In each of the following, give the justification of the construction also:

1. Draw a line segment of length 7.6 cm and divide it in the ratio 5 : 8. Measure the two parts.

   **Solution:**
   Steps of Construction:
   1. Draw line segment $AB$ of 7.6 cm and draw any ray $AX$ making an acute angle with $AB$.
   2. Locate $13(= 5 + 8)$ points $A_1, A_2, A_3, A_4, \ldots, A_{13}$ on $AX$ so that $AA_1 = A_1A_2 = A_2A_3 = \ldots = A_{12}A_{13}$
   3. Join $BA_{13}$.
   4. Through the point $A_5$, draw a line parallel to $A_{13}B$ (by making an angle equal to $\angle AA_{13}B$) at $A_5$ intersecting $AB$ at the point $C$.

   Now $C$ is the point dividing line segment $AB$ of 7.6 cm in the required ratio of 5 : 8.

   We can measure the approximate lengths of $AC$ and $CB$. The length of $AC$ and $CB$ comes to 2.9 cm and 4.7 cm respectively.

2. Construct a triangle of sides 4 cm, 5 cm and 6 cm and then a triangle similar to it whose sides are $\frac{2}{3}$ of the corresponding sides of the first triangle.

   **Solution:**
   The steps of construction are as follows:
(1) Draw a line segment \(AB = 4\) cm. Taking point A as centre draw an arc of 5 cm radius. Similarly, taking point B as its centre, draw an arc of 6 cm radius. These arcs will intersect each other at point C. Now \(AC = 5\) cm and \(BC = 6\) cm and \(\triangle ABC\) is the required triangle.

(2) Draw any ray AX making an acute angle with AB on opposite side of vertex C.

(3) Locate 3 points \(A_1, A_2, A_3\) (as 3 is greater between 2 and 3) on AX such that \(AA_1 = A_1A_2 = A_2A_3\)

(4) Join \(BA_3\) and draw a line through \(A_2\) parallel to \(BA_3\) to intersect \(AB\) at point \(B'\).

(5) Draw a line through \(B'\) parallel to the line \(BC\) to intersect \(AC\) at \(C'\). \(\triangle AB'C'\) is the required triangle.

Diagram:

3. Construct a triangle with sides 5 cm, 6 cm and 7 cm and then another triangle whose sides are \(\frac{7}{5}\) of the corresponding sides of the first triangle.

**Solution:**

The steps of construction are as follows:

(1) Draw a line segment \(AB\) of 5 cm. Taking A and B as centre, draw arcs of 6 cm and 7 cm radius respectively. Let these arcs intersect each other at point C. \(\triangle ABC\) is the required triangle having length of sides as 5 cm, 6 cm and 7 cm respectively.

(2) Draw any ray AX making an acute angle with line AB on opposite side of vertex C.

(3) Locate 7 points \(A_1, A_2, A_3, A_4, A_5, A_6, A_7\) (as 7 is greater between 5 and 7) on AX such that \(AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7\).

(4) Join \(BA_5\) and draw a line through \(A_7\) parallel to \(BA_5\) to intersect extended line segment \(AB\) at point \(B'\).
(5) Draw a line through $B'$ parallel to $BC$ intersecting the extended line segment $AC$ at $C'$. $\triangle AB'C'$ is the required triangle.

4. Construct an isosceles triangle whose base is 8 cm and altitude 4 cm and then another triangle whose sides are $\frac{1}{2}$ times the corresponding sides of the isosceles triangle.

Solution:

Let $\triangle ABC$ be an isosceles triangle having $CA$ and $CB$ of equal lengths, base $AB$ is 8 cm and $AD$ is the altitude of length 4 cm.

Now, the steps of construction are as follows:

(1) Draw a line segment $AB$ of 8 cm. Draw arcs of same radius on both sides of line segment while taking point $A$ and $B$ as its centre. Let these arcs intersect each other at $O$ and $O'$. Join $OO'$. Let $OO'$ intersect $AB$ at $D$.

(2) Take $D$ as centre and draw an arc of 4 cm radius which cuts the extended line segment $OO'$ at point $C$. Now an isosceles $\triangle ABC$ is formed, having $CD$ (altitude) as 4 cm and $AB$ (base) as 8 cm.

(3) Draw any ray $AX$ making an acute angle with line segment $AB$ on opposite side of vertex $C$.

(4) Locate 3 points (as 3 is greater between 3 and 2) on $AX$ such that $AA_1 = A_1A_2 = A_2A_3$.

(5) Join $BA_2$ and draw a line through $A_3$ parallel to $BA_2$ to intersect extended line segment $AB$ at point $B'$.

(6) Draw a line through $B'$ parallel to $BC$ intersecting the extended line segment $AC$ at $C'$. $\triangle AB'C'$ is the required triangle.
5. Draw a triangle ABC with side BC = 6 cm, AB = 5 cm and ∠ABC = 60°. Then construct a triangle whose sides are \( \frac{3}{4} \) of the corresponding sides of the triangle ABC.

