) of median class = 20

Cumulative frequency (cf) of class preceding the median class = $5 + x$

Frequency (f) of median class = 20

Class size (h) = 10

Now, median = $l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$

$$28.5 = 20 + \left[\frac{\frac{60}{2} - (5 + x)}{20}\right] \times 10$$

$$8.5 = \left(\frac{25 - x}{2}\right)$$

$$17 = 25 - x$$

$$x = 8$$

Put the value of x in equation (1)

$$8 + y = 15$$

$$y = 7$$

Hence values of x and y are 8 and 7 respectively.

3. A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 year.

Age (in years)	Number of policy holders
Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98

Below 60	100
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Solution:

We can see from above table that class width is not same. We don't need to adjust the frequencies according to class intervals. Now above frequency table is represented with upper class limits and is of less than type. As policies were given only to persons having age 18 years onwards but less than 60 years, we can define class intervals with their respective cumulative frequency as given below.

Age (in years)	Number of policy holders (f_i)	Cumulative frequency (cf)
18 – 20	2	2
20 – 25	$6 - 2 = 4$	6
25 – 30	$24 - 6 = 18$	24
30 - 35	$45 - 24 = 21$	45
35 - 40	$78 - 45 = 33$	78
40 - 45	$89 - 78 = 11$	89
45 – 50	$92 - 89 = 3$	92
50 – 55	$98 - 92 = 6$	98
55 – 60	$100 - 98 = 2$	100
Total (n)		

We can see from the table that $n = 100$.

$$\frac{n}{2} = 50$$

Cumulative frequency (cf) just greater than $\frac{n}{2}$ is 78. ($\frac{n}{2} = 50$)

It belongs to interval 35 – 40

So, median class = 35 – 40

Lower limit (l) of median class = 35

Class size (h) = 5

Frequency (f) of median class = 33

Cumulative frequency (cf) of class preceding median class = 45

$$\text{median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$= 35 + \left(\frac{50 - 45}{33} \right) \times 5$$

$$= 35 + \frac{25}{33}$$

$$= 35.76$$

So, median age is 35.76 years.

4. The lengths of 40 leaves of a plant are measured correct to the nearest millimeter, and the data obtained is represented in the following table:

Length (in mm)	Number or leaves (f_i)
118 – 126	3
127 – 135	5
136 – 144	9
145 – 153	12
154 – 162	5
163 - 171	4
172 - 180	2

Find the median length of the leaves.

(Hint: The formula is for continuous classes only so we have to convert the given data into continuous classes for finding the median. The classes then change to 117.5 - 126.5, 126.5 - 135.5 - 171.5 - 180.5)

Solution:

The given data does not have continuous class intervals. We can observe from the table that difference between two class intervals is 1. So, we have to add 0.5 to upper class limits and subtract 0.5 to lower class limits.

Now continuous class intervals with respective cumulative frequencies can be represented as below.

Length (in mm)	Number or leaves (f_i)	Cumulative frequency
117.5 - 126.5	3	3
126.5 - 135.5	5	3 + 5 = 8
135.5 - 144.5	9	8 + 9 = 17
144.5 - 153.5	12	17 + 12 = 29
153.5 - 162.5	5	29 + 5 = 34
162.5 - 171.5	4	34 + 4 = 38

171.5 - 180.5	2	$38 + 2 = 40$
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Here, $n = 40$

$$\text{So, } \frac{n}{2} = 20$$

We can observe from the table that cumulative frequency just greater than $\frac{n}{2}$ is 29.

It belongs to class interval 144.5 - 153.5.

So, Median class = 144.5 - 153.5

Lower limit (l) of median class = 144.5

Class size (h) = 9

Frequency (f) of median class = 12

Cumulative frequency (cf) of class preceding median class = 17

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$= 144.5 + \left(\frac{20 - 17}{12} \right) \times 9$$

$$= 144.5 + \frac{9}{4} = 146.75$$

So, median length of leaves is 146.75 mm.

5. Find the following tables gives the distribution of the life time of 400 neon lamps:

Life time (in hours)	Number of lamps
1500 - 2000	14
2000 - 2500	56
2500 - 3000	60
3000 - 3500	86
3500 - 4000	74
4000 - 4500	62
4500 - 5000	48

Find the median life time of a lamp.

Solution:

First of all, we have to find cumulative frequencies with their respective class intervals

Life time	Number of lamps (f_i)	Cumulative frequency
1500 - 2000	14	14
2000 - 2500	56	$14 + 56 = 70$
2500 - 3000	60	$70 + 60 = 130$
3000 - 3500	86	$130 + 86 = 216$
3500 - 4000	74	$216 + 74 = 290$
4000 - 4500	62	$290 + 62 = 352$
4500 - 5000	48	$352 + 48 = 400$
Total (n)	400	

Here, $n = 400$

$$\text{So, } \frac{n}{2} = 200$$

From the table we can observe that cumulative frequency just greater than $\frac{n}{2}$ (i.e., $\frac{400}{2} = 200$) is 216.

