CBSE NCERT Solutions for Class 12 Physics Chapter 13

Back of Chapter Questions

13.1. (a) Two stable isotopes of lithium ${}^{6}_{3}\text{Li}$ and ${}^{7}_{3}\text{Li}$ have respective abundances of 7.5% and 92.5%. These isotopes have masses 6.01512 u and 7.01600 u, respectively. Find the atomic mass of lithium.

(b) Boron has two stable isotopes, ${}^{10}_{5}\text{B}$ and ${}^{11}_{5}\text{B}$. Their respective masses are 10.01294 u and 11.00931 u, and the atomic mass of boron is 10.811 u. Find the abundances of ${}^{10}_{5}\text{B}$ and ${}^{11}_{5}\text{B}$

Solution:

Given that

Mass of ${}^{6}_{3}\text{Li}$ lithium isotope, $m_1 = 6.01512\text{ u}$

Mass of ${}^{7}_{3}\text{Li}$ lithium isotope, $m_2 = 7.01600\text{ u}$

Abundance of ${}^{6}_{3}\text{Li}$, $\eta_1 = 7.5\%$

Abundance of ${}^{7}_{3}\text{Li}$, $\eta_2 = 92.5\%$

The atomic mass of lithium atom is given as:

$$m = \frac{m_1\eta_1 + m_2\eta_2}{\eta_1 + \eta_2} = \frac{6.01512 \times 7.5 + 7.01600 \times 92.5}{7.5 + 92.5} = 6.940934\text{ u}$$

Mass of ${}^{10}_{5}\text{B}$ boron isotope, $m_1 = 10.01294\text{ u}$

Mass of ${}^{11}_{5}\text{B}$ boron isotope, $m_2 = 11.00931\text{ u}$

Assume that the abundance of ${}^{10}_{5}\text{B}$, $\eta_1 = x\%$

Therefore, the abundance of ${}^{11}_{5}\text{B}$, $\eta_2 = (100 - x)\%$

The atomic mass of boron $= 10.811\text{ u}$. The atomic mass of lithium atom is given as:

$$m = \frac{m_1\eta_1 + m_2\eta_2}{\eta_1 + \eta_2}$$

$$\Rightarrow 10.811 = \frac{10.01294 \times 11.00931 \times (100 - x)}{x + (100 - x)}$$

$$\Rightarrow 1081.1 = 10.1294\,x + 11009.931 + 1100.931\,x$$

$$\Rightarrow x = \frac{19.821}{0.99637} = 19.89\%$$

Therefore, $100 - x = 100\,\% - 19.89\,\% = 80.11\,\%$

Hence, the abundances of ${}^{10}_{5}B$ is 19.98 % and ${}^{11}_{5}B$ is 80.11 %.

13.2. The three stable isotopes of neon: ${}^{20}_{10}Ne$, ${}^{21}_{10}Ne$ and ${}^{22}_{10}Ne$ have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are 19.99 u, 20.99 u and 21.99 u, respectively. Obtain the average atomic mass of neon.

Solution:

Given that

Atomic mass of ${}^{20}_{10}Ne$, $m_1 = 19.99$ u

Abundance of ${}^{20}_{10}Ne$, $\eta_1 = 90.51$ %

Atomic mass of ${}^{21}_{10}Ne$, $m_2 = 20.99$ u

Abundance of ${}^{21}_{10}Ne$, $\eta_2 = 0.27$ %

Atomic mass of ${}^{22}_{10}Ne$, $m_3 = 21.99$ u

Abundance of ${}^{22}_{10}Ne$, $\eta_3 = 9.22$ %

The average atomic mass of neon is given as:

$$m = \frac{m_1\eta_1 + m_2\eta_2 + m_3\eta_3}{\eta_1 +_2 + \eta_3} = \frac{19.99 \times 90.51 + 20.99 \times 0.27 + 21.99 \times 9.22}{90.51 + 0.27 + 9.22}$$

$= 20.1771$ u

Hence, the average atomic mass of neon is 20.1771 u

13.3. Obtain the binding energy (in MeV) of a nitrogen nucleus (${}^{14}_{7}N$), given m(${}^{14}_{7}N$) = 14.00307 u

Solution:

Given that,

Atomic mass of ${}^{14}_{7}N$ nitrogen, m = 14.00307 u

We know that a nucleus of ${}^{14}_{7}N$ nitrogen contains 7 protons and 7 neutrons.

Hence, the mass defect of this nucleus, $\Delta m = 7m_H + 7m_n - m$

Where,

Mass of a proton, $m_H = 1.007825$ u

Mass of a neutron, $m_n = 1.008665$ u

$\therefore \Delta m = 7 \times 1.007825 + 7 \times 1.008665 - 14.00307$

$= 7.054775 + 7.06055 - 14.00307 = 0.11236$ u

But we know that 1 u = 931.5 MeV/c^2

$\therefore \Delta m = 0.11236 \times 931.5\ \mathrm{MeV/c^2}$

Hence, the binding energy of the nucleus $(E_b) = \Delta mc^2$

Where, c = Speed of light

$\therefore E_b = 0.11236 \times 931.5 \left(\frac{\mathrm{MeV}}{c^2}\right) c^2 = 104.66334\ \mathrm{MeV}.$

Hence, the binding energy of a nitrogen nucleus is 104.66334 MeV.

13.4. Obtain the binding energy of the nuclei ${}^{56}_{26}\mathrm{Fe}$ and ${}^{209}_{83}\mathrm{Bi}$ in units of MeV from the following data:

$m\left({}^{56}_{26}\mathrm{N}\right) = 55.934939\ \mathrm{u}, m\left({}^{209}_{83}\mathrm{Bi}\right) = 208.980388\ \mathrm{u}$

Solution:

Given that,

Atomic mass of ${}^{56}_{26}\mathrm{Fe}$, $m_1 = 55.934939\ \mathrm{u}$

Number of protons in ${}^{56}_{26}\mathrm{Fe}$ nucleus = 26

And number of neutrons in ${}^{56}_{26}\mathrm{Fe}$ nucleus = (56 − 26) = 30

Hence, the mass defect of the nucleus, $\Delta m = 26 \times m_H + 30 \times m_n - m_1$

Mass of a proton, $m_H = 1.007825\ \mathrm{u}$

Mass of a neutron, $m_n = 1.008665\ \mathrm{u}$

$\therefore \Delta m = 26 \times 1.007825 + 30 \times 1.008665 - 55.934939 = 26.20345 + 30.25995 - 55.934939 = 0.528461$

But we know that $1\ \mathrm{u} = 931.5\ \mathrm{MeV/c^2}$

$\therefore \Delta m = 0.528461 \times 931.5\ \mathrm{MeV/c^2}$

The binding energy of this nucleus is given as:

$E_{b1} = \Delta mc^2$ Where, c = Speed of light

$\therefore E_{b1} = 0.528461 \times 931.5 \left(\frac{\mathrm{MeV}}{c^2}\right) c^2 = 492.26\ \mathrm{MeV}$

Average binding energy per nucleon $= \frac{492.26}{56} = 8.79\ \mathrm{MeV}$

Given data for ${}^{209}_{83}\mathrm{Bi}$

Atomic mass of ${}^{209}_{83}\mathrm{Bi}$, $m_2 = 208.980388\ \mathrm{u}$

Number of protons in ${}^{209}_{83}\mathrm{Bi}$ nucleus = 83

And number of neutrons in ${}^{209}_{83}\mathrm{Bi}$ = (209 − 83) = 126

Hence, the expression for mass defect of this nucleus is given as:

$\Delta m' = 83 \times m_H + 126 \times m_n - m_2$

Where,

Mass of a proton, $m_H = 1.007825$ u

Mass of a neutron, $m_n = 1.008665$ u

$$\therefore \Delta m' = 83 \times 1.007825 + 126 \times 1.008665 - 208.980388$$
$$= 83.649475 + 127.091790 - 208.980388 = 1.760877 \text{ u}$$

