CBSE NCERT Solutions for Class 12 Physics Chapter 7

Back of Chapter Questions

7.1. A 100 Ω resistor is connected to a 220 V, 50 Hz ac supply. (a) What is the rms value of current in the circuit? (b) What is the net power consumed over a full cycle?

Solution:

Given,

The rms voltage, $V_{\text{rms}} = 220\text{ V}$

Resistance, $R = 100\ \Omega$

Frequency, $f = 50\text{ Hz}$

(a) As the circuit is purely resistive,

rms current, $I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{220\text{ V}}{100\ \Omega} = 2.2\text{ A}$

(b) We know, power for a purely resistive circuit,

$P = I_{\text{rms}}^2 \text{R} = (2.2\text{ A})^2 \times 100\ \Omega = 484\text{ W}$

The net power consumed over a full cycle is 484 W.

7.2. (a) The peak voltage of an ac supply is 300 V. What is the rms voltage?

(b) The rms value of current in an ac circuit is 10 A. What is the peak current?

Solution:

(a) Given,

Peak voltage, $V_{\text{m}} = 300\text{ V}$

We know,

rms voltage, $V_{\text{rms}} = \frac{V_{\text{m}}}{\sqrt{2}} = \frac{300\text{ V}}{\sqrt{2}} = 212.1\text{ V}$

(b) Given,

rms current, $I_{\text{rms}} = 10\text{ A}$

We know,

$$I_{\text{rms}} = \frac{I_{\text{m}}}{\sqrt{2}}$$

Hence, the peak current-

$$I_m = I_{rms} \times \sqrt{2} = 10\text{ A} \times \sqrt{2} = 14.1\text{ A}$$

7.3. A 44 mH inductor is connected to 220 V, 50 Hz ac supply. Determine the rms value of the current in the circuit.

Solution:

Given,

Inductance, $L = 44\text{ mH}$

Frequency, $f = 50\text{ Hz}$

rms voltage, $V = 220\text{ V}$

for a purely inductive circuit,

rms current, $I_{rms} = \frac{V_{rms}}{X_L}$

Where inductive reactance,

$X_L = 2\pi f L$ $\quad$ L: Inductance

$X_L = 2\pi \times 50\text{ Hz} \times 44\text{ mH} = 13.8\ \Omega$

The rms value of the current,

$$I_{rms} = \frac{220\text{ V}}{13.8\ \Omega} = 15.9\text{ A}$$

7.4. A 60 μF capacitor is connected to a 110 V, 60 Hz ac supply. Determine the rms value of the current in the circuit.

Solution:

Given,

Capacitance, $C = 60\ \mu\text{F}$

Frequency, $f = 60\text{ Hz}$

rms Voltage, $V = 110\text{ V}$

As for a purely inductive circuit,

rms current, $I_{rms} = \frac{V}{X_C}$

Where capacitive reactance-

$X_C = \frac{1}{2\pi f C}$ $\quad$ C: Capacitance

$$X_C = \frac{1}{2\pi \times 60\ \mu\text{F} \times 60\text{ Hz}} = 44.2\ \Omega$$

The rms value of the current,

$$I = \frac{110\text{ V}}{44.2\ \Omega} = 2.49\text{ A}$$

7.5. In Exercises 7.3 and 7.4, what is the net power absorbed by each circuit over a complete cycle. Explain your answer.

Solution:

In the inductive circuit,

rms current, $I_{\text{rms}} = 15.92\text{ A}$

rms voltage, $V_{\text{rms}} = 220\text{ V}$

Hence, the net power absorbed can be given by the relation,

$P = V_{\text{rms}} I_{\text{rms}} \cos\phi$

Where ϕ = Phase difference between V_{rms} and I_{rms}.

For a purely inductive circuit, the phase difference between alternating voltage and current is 90° i.e., $\phi = 90°$. Hence, $P = 0$ i.e., the net power is 0.

In the capacitive circuit,

rms current, $I_{\text{rms}} = 2.49\text{ A}$

rms voltage, $V_{\text{rms}} = 110\text{ V}$

Hence, the net power absorbed can be obtained as:

$P = V_{\text{rms}} I_{\text{rms}} \cos\phi$

For a purely capacitive circuit, the phase difference between alternating voltage and current is 90° i.e., $\phi = 90°$. Hence, $P = 0$ i.e., the net power is 0.

7.6. Obtain the resonant frequency ω_r of a series LCR circuit with $L = 2.0\text{H}$, $C = 32\ \mu\text{F}$ and $R = 10\ \Omega$. What is the Q-value of this circuit?

