2018 (II)
गणित विज्ञान
प्रश्न पत्र

1. आयत के बिंदुओं को मापक कुल है। इस प्रश्न के विकल्प में 20 मार्ग 'A' एवं 40 मार्ग 'B' 60 मार्ग 'C' में सुलभ सिलिंडर वर्ग (MCS) दिए गए हैं। आयत के मार्ग 'A' में से विकल्प 'B' का 15 मार्ग 'A' में एवं मार्ग 'B' में 25 मार्ग 'A' की समीकरण मार्ग 'C' में से 20 मार्ग 'A' की समीकरण है। कितने प्रश्न के अधीन हैं जो किसी प्रश्न के उत्तर देकर एक समीकरण पत्र मार्ग 'A' में 15, मार्ग 'B' में 25 तथा मार्ग 'C' में 20 मार्गों की समीकरण हैं।

2. क्रम में ठीक होगा कर दिया गया है। क्रम में हमें नकारा और क्रम का नकार दिया गया जो किसी तकनीकी दृष्टि से मुक्त हो और क्रम के नकार से नहीं है। क्रम की तरीका विकल्प में किसी भी क्रम का नकार नहीं है। क्रम में विकल्प ए के ही समान प्रश्न के समीकरण है।

3. प्रश्न पत्र के पंद्रह मार्ग उपरीत हैं। इस प्रश्न पत्र का प्रश्न क्रिया, जो क्रम में हमें विकल्प है। यदि प्रश्न पत्र के पंद्रह विकल्प हैं, तो प्रश्न के अधीन हैं।

4. एक क्रम की नकारात्मक एक विकल्प में होंगे नकार, दोनों विकल्प नकार हैं, जो एक विकल्प में होंगे नकार। प्रश्न पत्र के पंद्रह मार्ग हैं।

5. प्रश्न 'A' में प्रश्न पत्र 2 विकल्प, प्रश्न 'B' में प्रश्न पत्र 3 विकल्प, प्रश्न 'C' में प्रश्न पत्र 4.75 विकल्प रहेंगे। प्रत्येक पत्र का रूपांतरण नकारात्मक विकल्प अतिरिक्त 0.5 अवधि के 0.75 अवधि के नकार होगा। प्रश्न 'C' में अतिरिक्त है।

6. प्रश्न 'A' में प्रश्न पत्र 2 विकल्प, प्रश्न 'B' में प्रश्न पत्र 3 विकल्प, प्रश्न 'C' में प्रश्न पत्र 4.75 विकल्प रहेंगे। प्रत्येक पत्र का रूपांतरण नकारात्मक विकल्प अतिरिक्त 0.5 अवधि के 0.75 अवधि के नकार होगा। प्रश्न 'C' में अतिरिक्त है।

7. प्रश्न को आधार के अनुसार प्रश्न पत्र का नकार करना है। अन्य प्रश्न को 'A' और 'B' अनुसार प्रश्न पत्र का नकार करना है। जब 'B' में प्रश्न पत्र का 'A' अंक हो तो प्रश्न को 'A' अंक होंगे नकार।

8. प्रश्न को नकार करना है। प्रश्न पत्र के पंद्रह मार्गों में यह प्रश्न को नकार करना है।

9. विकल्प 'A' का नकार करना है। जब 'A' में प्रश्न पत्र का 'A' अंक हो तो प्रश्न को 'A' अंक होंगे नकार।

10. प्रश्न को नकार करना है। जब 'A' में प्रश्न पत्र का 'A' अंक हो तो प्रश्न को 'A' अंक होंगे नकार।

11. प्रश्न को नकार करना है। प्रश्न पत्र के पंद्रह मार्गों में यह प्रश्न को 'A' अंक होंगे नकार।

12. जब प्रश्न का नकार करना है, तब उसे अपने अंक पत्र को नकार करना है।
INSTRUCTIONS

1. This Test Booklet contains one hundred and twenty (20 Part 'A' = 60 Part 'B' = 60 Part 'C') Multiple Choice Questions (MCQs). You are required to answer a maximum of 15, 20 and 20 questions from Part 'A', 'B' and 'C' respectively. If more than required number of questions are answered, only first 15, 20 and 20 questions in Parts 'A', 'B' and 'C' respectively, will be taken up for evaluation.

2. OMR answer sheet has been provided separately. Before you start filling up your particulars, please ensure that the booklet contains requisite number of pages and that these are not torn or mutilated. If it is so, you may request the Invigilator to change the booklet of the same code. Likewise, check the OMR answer sheet also. Sheets for rough work have been appended to the test booklet.

3. Write your Roll No. Name and Serial Number of this Test Booklet on the OMR Answer sheet in the space provided. Also put your signatures in the space earmarked.

4. You must darken the appropriate circles with a black ball pen related to Roll Number, Subject Code, Booklet Code and Centre Code on the OMR answer sheet. It is the sole responsibility of the candidate to meticulously follow the instructions given on the OMR Answer Sheet, failing which, the computer shall not be able to decipher the correct details which may ultimately result in loss, including rejection of the OMR answer sheet.

5. Each question in Part 'A' carries 2 marks, Part 'B' 3 marks and Part 'C' 4.75 marks respectively. There will be negative marking @ 0.5 marks in Part 'A' and @ 0.75 marks in Part 'B' for each wrong answer and no negative marking for Part 'C'.