But we know that $1 \text{ u} = 931.5 \text{ MeV}/c^2$

$\therefore \Delta m' = 1.760877 \times 931.5 \text{MeV}/c^2$

Hence, the binding energy of the nucleus is given as:

$E_{b2} = \Delta m' c^2$

$= 1.760877 \times 931.5 \left(\frac{\text{MeV}}{c^2}\right) c^2 = 1640.26 \text{ MeV}$

Average binding energy per nucleon of ${}^{209}_{83}\text{Bi} = \frac{1640.26}{209} = 7.848 \text{ MeV}$

13.5. A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of ${}^{63}_{29}\text{Cu}$ atoms (of mass 62.92960 u).

Solution:

Given that,

Mass of copper coin $m' = 3$g

Atomic mass of ${}^{63}_{29}\text{Cu}$ atom, m = 62.92960 u

The total number of ${}^{63}_{29}\text{Cu}$ atoms in the coin, $N = \frac{N_A \times m'}{\text{Mass Number}}$

Where,

N_A =Avogadro's number $= 6.023 \times 10^{23}$ atoms/g and mass number = 63 g

$N = \frac{6.022 \times 10^{23} \times 3}{63} = 2.868 \times 10^{22}$ atoms

Number of protons in ${}^{63}_{29}\text{Cu}$ nucleus = 29

And the number of neutron in ${}^{63}_{29}\text{Cu}$ nucleus = (63-29) = 34

$\therefore$ Expression for mass defect of ${}^{63}_{29}\text{Cu}$ nucleus, $\Delta m' = 29 \times m_H + 34 \times m_n - m$

Where, Mass of the proton, $m_H = 1.007825$ u

Mass of a neutron, $m_n = 1.008665$ u

$\therefore \Delta m' = 29 \times 1.007825 + 34 \times 1.008665 - 62.9296 = 0.591935$ u

Mass defect of all the atoms present in the coin,

$\Delta m = 0.591935 \times 2.868 \times 10^{22} = 1.669766958 \times 10^{22}$u

But we know that 1 u = $931.5\ \text{MeV}/c^2$

$\therefore \Delta m = 1.69766958 \times 10^{22} \times 931.5 \left(\frac{\text{MeV}}{c^2}\right) c^2 = 1.581 \times 10^{25}\text{MeV}$

Hence, the binding energy of the nuclei of the coin is given as:

$E_b = 1.581 \times 10^{25} \times 10^6 \times 1.6 \times 10^{-19} = 2.5296 \times 10^{12}$ J

To separate all the neutrons and protons from the given coin, we need the energy of 2.5296×10^{12}J.

13.6. Write nuclear reaction equations for

(i) α-decay of ${}^{226}_{88}\text{Ra}$

(ii) α-decay of ${}^{242}_{94}\text{Pu}$

(iii) β^- -decay of ${}^{32}_{15}\text{P}$

(iv) β^- -decay of ${}^{210}_{83}\text{Bi}$

(v) β^+ -decay of ${}^{11}_{6}\text{C}$

(vi) β^+ -decay of ${}^{97}_{43}\text{Tc}$

(vii) Electron capture of ${}^{120}_{54}\text{Xe}$

Solution:

α is a nucleus of a helium ${}^{4}_{2}\text{He}$ and β is an electron (e- for β- and e+ for β). We know that in every α -decay, there is a loss of 2 protons and 2 neutrons. In every β^+decay, there is a loss of 1 proton, and a neutrino is emitted from the nucleus. In every β^-decay. There is a gain of 1 proton, and an antineutrino is emitted from the nucleus.

For the given cases, the various nuclear reactions can be written as:

$${}_{88}\text{Ra}^{226} \rightarrow {}_{86}\text{Rn}^{222} + {}_{2}\text{He}^4$$

$${}^{242}_{94}\text{Pu} \rightarrow {}^{238}_{92}\text{U} + {}^{4}_{2}\text{He}$$

$${}^{32}_{15}\text{P} \rightarrow {}^{32}_{16}\text{S} + e^- + \bar{\nu}$$

$${}^{210}_{83}\text{P} \rightarrow {}^{210}_{84}\text{S} + e^- + \bar{\nu}$$

$${}^{11}_{6}\text{C} \rightarrow {}^{11}_{5}\text{B} + e^+ + \nu$$

$${}^{97}_{43}\text{Tc} \rightarrow {}^{97}_{42}\text{Mo} + e^+ + \nu$$

$$^{120}_{54}Xe + e^- \rightarrow ^{120}_{53}I + \nu$$

13.7. A radioactive isotope has a half-life of T years. How long will it take the activity to reduce to a) 3.125%, b) 1% of its original value?

Solution:

Given that the half-life of the radioactive isotope = T years

Assume that the original amount of the radioactive isotope $= N_0$

(a) After the decay of original radioactive isotope, the amount of the radioactive isotope $= N$

It is given that only 3.125% of N_0 remains after decay. Hence, we can write:

$$\frac{N}{N_0} = 3.125\% = \frac{3.125}{100} = \frac{1}{32}$$

But we know that $\frac{N}{N_0} = e^{-\lambda t}$

Where, $\lambda =$ Decay constant and t= Time taken

$$\therefore e^{-\lambda t} = \frac{1}{32}$$

Taking ln both side

$$-\lambda t = \text{In } 1 - \text{In} 32$$

$$-\lambda t = 0 - 3.4657$$

$$t = \frac{3.4657}{\lambda}$$

Since $\lambda = \frac{0.693}{T}$

$$\therefore t = \frac{3.466}{\frac{0.693}{T}} \approx 5\,T \text{ year}$$

Hence, the isotope will take about 5 T years to reduce to 3.125% of its original value.

(b) Assume after decay, the amount of the radioactive isotope = N

It is given that only 1% of No remains after decay.

Hence, we can write:

$$\frac{N}{N_0} = 1\% = \frac{1}{100}$$

But $\frac{N}{N_0} = e^{-\lambda t}$

$$\therefore e^{-\lambda t} = \frac{1}{100}$$

$$-\lambda t = \text{In } I - \text{In } 100$$

$$-\lambda t = 0 - 4.6052$$

$$t = \frac{4.6052}{\lambda}$$

Since we know that $\lambda = 0.693/T$

$$\therefore t = \frac{4.6052}{\frac{0.693}{T}} = 6.645 \text{ T years}$$

Hence, the isotope will take about 6.645 T years to reduce to 1% of its original value.

13.8. The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. This activity arises from the small proportion of radioactive ${}^{14}_{6}C$ present with the stable carbon isotope ${}^{12}_{6}C$. When the organism is dead, its interaction with the atmosphere (which maintains the above equilibrium activity) ceases, and its activity begins to drop. From the known half-life (5730 years) of ${}^{14}_{6}C$, and the measured activity, the age of the specimen can be approximately estimated. This is the principle of ${}^{14}_{6}C$ dating used in archaeology. Suppose a specimen from Mohenjo-Daro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus-Valley civilisation.

Solution:

Given that,

The decay rate of living carbon-containing matter,

$R = 15$ decay/min

Assume that N is the number of radioactive atoms present in a normal carbon-containing matter.

Half-life of ${}^{14}_{6}C$, $T_{1/2} = 5730$ years (Given in question)

The decay rate of the specimen obtained from the Mohenjo-Daro:

$R' = 9$ decays/min

Let N' be the number of radioactive atoms present in the specimen during the Mohenjodaro period.