Solution:

Given,

Inductance, $L = 2.0\text{H}$

Capacitance, $C = 32\ \mu\text{F}$

Resistance, $R = 10\ \Omega$.

We know,

The resonating frequency,

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2\text{ H} \times 32\ \mu\text{F}}} = 125\ \text{s}^{-1}$$

Q-value of the circuit,

$$Q = \frac{\omega_r L}{R} = \frac{125\ \text{s}^{-1} \times 2\text{ H}}{10\ \Omega} = 25$$

7.7. A charged 30 μF capacitor is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit?

Solution:

Given,

Capacitance, $C = 30\ \mu\text{F}$

Inductance, $L = 27\text{ mH}$

The angular frequency of free oscillations,

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$\omega_r = \frac{1}{\sqrt{27\text{ mH} \times 30\ \mu\text{F}}} = 1.1 \times 10^3\ \text{rad s}^{-1}$$

7.8. Suppose the initial charge on the capacitor in Exercise 7.7 is 6 mC. What is the total energy stored in the circuit initially? What is the total energy at a later time?

Solution:

Given,

Charge on the capacitor, $\text{Q} = 6\text{ mC}$

Capacitance, $C = 30\ \mu\text{F}$

Total energy stored by a capacitive circuit having charge Q on the capacitor with capacitance C is given by,

$$E = \frac{\text{Q}^2}{2C}$$

$$E = \frac{(6\text{ mC})^2}{2 \times 30\ \mu\text{F}} = 0.6\text{ J}$$

In the case of LC oscillations, the total energy of the circuit remains constant over time. The energy in the system oscillates between the capacitor and the inductor, but their sum or the total energy is constant over time.

7.9. A series LCR circuit with $R = 20\ \Omega$, $L = 1.5\text{ H}$ and $C = 35\ \mu\text{F}$ is connected to a variable-frequency 200 V ac supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?

Solution:

Given,

Voltage, $V = 200$ V

Resistance, $R = 20\ \Omega$

Inductance, $L = 1.5$ H

Capacitance, $C = 35\ \mu$F

The power of a series LCR circuit is given by,

$P = I^2 Z \cos\phi$

Where, $Z = \sqrt{(X_L - X_C)^2 + R^2}$

$\cos\phi = \frac{R}{Z}$

When the frequency of the supply equals the natural frequency, it is the case of resonance in the circuit. At resonance, $X_C - X_L = 0$, $Z = R$ and $\phi = 0$. Therefore,

$\cos\phi = 1$ and $P = I^2 Z = I^2 R$.

That is, maximum power is dissipated in a circuit (through R) at resonance.

So, the average power transferred to the circuit in one complete cycle,

$$P = \frac{V^2}{R} = \frac{(200\text{ V})^2}{20\ \Omega} = 2000\text{ W}$$

7.10. A radio can tune over the frequency range of a portion of MW broadcast band: (800 kHz to 1200 kHz). If its LC circuit has an effective inductance of 200 μH, what must be the range of its variable capacitor? [Hint: For tuning, the natural frequency i.e., the frequency of free oscillations of the LC circuit should be equal to the frequency of the radiowave.]

Solution:

Given,

Inductance, $L = 200\ \mu$H

Range of frequency = 800 kHz to 1200 kHz

The frequency of the LC oscillation,

$$\omega_r = \frac{1}{\sqrt{LC}}$$

The maximum value of capacitance, i.e., at $f = 800$ kHz

$$C = \frac{1}{(2\pi f)^2 L} = \frac{1}{(2\pi \times 800\ \text{kHz})^2 \times 200\ \mu\text{H}} = 197.8\ \text{pF}$$

The minimum value of capacitance, i.e., at $f = 1200$ kHz

$$C = \frac{1}{(2\pi f)^2 L} = \frac{1}{(2\pi \times 1200\ \text{kHz})^2 \times 200\ \mu\text{H}} = 87.9\ \text{pF}$$

The value of capacitance must be in the range of 87.9 pF to 197.8 pF.

7.11. Figure 7.21 shows a series LCR circuit connected to a variable frequency 230 V source. $L = 5$ H, $C = 80\ \mu$F, $R = 40\ \Omega$.

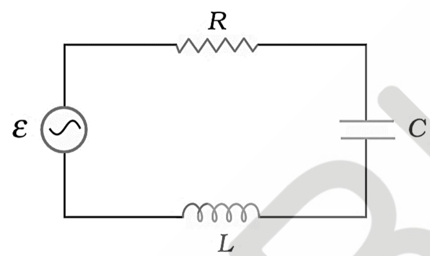

Figure 7.21

(a) Determine the source frequency which drives the circuit in resonance.