6. Below each question in Part 'A' and 'B', four alternatives or responses are given. Only one of these alternatives is the correct option to the question. You have to find, for each question, the correct or the best answer. In Part 'C' each question may have 'ONE' or 'MORE' correct options. Credit in a question shall be given only on identification of 'ALL' the correct options in Part 'C'.

7. Candidates found copying or restoring to any unfair means are liable to be disqualified from this and future examinations.

8. Candidate should not write anything anywhere except on OMR answer sheet or sheets for rough work.

9. Use of calculator is not permitted.

10. After the test is over, at the perforation point, tear the OMR answer sheet, hand over the original OMR answer sheet to the Invigilator and retain the carbonless copy for your record.

11. Candidates who sit for the entire duration of the exam will only be permitted to carry their Test booklet.
1. A mineral contains a cubic and a spherical cavity. The length of the side of the cube is the same as the diameter of the sphere. If the cubic cavity is half filled with a liquid and the spherical cavity is completely filled with liquid, what is the approximate ratio of the volume of liquid in the cubic cavity to that in the spherical cavity?
   - 1. 1:1
   - 2. 1:2
   - 3. 1:3
   - 4. 1:4

2. Out of 6 unbiased coins, 5 are tossed independently and they all result in heads. If the 6th is now independently tossed, the probability of getting head is:
   - 1. 1/16
   - 2. 0
   - 3. 1/2
   - 4. 1/6

3. What could the fourth figure in the sequence be?
   - 1. 
   - 2. 
   - 3. 
   - 4. 

4. A, B and C are 30 years old. The average age of A and B is 30 years.
   - 1. 31
   - 2. 32
   - 3. 35
   - 4. 37

5. The average age of A, B and C, whose ages are integers x, y and z respectively (x < y < z), is 30. If the age of C is exactly 5 more than that of A, what is the minimum possible value of z?
   - 1. 31
   - 2. 32
   - 3. 35
   - 4. 37
6. The total number of parallelograms in the given diagram?

1. 27  
2. 24  
3. 22  
4. 14

7. Election results of a city, which contains 3 segments (A, B and C) are given in the Table. Percentage votes obtained by parties X, Y and Z are also shown. Which party won the election?

<table>
<thead>
<tr>
<th>Segment</th>
<th>Total Voters</th>
<th>% of voting</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2,00,000</td>
<td>60</td>
<td>30</td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>B</td>
<td>2,50,000</td>
<td>70</td>
<td>40</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>C</td>
<td>3,00,000</td>
<td>80</td>
<td>30</td>
<td>40</td>
<td>30</td>
</tr>
</tbody>
</table>

1. Y  
2. X  
3. Z  
4. It was a tie between X and Y
8. The diagram shows the dimensions (in cm) of a prism having a square base and two identical square pyramids. What is the volume of this prism (in cm³)?

- 1. 3.2
- 2. 3.6
- 3. 6.4
- 4. 7.2

9. A boy throws a ball with a speed $v$ at a vehicle that is approaching him with a speed $V$. After bouncing from the vehicle, the ball hits the boy with a speed $V'$. Which of the following can be the value of $V'$?

- 1. $v$
- 2. $v + V$
- 3. $v + 2V$
- 4. $v + 4V$

10. Four friends were sharing a pizza. They decided that the oldest friend will get an extra piece of pizza. Bahubali is two months older than Kattappa, who in turn is three months younger than Bhalla. Devananda is one month older than Kattappa. Who should get the extra piece of pizza?

- 1. Bahubali
- 2. Devananda
- 3. Bhalla
- 4. Kattappa

11. A square prism $A$ has a base of side $a$. The height of the prism is $h$. A rectangular prism $B$ has a base of dimensions $b$ by $b$, and a height of $k$. Which of the following expressions gives the volume of prism $B$?

- 1. $abk$
- 2. $a^2b^2k$
- 3. $ab^2k$
- 4. $abh^2$
11. A funnel is connected to a cylindrical vessel of cross sectional area A as shown. To make an interconnected system of vessels, water is poured in the cylinder such that the height of water in the funnel is f as shown. If the level of water in the cylindrical vessel is pushed back by a distance $a < f$, the level of water in the funnel:

1. remains unchanged
2. rises by $\frac{Af}{a}$
3. rises by $\frac{Af}{a}$
4. rises by $\frac{Af}{a}$

12. सत्त्र श्रेणी के अंक (20 अंक में से) एक परीक्षा में 4, 15, 6, 7, 7, 3 तथा 8 है। यह पर $a = 0$ का गुणवत्ता है, तथा $b$ एक अनुदान अंक है।

इस संयुक्त में अंकों की देख (Range)
(अधिकतम अंक - न्यूनतम अंक) ने सर्वाधिक संयुक्त क्या है?

1. 25
2. 26
3. 27
4. 29

13. दो व्यक्ति A और B एक किन्द्र से निर्धारित दिशा में उसका प्रवास करते हैं। A की गति 3 km/hr है।

यदि 2 km दूरी में A का स्थल B का स्थल 5 km दूरी है, तो A प्रारंभिक किन्द्र से कितनी दूरी पर B से आगे कितना है?