Therefore, we can write the relation between the decay constant, λ and time, t as:

$$\frac{N}{N'} = \frac{R}{R'} = e^{-\lambda t}$$

$$e^{-\lambda t} = \frac{9}{15} = \frac{3}{5}$$

Take ln both side

$$-\lambda t = \log_e \frac{3}{5} = -0.5108$$

$$\therefore t = \frac{0.5108}{\lambda}$$

But we know that $\lambda = \frac{0.693}{T_{1_2}} = \frac{0.693}{5730}$

$$\therefore t = \frac{0.5108}{0.693} = 4223.5 \text{ years}$$

Hence, the approximate age of the Indus-Valley civilisation is 4223.5 year

13.9. Obtain the amount of $^{60}_{27}$Co Necessary to provide a radioactive source of 8.0 mCi strength. The half-life of $^{60}_{27}$Co is 5.3 years.

Solution:

The strength of the radioactive source is given as:

$$\frac{dN}{dt} = 8.0 \text{ mCi}$$

$$= 8 \times 10^{-3} \times 3.7 \times 10^{10} \text{ decay/s}$$

$$= 29.6 \times 10^{7} \text{ decay/s}$$

Where, N = Required number of atoms

Half-life of $^{60}_{27}$Co, $T_{1/2} = 5.3$ years $= 5.3 \times 365 \times 24 \times 60 \times 60 = 1.67 \times 10^{8}$ s

For decay constant λ, we have the formula of the rate of decay as:

$$\frac{dN}{dt} = \lambda N$$

Where,

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{1.67 \times 10^{8}} s^{-1}$$

$$\therefore N = \frac{1}{\lambda}\frac{dN}{dt}$$

$$= \frac{29.6 \times 10^{7} \times 1.67 \times 10^{8}}{0.693} = 7.133 \times 10^{16} \text{ atoms}$$

For $^{60}_{27}$Co, Mass of 023×10^{23} (Avogadro's number) atoms = 60 g

$\therefore$ Mass of 7.133×10^{16} atoms $= \frac{60 \times 1.133 \times 10^{16}}{6.023 \times 10^{23}} = 7.106 \times 10^{-6}$ g

Hence, the amount of $^{60}_{27}\text{Co}$ necessary for providing strength is 7.106×10^{-6}g

13.10. The half-life of $^{90}_{38}\text{Sr}$ is 28 years. What is the disintegration rate of 15 mg of this isotope?

Solution:

Given that,

Half-life of $^{90}_{38}\text{Sr}, t_{1/2} = 28$ years

$= 28 \times 365 \times 24 \times 60 \times 60$

$= 8.83 \times 10^{8}$s

Mass of the isotope, m=15 mg

90 g of $^{90}_{38}\text{Sr}$ atom contains 6.023×10^{23} (Avogadro's number) atoms.

Therefore, 15 mg of $^{90}_{38}\text{Sr}$ contains atoms $= \frac{6.023 \times 10^{23} \times 15 \times 10^{-3}}{90} = 1.0038 \times 10^{20}$

Rate of disintegration, $\frac{dN}{dt} = \lambda N$ where λ equals to decay constant $= \frac{0.693}{8.83 \times 10^{8}}\ s^{-1}$

Therefore, $\frac{dN}{dt} = \frac{0.693 \times 1.0038 \times 10^{20}}{8.83 \times 10^{8}} = 7.878 \times 10^{10}$atoms/s

Hence, the disintegration rate of 15 mg of the given isotope is 7.878×10^{10} atoms/s

13.11. Obtain approximately the ratio of the nuclear radii of the gold isotope $^{197}_{79}\text{Au}$ and the silver isotope $^{107}_{47}\text{Ag}$.

Solution:

Given that

The nuclear radius of the gold isotope $^{197}_{79}\text{Au} = R_{Au}$

The nuclear radius of the silver isotope $^{107}_{47}\text{Ag} = R_{Ag}$

The mass number of gold, $A_{Au} = 197$

The mass number of silver, $A_{Ag} = 107$

The ratio of the radii of the two nuclei in terms of their mass numbers is:

$$\frac{R_{Au}}{R_{Ag}} = \left(\frac{A_{Au}}{A_{Ag}}\right)^{\frac{1}{3}}$$

$$= \left(\frac{197}{107}\right)^{\frac{1}{3}} = 1.2256$$

Hence, the ratio of the nuclear radii of the gold and silver isotopes is approx 1.23

13.12. Find the Q-value and the kinetic energy of the emitted α-particle in the α-decay of

(a) $^{226}_{88}\text{Ra}$ and

(b) $^{220}_{86}\text{Rn}$

Given $\text{m}(\,^{226}_{88}\text{Ra}) = 226.02540\text{ u}, \text{m}(\,^{222}_{86}\text{Rn}) = 222.01750\text{ u},$

$\text{m}(\,^{220}_{86}\text{Rn}) = 220.01137\text{ u}, \text{m}(\,^{216}_{84}\text{Po}) = 216.00189\text{ u}.$

Solution:

(a) In alpha particle decay of $^{226}_{88}\text{Ra}$, it emits a helium nucleus.

Due to this, it's mass number reduces to $(226 - 4) = 222$

and its atomic number reduces to $(88 - 2) = 86.$

A nuclear reaction is as follows.

$^{226}_{88}\text{Ra} \longrightarrow \,^{222}_{86}\text{Rn} + \,^{4}_{2}\text{He}$

Q- the value of emitted α −particle = (Sum of initial mass – Sum of final mass) c^2

Where c= Speed of light

It is given that:

$\text{m}(\,^{226}_{88}\text{Ra}) = 226.02540\text{ u}$

$\text{m}(\,^{222}_{86}\text{Rn}) = 222.01750\text{ u}$

$\text{m}(\,^{4}_{2}\text{He}) = 4.002603\text{ u}$

Q-value $= [226.02540 - (222.01750 + 4.002603)]\text{u c}^2$

$= 0.005297\text{ u c}^2$

But we know that $1\text{ u} = 931.5\text{ MeV/c}^2$

$\therefore \text{Q} = 0.005297 \times 931.5 \approx 4.94\text{ MeV}$

The kinetic energy of the $\alpha-$ particle $= \frac{\text{Mass number after decay}}{\text{Mass number before decay}} \times \text{Q} = \frac{222}{226} \times 4.94 = 4.85\text{ MeV}$

(b) Alpha particle decay of $^{220}_{86}\text{Ra}$

$^{220}_{86}\text{Ra} \rightarrow \,^{216}_{84}\text{Po} + \,^{4}_{2}\text{He}$

It is given that:

Mass of $^{220}_{86}\text{Ra} \rightarrow 220.01137\text{ u}$ and mass of $^{216}_{84}\text{Po} = 216.00189\text{ u}$

$$\therefore Q-\text{value} = [220.01137 - (216.00189 + 4.00260)] \times 931.5$$
$$\approx 6.41 \text{ MeV}$$

The kinetic energy of the α − particle $= \left(\frac{220-4}{220}\right) \times 6.41 = 6.29\text{MeV}$

13.13. The radionuclide ^{11}C decays according to

$^{11}_{6}C \rightarrow {}^{11}_{5}B + e^{+} + \nu : T_{1/2} = 20.3$ min

The maximum energy of the emitted positron is 0.960 MeV. Given the mass values:

$m(^{11}_{6}C) = 11.011434$ u and $m(^{11}_{6}B) = 11.009305$ u,

calculate Q and compare it with the maximum energy of the positron emitted.

Solution:

The given value of masses is:

$m(^{11}_{6}C) = 11.011434$ u and $m(^{11}_{6}B) = 11.009305$ u

The given nuclear reaction is:

$^{11}_{6}C \rightarrow {}^{11}_{5}B + e^{+} + \nu$

Half-life of $^{11}_{6}C$ nuclei, $T_{1/2} = 20.3$ min

Maximum energy possessed by the emitted position = 0.960 MeV

The Q- value (ΔQ) of the nuclear masses of the $^{11}_{6}C$ reaction is

$Q = [m'(^{11}_{6}C) - \{m'(^{11}_{5}B) + m_e\}]c^2$

Where, m_e = Mass of an electron or positron = 0.000548 u

Where c= Speed of light and m' = Respective nuclear masses.