(b) Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.

(c) Determine the rms potential drops across the three elements of the circuit. Show that the potential drop across the LC combination is zero at the resonating frequency.

Solution:

Given,

Inductance, $L = 5$ H

Capacitance, $C = 80\ \mu$F

Resistance, $R = 40\ \Omega$

At resonance,

$$\omega_r = \frac{1}{\sqrt{LC}}$$

(a) Therefore, the resonant frequency

$$\omega_r = \frac{1}{\sqrt{5\ \text{H} \times 80\ \mu\text{F}}} = 50\ \text{rad s}^{-1}$$

(b) The impedance of the circuit,

$$Z = \sqrt{(X_L - X_C)^2 + R^2}$$

At resonating frequency, $X_C - X_L = 0$ and $Z = R$

Therefore, $Z = 40\ \Omega$

At resonance,

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}$$

$$= \frac{230\ \text{V}}{40\ \Omega} = 5.75\ \text{A}$$

$$I_m = \sqrt{2} \times I_{\text{rms}} = \sqrt{2} \times 5.75\ \text{A} = 8.1\ \text{A}$$

(c) Potential drop across the inductor,

$$V_{L\text{rms}} = I_{\text{rms}}\omega_r L = 5.75 \times 50 \times 5 = 1437.5\ \text{V}$$

Potential drop across the capacitor,

$$V_{C\text{rms}} = \frac{I_{\text{rms}}}{\omega_r C} = \frac{5.75}{50 \times 80 \times 10^{-6}} = 1437.5\ \text{V}$$

Potential drop across the resistor,

$$V_{R\text{rms}} = I_{\text{rms}}R = 5.75 \times 40 = 230\ \text{V}$$

Potential drop across the LC circuit,

$$V_{LC\text{rms}} = V_{L\text{rms}} - V_{C\text{rms}} = 0$$

Hence, the potential drop across the LC combination is zero at the resonating frequency.

Additional Exercises:

7.12. An LC circuit contains a 20 mH inductor and a 50 μF capacitor with an initial charge of 10 mC. The resistance of the circuit is negligible. Let the instant the circuit is closed be $t = 0$.

(a) What is the total energy stored initially? Is it conserved during LC oscillations?

(b) What is the natural frequency of the circuit?

(c) At what time is the energy stored

(i) completely electrical (i.e., stored in the capacitor)?

(ii) completely magnetic (i.e., stored in the inductor)?

(d) At what times is the total energy shared equally between the inductor and the capacitor?

(e) If a resistor is inserted in the circuit, how much energy is eventually dissipated as heat?

Solution:

Given,

Charge on the capacitor, $Q = 10$ mC,

Inductance, $L = 20$ mH

Capacitance, $C = 50$ μF

(a) The total energy stored initially at t = 0,

$$E = \frac{Q^2}{2C}$$

$$E = \frac{(10\text{ mC})^2}{2 \times 50\ \mu\text{F}} = 1\text{ J}$$

Yes, total energy will remain conserved during the oscillation.

(b) The natural frequency of the circuit,

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$\omega_r = \frac{1}{\sqrt{20\text{ mH} \times 50\ \mu\text{F}}} = 10^3\text{ rad s}^{-1}$$

$$f_r = \frac{\omega_r}{2\pi} = \frac{10^3}{2\pi}\text{ rad s}^{-1} = 159\text{ s}^{-1}$$

(c) (i) For time period ($T = \frac{1}{f} = \frac{1}{159} = 6.28$ ms), total charge on the capacitor at time t,

$$Q' = Q\cos\left(\frac{2\pi t}{T}\right)$$

For energy stored in electrical, we can write Q′ = Q.

Hence, it can be inferred that the energy stored in the capacitor is completely electrical at time, $t = 0, \frac{T}{2}, T, \frac{3T}{2}, \ldots \ldots$

(ii) Magnetic energy is maximum when electrical energy is minimum. Hence, it can be inferred that the energy stored in the circuit is completely magnetic at time $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4},$.