1. 2 km
2. 4 km
3. 6 km
4. 8 km

13. दो व्यक्ति A और B बिंदुओं पर बिंदुओं के द्वारा नियमित रूप से संबंधित हैं। A और B दोनों की गति में परीक्षण के लिए इकलों के लिए आवश्यक दूरी 63 cm है।

वर्ष 2002 के लिए आवश्यक दूरी 63 cm है। वर्ष 2004 के लिए आवश्यक दूरी 60 cm है। वर्ष 2005 की वातावरणीय दूरी 60 cm थी। वर्ष 2002 के लिए आवश्यक दूरी 63 cm कैसे प्राप्त करें?
15. The average rainfall over a given place during the three-year period of 2003-2005 was 65 cm. During the three-year period 2002-2004 the average rainfall was 63 cm. The actual rainfall during 2003 was 60 cm. What was the rainfall in 2002?

1. 55 cm  
2. 60 cm  
3. 54 cm  
4. 53 cm

16. A triangle has vertices at (1, 2), (3, 4), and (5, 6). If the area of the triangle is 10 square units, which of the following lines could be the equation of one of the sides of the triangle?

1. y = x + 3  
2. y = -x + 5  
3. y = 2x - 1  
4. y = -2x + 3

17. A tank contains 40 liters of oil. If it is filled at a rate of 8 liters per minute, how long will it take to fill the tank?

1. 50 minutes  
2. 60 minutes  
3. 40 minutes  
4. 20 minutes

18. A semicircle is inscribed in a circle. What fraction of the equilateral triangle shown below with three identical sections of a circle is shaded?

1. $\frac{1}{2}$  
2. $\frac{2}{3}$  
3. $\frac{2}{\sqrt{3}}$  
4. $\frac{1 - \sqrt{3}}{2}$

19. How many different salads can be made from cucumber, tomatoes, onions, beetroot, and carrots?

1. 16  
2. 28  
3. 31  
4. 32

18. किसी समबाहु त्रिकोण में कुल के तीन साथ बूटी बागी दिए जाने वाले क्षेत्र का घोटक वर्ग को उपयोगिता कर तिल्ह में देखा गया है। आस्तिकि भर्ता समबाहु त्रिकोण के कुल भूक्षण का विलय रूप है:

1. $\frac{1 - \pi}{2\sqrt{3}}$  
2. $\frac{\pi}{4\sqrt{3}}$  
3. $\frac{1 - \pi}{\sqrt{3}}$  
4. $\frac{1 - \sqrt{3}}{2}$

19. बीजगणित, द्वारक, ब्यांक, मृदुकार तथा योक्तर से जोड़-जोड़ एक एक के कितने संयोजन के संबंध आ लक्षित?

1. 16  
2. 28  
3. 31  
4. 32

19. How many different salads can be made from cucumber, tomatoes, onions, beetroot, and carrots?

1. 16  
2. 28  
3. 31  
4. 32
20. एक इलाक़ा की सीमा को पर 10 मी. की दूरी पर बढ़ाने के बाद शेषित को 10 मीटर मात्र बढ़ाए हुए हैं। 20 मीटर की दूरी पर बढ़ाने को क्या उपस्थिति निर्माण कर सकता है?

1. 25s
2. 40s
3. 14s
4. 80s

21. A bottle of perfume is opened and a person at a distance of 10 m gets the smell after 10 seconds. The time taken for a person 20 m away to get the smell is about
1. 20s
2. 40s
3. 14s
4. 80s

## भाग/PART - B

### अनुंशुक/UNIT - I

21. निम्नलिखित गणितीय परिशिष्टिक परिभाषा \( f : \mathbb{Q} \rightarrow \mathbb{R} \) के लिए

(i) \( f(0) = 0 \)

(ii) \( f(x) = \frac{p}{q} \) जहाँ \( x = \frac{p}{q} \) तथा \( p \in \mathbb{Z} \) तथा \( q \neq 0 \) एवं \( p, q \) यादृच्छिक

तब \( f(x) \) है

1. एक-तरीके लगे तथा
2. एक-तरीके नहीं, वर्तमान तथा
3. परिभाषा नहीं, एक-तरीके नहीं
4. नीचे एक तरीके, रूपरेखा

22. Consider the map \( f : \mathbb{Q} \rightarrow \mathbb{R} \) defined by

(i) \( f(0) = 0 \)

(ii) \( f(x) = \frac{p}{q} \) जहाँ \( x = \frac{p}{q} \) तथा \( p \in \mathbb{Z} \) तथा \( q \neq 0 \) एवं \( p, q \) यादृच्छिक

Then the map \( f \) is

1. एक-तरीके एवं तथा
2. नहीं, एक-तरीके, बल्के तथा
3. परिभाषा नहीं, एक-तरीके नहीं
4. नीचे एक-तरीके, रूपरेखा

23. यदि \( x_n \) एक वास्तविक संख्या तथा \( |x_n| < 1 \), तो \( \lim_{n \to \infty} x_n = 0 \) है।

24. \( A(n) = \int_{n}^{n+1} \frac{1}{x^2} dx \) जबकि \( n \geq 1 \)

\( 0 \leq x \leq 1 \) के लिए निम्नलिखित विकल्पों की सही है?

1. \( L = 0 \) तथा \( c = 2 \)
2. \( L = 1 \) तथा \( c = 3 \)
3. \( L = 2 \) तथा \( c = 3 \)
4. \( L = 3 \) तथा \( c = 0 \)
24. Let \( A(n) = \int \frac{1}{x^n} \, dx \) for \( n \geq 1 \).

For \( c \in \mathbb{R} \) let \( \lim_{n \to \infty} n^c A(n) = L \).