If atomic masses are used instead of nuclear masses, then we have to add 6 m_e in the case of $^{11}_{6}C$ and 5 m_e in the case of $^{11}_{5}B$.

Hence, equation (1) reduces to:

$Q = [m(^{11}_{6}C) - m(^{11}_{5}B) - 2m_e]c^2$

Here, $(^{11}_{6}C)$ and $m(^{11}_{6}B)$ are the atomic masses.

$\therefore Q = [11.011434 - 11.009305 - 2 \times 0.000548]\ c^2 = (0.001033\ c^2)$u

But 1 u = $931.5\text{MeV}/c^2$

$\therefore Q = 0.001033 \times 931.5 \approx 0.962$ MeV

The value of Q is almost comparable to the maximum energy of the emitted positron.

13.14. The nucleus $^{23}_{10}Ne$ decays by β^- emission. Write down the β-decay equation and determine the maximum kinetic energy of the electrons emitted. Given that:

$m(^{23}_{10}Ne) = 22.994466$ u

$m(^{23}_{11}Na) = 22.909770$ u.

Solution:

We know that in β^-emission, the number of protons increases by 1, and one electron and an antineutrino are emitted from the parent nucleus.

β^- emission of the nucleus $^{23}_{10}Ne$ is as follows

$^{23}_{10}Ne \rightarrow ^{23}_{11}Na + e^- + \bar{\nu} + Q$

It is given that:

The atomic mass of $m(^{23}_{20}Ne) = 22.994466$ u

The atomic mass of $m(^{23}_{11}Na) = 22.909770$ u

Mass of an electron, $m_e = 0.000548$ u

Q- the value of the given reaction is given as:

$Q = [m(^{23}_{10}Ne) - [m(^{23}_{11}Na) + m_e]]c^2$

There are 10 electrons in $^{23}_{10}Ne$ and 11 electrons in $^{23}_{11}Na$. Hence, the mass of the electron is cancelled in the Q-value equation.

Therefore, $Q = [22.994466 - 22.909770]c^2 = (0.004696\ c^2)$u

But $1\ u = 931.5\ MeV/c^2$

$\therefore \Delta Q = 0.004696 \times 931.5 = 4.374$ MeV

The daughter nucleus is too heavy as compared to $e^-$and $\bar{\nu}$. Hence, it carries negligible energy. The kinetic energy of the antineutrino is nearly zero. Hence, the maximum kinetic energy of the emitted electrons is almost equal to the Q-value, i.e., 4.374 MeV.

13.15. The Q value of a nuclear reaction $A + b \rightarrow C + d$ is defined by $Q = [m_A + m_b - m_C - m_d]c^2$ where the masses refer to the respective nuclei. Determine from the given data the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.

(i) $^1_1H + ^3_1H \rightarrow ^2_1H + ^2_1H$

(ii) $^{12}_6C + ^{12}_6C \rightarrow ^{20}_{10}Ne + ^4_2He$

Atomic masses are given to be

$m(^2_1H) = 2.014102$ u

$m({}^{3}_{1}H) = 3.016049$ u

$m({}^{12}_{6}C) = 12.000000$ u

$m({}^{20}_{10}Ne) = 19.992439$ u

Solution:

Given that

Atomic mass $m({}^{1}_{1}H) = 1.007825$ u

Atomic mass $m({}^{3}_{1}H) = 3.016049$ u

Atomic mass $m({}^{2}_{1}H) = 2.014102$ u

(i) Given the nuclear reaction is

$${}^{1}_{1}H + {}^{3}_{1}H \rightarrow {}^{2}_{1}H + {}^{2}_{1}H$$

According to the given question, the Q- the value of the reaction can be written as:

$$Q = [m({}^{1}_{1}H) + m({}^{3}_{1}H) - 2m({}^{2}_{1}H)]c^2$$

$$= [1.007825 + 3.016049 - 2 \times 2.014102]c^2$$

$$Q = (-0.00433\ c^2)\ u$$

But we know that $1\ u = 931.5\ Mev/c^2$

Hence, $Q = 0.00433 \times 931.5 = -4.0334$ MeV

The negative sign of the reaction shows that the reaction is endothermic in nature.

(ii) The given nuclear reaction is

$${}^{12}_{6}C + {}^{12}_{6}C \rightarrow {}^{20}_{10}Ne + {}^{4}_{2}He$$

The atomic mass of $m({}^{12}_{6}C) = 12.0$ u

The atomic mass of $m({}^{20}_{10}Ne) = 19.9924390$ u

The atomic mass of $m({}^{4}_{2}He) = 4.002603$ u

The Q value of this reaction is given as:

$$Q = [2m({}^{12}_{6}C) - m({}^{20}_{10}Ne) - m({}^{4}_{2}He)]\ c^2$$

$$= [2 \times 12.0 - 19.992439 - 4.002603]c^2$$

$$= (0.004958\ c^2)\ u$$

But we know that $1\ u = 931.5\ Mev/c^2$

$Q = 0.004958 \times 931.5 = 4.618377$ MeV

The positive value of Q shows that the reaction is exothermic in nature.

13.16. Suppose, we think of fission of a ${}^{56}_{26}\text{Fe}$ nucleus into two equal fragments, ${}^{28}_{13}\text{Al}$. Is the fission energetically possible? Argue by working out Q of the process. Given $\text{m}({}^{56}_{26}\text{Fe}) = 55.93494$ u and $\text{m}({}^{28}_{13}\text{Al}) = 27.98191$ u.

Solution:

It is given that

The atomic mass of $m({}^{56}_{26}\text{Fe}) = 55.93494.0$ u

The atomic mass of $m({}^{28}_{13}\text{Al}) = 27.98191$ u

The Q value of this nuclear reaction is given as:

$Q = [m({}^{56}_{26}\text{Fe}) - 2m({}^{28}_{13}\text{Al})]\,\text{c}^2$

$= [55.93494 - 2 \times 27.9819119]\text{c}^2$

$Q = (-0.02888\ \text{c}^2)$ u

But we know that $1\ \text{u} = 931.5\ \text{Mev/c}^2$

$Q = -0.02888 \times 931.5 = -26.902$ MeV

The Q value of the fission is negative. Therefore, fission is not possible. For a possible fission reaction, the Q value should be positive.

13.17. The fission properties of ${}^{239}_{94}\text{Pu}$ are very similar to those of ${}^{235}_{92}\text{U}$. The average energy released per fission is 180 MeV. How much energy, in MeV, is released if all the atoms in 1 kg of pure ${}^{239}_{94}\text{Pu}$ undergo fission?

Solution:

Given that

Average energy released per fission of ${}^{239}_{94}\text{Pu}$, $\text{E}_{\text{Avg}} = 180$ MeV

Amount of pure ${}^{239}_{94}\text{Pu}$, m = 1 kg = 1000 g

N_A = Avogadro number $= 6.023 \times 10^{23}$

Mass number of ${}^{239}_{94}\text{Pu} = 239$ g

m gram of ${}^{239}_{94}\text{Pu}$ contains $\frac{N_A}{Mass\ number} \times$ m atoms.

1000 g of ${}^{239}_{94}\text{Pu}$ contains $\frac{N_A}{Mass\ number} \times 1000$ atoms $= 2.52 \times 10^{24}$ atoms.

The total energy released during the fission of 1 kg of ${}^{239}_{94}\text{Pu}$ is calculated as:

$E = E_{Avg} \times 2.52 \times 10^{24}$

$E = 180 \times 2.52 \times 10^{24} = 4.536 \times 10^{26}$ MeV

Hence, if all the atoms in 1 kg of pure $^{239}_{94}Pu$ undergo fission then 4.536×10^{26} MeV is released

13.18. A 1000 MW fission reactor consumes half of its fuel in 5.00 y. How much $^{235}_{92}U$ did it contain initially? Assume that the reactor operates 80% of the time, that all the energy generated arises from the fission of $^{235}_{92}U$ and that this nuclide is consumed only by the fission process.