(d) The total energy is equally shared between the inductor and the capacitor. So, the energy stored in the capacitor is-

$$\Rightarrow \frac{Q'^2}{2C} = \frac{1}{2} \times \frac{Q^2}{2C}$$

$$\Rightarrow Q' = \frac{Q}{\sqrt{2}}$$

Also,

$$Q' = Q\cos\left(\frac{2\pi t}{T}\right) = \frac{Q}{\sqrt{2}}$$

$$\Rightarrow \quad \cos\left(\frac{2\pi t}{T}\right) = \frac{1}{\sqrt{2}} = \cos(2n+1)\frac{\pi}{4}$$

where n = 0 , 1 , 2…..

$$\Rightarrow \quad t = (2n+1)\left(\frac{\pi}{8}\right)$$

Hence, total energy is equally shared between the inductor and the capacitor at time,

$$t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8},$$

(e) Resistor damps out the LC oscillations. The whole of the initial energy 1 J, is eventually dissipated as heat.

7.13. A coil of inductance 0.50 H and resistance 100 Ω is connected to a 240 V, 50 Hz ac supply.

(a) What is the maximum current in the coil?

(b What is the time lag between the voltage maximum and the current maximum?

Solution:

Given,

Inductance, $L = 0.50$ H

Resistance, $R = 100\ \Omega$

rms voltage, $V_{\text{rms}} = 240$ V

frequency, $f = 50$ Hz

(a) The rms current in the coil,

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{\sqrt{X_L^2 + R^2}}$$

For an inductive circuit,

$X_L = 2\pi \times 50\text{ Hz} \times 0.5\text{ H} = 157\ \Omega$

$$I_{\text{rms}} = \frac{240\text{ V}}{\sqrt{(157\ \Omega)^2 + (100\ \Omega)^2}} = \frac{240\text{ V}}{186.1\ \Omega} = 1.2\text{ A}$$

So, The maximum current in the coil

$I_m = \sqrt{2} \times I_{\text{rms}} = \sqrt{2} \times 1.2\text{ A} = 1.82\text{ A}$

(b) In LR circuit,

If, $V = V_m \cos \omega t$ then,

$I = I_m \cos(\omega t - \phi)$

Voltage is maximum at $t = 0$ whereas, the current is maximum at $= \frac{\phi}{\omega}$,

Therefore, the time lag between the maximum voltage and maximum current $= \frac{\phi}{\omega}$.

Also, $\tan \phi = \frac{\omega L}{R} = \frac{2\pi \times 50\text{ Hz} \times 0.5\text{ H}}{100\ \Omega} = 1.57$

$$\phi = \tan^{-1} 1.57 = 57.5° = \frac{57.5\pi}{180}$$

Timelag $= \frac{\phi}{\omega} = \frac{57.5\pi}{180 \times 2\pi \times 50} = 3.2$ ms.

7.14. Obtain the answers (a) to (b) in Exercise 7.13 if the circuit is connected to a high frequency supply (240 V,10 kHz). Hence, explain the statement that at very high frequency, an inductor in a circuit nearly amounts to an open circuit. How does an inductor behave in a dc circuit after the steady state?

Solution:

Given,

Inductance, $L = 0.50$ H

Resistance, $R = 100\ \Omega$

rms voltage, $V_{\text{rms}} = 240$ V

frequency, $f = 10$ kHz

(a) The maximum current in the coil,

$$I_{\text{m}} = \frac{\sqrt{2} \times V_{\text{rms}}}{\sqrt{X_L^2 + R^2}}$$

For an inductive circuit,

$X_L = 2\pi \times 10 \text{ kHz} \times 0.5 \text{ H} = 31415.9\ \Omega$

So, The maximum current in the coil,

$$I_m = \frac{\sqrt{2} \times 240 \text{ V}}{\sqrt{(31415.9\ \Omega)^2 + (100\ \Omega)^2}} = 1.1 \times 10^{-2} \text{ A}$$

(b) $\tan\phi = \frac{\omega L}{R} = \frac{2\pi \times 10^4 \times 0.5}{100} = 100\pi$

$$\phi = \tan^{-1} 100\pi = 89.82° = \frac{89.82\pi}{180}$$

$$\text{Timelag} = \frac{\phi}{\omega} = \frac{89.82\pi}{180 \times 2\pi \times 50} = 25\ \mu\text{s}.$$

I_m in this case, is too small, so it can be concluded that at high frequencies an inductor behaves as the open circuit.

In a steady dc circuit $\omega = 0$, so inductor acts as a simple conductor.

7.15. A 100 μF capacitor in series with a 40 Ω resistance is connected to a110 V, 60 Hz supply.

(a) What is the maximum current in the circuit?

(b) What is the time lag between the current maximum and the voltage maximum?

