CBSE NCERT Solutions for Class 9 Mathematics Chapter 8

Back of Chapter Questions

Exercise: 8.1

1. The angles of a quadrilateral are in the ratio $3:5:9:13$. Find all the angles of the quadrilateral.

Solution:

The ratio of the angles $= 3:5:9:13$

Let the angles be $3x, 5x, 9x$ and $13x$

$3x + 5x + 9x + 13x = 360^o$ ($\because$ sum of all the angles of a quadrilateral equals 360^o)

$30x = 360^o$

$x = 12^o$

$\therefore 3x = 3 \times 12^o = 36^o$

$5x = 5 \times 12^o = 60^o$

$9x = 9 \times 12^o = 108^o$

$13x = 13 \times 12^o = 156^o$

Therefore, the angles of a quadrilateral are $36^o, 60^o, 108^o$ and 156^o

2. If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Solution:

ABCD is a parallelogram in which $AC = BD$ in ΔABC and ΔBCD

$AC = BD$ (given)

$BC = BC$ (common side)

$AB = DC$ (opposite sides of a parallelogram are equal)

$\Rightarrow \Delta ABC = \Delta BCD$ (SSS criteria)

$\therefore$ Their corresponding parts are equal.

$\Rightarrow \angle ABC = \angle DCB$...(1)

$\because AB \parallel DC$, this implies BC is a transversal. [$\because$ ABCD is a parallelogram]

$\therefore \angle ABC + \angle DCB = 180^o$...(2)

From (1) and (2), we have

$\angle ABC = \angle DCB = 90^{o}$

i.e. ABCD is a parallelogram having an angle equal to 90^{o}.

Therefore, ABCD is a rectangle

3. Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Solution:

In Quadrilateral ABCD, diagonals AC and BD bisect each other at right angles at O

$\therefore$ In ΔAOB and ΔAOD, we have

$AO = AO$ (Common)

$OB = OD$ (since O is the midpoint of BD)

$\angle AOB = \angle AOD$ (right angles, i.e.,90^{o}]

$\Delta\ AOB \cong \Delta\ AOD$ (SAS criteria)

$\Rightarrow$ Their corresponding parts are equal.

$AB = AD$...(1)

Similarly, $AB = BC$...(2)

$BC = CD$...(3)

$CD = AD$...(4)

$\therefore$ From (1), (2), (3) and (4), we get $AB = BC = CD = DA$

Therefore, the quadrilateral ABCD is a rhombus.

4. Show that the diagonals of a square are equal and bisect each other at right angles.

Solution:

ABCD is a square with its diagonals AC and BD intersecting at O.

(a) To prove that the diagonals are equal, i.e. $AC = BD$

In ΔABC and ΔBAD, $AB = BA$ (common side)

$BC = AD$ (opposite sides of the square ABCD]

$\angle ABC = \angle BAD$ (angles of a square are equal to 90^{o}]

$\therefore\ \Delta\ ABC \cong \Delta\ BAD$ (SAS criteria)

$\Rightarrow$ Their corresponding parts are equal.

$\Rightarrow AC = BD$ ...(1)

(b) To prove that $'O'$ is the midpoint of AC and BD.

$\because$ AD $\parallel$ BC and AC is a transversal. ($\because$ opposite sides of a square are parallel)

$\therefore \angle 1 = \angle 3$ (interior alternate angles)

Similarly, $\angle 2 = \angle 4$ (interior alternate angles)

Now, in ΔOAD and ΔOCB, $AD = CB$ (opposite sides of the square)

$\angle 1 = \angle 3$ (proved)

$\angle 2 = \angle 4$ (proved)

$\Delta OAD \cong \Delta\, OCB$ [ASA criteria]

$\therefore$ Their corresponding parts are equal.

$\Rightarrow OA = OC$ and $OD = OB$

$\Rightarrow O$ is the midpoint of AC and BD, i.e. the diagonals AC and BD bisect each other at O....(2)

(c) To prove that $AC \perp BD$

In $\Delta\, OBA$ and $\Delta\, ODA$, $OB = OD$ (proved)

$BA = DA$ (opposite sides of the square)

$OA = OA$ (common)

$\therefore \Delta\, OBA \cong \Delta\, ODA$ (SSS criteria)

$\Rightarrow$ Their corresponding parts are equal.

$\Rightarrow \angle AOB = \angle AOD$

And $\angle AOB$ and $\angle AOD$ form a linear pair.

