CBSE NCERT Solutions for Class 9 Mathematics Chapter 5

Back of Chapter Questions

Exercise: 5.1

1. Which of the following statements are true and which are false? Give reasons for your answers.

(i) Only one line can pass through a single point.

(ii) There are an infinite number of lines which pass through two distinct points.

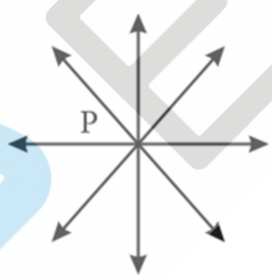
(iii) A terminated line can be produced indefinitely on both the sides.

(iv) If two circles are equal, then their radii are equal.

(v) In the following figure, if AB = PQ and PQ = XY, then AB = XY.

Solution:

(i) Single point infinite number of lines can pass. Here in the figure we may find that there are infinite numbers of lines, which are passing through a single point P.



(ii) False, as through two district points only one line can pass. Here in the following figure we may find that there is only one single line that can pass through two distinct points P and Q.

(iii) True, a terminated line can be produced indefinitely on both the sides. Let AB be a terminated line. We may find that it can be produced indefinitely on both the sides.

(iv) True, if two circles are equal, their center and circumference will coincide and hence radii will be equal.

(v) True. It is given that AB and XY are two terminated lines and both are equal to a third line PQ. Euclid's First axiom states that things which are equal to the same thing are equal to one another. So, the lines AB and XY will be equal.

2. Give a definition for each of the following terms. Are there other terms that need to be defined first? What are they, and how might you define them?

(i) Parallel lines

(ii) Perpendicular lines

(iii) Line segment

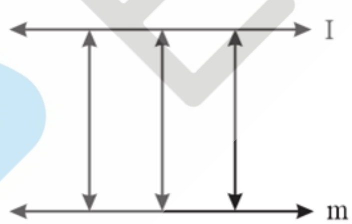
(iv) Radius of a circle

(v) Square

Solution:

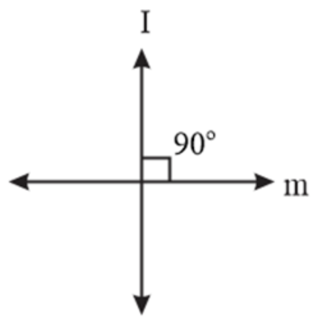
(i) Parallel Lines:

If the perpendicular distance between two lines is always constant, these are called parallel lines. In other words we may say that the lines which never intersect each other are called parallel lines. To define parallel line, we must know about point, lines and distance between lines and point of intersection.



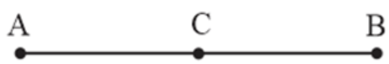
(ii) Perpendicular lines:

If two lines intersect each other at 90°, these are called perpendicular lines. We need to define line and the angle before defining perpendicular lines.



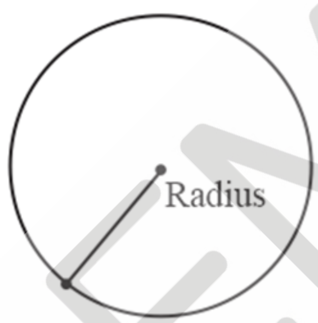
(iii) Line segment:

A straight line drawn from any point to any other point is called as line segment. To define line segment, we must know about point and line segment.



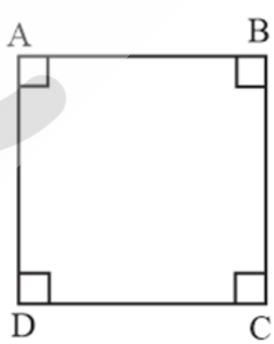
(iv) Radius of a circle:

It is the distance between the center of a circle to any point lying on the circle. To define radius of circle, we must know about point and circle.



(v) Square:

A square is a quadrilateral having all sides of equal length and all angles of same measure i.e. 90°. To define square, we must know about quadrilateral, side, and angle.



3. Consider two 'postulates' given below:

(i) Given any two distinct point A and B, there exists a third point C which is in between A and B.

(ii) There exist at least three points that are not on the same line.

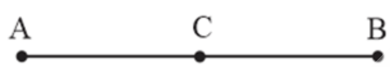
Do these postulates contain any undefined terms? Are these postulates consistent?

Do they follow from Euclid's postulates? Explain.

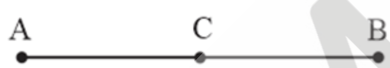
Solution:

There are undefined terms. They are consistent. As we have no information about line segment AB and we don't know about the position of third point C whether it is lying on the line segment AB or not, now two different cases are possible -

(i) The third point C lies on the line segment made by joining the points A and B.



(ii) The third point C does not lie on the line segment made by joining the points A and B.



Yes, they are consistent as these are two different situations;

(i) The third point C lies on the line segment made by joining points A and B.

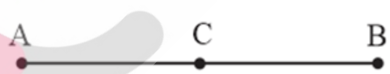
(ii) The third point does not lie on the line segment made by joining points A and B.

No, these postulates do not follow Euclid's postulates these are axiom.

4. If a point C lies between two points A and B such that $AC = BC$, then prove that $AC = \frac{1}{2}AB$. Explain by drawing the figure.

Solution:

Given that $AC = BC$



$AC + AC = BC + AC$ (equals are added on both sides) ...(i)

Here, $(BC + AC)$ coincides with AB. We know that things which coincide with one another are equal to one another.