Solution:

Given,

Capacitance, $C = 100\ \mu\text{F}$

Resistance, $R = 40\ \Omega$,

rms voltage, $V_{rms} = 110 \text{ V}$

frequency, $f = 60 \text{ Hz}$

(a) For a capacitive circuit, the maximum current-

$$I_m = \frac{\sqrt{2} \times V_{rms}}{\sqrt{X_C^2 + R^2}}$$

$$I_m = \frac{\sqrt{2} \times 110 \text{ V}}{\sqrt{\left(\frac{1}{2\pi \times 60 \text{ Hz} \times 100\ \mu\text{F}}\ \Omega\right)^2 + (40\ \Omega)^2}} = 3.23 \text{ A}$$

(b) In RC circuit,

If, $I = I_m \cos \omega t$ then,

$V = V_m \cos(\omega t - \phi)$

current is maximum at $t = 0$ whereas, voltage is maximum at $t = \frac{\phi}{\omega}$,

Therefore, the time lag between the maximum voltage and maximum current $= \frac{\phi}{\omega}$

Also, $\tan\phi = \frac{1}{\omega CR} = \frac{1}{2\pi \times 60\text{ Hz} \times 100\ \mu\text{F} \times 100\ \Omega} = 0.66$

$\phi = \tan^{-1} 0.66 = 33.56° = \frac{33.56\pi}{180}$

time lag $= \frac{\phi}{\omega} = \frac{33.56\pi}{180 \times 2\pi \times 60} = 1.55$ ms.

7.16. Obtain the answers to (a) and (b) in Exercise 7.15 if the circuit is connected to a 110 V, 12 kHz supply? Hence, explain the statement that a capacitor is a conductor at very high frequencies. Compare this behaviour with that of a capacitor in a dc circuit after the steady state.

Solution:

Given,

Capacitance, $C = 100\ \mu$F

Resistance, $R = 40\ \Omega$,

rms voltage, $V_{\text{rms}} = 110$ V

frequency, $f = 12$ kHz

(a) For a capacitive circuit, the maximum current-

$$I_{\text{m}} = \frac{\sqrt{2} \times V_{\text{rms}}}{\sqrt{X_C^2 + R^2}}$$

$$I_{\text{m}} = \frac{\sqrt{2} \times 110\text{ V}}{\sqrt{\left(\frac{1}{2\pi \times 12\text{ kHz} \times 100\ \mu\text{F}}\ \Omega\right)^2 + (40\ \Omega)^2}} = 3.88\text{ A}$$

(b) $\tan\phi = \frac{1}{\omega CR} = \frac{1}{2\pi \times 12\text{ kHz} \times 100\ \mu\text{F} \times 100\ \Omega} = \frac{1}{96\pi}$

$$\phi = \tan^{-1}\frac{1}{96\pi} = 0.2° = \frac{0.2\pi}{180}\text{ rad}$$

Timelag $= \frac{\phi}{\omega} = \frac{0.2\pi}{180 \times 2\pi \times 12 \times 10^3} = 0.04\ \mu$s.

Hence, ϕ tends to become 0 at high frequencies. At a high frequency, capacitor C acts as a conductor. In a dc circuit, after the steady state is achieved, $\omega = 0$. Hence, capacitor C amounts to an open circuit.

7.17. Keeping the source frequency equal to the resonating frequency of the series LCR circuit, if the three elements, L, C, and R are arranged in parallel, show that the total current in the parallel LCR circuit is minimum at this frequency. Obtain the current rms value in each branch of the circuit for the elements and source specified in Exercise 7.11 for this frequency.

Solution:

Given,

rms voltage, $V = 230$ V

Inductance, $L = 5$ H

Resistance, $R = 40\ \Omega$

Capacitance, $C = 80\ \mu$F

As the circuit elements are arranged in parallel, the impedance of the circuit is given by,

$$\frac{1}{Z^2} = \frac{1}{X_L^2} + \frac{1}{X_C^2}$$

Also, source frequency is equal to the resonance frequency-

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5\text{ H} \times 80\ \mu\text{F}}} = 50\text{ rad s}^{-1}$$

As the value of Z is maximum at this frequency. Hence, the total current will be minimum.

The rms current in the inductor,

$$I_L = \frac{V}{\omega L} = \frac{230}{50 \times 5} = 0.92\text{ A}$$

The rms current in the capacitor,

$$I_L = V\omega C = 230 \times 50 \times 80 \times 10^{-6} = 0.92\text{ A}$$

The rms current in the resistor,

$$I_R = \frac{V}{R} = \frac{230}{40} = 5.75\text{ A}$$

7.18. A circuit containing a 80 mH inductor and a 60 μF capacitor in series is connected to a 230 V, 50 Hz supply. The resistance of the circuit is negligible.

(a) Obtain the current amplitude and rms values.