$\angle AOB + \angle AOD = 180^o$

$\angle AOB = \angle AOD = 90^o$

$\Rightarrow AC \perp BD$...(3)

Therefore, from (1), (2) and (3), we get AC and BD are equal and bisect each other at right angles.

5. Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Solution:

Quadrilateral ABCD has O as the midpoint of AC and BD.

$\Rightarrow AC \perp BD$

In ΔAOD and ΔAOB, $AO = AO$ (Common)

$OD = OB$ ($\because$ O is the midpoint of BD)

$\angle AOD = \angle AOB$ (right angles)

$\therefore \Delta AOD \cong \Delta AOB$ (SAS criteria)

$\therefore$ Their corresponding parts are equal

$\Rightarrow AD = AB$...(1)

Similarly, $AB = BC$...(2)

$BC = CD$...(3)

$CD = DA$...(4)

From (1), (2), (3) and (4) we have: $AB = BC = CD = DA$

Therefore, Quadrilateral ABCD has all sides equal.

In ΔAOD and ΔCOB, $AO = CO$ (given)

$OD = OB$ (given)

$\angle AOD = \angle COB$ (vertically opposite angles)

$\therefore \Delta AOD \cong \Delta COB$ (SAS criteria)

$\Rightarrow$ Their corresponding parts are equal.

$\Rightarrow \angle 1 = \angle 2$ (form a pair of interior alternate angles)

$\therefore AD \parallel BC$

Similarly, $AB \parallel DC$ $\therefore$ ABCD is a parallelogram.

We know that parallelogram with all sides equal is a rhombus.

$\therefore$ ABCD is a rhombus.

In ΔABC and ΔBAD, $AC = BD$ (Given)

$BC = AD$ (as proved)

$AB = BA$ (Common)

$\therefore \Delta ABC \cong \Delta BAD$ (SSS criteria)

$\Rightarrow$ Their corresponding angles are equal.

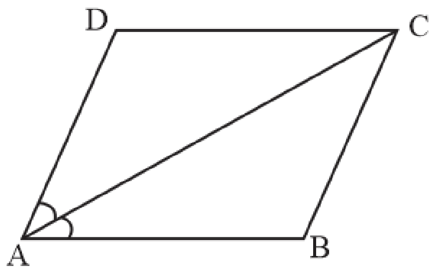
$\angle ABC = \angle BAD$

Now that $AD \parallel BC$ and AB is a transversal.

$\angle ABC + \angle BAD = 180^o$ (interior opposite angles are supplementary) i.e. The rhombus ABCD has an angle equal to 90^{o}.

Therefore, ABCD is a square.

6. Diagonal AC of a parallelogram ABCD bisects $\angle A$ (see Fig). Show that



(i) it bisects $\angle C$ also,

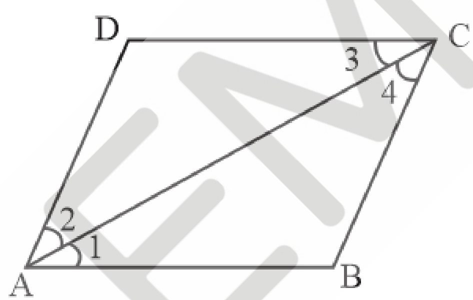
(ii) ABCD is a rhombus.

Solution:

In parallelogram ABCD, diagonal AC bisects $\angle A$.

$\Rightarrow \angle DAC = \angle BAC$

(i) To prove that AC bisects $\angle C$.



ABCD is a parallelogram,

$\Rightarrow AB \parallel DC$ and AC is a transversal.

$\therefore \angle 1 = \angle 3$ (interior alternate angles) ...(1)

$\Rightarrow BC \parallel AD$ and AC is a transversal.

$\therefore \angle 2 = \angle 4$ (interior alternate angles) ...(2)

We know, AC bisects $\angle A$. (given)

$\therefore \angle 1 = \angle 2$...(3)

From (1), (2) and (3), we get

$\angle 3 = \angle 4$

Therefore, this proves AC bisects ∠C.

(ii) To prove ABCD is a rhombus.

