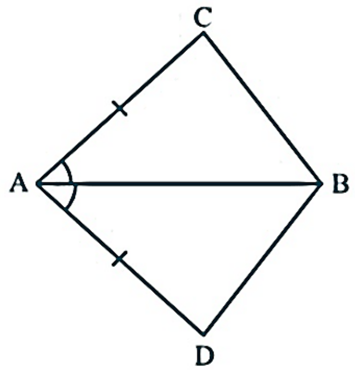
CBSE NCERT Solutions for Class 9 Mathematics Chapter 7

Back of Chapter Questions

Exercise: 7.1

1. In quadrilateral ACBD, $AC = AD$ and AB bisects $\angle A$ as shown in figure. Show that $\Delta ABC \cong \Delta ABD$. What can you say about BC and BD?



Solution:

In ΔABD and ΔABC

$\angle DAB = \angle CAB$(AB is a bisection of $\angle CAD$)

$AD = AC$(given)

$AB = AB$(common)

$\therefore \Delta ABD \cong \Delta ABC$ (by SAS postulate)

$\therefore BD = BC$ (by CPCT)

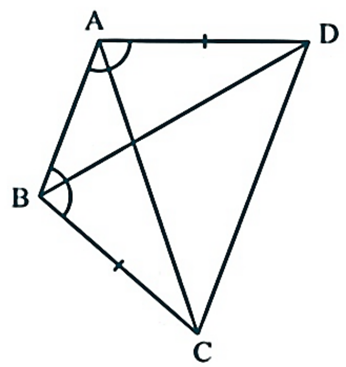
Hence, BC and BD are of equal length.

2. ABCD is a quadrilateral in which $AD = BC$ and $\angle CBA = \angle DAB$ as shown in figure. Prove that

(i) $\Delta ABD \cong \Delta BAC$

(ii) $BD = AC$

(iii) $\angle ABD = \angle BAC$



Solution:

(i) In ΔBAC and ΔABD

$BC = AD$(given)

$\angle CBA = \angle DAB$(given)

$BA = AB$(given)

$\Delta ABD \cong \Delta BAC$(by SAS postulate)

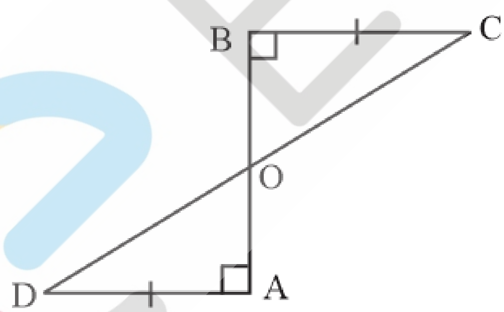
(ii) $BD = AC$(by CPCT rule)

(iii) $\angle ABD = \angle BAC$ (by CPCT rule)

Given $AD = BC$

$\angle ADC = \angle BCD$

3. AD and BC are equal perpendiculars to a line segment AB as shown in figure. Show that CD bisects AB.



Solution:

In ΔAOD and ΔBOC

$\angle AOD = \angle BOC$(verifying opposite angle)

$\angle DAO = \angle CBO(90^o)$

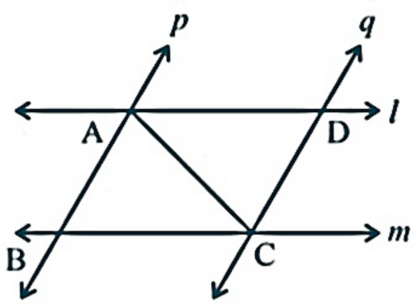
$AD = BC$ (given)

$\therefore \Delta AOD \cong \Delta BOC$(by AAS rule)

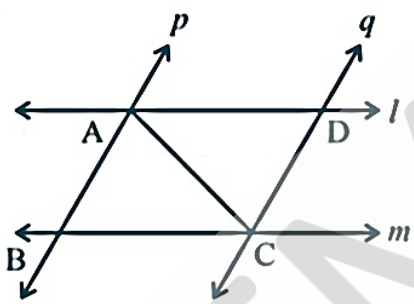
$BO = AO$ (by CPCT)

$\Rightarrow$ CD bisects AB

4. l and m are two parallel lines intersected by another pair of parallel lines p and q as shown in figure. Show that $\Delta ABC \cong \Delta CDA$.



Solution:



In ΔCDA and ΔABC

$\angle DCA = \angle BAC$(alternate interior angle of parallel line P and Q)

$CA = AC$ (common)

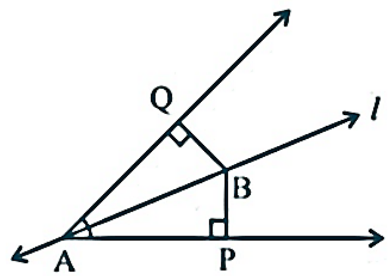
$\angle DAC = \angle BCA$(alternate interior angle of parallel line l and m)

$\Delta ABC \cong \Delta CDA$(by ASA rule).

5. Line l is the bisector of an angle $\angle A$ and B is any point on l. BP and BQ are perpendiculars from B to the arms of $\angle A$ as shown in figure. Show that

(i) $\Delta APB \cong \Delta AQB$

(ii) $BP = BQ$ or B is equidistant from the arms of $\angle A$.