Then
1. \( L = 0 \) if \( c > 1 \)
2. \( L = 1 \) if \( c = 1 \)
3. \( L = 2 \) if \( c = 0 \)
4. \( L = \infty \) if \( 0 < c < 1 \)

25. Define the Chernoff parameter \( \alpha \) by \( S = \{ x \in \mathbb{R} : x \geq 0, x \neq k \pi \} \).

(i) \( \alpha \) is the \( \mathbb{R} \) such that \( \mathbb{R} \) is the union of \( S \).

(ii) If \( \alpha \) is the \( \mathbb{R} \) such that \( \mathbb{R} \) is the union of \( S \), then there is exactly one fixed point.

26. Let \( x > 0 \) be such that \( f(x) = \frac{1}{\sqrt{x}} \).

(i) Define \( f(x) = \frac{1}{\sqrt{x}} \) for \( x > 0 \).

(ii) \( f(x) \) is uniformly continuous.

27. Consider the subspace \( W_1 \) and \( W_2 \) of \( \mathbb{R}^3 \) given by

\( W_1 = \{ (x, y, z) \in \mathbb{R}^3 : x + y + z = 0 \} \)

and

\( W_2 = \{ (x, y, z) \in \mathbb{R}^3 : x - y + z = 0 \} \).

Let \( W \) be the subspace of \( \mathbb{R}^3 \) such that \( W = \text{span} \{ (0,1,1) \} \).

(i) \( W \cup W_1 \) is spanned by \( \{ (0,1,1) \} \).

(ii) \( W \cap W_1 \) is spanned by \( \{ (0,1,1) \} \).

(iii) \( W \cup W_2 \) is spanned by \( \{ (1,0,1) \} \).

(iv) \( W \cap W_2 \) is spanned by \( \{ (1,0,1) \} \).

28. Consider the subspace \( C \) of \( \mathbb{R}^2 \) defined by

\( C = \left\{ \left( \frac{1}{3}, \frac{1}{3} \right) \right\} \).

Let \( T : \mathbb{R}^2 \to \mathbb{R}^2 \) be defined by

\( T(x, y) = \left( \frac{x+y}{x-2y} \right) \).

(a) Show that \( T \) is an isomorphism of \( \mathbb{R}^2 \) onto itself.

(b) Compute the matrix of \( T \) with respect to the standard basis of \( \mathbb{R}^2 \).

(c) Let \( \mathbb{R}^2 \) be the set of all real numbers.

(d) Compute the eigenvalues and eigenvectors of \( T \).

29. Let \( \mathbb{R}^2 \) be the set of all real numbers.

(a) Compute the eigenvalues and eigenvectors of \( T \).

(b) Compute the determinant of \( T \).

(c) Compute the trace of \( T \).

(d) Compute the matrix of \( T \) with respect to the standard basis of \( \mathbb{R}^2 \).

30. Let \( \mathbb{R}^2 \) be the set of all real numbers.

(a) Compute the eigenvalues and eigenvectors of \( T \).

(b) Compute the determinant of \( T \).

(c) Compute the trace of \( T \).

(d) Compute the matrix of \( T \) with respect to the standard basis of \( \mathbb{R}^2 \).
28. Let \( C = \begin{bmatrix} x \ 2y \end{bmatrix} \) be a basis of \( R^2 \) and
\( T : R^2 \rightarrow R^2 \) be defined by \( T^T(x) = x + 2y \). If \( T^T(C) \) represents the matrix of \( T \) with respect to the basis \( C \) then which among the following is true?
1. \( T^T(C) = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \)
2. \( T^T(C) = \begin{bmatrix} 3 & 2 \\ -3 & 1 \end{bmatrix} \)
3. \( T^T(C) = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix} \)
4. \( T^T(C) = \begin{bmatrix} 3 & -3 \\ 1 & 2 \end{bmatrix} \)

c28. Let \( A = \begin{bmatrix} 1 \ 1 \\ 0 \ 2 \end{bmatrix} \) be a matrix. Then all eigenvalues of \( A + iI \) are
1. purely imaginary
2. of modulus one
3. real
4. of modulus less than one

31. Let \( \mathbb{C}^n \) be the vector space and \( \{v_1, v_2, \ldots, v_k\} \) be a linearly independent set in \( \mathbb{C}^n \). Then \( P \) a linear transformation from \( \mathbb{C}^k \) to \( \mathbb{C}^n \). If \( M = (v_1, v_2, \ldots, v_k) \) and \( P \) a linear transformation from \( \mathbb{C}^k \) to \( \mathbb{C}^n \), then \( M^P = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_k \end{bmatrix} \) and \( P \) a linear transformation from \( \mathbb{C}^k \) to \( \mathbb{C}^n \). Which of the following is true?
1. \( P^*(M^P) = k \text{ dim } \mathbb{C} \)
2. \( P^*(M^P) = \sum_{i=1}^{k} a_i \)
3. \( P^*(M^P) = \min (k, n-k) \)
4. \( P^*(M^P) < n \)

32. Let \( (v_1, v_2, \ldots, v_k) \) be an orthonormal basis of \( \mathbb{C}^n \). Let \( M = (v_1, v_2, \ldots, v_k) \) and \( P \) be the diagonal \( k \times k \) matrix with entries \( a_1, a_2, \ldots, a_k \) in \( \mathbb{C} \). Then which of the following is true?
1. \( \text{rank} (M^P) = k \) whenever \( a_i = a_j \)
2. \( \text{Tr}(M^P) = \sum_{i=1}^{k} a_i \)
3. \( \text{rank} (M^P) = \min (k, n-k) \)
4. \( \text{Tr}(M^P) < n \)

33. Let \( B \) be a linear transformation from \( \mathbb{R}^n \) to \( \mathbb{R}^n \). Then which of the following is true?
1. \( B \) is a linear transformation
2. \( B \) is a linear transformation from \( \mathbb{R}^n \) to \( \mathbb{R}^n \)
3. \( B \) is a linear transformation from \( \mathbb{R}^n \) to \( \mathbb{R}^n \)
4. \( B \) is neither linear nor bilinear
33. \[ \sum_{k=0}^{\infty} kx^k \]

where \( x \) is a complex number.