(b) Obtain the rms values of potential drops across each element.

(c) What is the average power transferred to the inductor?

(d) What is the average power transferred to the capacitor?

(e) What is the total average power absorbed by the circuit? ['Average' implies 'averaged over one cycle'.]

Solution:

Here,

Inductance, $L = 80$ mH

Capacitance, $C = 60$ μF

rms voltage, $V = 230$ V

Frequency, $f = 50$ Hz

(a) $\omega = 2\pi f = 2\pi \times 50 \text{ Hz} = 100\pi \text{ rads}^{-1}$

current amplitude,

$$I_m = \sqrt{2} \times \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}\sqrt{2}}{X_L - X_C} = \frac{230 \times \sqrt{2}}{\left(100\pi \times 80 \times 10^{-3} - \frac{1}{100\pi \times 60 \times 10^{-6}}\right)}$$

$= -11.6$ A

rms current,

$$I_{\text{rms}} = \frac{I_{\text{m}}}{\sqrt{2}} = \frac{-11.63}{\sqrt{2}} = -8.24 \text{ A}$$

(b) rms value of potential drop across the inductor,

$$V_{L\text{rms}} = I_{\text{rms}} \times X_L = 8.24 \times 100\pi \times 80 \times 10^{-3} = 207 \text{ V}$$

rms voltage across the inductor,

$$V_{C\text{rms}} = I_{\text{rms}} \times X_C = 8.24 \times \frac{1}{100\pi \times 60 \times 10^{-6}} = 437 \text{ V}$$

(c) As the phase difference between voltage and current through the inductor is $\frac{\pi}{2}$, the average power transferred over a complete cycle by the source to inductor will always be zero.

$P_{\text{I,avg}} = 0.$

(d) For C, voltage lags by $\pi/2$. As the phase difference between voltage and current through the capacitor is $\pi/2$, the average power transferred over a complete cycle by the source to inductor will always be zero.

$P_{\text{I,avg}} = 0.$

(e) As there is no average power transfer over a complete cycle by the source to the circuit elements, so the average power absorbed by the circuit is zero.

7.19. Suppose the circuit in Exercise 7.18 has a resistance of 15 Ω. Obtain the average power transferred to each element of the circuit, and the total power absorbed.

Solution:

Here,

Inductance, $L = 80$ mH

Capacitance, $C = 60$ μF

rms voltage $V = 230$ V

Frequency, $f = 50$ Hz

$\omega = 2\pi f = 2\pi \times 50 \text{ Hz} = 100\pi \text{ rads}^{-1}$

Resistance, $R = 15\ \Omega$

The impedance of the circuit –

$$Z = \sqrt{(X_L - X_C)^2 + R^2}$$

$$= \sqrt{\left(100\pi \times 80 \times 10^{-3} - \frac{1}{100\pi \times 60 \times 10^{-6}}\right)^2 + 15^2} = 31.728\ \Omega$$

Current following in the circuit-

$$I_{\text{rms}} = \frac{V}{Z} = \frac{230 \text{ V}}{31.72\ \Omega} = 7.25 \text{ A}$$

The average power transferred to the resistor,

$$P_R = I_{\text{rms}}^2 R = 7.25^2 \times 15 = 791 \text{ W}$$

The average power transferred to the resistor and capacitor, $P_C = P_I = 0$

As the average power transferred to inductor and capacitor is zero in one complete cycle. Hence, the total transferred is 791 W.

7.20. A series LCR circuit with $L = 0.12$ H, $C = 480$ nF, $R = 23\ \Omega$ is connected to a 230 V variable frequency supply.

(a) What is the source frequency for which current amplitude is maximum? Obtain this maximum value.

(b) What is the source frequency for which average power absorbed by the circuit is maximum? Obtain the value of this maximum power.

(c) For which frequencies of the source is the power transferred to the circuit half the power at resonant frequency? What is the current amplitude at these frequencies?

(d) What is the Q − factor of the given circuit?