Solution:

In ΔAQB and ΔAPB

$AB = AB$(common)

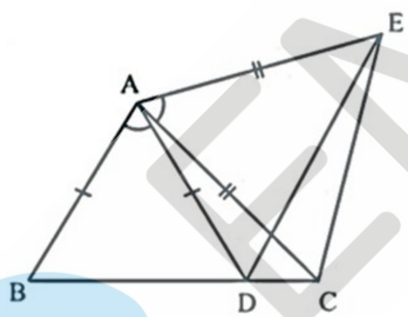
$\angle AQB = \angle APB$ (90^o)

$\angle QAB = \angle PAB$ (l is bisector)

$\Delta APB \cong \Delta AQB$(by AAS rule)

$\therefore BQ = BP$(by CPCT)

6. In figure shown, $AC = AE, AB = AD$ and $\angle BAD = \angle CAE$. Show that $BC = DE$.



Solution:

Given,

$AB = AD, AC = AE$ and

$\angle BAD = \angle CAE$

Now,

$\angle EAC + \angle DAC = \angle BAD + DAC$ ($\angle DAC$ is common)

Now in ΔDAE and ΔBAC

$AD = AB$

$\angle DAE = \angle BAC$

$AE = AC$

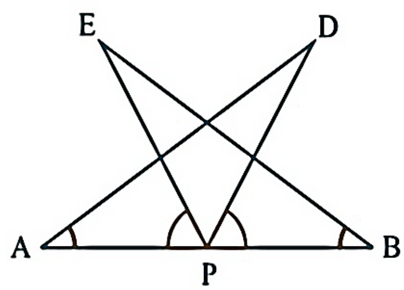
$\Delta DAF \cong \Delta BAC$(by SAS rule).

$DE = BC$(by CPCT).

7. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$ as shown in figure. Show that

(i) $\Delta DAP \cong \Delta EBP$

(ii) AD=BE



Solution:

Given,

$\angle DPB = \angle EPA$

$\Rightarrow \angle DPB + \angle DPE = \angle EPA + \angle DPE$

$\Rightarrow \angle DPA = \angle EPB$

In ΔDAP and ΔEBP

$\angle DAP = \angle EBP$(given)

$AP = BP$(P is mid-point of AB)

$\angle DPA = \angle EPB$(from above)

$\therefore \Delta DAP \cong \Delta EBP$ (ASA)

$BE = AD$(by CPCT)

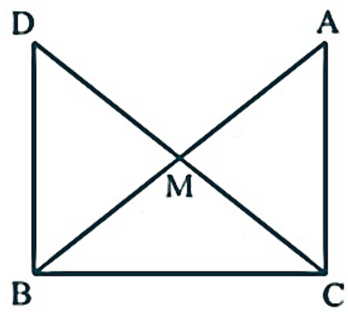
8. In right triangle ABC, right angled at C, M is the midpoint of hypotenuse AB. C is joined to M and produced to a point D such that $DM = CM$. Point D is joined to point B as shown in figure. Show that

(i) $\Delta BMD \cong \Delta AMC$

(ii) $\angle DBC$ is a right angle.

(iii) $\Delta DBC \cong \Delta ACB$

(iv) $CM = \frac{1}{2}AB$



Solution:

(i) In ΔBMD and ΔAMC

$BM = AM$(M is mid-point)

$\angle BMD = \angle AMC$ (vertically opposite angles)

$DM = CM$(given)

$\Delta BMD \cong \Delta AMC$ (by SAS rule)

$BD = AC$(by CPCT)

$\angle BDM = \angle ACM$ (by CPCT)

(ii) Given,

$\angle BDM = \angle ACM$ (alternate interior angles)

$DB \parallel AC$(alternate angles are equal)

$\Rightarrow \angle DBC + \angle ACB = 180^o$

$\Rightarrow \angle DBC + 90^o = 180^o$ (co-interior angles)

$\Rightarrow \angle DBC = 90^o$

(iii) Now in ΔDBC and ΔACB

$DB = AC$(proved)

$\angle DBC = \angle ACB(90^o)$

$CB = BC$(common)

$\Delta DBC \cong \Delta ACB$(by SAS rule)

(iv) We have, $\Delta DBC \cong \Delta ACB$

$AB = DC$(by CPCT)

$\Rightarrow AB = 2\ CM$

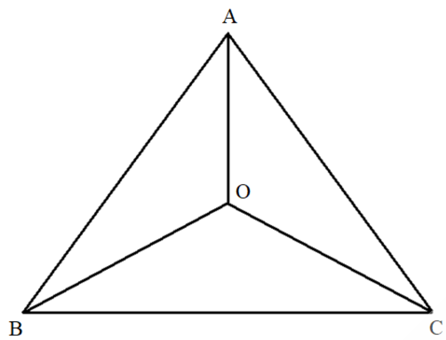
$$\text{CM} = \frac{1}{2}\text{AB}$$

Exercise: 7.2

1. In an isosceles triangle ABC, with AB = AC, the bisectors of $\angle B$ and $\angle C$ intersect each other at O. Join A to O. Show that:

(i) $\text{OB} = \text{OC}$

(ii) AO bisects $\angle A$



Solution:

(i) Given,

$\text{AB} = \text{AC}$

$\Rightarrow \angle ABC = \angle ACB$ (angle opposite to equal sides of a triangle are equal)

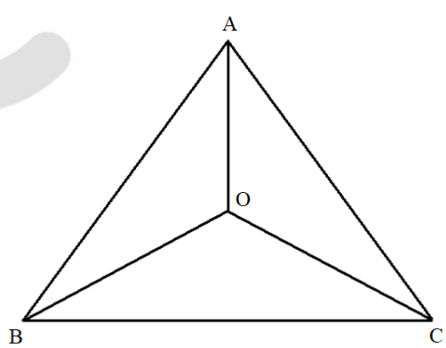
$\Rightarrow \frac{1}{2}\angle ACB = \frac{1}{2}\angle ABC$