Solution:

Given that

The half-life of the fuel of the fission reactor, $t_{\frac{1}{2}} = 5$ years $= 5 \times 365 \times 24 \times 60 \times 60$ s

We know that in the fission of 1 g of $^{235}_{92}U$ nucleus, the energy released is equal to 200 MeV.

1 mole, i.e., 235 g of $^{235}_{92}U$ contains 6.023×10^{23} atoms.

$\therefore$ 1 g $^{235}_{92}U$ contains $\frac{6.023\times10^{23}}{235}$ atoms.

The total energy generated per gram of $^{235}_{92}U$ is calculated as:

$$E = \frac{6.023 \times 10^{23}}{235} \times 200 \text{ MeV/g}$$

$$= \frac{200 \times 6.023 \times 10^{23} \times 1.6 \times 10^{-19} \times 10^{6}}{235} = 8.20 \times 10^{10} \text{ J/g}$$

It is given that the reactor operates only for 80% of the time.

Hence, the amount of $^{235}_{92}U$ consumed in 5 years by the 1000 MW fission reactor is calculated as:

$$m = \frac{5 \times 80 \times 60 \times 365 \times 24 \times 1000 \times 10^{6}}{100 \times 8.20 \times 10^{10}} = 1537 \text{ kg.}$$

Hence the initial amount of $^{235}_{92}U = 2 \times 1538 = 3076$ kg

13.19. How long can an electric lamp of 100W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as $^{2}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{2}He + n + 3.27$ MeV

Solution:

Given that

Amount of deuterium, m = 2 kg

a fusion reaction is:

$^{2}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{2}He + n + 3.27$ MeV

We know that 1 mole, i.e., 2 g of deuterium contains 6.023×10^{23} atoms.

Hence 2.0 kg of deuterium contains $\frac{2000}{2} \times 6.023 \times 10^{23} = 6.023 \times 10^{26} atoms.$

We can say that from the given reaction, when two atoms of deuterium fuse, 3.27 MeV energy is released.

Total energy per nucleus released in the fusion reaction:

$$E = \frac{3.27}{2} \times 6.023 \times 10^{26} \times 1.6 \times 10^{-19} \times 10^{6}.$$

$$E = 1.576 \times 10^{14} \text{ J}$$

Power of the electric lamp, P = 100 W = 100 J/s

Hence, the energy consumed by the lamp per second = 100 J

The total time for which the electric lamp will glow is calculated as:

$$= \frac{1.576 \times 10^{14}}{100} \text{sec} = \frac{1.576 \times 10^{14}}{100 \times 60 \times 60 \times 24 \times 365} = 4.9 \times 10^{14} \text{ years}$$

Hence time taken by the electric lamp is 4.9×10^{14} years

13.20. Calculate the height of the potential barrier for a head-on collision of two deuterons. (Hint: The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other. Assume that they can be taken as hard spheres of radius 2.0 fm.)

Solution:

When two deuterons collide head-on, the distance between their centres, d is equal to the sum of the radius of 1^{st} deuteron and 2^{nd} deuteron.

d = Radius of 1^{st} deuteron + Radius of 2^{nd} deuteron

The radius of a deuteron nucleus = 2 fm = 2×10^{-15}m

Hence, $d = 2 \times 10^{-15} + 2 \times 10^{-15} = 4 \times 10^{-15}$ m

Charge on a deuteron nucleus = Charge on an electron = $e = 1.6 \times 10^{-19}$ C

The potential energy of the two-deuteron system, $V = \frac{e^2}{4\pi\varepsilon_0 d}$

Hence $V = \frac{9\times10^{9}\times(1.6 \times 10^{-19})}{4 \times 10^{-15}}$ J

$$V = \frac{9 \times 10^{9} \times (1.6 \times 10^{-19})}{4 \times 10^{-15} \times 1.6 \times 10^{-19}} \text{ eV} = 360 \text{ keV}$$

Hence, the height of the potential barrier of the two-deuteron system is 360 keV.

13.21. From the relation $R = R_0A^{1/3}$, where R_0 is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e. independent of A).

Solution:

Given that the expression for the nuclear radius is

$R = R_0\, A^{\frac{1}{3}}$

Where, R_0 = Constant,

A = Mass number of the nucleus

Nuclear matter density, $\rho = \frac{Mass\ of\ nucleus}{Volume\ of\ nucleus}$

Assume m be the average mass of the nucleus.

Hence, the mass of the nucleus = mA

Hence, $\rho = \frac{mA}{\frac{4}{3}\times\pi\times R^3} = \frac{3mA}{4\times\pi\times\left(R_0\,A^{\frac{1}{3}}\right)^3} = \frac{3m}{4\pi {R_0}^3}$

Hence, we can see that nuclear matter density is independent of A. It is almost constant.

13.22. For the β^+ (positron) emission from a nucleus, there is another competing process known as electron capture (electron from an inner orbit, say, the K–shell, is captured by the nucleus and a neutrino is emitted).

$e^+ + {}^A_Z X \rightarrow {}_{Z-1}^{\ \ A}Y + \nu$

Show that if β^+ emission is energetically allowed; electron capture is necessarily allowed but not vice–versa.

Solution:

Assume that the amount of energy released during the electron capture process be Q_1 .

Hence, the nuclear reaction can be written as:

$e^+ + {}^A_Z X \rightarrow {}_{Z-1}^{\ \ A}Y + \nu + Q_1$ ----------(1)

Let the amount of energy released during the positron capture process be Q_2.

Hence, the nuclear reaction can be written as:

${}^A_Z X \rightarrow {}_{Z-1}^{\ \ A}Y + e^+ + \nu + Q_2$ ------------(2)

Where,

$m_N\left({}^A_Z X\right) = Nuclear\ mass\ of\ {}^A_Z X$

$m\left({}_{Z}^{A}X\right) = Atomic\ mass\ of\ {}_{Z}^{A}X$

$m_N\left({}_{Z-1}^{A}Y\right) = Nuclear\ mass\ of\ {}_{Z-1}^{A}Y$

$m\left({}_{Z-1}^{A}Y\right) = Atomic\ mass\ of\ {}_{Z-1}^{A}Y$

$m_e = Mass\ of\ an\ electron.$

c = Speed of light

Q-value of the electron capture reaction is given as

$Q_1 = \left[m_N\left({}_{Z}^{A}X\right) + m_e - m_N\left({}_{Z-1}^{A}Y\right)\right]c^2$

$= \left[m\left({}_{Z}^{A}X\right) - Zm_e + m_e - m\left({}_{Z-1}^{A}Y\right) + (Z-1)m_e\right]c^2$

$Q_1 = \left[m\left({}_{Z}^{A}X\right) - m\left({}_{Z-1}^{A}Y\right)\right]c^2$ ---------(3)

Q-value of the positron capture reaction is given as:

$Q_2 = \left[m_N\left({}_{Z}^{A}X\right) - m_e - m_N\left({}_{Z-1}^{A}Y\right)\right]c^2$

$= \left[m\left({}_{Z}^{A}X\right) - Zm_e - m_e - m\left({}_{Z-1}^{A}Y\right) + (Z-1)m_e\right]$

$= \left[m\left({}_{Z}^{A}X\right) - m\left({}_{Z-1}^{A}Y\right) - 2m_e\right]c^2$ ---------(4)

From the above equations, we can say that if $Q_2 > 0, then\ Q_1 > 0$;

But if $Q_1 > 0$ it does not necessarily mean that $Q_2 > 0$ or we can say that if β^+ emission is energetically allowed, then the electron capture process is necessarily allowed, but not vice-versa.

