Exercise: 6.1

1. In the given figure, lines AB and CD intersect at O. If \( \angle AOC + \angle BOE = 70^\circ \) and \( \angle BOD = 40^\circ \), find \( \angle BOE \) and reflex \( \angle COE \).

**Solution:**

AB is a straight line, OC and OE rays stand on it.

Therefore,

\[ \angle AOC + \angle COE + \angle BOE = 180^\circ \]

\[ \Rightarrow (\angle AOC + \angle BOE) + \angle COE = 180^\circ \]

\[ \Rightarrow 70^\circ + \angle COE = 180^\circ \]

\[ \Rightarrow \angle COE = 180^\circ - 70^\circ = 110^\circ \]

Now, reflex \( \angle COE = 360^\circ - 110^\circ = 250^\circ \)

CD is a straight line, OE and OB rays stand on it.

Therefore,

\[ \angle COE + \angle BOE + \angle BOD = 180^\circ \]

\[ \Rightarrow 110^\circ + \angle BOE + 40^\circ = 180^\circ \]

\[ \Rightarrow \angle BOE = 180^\circ - 150^\circ = 30^\circ \]

Hence \( \angle BOE = 30^\circ \) and reflex \( \angle COE = 250^\circ \).

2. In the given figure, lines XY and MN intersect at O. If \( \angle POY = 90^\circ \) and \( a:b = 2:3 \), find \( c \).


Solution:
Let the common ratio between \(a\) and \(b\) be \(x\),
\[ \therefore a = 2x \text{ and } b = 3x. \]

XY is a straight line, OM and OP rays stands on it.
Therefore,
\[ \angle XOM + \angle MOP + \angle POY = 180^\circ \]
\[ \Rightarrow b + a + \angle POY + 180^\circ \]
\[ \Rightarrow 3x + 2x + 90^\circ = 180^\circ \]
\[ \Rightarrow 5x = 90^\circ \]
\[ \Rightarrow x = 18^\circ \]
\[ a = 2x \]
\[ \Rightarrow 2 \times 18 \]
\[ \Rightarrow 36^\circ \]
\[ b = 3x \]
\[ \Rightarrow 3 \times 18 \]
\[ \Rightarrow 54^\circ \]

Now, MN is a straight line. OX ray stands on it.
\[ \angle b + \angle c = 180^\circ \]
\[ \Rightarrow 54^\circ + \angle c = 180^\circ \]
\[ \Rightarrow \angle c = 180^\circ - 54^\circ = 126^\circ \]
\[ \Rightarrow \angle c = 126^\circ \]

3. In the given Figure, \(\angle PQR = \angle PRQ\), then prove that \(\angle PQS = \angle PRT\)
**Solution:**

In the figure, it is given that ST is a straight line and QP ray stand on it.

Therefore,
\[ \angle PQS + \angle PQR = 180^\circ \text{ (Linear Pair of angles)} \]
\[ \Rightarrow \angle PQR = 180^\circ - \angle PQS \tag{1} \]
\[ \Rightarrow \angle PRT + \angle PRQ = 180^\circ \text{ (Linear Pair of angles)} \]
\[ \Rightarrow \angle PRQ = 180^\circ - \angle PRT \tag{2} \]

It is given that \( \angle PQR = \angle PRQ \).

Now, from equations (1) and (2), we have
\[ 180^\circ - \angle PQS = 180^\circ - \angle PRT \]
\[ \Rightarrow \angle PQS = \angle PRT \]

4. In the given figure, if \( x + y = w + z \), then prove that AOB is a line.

**Solution:**

From the figure, it can be observed that,
\[ x + y + z + w = 360^\circ \text{ (Complete angle)} \]

It is given that, \( x + y = z + w \)

\[ \therefore x + y + x + y = 360^\circ \]
\[ \Rightarrow 2(x + y) = 360^\circ \]
\[ \Rightarrow x + y = 180^\circ \]
Since \(x\) and \(y\) form a linear pair, \(AOB\) is a line.