$\Rightarrow \angle OBA = \angle OCA$

$\text{OA} = \text{OA}$

$\Delta OAB \cong \Delta OAC$ (by SAS)

$\text{OB} = \text{OC}$ (by CPCT)



(ii) In ΔOAB and ΔOAC

$AO = AO$(common)

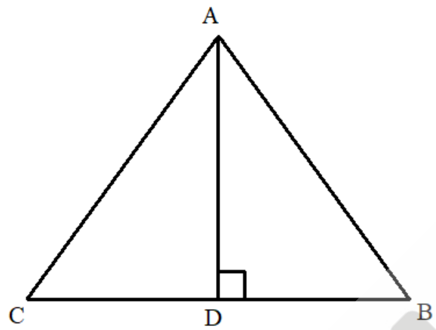
$AB = AC$(given)

$OB = OC$(proved)

$\therefore\ \Delta OAB \cong \Delta OAC$

$\Rightarrow \angle BAO = \angle CAO$

2. In ΔABC, AD is the perpendicular bisector of BC as shown in figure. Show that ΔABC is an isosceles triangle in which $AB = AC$.



Solution:

In ΔADB and ΔADC

$AD = AD$ (common)

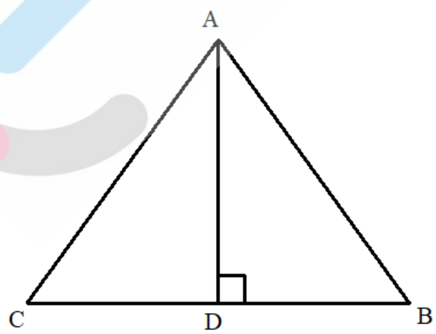
$\angle ADB = \angle ADC (90^o)$

$BD = CD$ (AD is perpendicular bisector of BC)

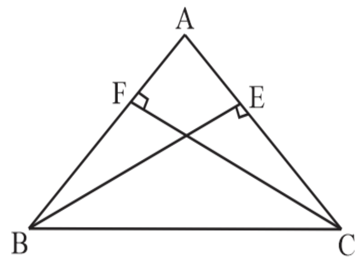
$\Delta ADC \cong \Delta ADB$(by SAS rule)

$AB = AC$ (by CPCT)

$\therefore\ \Delta ABC$ is an isosceles triangle being which $AB = AC$



3. ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively as shown in figure. Show that these altitudes are equal.



Solution:

In ΔAFC and ΔAEB

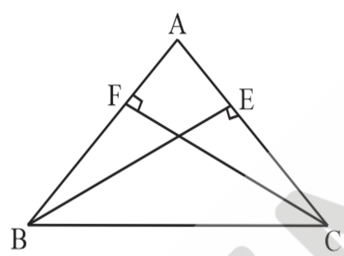
$\angle AFC = \angle AEB$. (90^o)

$\angle A = \angle A$ (common)

$AC = AB$ (given)

$\Delta AFC \cong \Delta AEB$(by AAS rule)

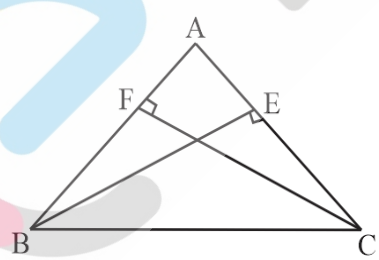
$\Rightarrow BE = CF$



4. ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal as shown in figure. Show that

(i) $\Delta AFC \cong \Delta AEB$

(ii) $AB = AC$, i.e ABC is an isosceles triangle.



Solution:

(i) In ΔAFC and ΔAEB

$\angle AFC = \angle AEB$ (90^o)

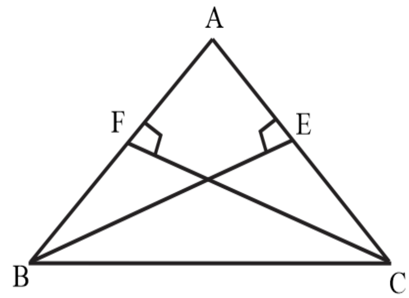
$\angle A = \angle A$ (common)

CF = BE (given)

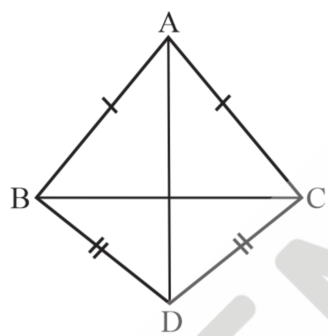
$\Delta AFC \cong \Delta AEB$ (by AAS rule)

(ii) $\Delta AFC \cong \Delta AEB$

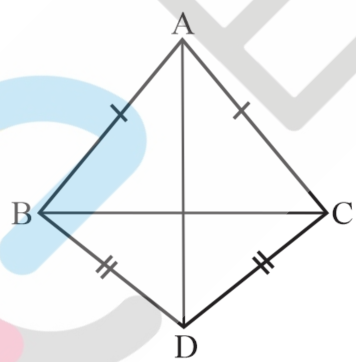
$\Rightarrow AB = AC$ (CPCT)



5. ABC and DBC are two isosceles triangles on the same base BC as shown in the figure. Show that $\angle ABD = \angle ACD$



Solution:



We join AD.

In ΔACD and ΔABD

$AC = AB$(given)

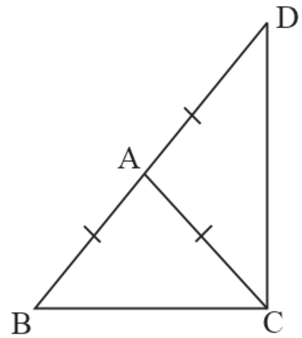
$DC = BD$(given)

$AD = AD(\text{common})$

$\Delta ABD \cong \Delta ACD$ (by SSS rule)

$\Rightarrow \angle ABD = \angle ACD$ (by CPCT)

6. ΔABC is an isosceles triangle in which $AB = AC$. Side BA is produced to D such that $AD = AB$ as shown in figure. Show that $\angle BCD$ is a right angle.