Additional Exercises

13.23. In a periodic table the average atomic mass of magnesium is given as 24.312 u. The average value is based on their relative natural abundance on earth. The three isotopes and their masses are ${}_{12}^{24}Mg(23.98504u)$, ${}_{12}^{25}Mg(24.98584u)$ and ${}_{12}^{26}Mg(25.98259u)$. The natural abundance of ${}_{12}^{24}Mg$ is 78.99% by mass. Calculate the abundances of other two isotopes.

Solution:

The average atomic mass of magnesium, m = 24.312 u

Mass of magnesium ${}_{12}^{24}Mg$ isotope, $m_1 = 23.98504$ u,

Mass of magnesium ${}_{12}^{25}Mg$, isotope, $m_2 = 24.98584$ u,

Mass of magnesium ${}_{12}^{26}Mg$ isotope, $m_3 = 25.98259$ u

Abundance of ${}_{12}^{24}Mg$, $\eta_1 = 78.99$ %

Let the abundance of ${}^{25}_{12}\text{Mg}$, $\eta_2 = x\ \%$

Therefore, the abundance of ${}^{26}_{12}\text{Mg}$, $\eta_3 = (100 - x - 78.99) = (21.01 - x)\%$

We have the relation for the average atomic mass as:

$$m = \frac{m_1\eta_1 + m_2\eta_2 + m_3\eta_3}{\eta_1 + \eta_2 + \eta_3}$$

$$24.312 = \frac{23.98504 \times 78.99 + 24.98584 \times x + 25.98259 \times (21.01 - x)}{78.99 + x + (21.01 - x)}$$

$0.99675x = 9.2725255$

Hence, $x = 9.3\ \% = \eta_2$

$And\ \eta_3 = (21.01 - x)\ \% = 11.71\ \%$

We can say that the abundance of ${}^{25}_{12}\text{Mg}$ is 9.3% and that of ${}^{26}_{12}\text{Mg}$ is 11.71%.

13.24. The neutron separation energy is defined as the energy required to remove a neutron from the nucleus. Obtain the neutron separation energies of the nuclei ${}^{41}_{20}\text{Ca}$ and ${}^{27}_{13}\text{Al}$ from the following data:

$\text{m}({}^{40}_{20}\text{Ca}) = 39.962591\ \text{u}$

$\text{m}({}^{41}_{20}\text{Ca}) = 40.962278\ \text{u}$

$\text{m}({}^{26}_{13}\text{Al}) = 25.986895\ \text{u}$

$\text{m}({}^{27}_{13}\text{Al}) = 26.981541\ \text{u}$

Solution:

If a neutron ${}^{1}_{0}\text{n}$ is removed from ${}^{41}_{20}\text{Ca}$, the corresponding nuclear reaction can be written as:

${}^{41}_{20}\text{Ca} \rightarrow {}^{40}_{20}\text{Ca} + {}^{1}_{0}\text{n}$

It is given that:

$\text{m}({}^{41}_{20}\text{Ca}) = 39.962591\ \text{u}$

$\text{m}({}^{41}_{20}\text{Ca}) = 40.962278\ \text{u}$

$\text{m}({}^{1}_{0}\text{n}) = 1.008665\ \text{u}$

The mass defect of this reaction is given as:

$\Delta\text{m} = \text{m}({}^{40}_{20}\text{Ca}) + ({}^{1}_{0}\text{n}) - \text{m}({}^{41}_{20}\text{Ca})$

$= 39.96259 + 1.008665 - 40.962278 = 0.008978\ \text{u}$

But $1\ \text{u} = 931.5\ \text{MeV/c}^2$

$\therefore \Delta\text{m} = 0.008978 \times 931.5\ \text{MeV/c}^2$

Hence, the energy required for neutron removal is calculated as:

$E = \Delta mc^2$

$= 0.008978 \times \frac{931.5\ \text{MeV}}{c^2} = 8.363007\ \text{MeV}$

For ${}^{27}_{13}\text{Al}$, the neutron removal reaction can be written as:

${}^{27}_{13}\text{Al} \rightarrow {}^{26}_{13}\text{Al} + {}^{1}_{0}\text{n}$

It is given that:

$m({}^{26}_{13}\text{Al}) = 25.986895\ \text{u}$

$m({}^{27}_{13}\text{Al}) = 26.981541\ \text{u}$

The mass defect of this reaction is given as:

$\Delta m = m({}^{26}_{13}\text{Al}) + m({}^{1}_{0}\text{n}) - m({}^{27}_{13}\text{Al})$

$= 25{,}986895 + 1.008665 - 26.981541$

$= 0.014019 \times 931.5\ \text{MeV}/c^2$

Hence, the energy required for neutron removal is calculated as:

$E = \Delta mc^2$

$= 0.014019 \times 931.5 = 13.059\ \text{MeV}$

Hence, For ${}^{41}_{20}\text{Ca}$: Separation energy = 8.363007 MeV

For ${}^{27}_{13}\text{Al}$: Separation energy = 13.059 MeV

13.25. A source contains two phosphorous radio nuclides ${}^{32}_{15}\text{P}(T_{1/2} = 14.3\text{d})$ and ${}^{33}_{15}\text{P}(T_{1/2} = 25.3\text{d})$. Initially, 10% of the decays come from ${}^{33}_{15}\text{P}$. How long one must wait until 90% do so?

Solution:

Given that,

Half-life ${}^{32}_{15}\text{P}$, $T_{1/2} = 14.3$ days

Half-life of ${}^{33}_{15}\text{P}$, $T'_{1/2} = 25.3$ days and ${}^{33}_{15}\text{P}$ nucleus is 10 % of the total amount of decay

The source has initially 10% of ${}^{33}_{15}\text{P}$ nucleus and 90% of ${}^{32}_{15}\text{P}$ nucleus.

Suppose after t days; the source has 10% of ${}^{32}_{15}\text{P}$ nucleus and 90% of ${}^{33}_{15}\text{P}$ nucleus.

Assume that Initially the number of ${}^{33}_{15}\text{P}$ nucleus = N

Number of ${}^{32}_{15}\text{P}$ nucleus = 9 N

Finally:

Number of $^{33}_{15}P$ nucleus $= 9N'$

Number of $^{32}_{15}P$ nucleus $= N'$

For $^{32}_{15}P$ nucleus, we can write the number ratio as:

$$\frac{N'}{9N} = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

$$N' = 9N(2)^{\frac{-t}{14.3}} \qquad \ldots\ldots (1)$$

For $^{33}_{15}P$ nucleus, we can write the number ration as:

$$\frac{9N'}{N} = \left(\frac{1}{2}\right)\frac{t}{T_{1/2}}$$

$$9N' = N(2)^{\frac{-t}{25.3}} \qquad \ldots\ldots (2)$$

On dividing equation (1) by equation (2), we get:

$$\frac{1}{9} = 9 \times 2^{\left(\frac{t}{25.3} - \frac{t}{14.3}\right)}$$

$$\frac{1}{81} = 2^{-\left(\frac{11t}{25.3\times 14.3}\right)}$$

$$\log 1 - \log 81 = \frac{-11t}{25.3 \times 14.3}\log 2$$

$$\frac{-11t}{25.3 \times 14.3} = \frac{0 - 1.908}{0.301}$$

$$t = \frac{25.3\times 14.3\times 1.908}{11\times 0.301} \approx 208.5 \text{ days}$$

Hence, it will take about 208.5 days for 90% decay of $^{33}_{15}P$.

3.26. Under certain circumstances, a nucleus can decay by emitting a particle more massive than an α-particle. Consider the following decay processes:

$$^{223}_{88}Ra \rightarrow {}^{209}_{82}Pb + {}^{14}_{6}C$$

$$^{223}_{88}Ra \rightarrow {}^{219}_{86}Rn + {}^{4}_{2}He$$

Calculate the Q-values for these decays and determine that both are energetically allowed.