1. \( x > 0 \) and the series converges.
2. \( x = 0 \) and the series does not converge.
3. \( x < 0 \) and the series is divergent.
4. \( R = 0 \).

34. \( f : C \to C \) is a non-constant entire function, and let \( \text{Image}(f) \) be a subset of \( C \).

1. \( \text{Image}(f) \) is open.
2. \( \text{Image}(f) \) is connected.
3. \( \text{Image}(f) \) is closed.
4. \( \text{Image}(f) \) is bounded.

35. The sum \( \sum_{k=0}^{\infty} kx^k \) converges for \( \Re(x) > 0 \).

1. \( R = 0 \).
2. \( R < 0 \) and the series is divergent.
3. \( R > 0 \) and the series is convergent.

36. \( \gamma(t) = \frac{1}{2} \) for all \( t \) in \( [0, 2\pi] \).

1. \( \lambda = -1/3 \)
2. \( \lambda = 0 \)
3. \( \lambda = 1/3 \)
4. \( \lambda = 2 \)
37. The number of group homomorphisms from the alternating group $A_5$ to the symmetric group $S_5$ is $1$.

38. Given integers $a$ and $b$, let $N_{a,b}$ denote the number of positive integers $k < 100$ such that $k \equiv a \pmod{9}$ and $k \equiv b \pmod{11}$. Which of the following statements is true?

1. $N_{a,b} = 1$ for all integers $a$ and $b$.
2. There exist integers $a$ and $b$ satisfying $N_{a,b} > 1$.
3. There exist integers $a$ and $b$ satisfying $N_{a,b} = 0$.
4. There exist integers $a$ and $b$ satisfying $N_{a,b} = 0$ and there exist integers $c$ and $d$ satisfying $N_{c,d} > 1$.

39. Let $p$ be a prime number, and let $L$ be the group of integers modulo $p$. Then which of the following statements is true?

1. The order of $10 \in (Z/pZ)^*$ is a proper divisor of $(p-1)$.
2. The order of $10 \in (Z/pZ)^*$ is a generator of the group $(Z/pZ)^*$.
3. The group $(Z/pZ)^*$ is cyclic and has order $p-1$.
4. The group $(Z/pZ)^*$ is cyclic but not generated by the element $10$.

40. Let $X$ be a topological space and $U$ be a proper dense open subset of $X$. Pick the correct statement from the following:

1. If $X$ is connected then $U$ is connected.
2. If $X$ is compact then $U$ is compact.
3. If $X \setminus U$ is compact then $X$ is compact.
4. If $X$ is compact, then $X \setminus U$ is compact.
41. If \( y_1(x) \) and \( y_2(x) \) are two solutions of the differential equation
\[
\cos x \cdot y'' + (\sin x) y' - (1 + e^{-x}) y = 0
\]
then \( y_1(0) = \sqrt{2}, y_2(0) = 2 \) and the Wronskian of \( y_1(x) \) and \( y_2(x) \) at \( x = \frac{\pi}{4} \) is
\[
1. \quad 3\sqrt{2} \quad 2. \quad 6 \quad 3. \quad 3 \quad 4. \quad -3\sqrt{2}
\]

42. The critical point \( (0,0) \) for the system
\[
x'(t) = x - 2y + y^2 \sin(x) \\
y'(t) = 2x - 2y - 3y \cos(y)
\]
is
1. stable spiral point
2. unstable spiral point
3. saddle point
4. stable node

43. Let \( u(x,t) \) be a function that satisfies the PDE
\[
u_{tt} - \Delta u = x^2 + 6t, \quad x \in \mathbb{R}, \quad t > 0
\]
with the initial conditions
\[
u(x,0) = \sin(x), \quad \nu_t(x,0) = 0
\]
for every \( x \in \mathbb{R} \)

44. Let \( u(x,t) \) satisfy the IVP:
\[
u_{tt} - \Delta u = e^x, \quad x \in \mathbb{R}, \quad t > 0
\]
with the initial conditions
\[
u(x,0) = 1, \quad 0 \leq x \leq 1
\]
and the boundary conditions
\[
u(\pm 1,t) = 0, \quad 0 < t < \infty
\]
Then the value of \( \lim_{t \to 0^+} u(1, t) \) equals
1. \( e \) \quad 2. \( \pi \)
3. \( 1/2 \) \quad 4. \( 1 \)
45. समा कि \( f(x) \) एक अभिविन्य संख्य (हिन्दी) का वंशय दी जिसका मूल दी गई सर्वनाम के अनुसार हैं

\[
\begin{array}{c|cccc}
\text{स्थ} & 0 & 1 & 2 & 3 \\
\hline
0 & 2 & 1 & 1 & 1
\end{array}
\]