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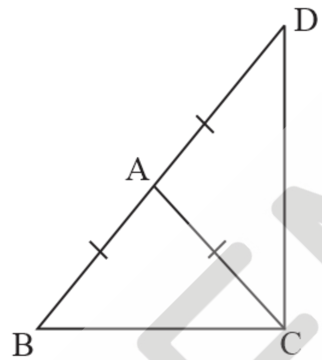


In ΔABC

$AC = AB(\text{given})$

$\angle ABC = \angle ACB$ (angles opposite to equal sides of a triangle are also equal)

Now, in ΔACD

$AD = AC$

$\Rightarrow \angle ACD = \angle ADC$ (angles opposite to equal sides of a triangle are equal)

Now, in ΔBCD

$\angle ABC + \angle BCD + \angle ADC = 180^o$ (angle sum property)

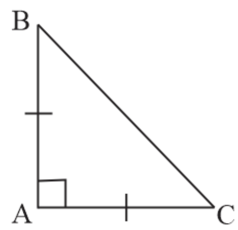
$\Rightarrow \angle ACB + \angle ACB + \angle ACD + \angle ADC = 180^o$

$\Rightarrow 2(\angle ACB + \angle ACD) = 180^o$

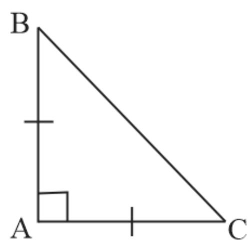
$\Rightarrow 2\angle BCD = 180^o$

$\Rightarrow \angle BCD = 90^o$

7. ABC is a right angled triangle in which $\angle A = 90^o$ and $AB = AC$. Find $\angle B$ and $\angle C$.



Solution:



$\angle B = \angle C$(angles opposite to equal sides are also equal)

In ΔABC,

$\angle A + \angle B + \angle C = 180^o$(angle sum property of triangle)

$\Rightarrow \angle B + \angle B + 90^o = 180^o(LC = LB)$

$\Rightarrow 2\angle B = 90^o$

$\Rightarrow \angle B = 45^o$

i.e. $\Rightarrow \angle B = \angle C = 45^o$

8. Show that the angles of an equilateral triangle are 60^o each.

Solution:

Since triangle is an equilateral triangle are equal.

$\angle A = \angle B = \angle C$

$\angle A + \angle B + \angle C = 180^o$ (angle sum property of a triangle)

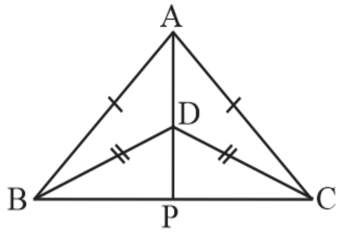
$\Rightarrow 3\angle A = 180^o$

$\Rightarrow \angle A = 60^o$

$\therefore \angle A = \angle B = \angle C = 60^o$

Exercise: 7.3

1. ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC as shown in figure. If AD is extended to intersect BC at P, show that



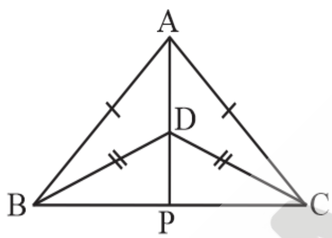
(i) $\Delta ABD \cong \Delta ACD$

(ii) $\Delta ACP \cong \Delta ABP$

(iii) AP bisects $\angle A$ as well as $\angle D$.

(iv) AP is the perpendicular bisector of BC.

Solution:



(i) In ΔACD and ΔABD

$AC = AB(\text{given})$

$CD = BD(\text{given})$

$AD = AD(\text{common})$

$\therefore\ \Delta ABD \cong \Delta ACD$ (by SSS rule)

$\Rightarrow \angle BAP = \angle CAP$ (by CPCT)

(ii) In ΔACP and ΔABP

$AC = AB(\text{given})$

$\angle CAP = \angle BAP$

$AP = AP(\text{common})$

$\therefore \Delta ACP \cong \Delta ABP$ (by CPCT)(2)

(iii) $\angle CAP = \angle BAP$

Hence, AP bisects $\angle A$

Now, in ΔCDP and ΔBDP

$CD = BD$(given)

$DP = DP$(common)

$CP = BP$(from equation (2))

$\Delta CDP \cong \Delta BDP$(by SSS rule)

$\Rightarrow \angle BDP = \Delta CDP$ (by CPCT)

(iv) We have, $\Delta CDP \cong \Delta BDP$

$\therefore \angle CDP = \angle BDP$(by CPCT)

Now, $\angle CPD + \angle BPD = 180^0$ (linear pair angles)

$= \angle BPD + \angle BPD = 180^0$

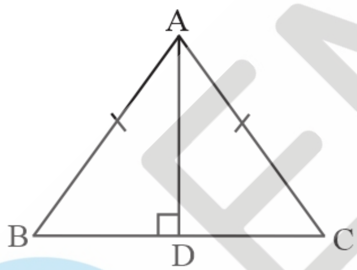
$= BPD = 90^0 (3)$

From equations (2) and (3) we can say that AP is bisector of BC.

2. AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that

(i) AD bisects BC

(ii) AD bisects $\angle A$



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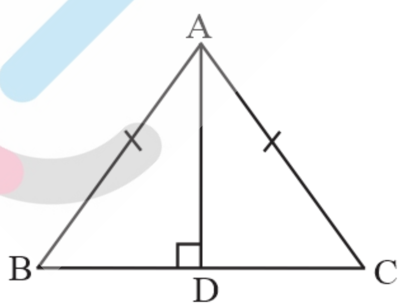


(i) In ΔCAD and ΔBAD

$\angle ADB = \angle ADC(90^0$ as AD is altitude)

$AC = AB$ (given)

$AD = AD$ (common)

$\Rightarrow \Delta CAD \cong \Delta BAD$(by RHS rule)

Hence, $BD = CD$(by CPCT)

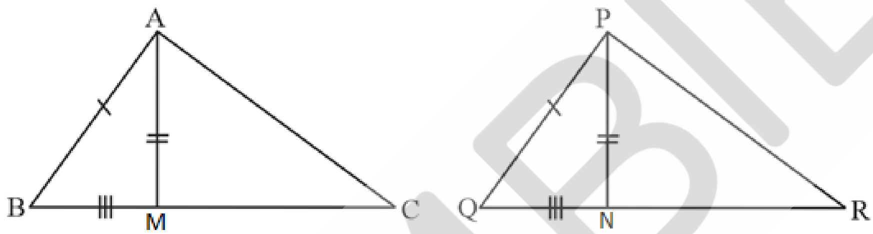
$\therefore$ AD bisects BC

(ii) Also, by CPCT

$\angle BAD = \angle CAD$

Hence, AD bisects $\angle A$

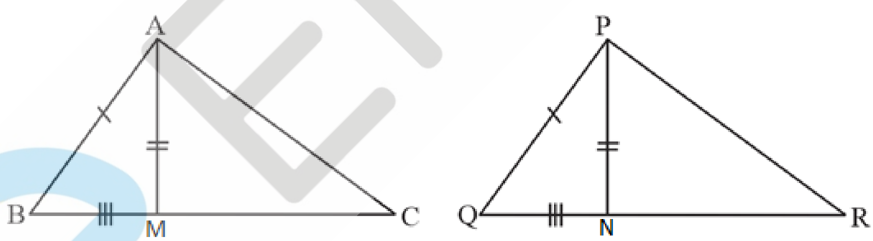
3. Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of ΔPQR as shown in figure. Show that



(i) $\Delta PQN \cong \Delta ABM$

(ii) $\Delta ABC \cong \Delta PQR$

Solution:



(i) In ΔABC, AM is median to BC

$$\frac{1}{2}BC = BM$$

In ΔPQR, PN is median to QR

$$\frac{1}{2}QR = \frac{1}{2}BC = QN$$

$$\Rightarrow \frac{1}{2}QR = \frac{1}{2}BC \qquad \ldots.(1)$$

$$\Rightarrow BN = QN$$

Now in ΔPQN and ΔABM

$PQ = AB$ (given)

$QN = BM$ (from equation 1)

$PN = AM$ (given)

$\therefore \Delta PQN \cong \Delta ABM$(by SSS rule)

$\angle ABM = \angle PQN$(by CPCT)

$\angle ABC = \angle PQR$ ….(2)

(ii) Now in ΔABC and ΔPQR

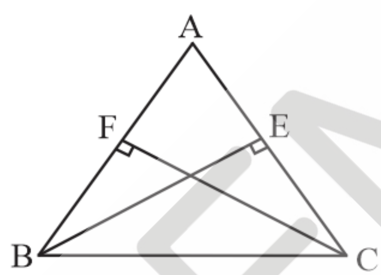
$BC = QR$ (given)

$AB = PQ$ (given)

$\angle ABC = \angle PQR$ (from equation 2)

$\Rightarrow \Delta ABC \cong \Delta PQR$(by SAS rule)

4. BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.



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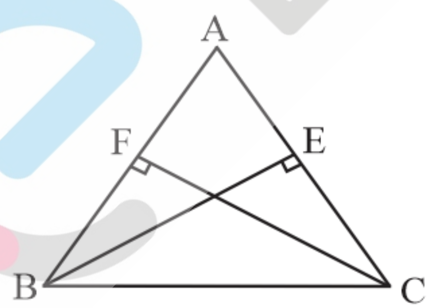


In ΔCFB and ΔBEC

$\angle CFB = \angle BEC$ (90^o)

$CB = BC$ (common)

$CF = BE$ (given)

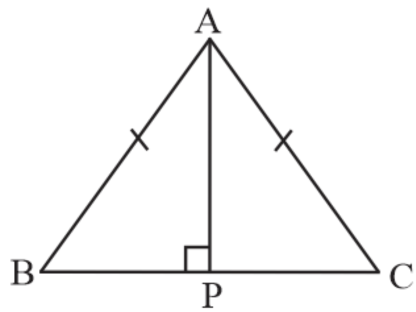
$\therefore \Delta CFB \cong \Delta BEC$(by RHS rule)

$\Rightarrow \angle BCE = \angle CBF$(by CPCT)

$\therefore AB = AC$ (sides opposite to equal angles of a triangle are equal)

$\therefore \Delta ABC$ is isosceles.