Solution:

For the emission of $^{14}_{6}C$, the nuclear reaction: $^{223}_{88}Ra \rightarrow {}^{209}_{82}Pb + {}^{14}_{6}C$

We know that:

Mass of $^{223}_{88}Ra$, $m_1 = 223.01850$ u

Mass of $^{209}_{82}\text{Pa}, m_2 = 208.98107\text{ u}$

Mass of $^{14}_{6}\text{C}, m_3 = 14.00324\text{ u}$

Hence, the Q-value of the reaction is given as:

$$Q = (m_1 - m_2 - m_3)c^2$$
$$= (223.01850 - 208.98107 - 14.00324 - 14.00324)c^2$$

$= (0.03419\ c^2)\text{u}$

But $1\text{ u} = 931.5\text{ MeV}/c^2$

$\therefore Q = 0.03419 \times 931.5 = 31.848\text{ MeV}$

Hence, the Q-value of the nuclear reaction is 31.848 MeV. Since the Q-value is positive, the reaction is energetically possible and allowed.

For the emission of $^{4}_{2}\text{He}$, the nuclear reaction: $^{223}_{88}\text{Ra} \rightarrow {}^{219}_{86}\text{Rn} + {}^{4}_{2}\text{He}$

We know that:

Mass of $^{223}_{88}\text{Ra}, m_1 = 223.01850\text{ u}$

Mass of $^{219}_{86}\text{Rn}, m_2 = 219.00948\text{ u}$

Mass of $^{4}_{2}\text{He}, m_3 = 4.00260\text{ u}$

Q-value of this nuclear reaction is given as:

$$Q = (m_1 - m_2 - m_3)c^2 = (223.01850 - 219.00948 - 4.00260)C^2$$
$$= (0.00642\ c^2)$$

$u = 0.00642 \times 931.5 = 5.98\text{ MeV}$

Hence, the Q value of the second nuclear reaction is 5.98 MeV. Since the value is positive, the reaction is energetically possible and allowed.

13.27. Consider the fission of $^{238}_{92}\text{U}$ by fast neutrons. In one fission event, no neutrons are emitted and the final end products, after the beta decay of the primary fragments, are $^{140}_{58}\text{Ce}$ and $^{99}_{44}\text{RU}$. Calculate Q for this fission process. The relevant atomic and particle masses are

$m(^{238}_{92}\text{U}) = 238.05079\text{ u}$

$m(^{140}_{58}\text{Ce}) = 139.90543\text{ u}$

$m(^{99}_{44}\text{RU}) = 98.90594\text{ u}$

Solution:

In the fission of $^{238}_{92}\text{U}$, 10 β^- particle decay from the parent nucleus. The nuclear reaction can be written as:

$$^{238}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{140}_{58}\text{Ce} + {}^{99}_{44}\text{Ru} + 10\ e^{-1}$$

It is given that:

Mass of a $^{238}_{92}\mathrm{U}$ nucleus, $m_1 = 238.05079$ u

Mass of a $^{140}_{58}\mathrm{Ce}$ nucleus $m_2 = 139.90543$ u

Mass of a $^{99}_{44}\mathrm{Ru}$ nucleus, $m_3 = 98.90594$ u

Mass of neutron $^{1}_{0}\mathrm{n}$, $m_4 = 1.008665$ u

Q-value of the above equation,

$Q = [m'(^{238}_{92}\mathrm{U}) + m'(^{1}_{0}\mathrm{n}) - m'(^{140}_{58}\mathrm{Ce}) - m'(^{99}_{44}\mathrm{Ru}) - 10m_e]c^2$

Where,

m' = represents the corresponding atomic masses of the nuclei.

$(m'(^{238}_{92}\mathrm{U}) = m_1 - 92m_e$

$m'(^{140}_{58}\mathrm{Ce}) = m_2 - 58m_e$

$m'(^{99}_{44}\mathrm{Ru}) = m_3 - 44\, m_e$

and $m'(^{1}_{0}\mathrm{n}) = m_4$

$Q = [m_1 - 92m_e + m_4 - m_2 + 58m_e - m_3 + 44m_e - 10m_e]c^2$

$= [m_1 + m_4 - m_2 - m_3]c^2$

$= [238.0507 + 1.008005 - 139.90543 - 98.90594]c^2$

$= [0.247995\ c^2]$u

But we know that $1\ \mathrm{u} = 931.5\ \mathrm{MeV}/c^2$

$\therefore Q = 0.247995 \times 931.5 = 231.007$ MeV.

Hence, the Q-value of the fission process is 231.007 MeV

13.28. Consider the D–T reaction (deuterium-tritium fusion)

$^{2}_{1}\mathrm{H} + ^{3}_{1}\mathrm{H} \rightarrow ^{4}_{2}\mathrm{He} + \mathrm{n}$

(a) Calculate the energy released in MeV in this reaction from the data:

$m(^{2}_{1}\mathrm{H}) = 2.014102$ u

$m(^{3}_{1}\mathrm{H}) = 3.016049$ u

(b) Consider the radius of both deuterium and tritium to be approximately 2.0 fm. What is the kinetic energy needed to overcome the Coulomb repulsion between the two nuclei? To what temperature must the gas be heated to initiate the reaction?

(Hint: Kinetic energy required for one fusion event =average thermal kinetic energy available with the interacting particles $= 2(3kT/2); k =$ Boltzmann's constant, $T =$ absolute temperature.)

Solution:

(a) Take the D-T nuclear reaction: ${}_1^2H + {}_1^3H \rightarrow {}_2^4He + n$

It is given that:

Mass of ${}_1^2H$, $m_1 = 2.014102$ u

Mass of ${}_1^3H$, $m_2 = 3.016049$ u

Mass of ${}_2^4He$, $m_3 = 4.002603$ u

Mass of ${}_0^1n$, $m_4 = 1.008665$ u

Q-value of the given D-T reaction is:

$$Q = [m_1 + m_2 - m_3 - m_4]c^2$$
$$= [2.014102 + 3.016049 - 4.002603 - 1.008665]c^2$$
$$= [0.018883\ c^2]u$$

But we know that $1\ u = 931.5\ MeV/c^2$

$\therefore Q = 0.018883 \times 931.5 = 17.59$ MeV

The radius of deuterium and tritium, $r \approx 2.0$ fm $= 2 \times 10^{-15}$ m

Distance between the two nuclei when they touch each other,

$d = r + r = 4 \times 10^{-15}$ m

Charge on the deuterium nucleus =e

Charge on the tritium nucleus =e

Hence, the repulsive potential energy between the two nuclei is given as:

$$V = \frac{e^2}{4\pi\varepsilon_0(d)}$$

Where,

ε_0 =Permittivity of free space

Kinetic energy (KE) is needed to overcome the Coulomb repulsive between the two nuclei.

However, it is given that:

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ N\ m^2\ C^{-2}$$

$$\therefore V = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{4 \times 10^{-15}} = 5.76 \times 10^{-14}\ J$$

$$= \frac{5.76 \times 10^{-14}}{1.6 \times 10^{-19}} eV = 3.6 \times 10^5\ eV = 360\ keV$$

Hence, 5.76×10^{-14}J or 360 keV of kinetic energy (KE) is needed to overcome the Coulomb repulsion between the two nuclei. It is given that:

$$KE = 2 \times \frac{3}{2} kT$$

Where,

$K =$ Boltzmann constant $= 1.38 \times 10^{-23}\ m^2\ kg\ s^{-2}\ K^{-1}$

$T =$ Temperature required to start the reaction is

$$\therefore T = \frac{KE}{3k}$$

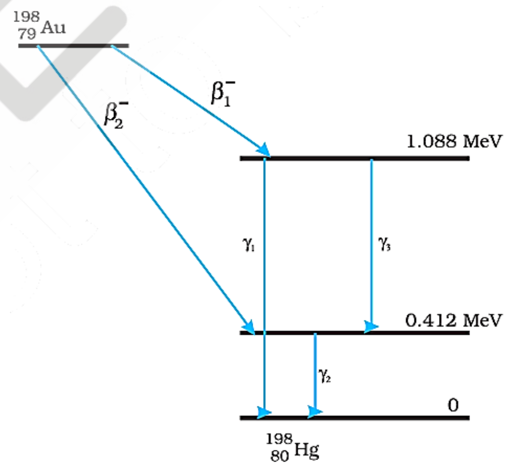
$$= \frac{5.76 \times 10^{-14}}{3 \times 1.38 \times 10^{-23}} = 1.39 \times 10^9\ K$$

Hence, the gas must be heated to a temperature of 1.39×10^9 K to initiate the reaction.