**Solution:**

The steps of construction are as follows:

1. Draw a line segment BC of length 6 cm. Draw an arc of any radius while taking B as centre. Let it intersect line BC at point O. Now taking O as centre draw another arc to cut the previous arc at point O'. Join BO' which is the ray making 60° with line BC.

2. Now draw an arc of 5 cm radius while taking B as centre, intersecting extended line segment BO' at point A. Join AC. ΔABC is having AB = 5 cm. BC = 6 cm and ∠ABC = 60°.

3. Draw any ray BX making an acute angle with BC on opposite side of vertex A.

4. Locate 4 points (as 4 is greater in 3 and 4). B_1, B_2, B_3, B_4 on line segment BX.

5. Join B_4C and draw a line through B_3, parallel to B_4C intersecting BC at C'.

6. Draw a line through C' parallel to AC intersecting AB at A'. ΔA'BC' is the required triangle.
6. Draw a triangle ABC with side BC = 7 cm, \( \angle B = 45^\circ \), \( \angle A = 105^\circ \). Then, construct a triangle whose sides are \( \frac{4}{3} \) times the corresponding sides of \( \triangle ABC \).

**Solution:**

\[ \angle B = 45^\circ, \angle A = 105^\circ \]

It is known that the sum of all interior angles in a triangle is \( 180^\circ \)

\[ \angle A + \angle B + \angle C = 180^\circ \]

\[ \Rightarrow 105^\circ + 45^\circ + \angle C = 180^\circ \]

\[ \Rightarrow \angle C = 180^\circ - 150^\circ = 30^\circ \]

Now, the steps of construction are as follows:

1. **Draw a line segment BC = 7 cm.** Draw an arc of any radius while taking B as centre. Let it intersects BC at P. Draw an arc from P, of same radius as before, to intersect this arc at Q. From Q, again draw an arc, of same radius as before, to cut the arc at R. Now from points Q and R draw arcs of same radius as before, to intersect each other at S. Join BS.

   Let BS intersect the arc at T. From T and P draw arcs of same radius as before to intersect each other at U. Join BU which is making \( 45^\circ \) with BC.

2. **Draw an arc of any radius taking C as its centre.** Let it intersects BC at O. Taking O as centre, draw an arc of same radius intersecting the previous arc at O'. Now taking O and O' s centre, draw arcs of same radius as before, to intersect each at Y. Join CY which is making \( 30^\circ \) to BC.

3. **Extend line segment CY and BU.** Let they intersect each other at A. \( \triangle ABC \) is the triangle having \( \angle A = 105^\circ \), \( \angle B = 45^\circ \) and BC = 7 cm.

4. **Draw any ray BX making an acute angle with BC on opposite side of vertex A.**

5. **Locate 4 points (as 4 is greater in 4 and 3) B_1, B_2, B_3 and B_4 on BX.**
(6) Join \( B_3C \). Draw a line through \( B_4 \) parallel to \( B_3C \) intersecting extended \( BC \) at \( C' \).

(7) Through \( C' \) draw a line parallel to \( AC \) intersecting extended line segment \( BA \) at \( A' \). \( \Delta A'BC' \) is required triangle.

7. Draw a right triangle in which the sides (other than hypotenuse) are of lengths 4 cm and 3 cm. Then construct another triangle whose sides are \( \frac{5}{3} \) times the corresponding sides of the given triangle.

**Solution:**

The steps of construction are as follows:

(1) Draw a line segment \( AB = 4\) cm and draw a ray \( SA \) making 90\(^\circ\) with it.

(2) Draw an arc of 3 cm radius while taking \( A \) as its centre to intersect \( SA \) at \( C \). Join \( BC \). \( \Delta ABC \) is required triangle.

(3) Draw any ray \( AX \) making an acute angle with \( AB \) on the side opposite to vertex \( C \).

(4) Locate 5 points (as \( 5 \) is greater in \( 5 \) and \( 3 \)) \( A_1, A_2, A_3, A_4, A_5 \) on line segment \( AX \).

(5) Join \( A_3B \). Draw a line through \( A_5 \) parallel to \( A_3B \) intersecting extended line segment \( AB \) at \( B' \).

(6) Through \( B' \), draw a line parallel to \( BC \) intersecting extended line segment \( AC \) at \( C' \). \( \Delta AB'C' \) is required triangle.
EXERCISE 11.2

In each of the following, give also the justification of the construction:

1. Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths.

Solution:

The steps of construction are as follows:

(1) Taking any point O of the given plane as centre. Draw a circle of 6 cm radius. Locate a point P, 10 cm away from O. Join OP.

(2) Bisect OP. Let M be the midpoint of PO.

(3) Taking M as centre and MO as radius, draw a circle.

(4) Let this circle intersect our first circle at point Q and R.

(5) Join PQ and PR. PQ and PR are the required tangents. The length of tangents PQ and PR are 8 cm each.
2. Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also verify the measurement by actual calculation.