इसके समय अग्र विशिष्ट अन्तर का मूल

\[
-\frac{1}{6} \quad \text{का मूल है}
\]

1. \( \frac{1}{3} \)
2. \( -\frac{2}{3} \)
3. \( 16 \)
4. \( -1 \)

45. Let \( f(x) \) be a polynomial of unknown degree taking the values

\[
\begin{array}{c|cccc}
\text{x} & 0 & 1 & 2 & 3 \\
\hline
f(x) & 2 & 1 & 1 & 1
\end{array}
\]

All the fourth divided differences are \(-\frac{1}{16}\). Then the coefficient of \( x^3 \) is

1. \( \frac{1}{3} \)
2. \( -\frac{2}{3} \)
3. \( 16 \)
4. \( -1 \)

46. विषय की लिए कारक

\[
\int \frac{2}{3} (1 - x^2)^2 dx
\]

जो \( y(x) = 0 \) पर परिभाषित है। तथा \( y(0) = y(2) = 0 \)

1. एक अवगत कोण बिंदु (corner point)
2. दो बनता बिंदु (corner point)
3. दो से अधिक कोण बिंदु (corner point)
4. कोई भी कोण बिंदु (corner point) नहीं

46. Consider the functional

\[
\int (1 - y^2)^2 dx
\]

defined on \( y(y) = 0 \); \( y \) is piecewise \( C^2 \)

\[ y(0) = y(x) = 0 \]

Let \( \dot{y} \) be a minimizer of the above functional. Then \( \dot{y} \) has

1. a unique corner point
2. two corner points
3. more than two corner points
4. no corner points

47. विदि

\[
\int \frac{1}{2} (1 - x^2 + t^2) \psi(x) dx = \frac{1}{2} \int x^2 \psi \quad \text{का है अब \( \psi \) है}
\]

1. \( \sqrt{\frac{2}{\alpha}} \psi \)
2. \( \sqrt{\frac{2}{\alpha}} \psi \)
3. \( \sqrt{\frac{2}{\alpha}} \n \)
4. \( \psi \)

47. If \( \psi \) is the solution of

\[
\int \frac{1}{2} (1 - x^2 + t^2) \psi(x) dx = \frac{1}{2} \int x^2 \psi \quad \text{then \( \psi(\sqrt{2}) \) is equal to}
\]

1. \( \sqrt{\frac{2}{\alpha}} \psi \)
2. \( \sqrt{\frac{2}{\alpha}} \psi \)
3. \( \sqrt{\frac{2}{\alpha}} \n \)
4. \( \psi \)

48. एक अवधार के विषयों से जुड़े इतिहास में की अवधारणायें गलत हैं विचार करो, जिसका दूरस्थ लिख लिखता है। इसमें लेखक के विद्यमान का मूल \( \alpha \) है। इस संख्य की गणिती अवधारणा के अर्थ \( \alpha \) विचार विद्यमानों के दूर नहीं है।

\[
T = \frac{1}{2} m \left( \dot{v}^2 + (\dot{t})^2 \right) \quad \text{विध्यमान} \quad V = \frac{1}{2} m v^2, \text{के लिए}
\]

\[
\varphi = \frac{1}{2} \text{मध्य} \quad \text{तथा} \quad \dot{v} = \frac{1}{2} \text{मध्य} \text{ली} \text{समय} \text{करता} \text{है।}
\]

1. \( \alpha \) एक विद्यमान विद्यमान है
2. \( b \) एक विद्यमान विद्यमान नहीं है
3. स्पष्टतः गलत के रूप में \( \pi \) का मूल विद्यमान \( \text{है}
4. स्पष्टतः गलत के रूप में \( \pi \) का मूल विद्यमान \( \text{है}

48. Consider the two dimensional motion of a mass m attached to one end of a spring whose other end is fixed. Let \( k \) be the spring constant. The kinetic energy \( T \) and the potential energy \( V \) of the system are given by

\[
T = \frac{1}{2} m \dot{x}^2 + (\dot{y})^2 \quad \text{विध्यमान} \quad V = \frac{1}{2} k x^2
\]

where \( \dot{x} = \frac{dx}{dt} \) and \( \dot{y} = \frac{dy}{dt} \) with \( k \) as that.

Then which of the following statements is correct?

1. \( \dot{x} \) is an ignorable coordinate
2. \( \dot{y} \) is not an ignorable coordinate
3. \( r^2 \theta \) remains constant throughout the motion.
4. \( r \theta \) remains constant throughout the motion.

Unit-4

49. Suppose \( X \geq 0 \) is a random variable on \((\Omega, F, P)\) with \( E(X) = 1 \). Let \( Z \in F \) be an event with \( 0 < P(Z) < 1 \). Which of the following defines another probability measure on \((\Omega, F)\)?
   1. \( Q(B) = P(A \cap B) \quad \forall B \in F \)
   2. \( Q(B) = P(A \cup B) \quad \forall B \in F \)
   3. \( Q(B) = P(\Omega \setminus B) \quad \forall B \in F \)
   4. \( Q(B) = \begin{cases} \frac{P(A \setminus B)}{P(B)} & \text{if } P(B) > 0 \\ 0 & \text{if } P(B) = 0 \end{cases} \)

50. Suppose \( X \) and \( Y \) are independent random variables on \((\Omega, F, P)\).

51. Suppose \( X_n \) is a Markov Chain with 3 states and transition probability matrix

\[
\begin{pmatrix}
1 & 1 & 1 \\
1 & 2 & 1 \\
0 & 0 & 1
\end{pmatrix}
\]

Then which of the following statements is true?
   1. \( X_n \) is irreducible.
   2. \( X_n \) is recurrent.
   3. \( X_n \) does not admit a stationary distribution.
   4. \( X_n \) has an absorbing state.