In Δ ABC, ∠1 = ∠4 [∵ ∠1 = ∠2 = ∠4]

⇒ BC = AB (sides opposite to equal angles) ...(4)

Similarly, AD = DC...(5)

Since it is given that ABCD is a parallelogram

AB = DC (opposite sides of a parallelogram]....(6)

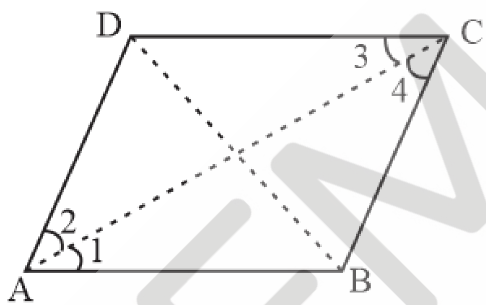
From (4), (5) and (6), we get AB = BC = CD = DA

Therefore, ABCD is a rhombus.

7. ABCD is a rhombus. Show that diagonal AC bisects ∠A as well as ∠C and diagonal BD bisects ∠B as well as ∠D

Solution:

ABCD is a rhombus.



∴ AB = BC = CD = AD

Then AB ∥ CD and AD ∥ BC

Now, AD = CD

⇒ ∠1 = ∠2.....(1)(angles opposite to equal sides are equal]

Also, CD ∥ AB (opposite sides of the parallelogram]

∴ ∠1 = ∠3……..(2)

And AC is transversal.

∴ ∠1 = ∠4……..(3)

From (1), (2) and (3), we have

∠2 = ∠3 and ∠1 = ∠4

This shows that AC bisects ∠C as well as ∠A.

Similarly, it is proved that BD bisects $\angle B$ as well as $\angle D$.

8. ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$. Show that:

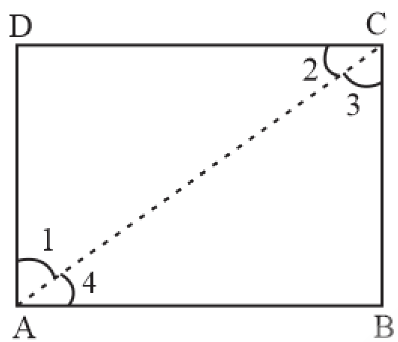
(i) ABCD is a square

(ii) Diagonal BD bisects $\angle B$ as well as $\angle D$.

Solution:

In rectangle ABCD, AC bisects $\angle A$ and $\angle C$.

$\Rightarrow \angle 1 = \angle 4$ and $\angle 2 = \angle 3$...(1)



(i) ABCD is a parallelogram.(rectangle is a parallelogram)

$\Rightarrow AB \parallel CD$ and AC is a transversal.

$\therefore \angle 2 = \angle 4$...(2) (alternate interior angles)

From (1) and (2), we get $\angle 3 = \angle 4$

$\Rightarrow AB = BC$ (sides opposite to equal angles in ΔABC are equal.)

$$AB = BC = CD = AD$$

$\Rightarrow$ ABCD is a rectangle having with all sides equal.

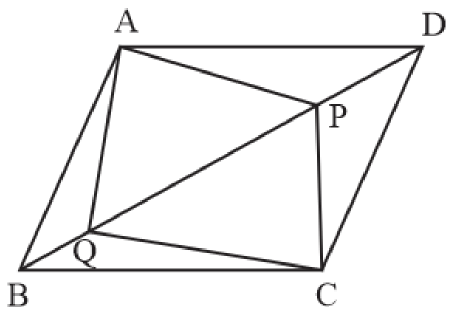
Therefore it is proved that ABCD is a square.

(ii) ABCD is a square, and diagonals of a square bisect the opposite angles.

$\therefore$ BD bisects $\angle B$ as well as $\angle D$

9. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ

(see fig) Show that:



(i) $\Delta APD \cong \Delta CQB$

(ii) $AP = CQ$

(iii) $\Delta AQB \cong \Delta CPD$

(iv) $AQ = CP$

(v) APCQ is a parallelogram

Solution:

(i) To prove that $\Delta\ APD \cong \Delta\ CQB$

AD ∥ BC and BD is a transversal. (∵ ABCD is a parallelogram)

∴ $\angle ADB = \angle CBD$ (interior alternate angles)

⇒ $\angle ADP = \angle CBQ$

In ΔAPD and ΔCQ,

$AD = CB$ (opposite sides of the parallelogram)

$DP = BQ$ (given)

$\angle CBQ = \angle ADP$ (already proved)

Therefore, $\Delta APD \cong \Delta CQB$ (SAS criteria)

(ii) To prove that $AP = CQ$

Since $\Delta APD \cong \Delta\ CQB$ (as proved)

∴ Their corresponding parts are equal.