5. In the given fig., \(POQ\) is a line. Ray OR is perpendicular to line \(PQ\). OS is another ray lying between rays OP and OR. Prove that \(\angle ROS = \frac{1}{2} (\angle QOS − \angle POS)\).

\[
\text{Solution:}
\]

Here, it is given that \(OR \perp PQ\)

\(\angle POR = 90^\circ\)

\(\Rightarrow \angle POS + \angle SOR = 90^\circ\)

\(\Rightarrow \angle ROS = 90^\circ − \angle POS \ldots (1)\)

\(\angle QOR = 90^\circ \) (As \(OR \perp PQ\))

\(\Rightarrow \angle QOS − \angle ROS = 90^\circ\)

\(\Rightarrow \angle ROS = \angle QOS − 90^\circ \ldots (2)\)

On adding equations (1) and (2), we have

\(2 \angle ROS = \angle QOS − \angle POS\)

\(\Rightarrow \angle ROS = \frac{1}{2} (\angle QOS − \angle POS)\)

6. It is given that \(\angle XYZ = 64^\circ\) and \(XY\) is produced to point \(P\). Draw a figure from the given information. If ray \(YQ\) bisects \(\angle ZYP\), find \(\angle XYQ\) and reflex \(\angle QYP\).

\[
\text{Solution:}
\]

It is given that the line \(YQ\) bisects \(\angle PYZ\).

So, \(\angle QYP = \angle ZYQ\)

Now \(PX\) is a line. \(YQ\) and \(YZ\) rays stand on it.

Therefore,
\[ \angle XYZ + \angle ZYQ + \angle QYP = 180^\circ \]
\[ \Rightarrow 64^\circ + 2\angle QYP = 180^\circ \]
\[ \Rightarrow 2\angle QYP = 180^\circ - 64^\circ = 116^\circ \]
\[ \Rightarrow \angle QYP = 58^\circ \]
Also, \( \angle ZYQ = \angle QYP = 58^\circ \)
\[ \Rightarrow \text{Reflex } \angle QYP = 360^\circ - 58^\circ = 302^\circ \]

Exercise: 6.2

1. In the given figure, find the values of \( x \) and \( y \) and then show that \( AB \parallel CD \).

Solution:
Here, we can see that,
\[ 50^\circ + x = 180^\circ \] (Linear pair of angles)
\[ \Rightarrow x = 130^\circ \]
\[ \therefore y = 130^\circ \] (vertically opposite angles)
As \( x \) and \( y \) are alternate interior angles for lines \( AB \) and \( CD \) and the measures of these angles are equal to each other.
Therefore, line \( AB \parallel CD \).

2. In the given figure, if \( AB \parallel CD, CD \parallel EF \) and \( y:z = 3:7 \), find \( x \).

Solution:
It is given that \( AB \parallel CD \) and \( CD \parallel EF \).
Therefore,
\[ AB \parallel CD \parallel EF \quad \text{(Lines parallel to a same line are parallel to each other)} \]
Now, we can see that,
\[ x = z \quad \text{(alternate interior angles)} \] \( \ldots (1) \)
Also it is given that,
\[ y: z = 3:7 \]
Let the common ratio between \( y \) and \( z \) be \( a \)
Therefore,
\[ y = 3a \text{ and } z = 7a \]
Also, \( x + y = 180^\circ \) \( \text{(co-interior angles on the same side of the transversal)} \)
\[ \Rightarrow z + y = 180^\circ \quad \text{[Using equation (1)]} \]
\[ \Rightarrow 7a + 3a = 180^\circ \]
\[ \Rightarrow 10a = 180^\circ \]
\[ \Rightarrow a = 18^\circ \]
Therefore,
\[ x = 7a \]
\[ \Rightarrow 7 \times 18^\circ = 126^\circ \]

3. In the given figure, if \( AB \parallel CD, EF \perp CD \) and \( \angle GED = 126^\circ \), find \( \angle AGE, \angle GEF \\text{and } \angle FGE. \)

\[ \text{Solution:} \]
It is given that,
\[ AB \parallel CD, EF \perp CD \text{ and } \angle GED = 126^\circ \]
Now,
\[ \angle GED = \angle AGE = 126^\circ \quad \text{[alternate interior angles]} \]
\[ \Rightarrow \angle GEF + \angle FED = 126^\circ \]
\[ \Rightarrow \angle GEF + 90^\circ = 126^\circ \]
\[ \Rightarrow \angle GEF = 36^\circ \]
Now, \( \angle AGE \) and \( \angle GED \) are alternate interior angles
⇒ ∠AGE = ∠GED = 126°

But ∠AGE + ∠FGE = 180° (linear pair)
⇒ 126° + ∠FGE = 180°
⇒ ∠FGE = 180° − 126° = 54°

4. In the given figure, if PQ ∥ ST, ∠PQR = 110° and ∠RST = 130°, find ∠QRS.

Solution:
Construction: Let us draw a line XY parallel to ST and passing through point R.