52. Suppose \( X \sim \text{Cauchy}(0, 1) \). Then the distribution of \( \frac{X}{1+X} \) is
   1. Uniform \((0, 1)\)
   2. Normal \((0, 1)\)
   3. Double exponential \((0, 1)\)
   4. Cauchy \((0, 1)\)

53. Let \( X \) be a random variable uniformly distributed on \((0, 1)\). Then the distribution of \( \frac{X}{1+X} \) is
   1. Uniform \((0, 1)\)
   2. Normal \((0, 1)\)
   3. Double exponential \((0, 1)\)
   4. Cauchy \((0, 1)\)

54. \( \theta = \frac{0.8 + 0.7 + 0.9 + 1.2 + 1.68 + 1.4 + 0.88 + 1.62}{8} \) is the mean of the data points. Which of the following statements is true?
   1. \( \theta \) is a positive number.
   2. \( \theta \) is a negative number.
   3. \( \theta \) is a zero number.
   4. \( \theta \) is a non-zero number.

55. Let \( X \) and \( Y \) be independent random variables uniformly distributed on \((0, 1)\). Then the distribution of \( X \) is
   1. Uniform \((0, 1)\)
   2. Normal \((0, 1)\)
   3. Double exponential \((0, 1)\)
   4. Cauchy \((0, 1)\)
53. Given the observations 0.8, 0.71, 0.9, 1.2, 1.68, 1.4, 0.88, 1.62 from the uniform distribution on $(-0.2, 0.6 + 0.0)$ with $-\infty < \theta < \infty$, which of the following is a maximum likelihood estimate for $\theta$?

1. $\theta = 0.7$
2. $\theta = 0.9$
3. $\theta = 1.1$
4. $\theta = 1.3$

54. Variance $\text{var}(T)$ for any measure $\mu$ depends on the measures $\mu_0, \mu_1$ of $T$. $\mu_0$ is the measure of $T$ if $\mu_0$ is the measure of $p - \text{var}(T)$ and $\mu_1$ is the measure of $p - \text{var}(T)$.

1. $p = 0.05 \Rightarrow \text{var}(T) = 0$
2. $p = 0.05 \Rightarrow \text{var}(T) = 1$
3. $\mu_0 = 0.05 \Rightarrow \text{var}(T) = 1$
4. $\mu_1 = 0.05 \Rightarrow \text{var}(T) = 1$

55. Let $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ be $n$ independent observations from a bivariate continuous distribution. Let $r_p$ be the product moment correlation coefficient and $r_p$ be the rank correlation coefficient computed based on these $n$ observations. Which of the following statements is correct?

1. $r_p \geq 0$ implies $r_p \geq 0$
2. $r_p \geq 0$ implies $r_p \geq 0$
3. $r_p = 1$ implies $r_p = 1$
4. $r_p = 1$ implies $r_p = 1$

56. Consider a linear model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$, where $i = 1, 2, \ldots, n$ are independent with $E(\epsilon_i) = 0$, $\text{var}(\epsilon_i) = \sigma^2 > 0$, and $\beta_0, \beta_1, \sigma^2 \in R$ which of the following parameters functions is estimable?

1. $\beta_0 + \beta_1$
2. $\beta_0 - \beta_1$
3. $2\beta_0 + \beta_1$
4. $2\beta_0 + 2\beta_1$
57. If $X = N_1(0,1)$ and $A_{pp}$ is an idempotent matrix with rank $(A) = k < p$, then which of the following statements are correct?

1. $\frac{X^T X}{k} = F_{kp}$
2. $\frac{X^T X}{k} - \frac{X^T P X}{k} = F_{kp-k}$
3. $\frac{X^T X}{k} \sim \text{Beta} \left( \frac{k}{2}, \frac{k}{2} \right)$
4. $\frac{X^T X}{k} \sim \text{Beta} \left( \frac{k-1}{2}, \frac{k-1}{2} \right)$

58. A sample of size $n \geq 3$ is drawn from a population of $N \geq 3$ units using PPSWR sampling scheme, where $p_i$ is the probability of selecting $i^{th}$ unit in a draw. $0 \leq p_i \leq 1$ and $\sum p_i = 1$. Then the inclusion probability $v_i$ is

1. $1 - p_i^2 - p_i \left( p_i + p_i \right)$
2. $1 - (p_i + p_i - p_i \cdot p_i)$
3. $1 - (1 - p_i) - (1 - p_i) \left( p_i + p_i \right)$
4. $1 - (1 - p_i) - (1 - p_i) \left( p_i + p_i \right)$

59. In a $2^k$ experiment with two blocks and factors $A, B, C$ and $D$, one block contains the following treatment combinations

1. $ABC$
2. $ABD$
3. $BCD$
4. $ABCD$

60. In an airport, domestic passengers and international passengers arrive independently according to Poisson processes with rates 100 and 70 per hour, respectively. If it is given that the total number of passengers arriving in a 90-110 AM period was 520, then what is the conditional distribution of the number of domestic passengers arriving in this period?