5. ABC is an isosceles triangle with $AB = AC$. Draw AP so that AP is perpendicular to BC and show that $\angle B = \angle C$



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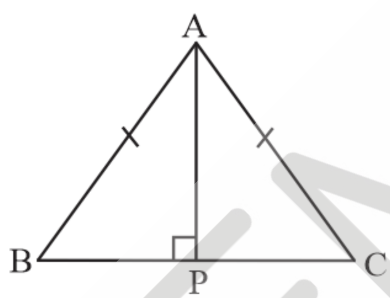


In ΔAPC and ΔAPB

$\angle APC = \angle APB$ (90^{o})

$AC = AB$ (given)

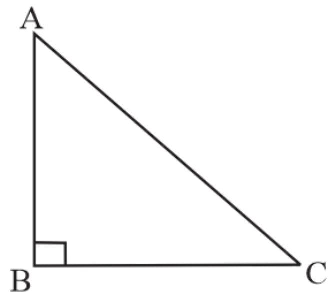
$AP = AP$ (common)

$\Delta APC \cong \Delta APB$ (by RHS rule)

$\Rightarrow \angle B = \angle C$ (CPCT)

Exercise: 7.4

1. Show that in a right-angled triangle, the hypotenuse is the longest side.



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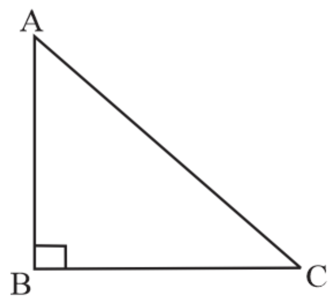


Let us consider a triangle ABC to be right angled triangle.

In ΔABC

$\angle A + \angle B + \angle C = 180^o$ (angle sum property of triangle)

$\angle A + \angle C = 90^o$

Hence, other two angles need to be acute.

$\angle B$ is larger in ΔABC

$\Rightarrow \angle B > \angle A$ and $\angle B > \angle C$

$\Rightarrow AC > BC$ and $AC > AB$

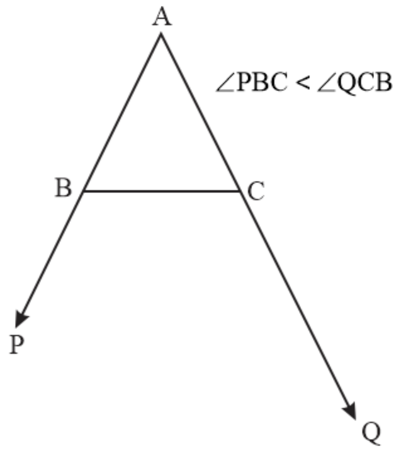
(In any triangle the side opposite to the larger angle is longer)

So AC is the largest side in ΔABC.

But AC is the hypotenuse of ΔABC.

Therefore, hypotenuse is the largest side in a right-angled triangle.

2. In figure shown below, sides AB and AC of ΔABC are extended to points P and Q respectively. Also, $\angle PBC < \angle QCB$. Show that $AC > AB$.



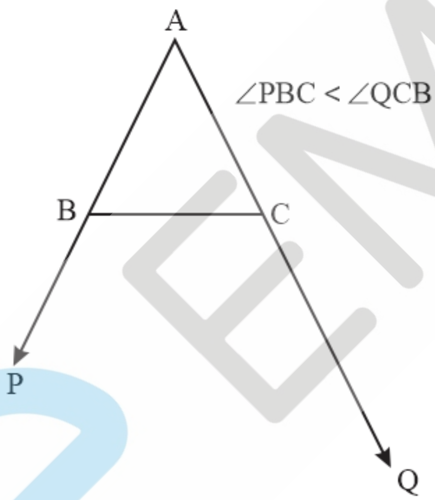
Solution:

In the given figure

$\angle PBC + \angle ABC = 180^o$ (linear)

$\Rightarrow \angle ABC = 180^o - \angle PBC \ldots (1)$

Also,



$\angle ACB + \angle QCB = 180^o$ (linear)

$\angle ACB = 180^o - \angle QCB$ $\ldots(2)$

As, $\angle PBC < \angle QCB$

$\Rightarrow 180 - \angle ABC < 180^o - \angle ACB$

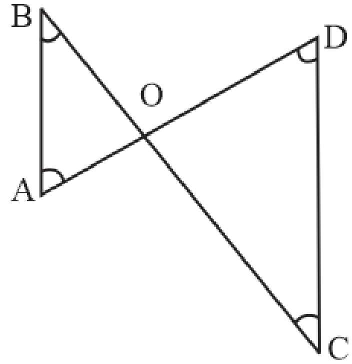
$\Rightarrow \angle ABC > \angle ACB$(from equation 1 and 2)

$\Rightarrow AC > AB$ (side opposite to larger side is equal)

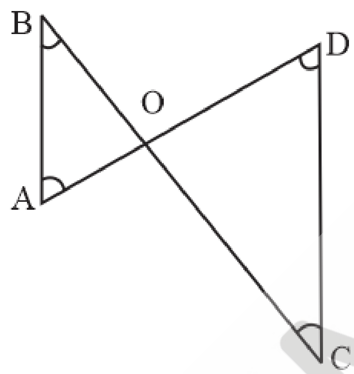
3. In figure shown below,

$\angle B < \angle A$ and

$\angle C < \angle D$. Show that $AD < BC$.



Solution:



In ΔAOB

$\angle B < \angle A$

$\Rightarrow AO < BO$ (side opposite to smaller angle is smaller)… (1)

Now in ΔCOD

$\angle C < \angle D$

$\Rightarrow OD < OC$ (Side opposite to smaller angle is smaller)… (2)

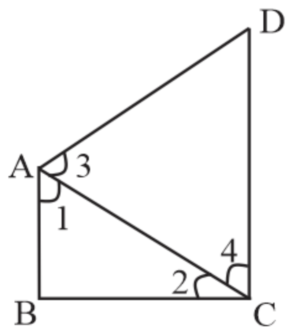
On adding equation 1 and 2

$AO + OD < BO + OC$

$AD < BC$

4. AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD as shown in figure.

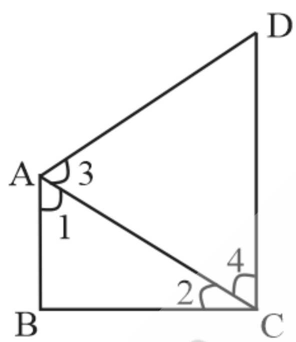
Show that $\angle A > \angle C$ and $\angle B > \angle D$.