Solution:

Given,

Inductance, $L = 0.12$ H

Capacitance, $C = 480$ nF

rms voltage, $V = 230$ V

Resistance, $R = 23\ \Omega$

(a) At resonance the impedance of the circuit will be minimum, resulting in maximum current in the circuit. This frequency is given by-

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.12 \times 480 \times 10^{-9}}} = 4167 \text{ rad s}^{-1}$$

$$f_r = \frac{\omega_r}{2\pi} = 663 \text{ s}^{-1}$$

The maximum current is given by,

$$I_{\text{m}} = \sqrt{2} \times \frac{V}{R} = \sqrt{2} \times \frac{230}{23} = 10\sqrt{2}\text{A}$$

(b) The maximum average power is given by,

$$P_{\text{avg,max}} = \frac{1}{2} I_{\text{m}}^2 R = \frac{1}{2} \times \left(10\sqrt{2}\right)^2 \times 23 = 2300 \text{ W}$$

(c) The two angular frequencies for which the power transferred to the circuit is half the power at the resonant frequency,

$$\omega = \omega_r \pm \Delta\omega$$

Where,

$$\Delta\omega = \frac{R}{2L} = \frac{23}{2 \times 0.12} = 95.8 \text{ rad s}^{-1}$$

$$\Delta f = \frac{\Delta\omega}{2\pi} = 15.2 \text{ Hz}$$

Hence, at frequencies 648 Hz and 678 Hz the power absorbed is half the peak power.

The current amplitude at these frequencies can be given as,

$$I' = \frac{I_m}{\sqrt{2}} = \frac{10\sqrt{2}}{\sqrt{2}} = 10$$

(d) Q-factor of the circuit is given by,

$$Q = \frac{\omega_r L}{R} = \frac{4167 \times 0.12}{23} = 21.7$$

7.21. Obtain the resonant frequency and $Q -$ factor of a series LCR circuit with $L = 3$ H, $C = 27\ \mu$F, and $R = 7.4\ \Omega$. It is desired to improve the sharpness of the resonance of the circuit by reducing its 'full width at half maximum' by a factor of 2. Suggest a suitable way.

Solution:

Given,

Inductance, $L = 3$ H

Capacitance, $C = 27\ \mu$F

Resistance, $R = 7.4\ \Omega$

Resonant frequency,

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3 \times 27 \times 10^{-6}}} = 111 \text{ rad s}^{-1}$$

$Q -$ factor of the circuit,

$$Q = \frac{\omega_r L}{R} = \frac{111 \times 3}{7.4} = 45$$

For improvement in sharpness of resonance by a factor of 2, Q should be doubled. To double Q without changing frequency R should be reduced to half, i.e., to $3.7\ \Omega$.

7.22. Answer the following questions:

(a) In any ac circuit, is the applied instantaneous voltage equal to the algebraic sum of the instantaneous voltages across the series elements of the circuit? Is the same true for rms voltage?

(b) A capacitor is used in the primary circuit of an induction coil.

(c) An applied voltage signal consists of a superposition of a dc voltage and an ac voltage of high frequency. The circuit consists of an inductor and a capacitor in series. Show that the dc signal will appear across C and the ac signal across L.

(d) A choke coil in series with a lamp is connected to a dc line. The lamp is seen to shine brightly. Insertion of an iron core in the choke causes no change in the lamp's brightness. Predict the corresponding observations if the connection is to an ac line.

(e) Why is choke coil needed in the use of fluorescent tubes with ac mains? Why can we not use an ordinary resistor instead of the choke coil?

Solution:

(a) The voltage applied will be equal to the average sum of the instantaneous voltage across the circu it's series elements and will be true for any ac circuit. But the voltages across different elements may not be in phase in the case of rms Voltage In the case of rms voltage, therefore, the statement is not true.

(b) When the circuit is broken, the capacitor is charged with a high induced voltage and the capaci tor is used in an induction coil's primary circuit to avoid sparks in the circuit.

(c) For dc signals, an inductor (L) impedance is negligible, but a capacitor impedance is very high. The dc signal will be appearing across the capacitor (C).In the case of a high frequency ac signal, an inductor (L) impedance will be high and a capacitor (C) impedance will be very low. A high frquency ac signal will therefore appear across the inductor (L).

(d) When an iron core is inserted into the choke coil in series with a lamp connected to the ac line,the lamp will glow dimly, and this is because the iron core and the choke coil increase the circuit impedance.

(e) The choke coil reduces the tube voltage without a lot of power being wasted. So, when using fluorescent tubes with ac mains, the choke coil is needed.An ordinary resistor wastes power in the form of heat so that the use of fluorescent tubes with ac mains cannot use an ordinary resistor instead of a choke coil.

7.23. A power transmission line feeds input power at 2300 V to a stepdown transformer with its primary windings having 4000 turns. What should be the number of turns in the secondary in order to get output power at 230 V?