It belongs to class interval 3000 – 3500.

So, Median class = 3000 - 3500

Lower limit (l) of median class = 3000

Frequency (f) of median class = 86

Cumulative frequency (cf) of class preceding median class = 130

Class size (h) = 500

$$\begin{aligned} \text{Median} &= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h \\ &= 3000 + \left(\frac{200 - 130}{86} \right) \times 500 \\ &= 3000 + \frac{70 \times 500}{86} \\ &= 3000 + 406.976 \\ &= 3406.976 \end{aligned}$$

So, median life time of lamps is 3406.98 hours.

6. 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows:

Number of letters	1 - 4	4 - 7	7 - 10	10 - 13	13 - 16	16 - 19
Number of surnames	6	30	40	16	4	4

Determine the median number of letters in the surnames. Find the mean number of letters in the surnames? Also, find the modal size of the surnames.

Solution:

First of all, we have to find cumulative frequencies with their respective class intervals

Number of letters	Frequency (f_i)	Cumulative frequency
1 - 4	6	6
4 - 7	30	$30 + 6 = 36$
7 - 10	40	$36 + 40 = 76$
10 - 13	16	$76 + 16 = 92$
13 - 16	4	$92 + 4 = 96$
16 - 19	4	$96 + 4 = 100$
Total (n)	100	

Here, $n = 100$.

So, $\frac{n}{2} = 50$

From the table, we can observe that cumulative frequency just greater than is 76. It belongs to class interval 7 - 10.

So, Median class = 7 - 10

Lower limit (l) of median class = 7

Cumulative frequency (cf) of class preceding median class = 36

Frequency (f) of median class = 40

Class size (h) = 3

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$= 7 + \left(\frac{50 - 36}{40} \right) \times 3$$

$$= 7 + \frac{14 \times 3}{40}$$

$$= 7 + 1.05$$

$$= 8.05$$

Now we have to find class marks of given class intervals by using relation

$$\text{Class mark} = \frac{\text{Upper class limit} + \text{lower class limit}}{2}$$

Taking 11.5 as assumed mean (a) we can find d_i , u_i and $f_i u_i$ according to step deviation method as below.

Number of letters	Number of surnames f_i	x_i	$x_i - a$	$u_i = \frac{x_i - a}{3}$	$f_i u_i$
1 - 4	6	2.5	-9	-3	-18
4 - 7	30	5.5	-6	-2	-60
7 - 10	40	8.5	-3	-1	-40
10 - 13	16	11.5	0	0	0
13 - 16	4	14.5	3	1	4
16 - 19	4	17.5	6	2	8
Total	100				-106

$$\sum f_i u_i = -106$$

$$\sum f_i = 100$$

$$\text{Mean } \bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) h$$

$$= 11.5 + \left(\frac{-106}{100} \right) \times 3$$

$$= 11.5 - 3.18 = 8.32$$

We know that

$$3 \text{ median} = \text{mode} + 2 \text{ mean}$$

$$3(8.05) = \text{mode} + 2(8.32)$$

$$24.15 - 16.64 = \text{mode}$$

$$7.51 = \text{mode}$$

So, median number and mean number of letters in surnames is 8.05 and 8.32 respectively while modal size of surnames is 7.51.

7. The distribution below gives the weights of 30 students of a class. Find the median weight of the students.

Weight (in kg)	40 - 45	45 - 50	50 - 55	55 - 60	60 - 65	65 - 70	70 - 75
Number of students	2	3	8	6	6	3	2

Solution:

We may find cumulative frequencies with their respective class intervals as below

Weight (in kgs)	Frequency (f_i)	Cumulative frequency
40-45	2	2
45-50	3	$2 + 3 = 5$
50-55	8	$5 + 8 = 13$
55-60	6	$13 + 6 = 19$
60-65	6	$19 + 6 = 25$
65-70	3	$25 + 3 = 28$
70-75	2	$28 + 2 = 30$
Total (n)	30	

Here, $n = 30$

$$\text{So, } \frac{n}{2} = 15$$

Cumulative frequency just greater than $\frac{n}{2}$ (i. e., $\frac{30}{2} - 15$) is 19.

It belongs to class interval 55 - 60.

So, Median class = 55 - 60

Lower limit (l) of median class = 55

Frequency (f) of median class = 6

Cumulative frequency (cf) of median class = 13

Class size (h) = 5

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$= 55 + \left(\frac{15 - 13}{6} \right) \times 5$$

$$= 55 + \frac{10}{6}$$

$$= 56.666$$

So, median weight is 56.67 kg.

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EXERCISE 14.4

1. The following distribution gives the daily income of 50 workers of a factory.

Daily income (in ₹)	100-120	120 – 140	140 – 160	160 – 180	180 – 200
Number of workers	12	14	8	6	10

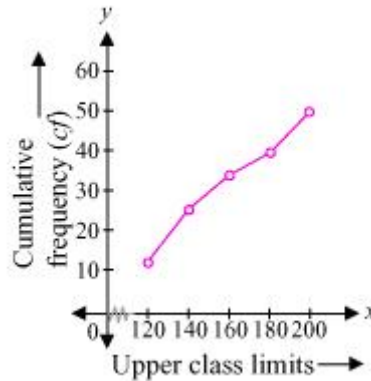
Convert the above distribution to a less than type cumulative frequency distribution, and draw its ogive.