Solution:

(i) Lets join AC

In ΔABC



$AB < BC$ (AB is smaller side of quadrilateral ABCD)

$\therefore \angle 2 < \angle 1$(angle opposite to smaller side is smaller)... (1)

In ΔADC

$AD < CD$ (CD is the largest side of quadrilateral ABCD)

$\therefore \angle 4 < \angle 3$ (angle opposite to smaller side is smaller)

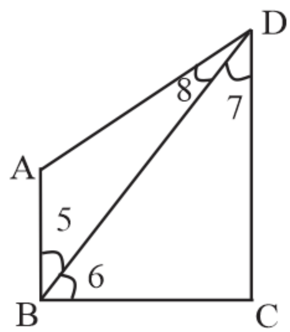
On adding (1) and (2) we have

$\angle 2 + \angle 4 < 1\angle + \angle 3$

$\Rightarrow \angle C < \angle A$

$\Rightarrow \angle A > \angle C$

(ii) Lets join BD



In ΔABD

$AB < AD$ (AB is smaller side of quadrilateral ABCD)

$\therefore \angle 8 < \angle 5$ (angle opposite to smaller side is smaller)... (3)

In ΔBDC

$\angle 7 < \angle 6$ (CD is the largest side of quadrilateral ABCD)... (4)

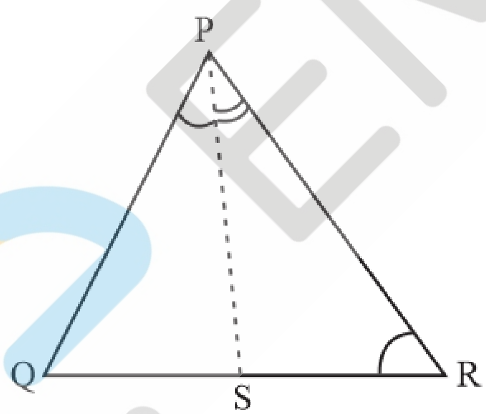
On adding equations (3) and (4)

$\angle 8 + \angle 7 < \angle 5 + \angle 6$

$\Rightarrow \angle D < \angle B$

$\Rightarrow \angle B > \angle D$

5. In shown figure, $PR > PQ$ and PS bisects $\angle QPR$. Prove that $\angle PSR > \angle PSQ$.



Solution:

Given $PR > PQ$

$\angle PQR > \angle PRQ$(angle opposite to larger side is larger)... (1)

PS is the bisector of $\angle QPR$

$\therefore \angle QPS = \angle RPS$ ... (2)

Now $\angle PSR$ is the exterior angle of ΔPQS

$\therefore \angle PSR = \angle PQR + \angle QPS \ldots (3)$

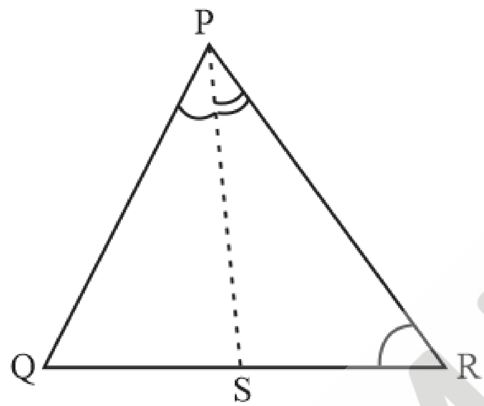
Now $\angle PSQ$ is the exterior angle of ΔPRS

$\therefore \angle PSQ = \angle PRQ + \angle RPS \ldots (4)$

Now, adding equations (1) and (2) we have

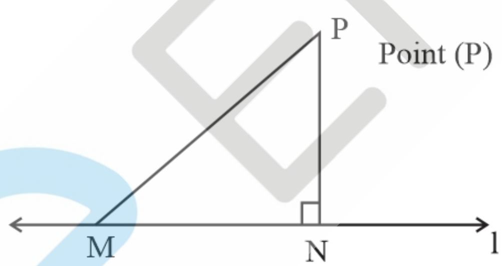
$\angle PQR + \angle QPS > \angle PRQ + \angle RPS$

$\Rightarrow \angle PSR > \angle PSQ$(using values of equation (3) and (4))



6. Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

Solution:



In ΔPNM

$\angle N = 90^o$

Now, $\angle P + \angle N + \angle M = 180^o$(angle sum property of a triangle)

$\angle P + \angle M = 90^o$

Clearly $\angle M$ is an acute angle

$\therefore \angle M < \angle N$

$\Rightarrow PN < PM$ (side opposite to smaller angle is smaller)

Similarly, by drawing different line segments from P to l we can prove that PN is smaller as comparison to then. So, we may observe that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

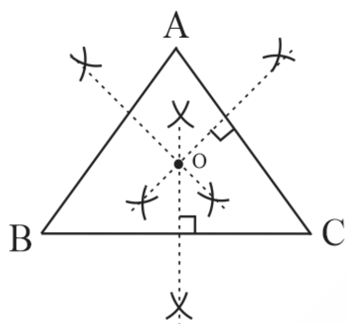
Exercise: 7.5

1. ABC is a triangle. Locate a point in the interior of ∠ABC which is equidistant from all the vertices of ∠ABC

Solution:

Triangle's circumcenter is always equidistant from all its vertices.