Therefore $AP = CQ$

(iii) To prove that $\Delta AQB \cong \Delta CPD$.

BD is a transversal

⇒ AB ∥ CD (∵ ABCD is a parallelogram)

∴ $\angle ABD = \angle CDB$

$\Rightarrow \angle ABQ = \angle CDP$

So, in ΔAQB and ΔCPD, $QB = PD$ (given)

$\angle ABQ = \angle CDP$ (as proved)

$AB = CD$ (opposite sides of parallelogram ABCD)

Therefore $\Delta AQB \cong \Delta CPD$ (SAS criteria)

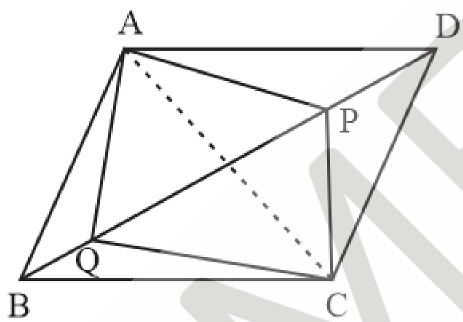
(iv) To prove that $AQ = CP$.

Now we have proved that $\Delta AQB \cong \Delta CPD$

$\therefore$ Their corresponding parts are equal.

$\Rightarrow$ this proves that $AQ = CP$.

(v) To prove that APCQ is a parallelogram.



By joining AC, the diagonals of a parallelogram bisect each other

$\Rightarrow AO = CO$

And $BO = DO$

$\Rightarrow (BO - BQ) = (DO - DP)$ ($\because BQ = DP$)

$\Rightarrow QO = PO$

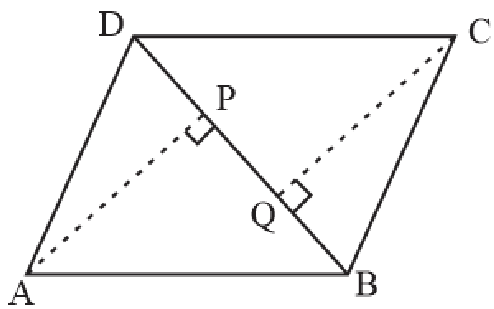
Now, in quadrilateral APCQ, we get

$AO = CO$ and $QO = PO$

$\Rightarrow$ AC and QP bisect each other at O.

Therefore, APCQ is a parallelogram.

10. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see Fig.). Show that



(i) $\Delta APB \cong \Delta CQD$

(ii) $AP = CQ$

Solution:

(i) To prove $\Delta APB \cong \Delta CQD$

In ΔAPB and ΔCQD, $\angle APB = \angle CQD$ (90^o each)

$AB = CD$ (opposite sides of parallelogram ABCD)

$\angle ABP = \angle CDQ$

Therefore, $\Delta APB \cong \Delta CQD$ (AAS criteria)

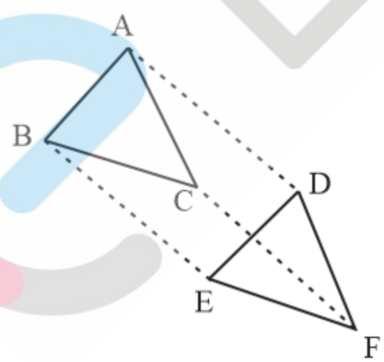
(ii) To prove $AP = CQ$

Now that $\Delta APB \cong \Delta CQD$

$\therefore$ Their corresponding parts are equal.

Therefore, $AP = CQ$

11. In ΔABC and ΔDEF, $AB = DE$, $AB \parallel DE$, $BC = EF$ and $BC \parallel EF$. Vertices A, B and C are joined to vertices D, E and F respectively (see Fig.) Show that



(i) quadrilateral ABED is a parallelogram

(ii) quadrilateral BEFC is a parallelogram

(iii) $AD \parallel CF$ and $AD = CF$

(iv) quadrilateral ACFD is a parallelogram

(v) AC = DF

(vi) ΔABC ≅ ΔDEF.

Solution:

(i) To prove that ABED is a parallelogram.

We know that "A quadrilateral is a parallelogram if a pair of opposite sides are of equal length."

Here, AB = DE (given)

AB ∥ DE (given)

Therefore, a pair of opposite sides of quadrilateral ABED is of equal length.

And this proves that ABED is a parallelogram.

To prove that ABED is a parallelogram.