Solution:

We can convert the given distribution table into frequency distribution table of less than type as following

Daily income (in ₹) (upper class limits)	Cumulative frequency
Less than 120	12
Less than 140	12 + 14 = 26
Less than 160	26 + 8 = 34
Less than 180	34 + 6 = 40
Less than 200	40 + 10 = 50

Now, on x-axis we take upper class limits of class intervals and their respective cumulative frequencies on y- axis we can draw its ogive as following



2. During the medical check-up of 35 students of a class, their weights were recorded as follows:

Weight (in kg)	Number of students
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

Draw a less than type ogive for the given data. Hence obtain the median weight from the graph verify the result by using the formula.

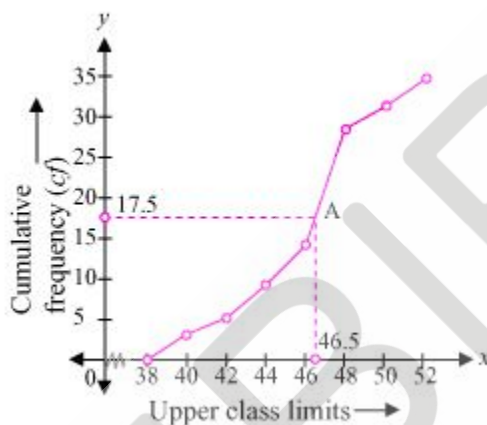
Solution:

The cumulative frequency distributions of less than type is given as

Weight (in kg) upper class limits	Number of students (cumulative frequency)
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9

Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

Now, on x-axis we take upper class limits of class intervals and their respective cumulative frequencies on y-axis we can draw its ogive as following



Now mark the point A whose ordinate is 17.5 its abscissa is 46.5. So median of this data is 46.5.

We can see from the table that difference between two consecutive upper-class limits is 2. Now we have to obtain class marks with their respective frequencies as below

Weight (in kg)	Frequency (f)	Cumulative frequency
Less than 38	0	0
38 - 40	$3 - 0 = 3$	3
40 - 42	$5 - 3 = 2$	5
42 - 44	$9 - 5 = 4$	9
44 - 46	$14 - 9 = 5$	14
46 - 48	$28 - 14 = 14$	28
48 - 50	$32 - 28 = 4$	32
50 - 52	$35 - 32 = 3$	35

Total (n)	35	
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Here, $n=35$.

$$\text{So, } \frac{n}{2} = 17.5$$

Now the cumulative frequency just greater than $\frac{n}{2}$ (i. e., $\frac{35}{2} = 17.5$) is 28.

It belongs to class interval 46 – 48

So, Median class = 46 – 48

Lower class limit (l) of median class = 46

Frequency (f) of median class = 14

Cumulative frequency (cf) of class preceding median class = 14

Class size (h) = 2

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$= 46 + \left(\frac{17.5 - 14}{14} \right) \times 2$$

$$= 46 + \frac{3.5}{7}$$

$$= 46 + 0.5$$

$$= 46.5$$

So median of this data is 46.5

Hence, value of median is verified.

3. The following table gives production yield per hectare of wheat of 100 farms of a village.

Production yield (in kg/ha)	50 - 55	55 - 60	60 - 65	65 - 70	70 - 75	75 - 80
Number of farms	2	8	12	24	38	16

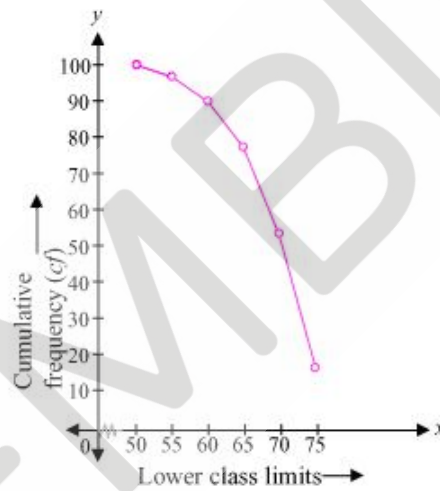
Change the distribution to a more than type distribution and draw ogive.

Solution:

We can obtain cumulative frequency distribution of more than type as following

Production yield (lower class limits)	Cumulative frequency
more than or equal to 50	100
more than or equal to 55	$100 - 2 = 98$
more than or equal to 60	$98 - 8 = 90$
more than or equal to 65	$90 - 12 = 78$
more than or equal to 70	$78 - 24 = 54$
more than or equal to 75	$54 - 38 = 16$

Now, on x-axis we take upper class limits of class intervals and their respective cumulative frequencies on y-axis we can draw its ogive as following.



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