Circumcenter is the point where perpendicular bisectors, of all the sides of triangles meet.



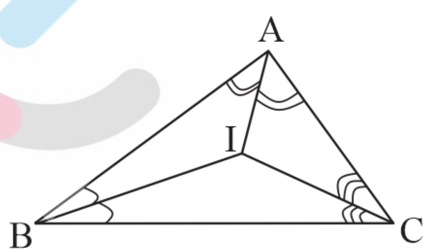
By drawing perpendicular bisectors of sides AB, BC and CA of this triangle, we can find circumcenter of ΔABC. O is the point where these bisectors are meeting together. Therefore O is a point which is equidistant from all the vertices of ΔABC.

2. In a triangle locate a point in its interior which is equidistant from all the sides of the triangle.

Solution:

Incenter of triangle is the point which is equidistant from all sides of a triangle.

The intersection point of angle bisectors of interior angles of triangle is called incenter of triangle.



We can find incenter of ΔABC by drawing angle bisectors of interior angles of this triangle.

All the angle bisectors are intersecting each other at point I. Therefore, I is equidistant from all sides of ΔABC.

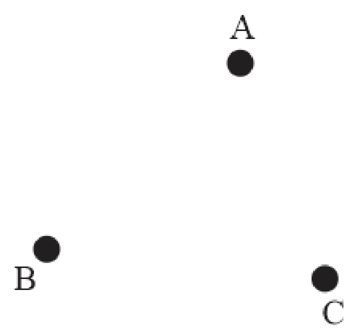
3. In a huge park, people are concentrated at three points as shown in the figure.

A: Where there are different slides and swings for children.

B: near which a manmade lake is situated.

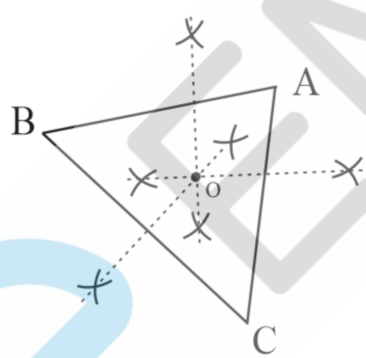
C: Which is near to a large parking and exit.

Where should an ice-cream parlor be set up so that maximum number of persons can approach it?



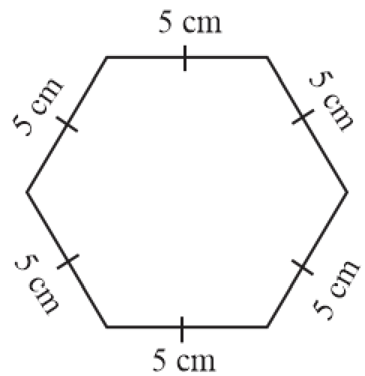
Solution:

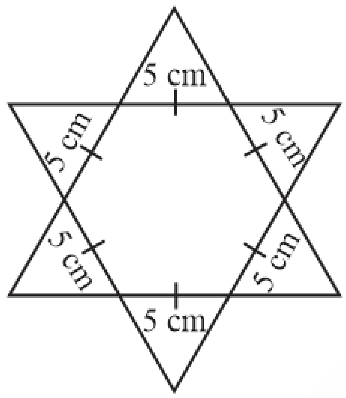
Ice-cream parlor must be set up at circumcenter O of ΔABC.



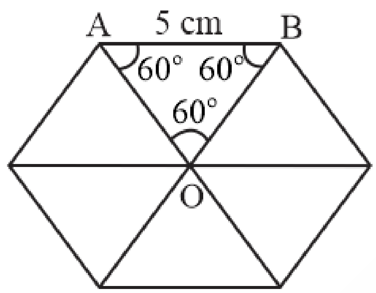
In this situation maximum number of persons can approach to it. Circumcenter O of this triangle can be found by drawing perpendicular bisectors of sides of this triangle.

4. Complete the hexagonal and star shaped rangolis by filling them with as many equilaterals triangles of side 1 cm as you can. Count the number of triangles in each case. Which has more triangles?





Solution:



We may observe that hexagonal shaped rangoli is having 6 equilateral triangles in it.

Area of $\Delta OAB = \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4} \times (5)^2$

$= \frac{25\sqrt{3}}{4}\ \text{cm}^2$

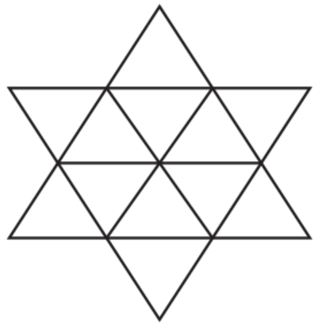
$\therefore$ Area of hexagonal shaped rangoly $= 6 \times \frac{25\sqrt{3}}{4} = \frac{75\sqrt{3}}{2}\ \text{cm}^2$

Area of equilateral triangle of side 1 cm $= \frac{\sqrt{3}}{4}(1)^2 = \frac{\sqrt{3}}{4}\ \text{cm}^2$

Number of equilateral triangles of 1 cm side that can be filled in this hexagonal shaped rangoly $= \frac{\frac{75\sqrt{3}}{2}}{\frac{\sqrt{3}}{4}}$

$= 150$

Star shaped rangoli is having 12 equilateral triangles of side 5 cm in it.



Area of star shaped rangoli $= 12 \times \frac{\sqrt{3}}{4} \times (5)^2 = 75\sqrt{3}$

Number of equilateral triangle of 1 cm side that can be filled in this star shaped rangoly $= \frac{75\sqrt{3}}{\frac{\sqrt{3}}{4}}$

$= 300$

So, star shaped rangoli has more equilateral triangles in it.