(ii) To prove that BEFC is a parallelogram.

BC = EF (given)

And BC ∥ EF (given)

Therefore, BECF is a quadrilateral in which a pair of opposite sides (BC and EF) is parallel and of equal length.

This proves that BECF is a parallelogram.

(iii) To prove that AD ∥ CF and AD = CF

ABED is a parallelogram. (proved)

∴ Its opposite sides are parallel and equal.

⇒ AD ∥ BE and AD = BE ...(1)

Also BEFC is a parallelogram. (proved)

∴ BE ∥ CF and BE = CF...(2) (opposite sides of a parallelogram are parallel and equal)

From (1) and (2), it is proved AD ∥ CF and AD = CF

(iv) To prove that ACFD is a parallelogram.

AD ∥ CF (proved)

and AD = CF (proved)

Therefore, ACFD is aquadrilateral which has one pair of opposite sides (AD and CF) parallel and of equal length.

This proves that Quadrilateral ACFD is a parallelogram.

(v) To prove that $AC = DF$.

ACFD is a parallelogram(proved)

$\Rightarrow$ Its opposite sides are parallel and of equal length.

Therefore $AC = DF$

(vi) To prove that $\Delta ABC = \Delta DEF$

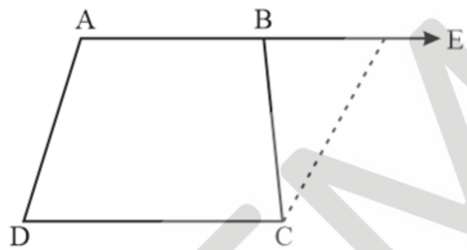
In ΔABC and ΔDEF, $AB = DE$ (opposite sides of a parallelogram)

$BC = EF$ (opposite sides of a parallelogram)

$AC = DF$ (proved)

Therefore $\Delta ABC \cong \Delta DEF$. (SSS criteria)

12. ABCD is a trapezium in which $AB \parallel CD$ and $AD = BC$ (see Fig.) Show that



(i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

(iii) $\Delta ABC \cong \Delta BAD$

(iv) diagonal AC = diagonal BD

Solution:

Given that $AB \parallel CD$ and $AD = BC$

(i) To prove that $\angle A = \angle B$.

Extend AB to E and draw $CE \parallel AD$.

$\therefore AB \parallel DC$

$\Rightarrow AE \parallel DC$ (given)

And $AD \parallel CE$

This shows that AECD is a parallelogram.

$\Rightarrow AD = CE$ (opposite sides of the parallelogram AECD)

But we know that $AD = BC$ (given)

$\therefore BC = CE$

In $\Delta BCE, BC = CE$

$\Rightarrow \angle CBE = \angle CEB \ldots(1)$ (angles opposite to equal sides of a triangle are equal)

Also, $\angle ABC + \angle CBE = 180^o \ldots(2)$ (linear pair)

and $\angle A + \angle CEB = 180^o \ldots(3)$(adjacent angles of a parallelogram are supplementary)

From (2) and (3), we get

$\angle ABC + \angle CBE = \angle A + \angle CEB$

We know, $\angle CBE = \angle CEB$

$\therefore \angle ABC = \angle A$

or $\angle B = \angle A$

i.e., $\angle A = \angle B$

(ii) To prove that $\angle C = \angle D$.

$AB \parallel CD$ and AD is a transversal.

$\angle A + \angle D = 180^o$ (sum of interior opposite angles is 180^o)

Similarly, $\angle B + \angle C = 180^o$

$\Rightarrow \angle A + \angle D = \angle B + \angle C$

We know, $\angle A = \angle B$ (as proved in (i)

Therefore $\angle C = \angle D$

(iii) To prove $\Delta ABC \cong \Delta BAD$

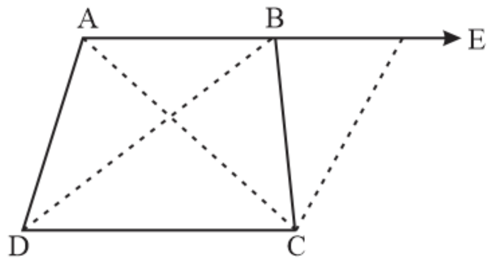
In ΔABC and $\Delta BAD, AB = BA$ (common side)

$BC = AD$ (given)

$\angle ABC = \angle BAD$ (proved)

Therefore $\Delta ABC \cong \Delta BAD$ (SAS criteria)

(iv) To prove that diagonal $AC =$ diagonal BD



$\Delta ABC \cong \Delta BAD$ (as proved in (iii))

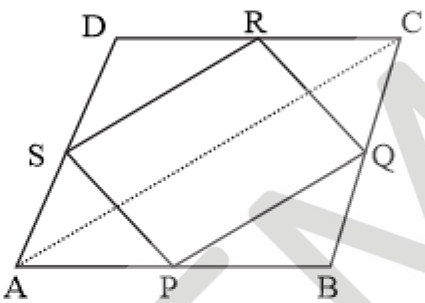
$\therefore$ Their corresponding parts are equal.