$BC + AC = AB$ ...(ii)

We know that things which are equal to the same thing are equal to one another. So, from equation (i) and equation (ii), we have

$\Rightarrow AC + AC = AB$

$\Rightarrow 2AC = AB$

$\therefore AC = \frac{1}{2}AB$

5. Point C is called a mid-point of line segment AB, prove that every line segment has one and only one mid-point.

Solution:

Let us assume there are two mid-points C and D

C is mid-point of AB

$AC = CB$

$AC + AC = BC + AC$ (equals are added on both sides) ... (1)

Here, $(BC + AC)$ coincides with AB. We know that things which coincide with one another are

equal.

$BC + AC = AB$ ... (2)

We know that things which are equal to the same thing are equal to one another. So, from

equation (1) and equation (2), we have

$AC + AC = AB$

$2AC = AB$ ... (3)

Similarly by taking D as the mid-point of AB, we can prove that

$2AD = AB$ ... (4)

From equation (3) and (4), we have

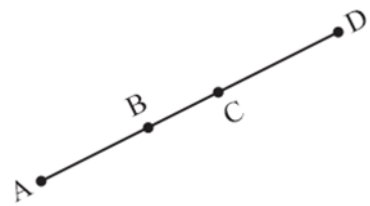
$2AC = 2AD$ (Things which are equal to the same thing are equal to one another.)

$AC = AD$ (Things which are double of the same things are equal to one another.)

It is possible only when point C and D are representing a single point.

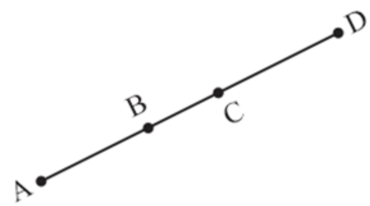
Hence our assumption is wrong and there can be only one mid-point of a given line segment.

6. In the following figure, if $AC = BD$, then prove that $AB = CD$.



Solution:

From the figure we may observe that



$AC = AB + BC$

$BD = BC + CD$

Given that $AC = BD$

$\Rightarrow AB + BC = BC + CD \quad \dots (1)$

According to Euclid's axiom, we know that when equals are subtracted from equals, remainders are also equal.

Subtracting BC from the equation (1), we have

$\Rightarrow AB + BC - BC = BC + CD - BC$

$\therefore AB = CD$

7. Why is Axiom 5, in the list of Euclid's axioms, considered a universal truth? (Note that the question is not about the fifth postulate.)

Solution:

Axiom 5: The whole is greater than the part.

Let t is representing a whole quantity and only a, b, c are parts of it.

Now $t = a + b + c$

Clearly t will be greater than all its parts a, b and c

So, it is rightly said that the whole is greater than the part.

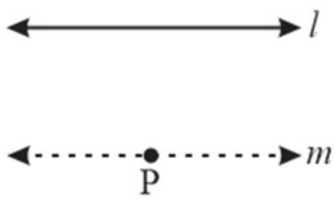
Exercise: 5.2

1. How would you rewrite Euclid's fifth postulate so that it would be easier to understand?

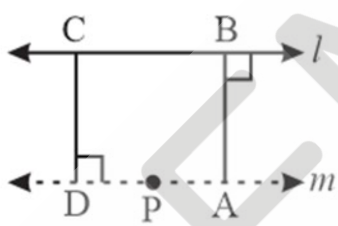
Solution:

Two lines are said to be parallel if they are equidistant from one other and they do not have any point of intersection. In order to understand it easily let us take any line l and a point P not on l.

Then, by Play fair's axiom (equivalent to the fifth postulate), we know that there is a unique line m through P which is parallel to l.



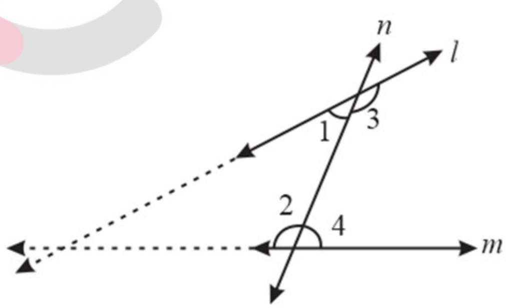
Now, the distance of a point from a line is the length of the perpendicular from the point to the line. Let AB be the distance of any point on m from l and CD be the distance of any point on l from m. We can observe that AB $=$ CD. In this way the distance will be the same for any point on m from l and any point on l from m. So, these two lines are everywhere equidistant from one another.



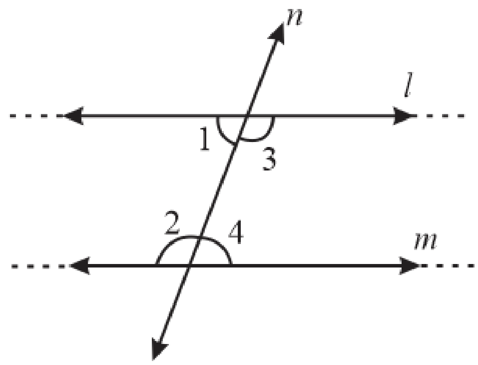
2. Does Euclid's fifth postulate imply the existence of parallel lines? Explain.

Solution:

According to Euclid's 5^{th} postulates when n line falls on l and m and if, then producing line l and m further will meet in the side of 1 and 2 which is less than 180^o



If $\angle 1 + \angle 2 < 180^{o}$ then $\angle 3 + \angle 4 > 180^{o}$



Now, the lines l and m neither meet at the side of 1 and 2 nor at the side of 3 and 4.

That means the lines l and m will never intersect each other. Therefore, we can say that the lines are parallel.