Solution:

Given,

Input voltage, $V_1 = 2300$ V

Input voltage, $V_2 = 230$ V

No. of turns in the primary coil, $n_1 = 4000$

Using the formula for transformers, we get

$$\frac{V_2}{V_1} = \frac{n_2}{n_1}$$

Or, $n_2 = n_1 \times \frac{V_2}{V_1} = 4000 \times \frac{230}{2300} = 400$ turns

7.24. At a hydroelectric power plant, the water pressure head is at a height of 300 m, and the water flow available is $100 \text{ m}^3\text{s}^{-1}$. If the turbine generator efficiency is 60%, estimate the electric power available from the plant ($g = 9.8 \text{ ms}^{-2}$).

Solution:

Given,

Height of water pressure head, $h = 300$ m

The volume of water flown in 1 sec, $\text{V} = 100 \text{ m}^3$

Mass of water, $\text{m} = 100 \times 10^3 = 10^5$ kg

Efficiency, $e = 60\%$

The potential energy of water during one second $= mgh = 10^5 \times 9.8 \times 300 = 29.4 \times 10^7 \text{ J s}^{-1}$

Efficiency,

$$e = \frac{\text{output power}}{\text{input power}}$$

output power $= e \times$ input power $= 0.6 \times 294 \times 10^6 = 176$ MW

Hence, 176 MW of power is available from the plant.

7.25. A small town with a demand of 800 kW of electric power at 220 V is situated 15 km away from an electric plant generating power at 440 V. The resistance of the two wire line carrying power is 0.5 Ω per km. The town gets power from the line through a $4000 - 220$ V step-down transformer at a sub-station in the town.

(a) Estimate the line power loss in the form of heat.

(b) How much power must the plant supply, assuming there is negligible power loss due to leakage?

(c) Characterise the step-up transformer at the plant.

Solution:

Given,

The power required, $P = 800 \times 10^3$ W

Voltage to be supplied, $V = 220$ V

Voltage at power generating plant, $V' = 440$ V

The total resistance of two wire line, $R = 2 \times 15 \text{ km} \times 0.5\ \Omega$ per km $= 15\ \Omega$

A step-down transformer of rating $4000 - 220$ V is used in the sub-station.

Input Voltage, $V_1 = 4000$ V

Output Voltage, $V_2 = 220$ V

rms current in the wire lines can be given by,

$$I = \frac{P}{V_1} = \frac{800 \times 10^3}{4000} = 200 \text{ A}$$

(a) Line power loss in the form of heat,

$P' = I^2R = 200^2 \times 15 = 600$ kW

(b) As there is no power loss due to leakage current. Total power,

$P_T = P + P' = 800 \text{ kW} + 600 \text{ kW} = 1400 \text{ kW}$

(c) Drop across the power line $= IR = 200 \times 15 = 3000\ \Omega$

Total voltage transmitted by the plant $= 3000 + 4000 = 7000$ V

Voltage at power generating plant, $V' = 440$ V

Hence, the rating of the step-up transformer at the power plant is $440 \text{ V} - 7000$ V.

7.26. Do the same exercise as above with the replacement of the earlier transformer by a $40{,}000 - 220$ V step-down transformer (Neglect, as before, leakage losses though this may not be a good assumption any longer because of the very high voltage transmission involved). Hence, explain why high voltage transmission is preferred?

Solution:

Given,

The power required, $P = 800 \times 10^3$ W

Voltage to be supplied, $V = 220$ V

Voltage at power generating plant, $V' = 440$ V

The total resistance of two wires, $R = 2 \times 15 \text{ km} \times 0.5\ \Omega \text{ per km} = 15\ \Omega$

A step-down transformer of rating $40000 - 220$ V is used in the sub-station.

Input Voltage, $V_1 = 40000$ V

Output Voltage, $V_2 = 220$ V

rms current in the wire lines can be given by,

$$I = \frac{P}{V_1} = \frac{800 \times 10^3}{40000} = 20 \text{ A}$$

(a) Line power loss in the form of heat,

$P' = I^2R = 20^2 \times 15 = 6\text{ kW}$

(b) As there is no power loss due to leakage current. Total power,

$P_T = P + P' = 800\text{ kW} + 6\text{ kW} = 806\text{ kW}$

(c) Drop across the power line $= IR = 20 \times 15 = 300\text{ V}$

Total voltage transmitted by the plant $= 300 + 40000 = 40300\text{ V}$

Voltage at power generating plant, $V' = 440\text{ V}$

Hence, the rating of the step-up transformer at the power plant is $440\text{ V} - 40300\text{ V}$.

Percentage loss during transmission $= \frac{6}{806} \times 100 = 0.74\%$

Percentage loss in the previous exercise $= \frac{600}{1400} \times 100 = 43\%$

The percentage of power loss greatly reduced by high voltage transmission.

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