This proves that diagonal $AC =$ diagonal BD.

Hint: Extend AB and draw a line through C parallel to DA intersecting AB produced at E.

Exercise: 8.2

13. ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see Fig.) AC is a diagonal. Show that:



(i) $SR \parallel AC$ and $SR = \frac{1}{2}AC$

(ii) $PQ = SR$

(iii) PQRS is a parallelogram.

Solution:

P is the midpoint of AB, Q is the midpoint of BC

(i) To prove that $SR \parallel AC$ and $SR = \frac{1}{2}AC$ and $SR \parallel AC$

ABCD is a quadrilateral with R as the midpoint of CD, S as the mid-point of DA, and AC as the diagonal of a quadrilateral ABCD.

In ΔACD, S is the midpoint of AD,

R is the midpoint of CD.

$\Rightarrow$ The line segment joining the mid-point of any two sides of a triangle is parallel to the third side and half of it.

$SR = \frac{1}{2}AC$ and $SR \parallel AC$

(ii) To prove that $PQ = SR$.

In ΔABC, we have

P is the midpoint of AB

Q is the midpoint of BC.

$\therefore PQ = \frac{1}{2}AC \ldots\ldots(1)$

And, $SR = \frac{1}{2}AC \ldots\ldots..(2)$ (as proved in (i))

Therefore From (1) and (2), it is proved

$PQ = SR$

(iii) To prove that PQRS is a parallelogram.

In ΔABC, P is the midpoint of AB and Q is the midpoint of BC.

$\therefore PQ = \frac{1}{2}AC$ and $PQ \parallel AC \ldots\ldots(3)$

In ΔACD, S is the midpoint of DA and R is the midpoint of CD

$\therefore SR = \frac{1}{2}AC$ and $SR \parallel AC \ldots\ldots.(4)$

From (3) and (4), we get

$PQ = SR = \frac{1}{2}AC$ and $PQ \parallel AC \parallel SR$

$\Rightarrow PQ = SR$ and $PQ \parallel SR$

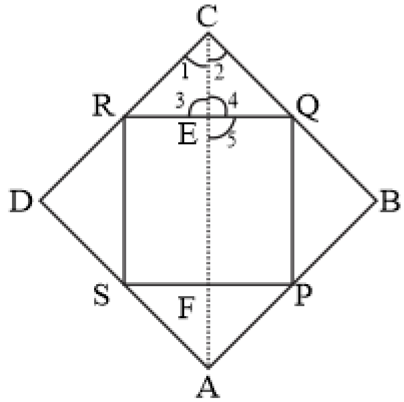
We know that one pair of opposite sides in a quadrilateral PQRS is equal and parallel.

Therefore, PQRS is a parallelogram.

14. ABCD is a rhombus and P, Q, R and S are the midpoints of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

Solution:

To prove that PQRS is a rectangle



In Rhombus ABCD, P, Q, R, and S are the mid points of AB, BC, CD, and DA.

By joining AC, In Δ ABC, P and Q are the mid-points of AB and BC.

$\therefore PQ = \frac{1}{2}AC$ and $PQ \parallel AC$……(1)

In ΔADC, R and S are the mid-points of CD and DA.

$\therefore SR = \frac{1}{2}AC$ and $SR \parallel AC$……(2)

From (1) and (2), we get

$PQ = \frac{1}{2}AC = SR$ and $PQ \parallel AC \parallel SR$

$\Rightarrow PQ = SR$ and $PQ \parallel SR$

We know that, one pair of opposite sides of quadrilateral PQRS is equal and parallel.

Therefore PQRS is a parallelogram.