**Solution:**

The steps of construction are as follows:

1. Draw a circle of 4 cm radius with centre as O on the given plane.
2. Draw a circle of 6 cm radius taking O as its centre. Locate a point P on this circle and join OP.
3. Bisect OP. Let M be the midpoint of PO.
4. Taking M as its centre and MO as its radius draw a circle. Let it intersect the given circle at the points Q and R.
5. Join PQ and PR. PQ and PR are the required tangents.

Now, PQ and PR are of length 4.47 cm each.

In ΔPQO, since PQ is tangent, \( \angle PQO = 90^\circ \).

PO = 6 cm
QO = 4 cm

Applying Pythagoras theorem in ΔPQO,

\[
PQ^2 + QO^2 = PO^2
\]
\[
\Rightarrow PQ^2 + (4)^2 = (6)^2
\]
\[
\Rightarrow PQ^2 = 20
\]
\[
\therefore PQ = 2\sqrt{5} = 4.47 \text{ cm}
\]
3. Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q.

**Solution:**

The steps of construction are as follows:

1. Taking any point O on given plane as centre, draw a circle of 3 cm radius.
2. Take one of its diameters, RS, extended it on both sides. Locate two points on this diameter such that OP = OQ = 7 cm.
3. Bisect OP and OQ. Let T and U be the midpoints of OP and OQ respectively.
4. Taking T and U as its centre, with TO and UO as radius, draw two circles. These two circles will intersect our circle at point V, W, X and Y respectively. Join PV, PW, QX and QY. These are required tangents.

4. Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°.

Consider the above figure. PQ and PR are the tangents to the given circle.

If they are inclined at 60°, then ∠QPO = ∠OPR = 30°

Hence, ∠POQ = ∠POR = 60°

Consider ΔQSO,

∠QOS = 60°

OQ = OS (radius)
So, \( \angle OQS = \angle OSQ = 60^\circ \)
\[ \therefore \Delta QSO \text{ is an equilateral triangle} \]
So, \( QS = SO = QO = \text{radius} \)
\[ \angle PQS = 90^\circ - \angle OQS = 90^\circ - 60^\circ = 30^\circ \]
\[ \angle QPS = 30^\circ \]
\( PS = SQ \) (Isosceles triangle)
Hence, \( PS = SQ = OS \) (radius)

Now, the steps of construction are as follows:

1. Draw a circle of 5 cm radius and with centre \( O \).
2. Take a point \( P \) on circumference of this circle. Extend \( OP \) to \( Q \) such that \( OP = PQ \).
3. Midpoint of \( OQ \) is \( P \). Draw a circle with radius \( OP \) with centre as \( P \). Let it intersect our circle at \( R \) and \( S \). Join \( QR \) and \( QS \). \( QR \) and \( QS \) are required tangents.

5. Draw a line segment \( AB \) of length 8 cm. Taking \( A \) as centre, draw a circle of radius 4 cm and taking \( B \) as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle.

**Solution:**

The steps of construction are as follows:

1. Draw a line segment \( AB \) of 8 cm. Taking \( A \) and \( B \) as centre, draw two circles of 4 cm and 3 cm radius.
2. Bisect the line \( AB \). Let midpoint of \( AB \) is \( C \). Taking \( C \) as centre draw a circle of radius \( AC \) which will intersect our circles at point \( P, Q, R \) and \( S \). Join \( BP, BQ, AS \) and \( AR \). These are our required tangents.
6. Let \(ABC\) be a right triangle in which \(AB = 6\) cm, \(BC = 8\) cm and \(\angle B = 90^\circ\). \(BD\) is the perpendicular from \(B\) on \(AC\). The circle through \(B, C, D\) is drawn. Construct the tangents from \(A\) to this circle.

**Solution:**

In the following figure, it can be seen that if a circle is drawn through \(B, D\) and \(C\), then \(BC\) will be its diameter as \(\angle BDC\) is \(90^\circ\). The centre \(E\) of this circle will be the midpoint of \(BC\).

The steps of construction are as follows:

1. Join \(AE\) and bisect it. Let \(F\) be the midpoint of \(AE\).
2. Now with \(F\) as centre and radius \(FE\), draw a circle intersecting the first circle at point \(B\) and \(G\).
3. Join \(AG\).

Thus, \(AB\) and \(AG\) are the required tangents.
7. Draw a circle with the help of a bangle. Take a point outside the circle. Construct the pair of tangents from this point to the circle.

**Solution:**

The steps of construction are as follows:

1. Draw a circle with bangle.
2. Take a point \( P \) outside this circle and take two non-parallel chords \( QR \) and \( ST \).
3. Draw perpendicular bisectors of these chords intersecting each other at point \( O \) which is centre of the given circle.
4. Join \( OP \) and bisect it. Let \( U \) be the midpoint of \( OP \). With \( U \) as centre and radius \( OU \), draw a circle, intersecting our first circle at \( V \) and \( W \). Join \( PV \) and \( PW \).

**Thus,** \( PV \) and \( PW \) are the required tangents.