13.29. Obtain the maximum kinetic energy of β-particles and the radiation frequencies of γ decay in the decay scheme shown in Figure. You are given that

$m(^{198}Au) = 197.968233\ u$

$m(^{198}Hg) = 197.966760\ u$



Solution:

We can see from γ –decay diagram that γ_1 decays from the 1.088 MeV Energy level to the 0 MeV energy level.

Hence, the energy corresponding to $\gamma_1 -$ decay is given as:

$E_1 = 1.088 - 0 = 1.088$ MeV, $h\nu_1 = 1.6 \times 10^{-19} \times 10^6$ J

Where,

h = Planck's constant $= 6.6 \times 10^{-34}$ J s

$\nu_1 =$ Frequency of radiation radiated by $\gamma_1 -$ decay

$\therefore \nu_1 = \frac{E_1}{h}$

$= \frac{1.088 \times 1.6 \times 10^{-19} \times 10^6}{6.6 \times 10} = 2.637 \times 10^{20}$ Hz

It can be observed from the given $\gamma -$ decay diagram that γ_2 decays from the 0.412 MeV energy level to the 0 MeV energy level.

Hence, the energy corresponding to $\gamma_2 -$ decay is given as:

$E_2 = 0.412 - 0 = 0{,}412$ MeV $h\nu_2 = 0.412 \times 1.6 \times 10^{-19} \times 10^6$ J

Where, $\nu_2 =$ Frequency of radiation radiated by $\gamma_2 -$ decay

$\therefore \nu_2 = \frac{E_2}{h}$

$= \frac{0.412 \times 1.6 \times 10^{-19} \times 10^6}{6.6 \times 10^{-34}} = 9.988 \times 10^{19}$ Hz

It can be observed from the given $\gamma -$ decay diagram that γ_3 decays from the 1.088MeV energy level to the 0.412 MeV energy level.

Hence, the energy corresponding to γ_3 −decay is given as:

$E_3 = 1.088 - 0.412 - 0.676$ MeV

$h\nu_3 = 0.676 \times 10^6$ J

Where, $\nu_3 =$ Frequency of radiation radiated by $\gamma_3 -$ decay

$\therefore \nu_3 = \frac{E_3}{h}$

$= \frac{0.676 \times 1.6 \times 10^{-19} \times 10^6}{6.6 \times 10^{-34}} = 1.639 \times 10^{20}$ Hz

Mass of $m({}^{198}_{78}Au) = 197.968233$ u

Mass of $m({}^{198}_{80}Hg) = 197.966760$ u

we know that 1 u = 931.5 MeV/c^2

The energy of the highest level is given as:

$E = [m({}^{198}_{78}Au) - m({}^{190}_{80}Hg)]$

$= 197.968233 - 197.966760 = 0.001473$ u

$= 0.001473 \times 931.5 = 1.3720995$ MeV

β_1 decays from the 1.3720995 MeV level to the 1.088 MeV level

$\therefore$ The maximum kinetic energy of the β_1 particle $= 1.2720995 - 1.088 = 0.2840995$ MeV

β_2 decays from the 1.3720995 MeV level to the 0.412 MeV level

$\therefore$ The maximum kinetic energy of the β_2 particle $= 1.3720995 - 0.412 = 0.9600995$ MeV

13.30. Calculate and compare the energy released by a) fusion of 1.0 kg of hydrogen deep within Sun and b) the fission of 1.0 kg of ${}^{235}U$ in a fission reactor.

Solution:

Given that

(a) Amount of hydrogen, m=1 kg = 1000 g

1 mole, i.e., 1 g of hydrogen (${}^{1}_{1}H$) contains 6.023×10^{23} atoms.

$\therefore$ 1000 g of ${}^{1}_{1}H$ contains $6.023 \times 10^{23} \times 1000$ atoms.

In the sun four ${}^{1}_{1}H$ nuclei combine and form one ${}^{4}_{2}He$ nucleus. In this process, 26 MeV of energy is released.

Hence, the energy released from the fusion of 1 kg ${}^{1}_{1}H$ is: $E_1 = \dfrac{6.023 \times 10^{23} \times 26 \times 10^3}{4} = 39.1495 \times 10^{26}$ MeV

(b) Amount of ${}^{235}_{92}U = 1\text{kg} = 1000$ g

1 mole of, i.e., 235 g of ${}^{235}_{92}U$ contains 6.023×10^{23} atoms.

$\therefore$ 1000 g of ${}^{235}_{92}U$ contains $\frac{6.023 \times 10^{23} \times 1000}{235}$ atoms

It is known that the amount of energy released in the fission of one atom of ${}^{235}_{92}U$ is 200 MeV.

Hence, the energy released from the fission of 1 kg ${}^{235}_{92}U$ is:

$$E_2 = \frac{6 \times 10^{23} \times 1000 \times 200}{235}$$

$= 5.106 \times 10^{26}$ MeV

$$\therefore \frac{E_1}{E_2} = \frac{39.1495 \times 10^{26}}{5.106 \times 10^{26}} = 7.67 \approx 8$$

Therefore, the energy released in the fusion of 1 kg of hydrogen is nearly 8 times the energy released in the fission of 1 kg of uranium.

13.31. Suppose India had a target of producing by 2020 AD, 200,000 MW of electric power, ten percent of which was to be obtained from nuclear power plants. Suppose we are given that, on an average, the efficiency of utilization (i.e. conversion to electric energy) of thermal energy produced in a reactor was 25%. How much amount of fissionable uranium would our country need per year by 2020? Take the heat energy per fission of ^{235}U to be about 200MeV.

Solution:

Given that amount of electric power to be generated, $P = 2 \times 10^5$ MW

10% of this amount has to be obtained from nuclear power plants.

$\therefore$ Amount of nuclear power, $P_1 = \frac{10}{100} \times 2 \times 10^5 = 2 \times 10^4$ MW $= 2 \times 10^4 \times 10^6$ J/s

$= 2 \times 10^{10} \times 60 \times 60 \times 24 \times 365$ J/yr

The heat energy released per fission of a ^{235}U nucleus, $E = 200$ MeV

Given that efficiency of a reactor =25%

Hence, the amount of energy converted into electrical energy per fission is calculated as:

$$= \frac{25}{100} \times 200 = 50 \text{ MeV}$$

$= 50 \times 1.6 \times 10^{19} \times 8 \times 10^{-12}$ J

A number of atoms required for fission per year:

$$= \frac{2 \times 10^{10} \times 60 \times 6024 \times 365}{8 \times 10^{-12}} = 78840 \times 10^{24} \text{ atoms}$$

1 mole, i.e., 235 g of ^{235}U contains 6.023×10^{23} atoms.

$\therefore$ mass of 6.023×10^{23} atoms $^{235}U = 235 \text{ g} = 235 \times 10^{-3}$ kg

$\therefore$ Mass of 78840×10^{24} atoms of U^{235}

$$= \frac{235 \times 10^{-3}}{6.023 \times 10^{23}} \times 78840 \times 10^{24}$$

$= 3.076 \times 10^4$ kg

Hence, the amount of uranium needed per year is 3.076×10^4 kg

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