Now, in ΔERC and ΔEQC,

$\angle 1 = \angle 2$ (the diagonal of a rhombus bisects the opposite angles)

$CR = CQ$ (each is equal to $\frac{1}{2}$ of a side of rhombus)

$CE = CE$ (common)

$\Rightarrow \Delta ERC \cong \Delta EQC$ (SAS criteria)

So we have $\angle 3 = \angle 4$ (corresponding parts of congruent triangles)

But $\angle 3 + \angle 4 = 180^o$ (linear pair)

$\Rightarrow \angle 3 = \angle 4 = 90^o$

But $\angle 5 = \angle 3$ (vertically opposite angles)

$\angle 5 = 90^o$

This gives $PQ \parallel AC \Rightarrow PQ \parallel EF$

$\therefore$ PQEF is a quadrilateral having a pair of opposite sides parallel and one of the angles is 90^o.

Therefore PQEF is a rectangle.

$\Rightarrow \angle RQP = 90^o$

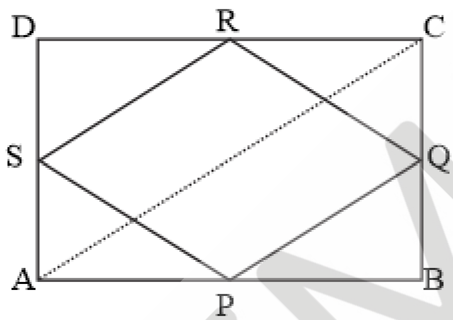
Now that one angle of parallelogram PQRS is 90^o.

It is proved that PQRS is a rectangle.

15. ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Solution:

ABCD is a rectangle which has P, Q, R and S as the midpoints of AB, BC, CD and DA, AC is the diagonal.



To prove that PQRS is a rhombus

By joining AC, in ΔABC, $PQ = \frac{1}{2}AC$ and $PQ \parallel AC$...(1) (midpoint theorem)

And in ΔACD, $SR = \frac{1}{2}AC$ and $SR \parallel AC$...(2) (midpoint theorem)

From (1) and (2), we get

$PQ = SR$ and $PQ \parallel SR$

Similarly, by joining BD, we'll get

$PS = QR$ and $PS \parallel QR$

$\Rightarrow$ Both pairs of opposite sides of quadrilateral PQRS are equal and parallel.

$\therefore$ PQRS is a parallelogram.

Now, in ΔPAS and ΔPBQ,

$\angle A = \angle B$ (each angle $= 90^o$)

$AP = BP$ (each side $= \frac{1}{2}AB$)

$AS = BQ$ (each side $= \frac{1}{2}$ of opposite sides of a rectangle)

$\Rightarrow \Delta PAS \cong \Delta PBQ$ (SAS criteria)

$\therefore$ Their corresponding parts are equal.

$\Rightarrow PS = PQ$

$PS = QR$ (proved already)

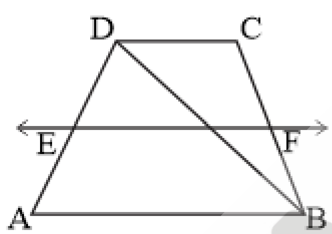
And $PQ = SR$ (proved already)

So, $PQ = QR = RS = SP$

i.e., PQRS is a parallelogram having all sides equal.

This proves that PQRS is a rhombus.

16. ABCD is a trapezium in which $AB \parallel DC$, BD is a diagonal and E is the midpoint of AD. A line is drawn through E parallel to AB intersecting BC at F (see Fig). Show that F is the midpoint of BC.



Solution:

In trapezium ABCD, $AB \parallel DC$. E is the midpoint of AD. EF is drawn parallel to AB.

To prove that F is the midpoint of BC.

Joining BD, In ΔDAB, E is the midpoint of AD

$EG \parallel AB$

So we get that G is the midpoint BD (converse of midpoint theorem)

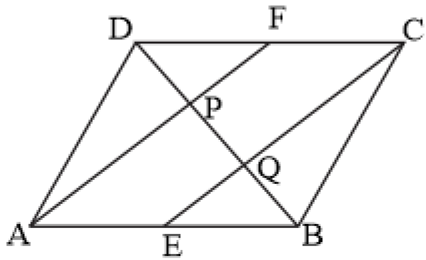
Now in ΔBDC, since G is the midpoint of BD (as proved already)

$GF \parallel DC$ ($\because AB \parallel DC$ and $EF \parallel AB$ and GF is a part of EF)

Therefore, by the converse of the mid-point theorem,

It is proved that F is the midpoint of BC

17. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see Fig) Show that the line segments AF and EC trisect the diagonal BD.