Now,
∠PQR + ∠QRX = 180° (co-interior angles on the same side of transversal QR)
⇒ 110° + ∠QRX = 180°
⇒ ∠QRX = 70°

Also,
⇒ ∠RST + ∠SRY = 180° (co-interior angles on the same side of transversal SR)
⇒ 130° + ∠SRY = 180°
⇒ ∠SRY = 50°

XY is a straight line. RQ and RS stand on it.

Therefore,
∠QRX + ∠QRS + ∠SRY = 180°
⇒ 70° + ∠QRS + 50° = 180°
⇒ ∠QRS = 180° − 120° = 60°

5. In the given figure, if AB ∥ CD, ∠APQ = 50° and ∠PRD = 127°, find x and y.
Solution:

Given $\angle PRD = 127^\circ$

$\angle APQ = \angle PQR$  \hspace{1cm} (alternate interior angles)

$\Rightarrow 50^\circ = x$

$\Rightarrow \angle APR = \angle PRD$  \hspace{1cm} (alternate interior angles)

$x + y = \angle PRD$

$\Rightarrow 50^\circ + y = 127^\circ$

$\Rightarrow y = 127^\circ - 50^\circ$

$\Rightarrow y = 77^\circ$

6. In the given figure, PQ and RS are two mirrors placed parallel to each other. An incident ray $AB$ strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that $AB \parallel CD$.

Solution:

Let us draw $BM \perp PQ$ and $CN \perp RS$. 
As PQ \parallel RS

So, BM \parallel CN

Therefore, BM and CN are two parallel lines and a transversal line BC cuts them at B and C respectively.

\( \angle 2 = \angle 3 \) \hspace{1cm} (alternate interior angles)

But

\( \angle 1 = \angle 2 \) and \( \angle 3 = \angle 4 \) \hspace{1cm} (By laws of reflection)

\( \Rightarrow \angle 1 = \angle 2 = \angle 3 = \angle 4 \)

Now, \( \angle 1 + \angle 2 = \angle 3 + \angle 4 \)

\( \Rightarrow \angle ABC = \angle DCB \)

But, these are alternate interior angles

Therefore,

AB \parallel CD

**Exercise: 6.3**

1. In the given figure, sides QP and RQ of \( \triangle PQR \) are produced to points S and T respectively. If \( \angle SPR = 135^\circ \) and \( \angle PQT = 110^\circ \), find \( \angle PRQ \).

\[ \angle SPR = 135^\circ \] and \( \angle PQT = 110^\circ \)

Now,

\( \angle SPR + \angle QPR = 180^\circ \) \hspace{1cm} (linear pair angles)

\( \Rightarrow 135^\circ + \angle QPR = 180^\circ \)

\( \Rightarrow \angle QPR = 45^\circ \)

Also,

\( \angle PQT + \angle PQR = 180^\circ \) \hspace{1cm} (linear pair angles)

\( \Rightarrow 110^\circ + \angle PQR = 180^\circ \)
\[ \Rightarrow \angle PQR = 70^\circ \]

As we know that sum of all interior angles of a triangle is 180°,

Therefore in \( \triangle PQR \)

\[ \angle QPR + \angle PQR + \angle PRQ = 180^\circ \]

\[ \Rightarrow 45^\circ + 70^\circ + \angle PRQ = 180^\circ \]

\[ \Rightarrow \angle PRQ = 180^\circ - 115^\circ \]

\[ \Rightarrow \angle PRQ = 65^\circ \]

2. In the given Figure, \( \angle X = 62^\circ, \angle XYZ = 54^\circ \). If \( \text{YO} \) and \( \text{ZO} \) are the bisectors of \( \angle XYZ \) and \( \angle XZY \) respectively of \( \triangle XYZ \), find \( \angle OZY \) and \( \angle YOZ \).