Solution:

ABCD is a parallelogram with E and F as the midpoints of AB and CD.

To prove that the line segments AF and EC trisect the diagonal BD.

By joining the opposite vertices B and D.

We know that the opposite sides of a parallelogram are parallel and equal.

$\therefore$ AB $\parallel$ DC $\Rightarrow$ AE $\parallel$ FC...(1)

And also AB = DC

i.e., $\frac{1}{2}AB = \frac{1}{2}DC \Rightarrow AE = FC$...(2)

From (1) and (2), we get that AECF is a quadrilateral with a pair of opposite sides parallel and equal.

$\therefore$ we can say AEFC is a parallelogram.

$\Rightarrow$ AE $\parallel$ CF

Now, in ΔDBC, F is the midpoint of DC (given)

FP $\parallel$ CQ (since AF $\parallel$ CE)

$\Rightarrow$ P is the mid-point of DQ (converse of midpoint theorem)

$\Rightarrow$ DP = PQ...(3)

Similarly, in ΔBAP, BQ = PQ ...(4)

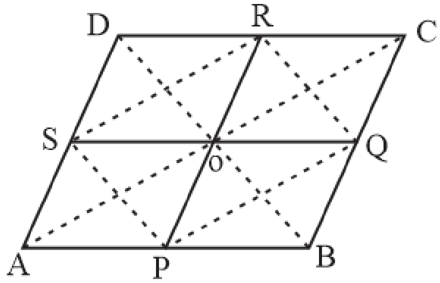
Therefore from (3) and (4), we have DP = PQ = BQ

This proves that the line segments AF and EC trisect the diagonal BD.

18. Show that the line segments joining the midpoints of the opposite sides of a quadrilateral bisect each other.

Solution:

ABCD is a quadrilateral with P, Q, R and S as the midpoints of AB, BC, CD and DA respectively,



To prove that the diagonals of PQRS bisect the sides at O.

By joining PQ, QR, RS, SP and also PR and SQ,

In Δ ABC, P and Q are the midpoints of its sides AB and BC respectively.

∴ PQ ∥ AC and $PQ = \frac{1}{2}AC$

Similarly, RS ∥ AC and $RS = \frac{1}{2}AC$

⇒ PQRS is a quadrilateral with PQ = RS

and PQ ∥ RS

∴ PQRS is a parallelogram.

Now we know that the diagonals of a parallelogram bisect each other.

i.e., PR and SQ bisect each other.

Therefore, it is proved that the line segments joining the mid-points of opposite sides of a quadrilateral ABCD bisect each other.

19. ABC is a triangle right angled at C. A line through the midpoint M of hypotenuse AB and parallel to BC intersects AC at D. Show that

(i) D is the midpoint of AC

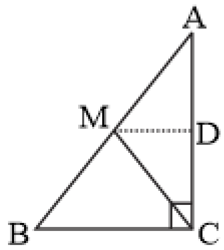
(ii) MD ⊥ AC

(iv) $CM = MA = \frac{1}{2}AB$

Solution:

ABC is a triangle, such that $\angle C = 90^o$

M is the mid-point of AB and MD ∥ BC.



(i) To prove that D is the midpoint of AC.

In ΔABC, M is the midpoint of AB (given)

$MD \parallel BC$ (given)

Therefore, it is proved that D is the midpoint of AC. (by converse of midpoint theorem)

(ii) To prove that $MD \perp AC$.

$MD \parallel BC$ (given)

and AC is a transversal.

$\angle MDA = \angle BCA$ (corresponding angles)

We know that $\angle BCA = 90^o$ (given)

$\Rightarrow \angle MDA = 90^o$

Therefore, it is proved $MD \perp AC$.

(iii) To prove that $CM = MA = \frac{1}{2}AB$

In ΔADM and ΔCDM, $\angle ADM = \angle CDM$ (each angle $= 90^o$)

$MD = MD$ (common)

$AD = CD$ ($\because$ it is proved that M is the midpoint of AC)

$\therefore \Delta ADM \cong \Delta CDM$ (SAS criteria)

$\therefore$ Their corresponding parts are equal

$\Rightarrow MA = MC$...(1)

$\because$ M is the midpoint AB. (given)

$\therefore$ we get $MA = \frac{1}{2}AB$...(2)

Therefore, from (1) and (2)

It is proved that $CM = MA = \frac{1}{2}AB$