![Figure](image)

**Solution:**

We know that the sum of interior angles of a triangle is 180°,

Therefore in \( \triangle XYZ \)

\[ \angle X + \angle XYZ + \angle XZY = 180^\circ \]

\[ \Rightarrow 62^\circ + 54^\circ + \angle XZY = 180^\circ \]

\[ \Rightarrow \angle XZY = 180^\circ - 116^\circ \]

\[ \Rightarrow \angle XZY = 64^\circ \]

\[ \Rightarrow \angle OZY = \frac{64^\circ}{2} = 32^\circ \quad (\text{OZ is angle bisector of} \ \angle XZY) \]

Similarly, \n
\[ \Rightarrow \angle OYZ = \frac{54^\circ}{2} = 27^\circ \quad (\text{OY is angle bisector of} \ \angle XYZ) \]

Now, using angle sum property for \( \triangle OYZ \), we have

\[ \angle OYZ + \angle YOZ + \angle OZY = 180^\circ \]

\[ \Rightarrow 27^\circ + \angle YOZ + 32^\circ = 180^\circ \]

\[ \Rightarrow \angle YOZ = 180^\circ - 59^\circ \]

\[ \Rightarrow \angle YOZ = 121^\circ \]

3. In the given Figure, if \( AB \parallel DE, \angle BAC = 35^\circ \) and \( \angle CDE = 53^\circ \), find \( \angle DCE \)

...
Solution:
Here, AB \parallel DE and AE is a transversal
⇒ \angle BAC = \angle CED \hspace{1cm} \text{(alternate interior angle)}
⇒ \angle CED = 35°

In \triangle CDE,
\angle CDE + \angle CED + \angle DCE = 180° \hspace{1cm} \text{(Angle sum property of a triangle)}
⇒ 53° + 35° + \angle DCE = 180°
⇒ \angle DCE = 180° - 88°
⇒ \angle DCE = 92°

4. In the given Figure, if lines PQ and RS intersect at point T, such that \angle PRT = 40°, \angle RPT = 95° and \angle TSQ = 75°, find \angle SQT.

Solution:
Here, using angle sum property for \triangle we have,
\angle PRT + \angle RPT + \angle PTR = 180°
⇒ 40° + 95° + \angle PTR = 180°
⇒ \angle PTR = 180° - 135°
⇒ \angle PTR = 45°
⇒ \angle STQ = \angle PTR = 45° \hspace{1cm} \text{(vertically opposite angles)}
⇒ \angle STQ = 45°

Now, by using angle sum property for \triangle STQ, we have
\[ \Rightarrow \angle STQ + \angle SQT + \angle QST = 180^\circ \]
\[ \Rightarrow 45^\circ + \angle SQT + 75^\circ = 180^\circ \]
\[ \Rightarrow \angle SQT = 180^\circ - 120^\circ \]
\[ \Rightarrow \angle SQT = 60^\circ \]

5. In the given Figure, if PQ \perp PS, PQ \parallel SR, \angle SQR = 28^\circ \text{ and } \angle QRT = 65^\circ, \text{ then find the values of } x \text{ and } y.

![Diagram](image)

**Solution:**

It is given that PQ \parallel SR and QR is a transversal line.

\[ \angle PQR = \angle QRT \] (alternate interior angles)

\[ \Rightarrow x + 28^\circ = 65^\circ \]
\[ \Rightarrow x = 65^\circ - 28^\circ \]
\[ \Rightarrow x = 37^\circ \]

Now, By using angle sum property for \( \triangle SPQ \), we have

\[ \Rightarrow \angle SPQ + x + y = 180^\circ \]
\[ \Rightarrow 90^\circ + 37^\circ + y = 180^\circ \]
\[ \Rightarrow y = 180^\circ - 127^\circ \]
\[ \Rightarrow y = 53^\circ \]

6. In the given Figure, the side QR of \( \triangle PQR \) is produced to a point S. If the bisectors of \( \angle PQR \) and \( \angle PRS \) meet at point T, then prove that \( \angle QTR = \frac{1}{2} \angle QPR \).

![Diagram](image)

**Solution:**

Here, in \( \triangle QTR \), \( \angle TRS \) is an exterior angle.
⇒ \( \angle QTR + \angle TQR = \angle TRS \)
⇒ \( \angle QTR = \angle TRS - \angle TQR \)  ...(1)

Now, in \( \triangle PQR \), \( \angle PRS \) is external angle
⇒ \( \angle QPR + \angle PQR = \angle PRS \)
⇒ \( \angle QPR + 2\angle TQR = 2\angle TRS \)  (As QT and RT are angle bisectors)
⇒ \( \angle QPR = 2(\angle TRS - \angle TQR) \)
⇒ \( \angle QPR = 2\angle QTR \)  [By using equation (1)]
⇒ \( \angle QTR = \frac{1}{2} \angle QPR \)