1. Poisson (200)
2. Poisson (100)
3. Binomial $\left( 520, \frac{200}{170} \right)$
4. Binomial $\left( 520, \frac{100}{170} \right)$
61. Let \( \{u_n\}_{n=0}^\infty \) be a sequence of real numbers satisfying the following conditions:

(1) \((-1)^n u_n \geq 0 \), for all \( n \geq 1 \).
(2) \( |u_{n+1}| < \frac{1}{2} |u_n| \), for all \( n \geq 13 \).

Which of the following statements are necessarily true?
1. \( \sum_{n=0}^{\infty} u_n \) converges in \( \mathbb{R} \).
2. \( \sum_{n=0}^{\infty} u_n \) converges to zero.
3. \( \sum_{n=13}^{\infty} u_n \) converges to a non-zero real number.
4. If \( |u_n| < \frac{1}{2} |u_{n-1}| \), for all \( 2 \leq n \leq 13 \), then \( u_n \) is a negative real number.

62. Let \( S \) be an infinite set. Which of the following statements are true?
1. If there is an injection from \( S \) to \( \mathbb{N} \), then \( S \) is countable.
2. If there is a surjection from \( S \) to \( \mathbb{N} \), then \( S \) is countable.
3. If there is an injection from \( \mathbb{N} \) to \( S \), then \( S \) is countable.
4. If there is a surjection from \( \mathbb{N} \) to \( S \), then \( S \) is countable.

63. Let \( \{p_n\}_{n=1}^\infty \) be the sequence of odd prime numbers, when we enumerate the prime numbers in the increasing order. For example, \( p_1 = 2, p_2 = 3, p_3 = 5 \), and so on. Let \( S = \{p_n = p_{n+1} - p_n : n \in \mathbb{N}, n \geq 1\} \).

Which of the following are correct?
1. \( \sup S = \infty \).
2. \( \inf \{p_{n+1} - p_n : n \in \mathbb{N}, n \geq 1\} = \infty \).
3. \( \inf S < \infty \) and \( \inf S = 1 \).
4. \( \lim_{n \to \infty} p_{n+1} - p_n = \infty \).

64. Let \( f(x) = \frac{1}{2} x + \frac{1}{4} \), where \( x \) is a real number.

Which of the following statements are correct?
(1) \( f(x) \) is an even function.
(2) \( f(x) \) is differentiable at \( x = 0 \).
(3) \( f(x) \) is continuous at \( x = 0 \).
(4) \( f(x) \) is an odd function.
64. For \( n \geq 1 \), consider the sequence of functions
\[
g_n(x) = \sum_{k=1}^{n} (-1)^{k+1} k x^k,
\]
on the open interval \((0, 1)\). Consider the statements:
(I) The sequence \( (g_n) \) converges uniformly on \((0, 1)\).
(II) The sequence \( (g_n) \) converges uniformly on \([0, 1)\).
Then,
1. (I) is true
2. (I) is false
3. (I) is false and (II) is true
4. Both (I) and (II) are true.

65. Suppose \( f(x) \) is a continuous real valued functions on \([0,1]\) satisfying the following:
(A) \( \forall x \in \mathbb{R}, \quad f(x) \) is a decreasing sequence.
(B) \( \forall x \in \mathbb{R}, \quad f(x) \) converges uniformly to 0.
Then
1. \( (g_n) \) is Cauchy with respect to the sup norm.
2. \( (g_n) \) is uniformly convergent.
3. \( (g_n) \) need not converge pointwise.
4. \( M > 0 \) such that \( |g_n(x)| \leq M, \forall x \in [0,1], \forall n \in \mathbb{N} \).

66. Given \( f: [1/2, 2] \rightarrow \mathbb{R} \), a strictly increasing function, we put \( g(x) = f(x) + f(1/x), x \in [1, 2] \).

67. \( f \) is a continuous real valued function on \([0,1]\) satisfying the following:
(A) \( \forall x \in \mathbb{R}, \quad f(x) \) is a decreasing sequence.
(B) \( \forall x \in \mathbb{R}, \quad f(x) \) converges uniformly to 0.
Let \( g_n(x) = \sum_{k=1}^{n} (-1)^{k+1} k x^k \), \( \forall x \in \mathbb{R} \).
Then
1. \( (g_n) \) is Cauchy with respect to the sup norm.
2. \( (g_n) \) is uniformly convergent.
3. \( (g_n) \) need not converge pointwise.
4. \( M > 0 \) such that \( |g_n(x)| \leq M, \forall x \in [0,1], \forall n \in \mathbb{N} \).
67. Let $f$ be a real valued continuous differentiable function of $(0, 1)$. Set 
$y = f + 1f$, where $t^2 = -1$ and $f'$ is the derivative of $f$. Let $a, b \in (0, 1)$ be two consecutive zeros of $f$. Which of the 
following statements are necessarily true?
1. If $g(a) > 0$, then $g$ crosses the real line from upper half plane to lower half plane at $a$.
2. If $g(a) > 0$, then $g$ crosses the real line from lower half plane to upper half plane at $a$.
3. If $g(a)g(b) \neq 0$, then $g(a), g(b)$ have the same sign.
4. If $g(a)g(b) = 0$, then $g(a), g(b)$ have opposite signs.

68. Let $A$ be a $n \times n$ matrix.
Define a function $F: \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ by 
$F(x, y) = (x, y)$, where $(x, y)$ denotes the inner product of $x$ and $y$. Let $DF(x, y)$ denote the derivative of $F$ at $(x, y)$ which is 
a linear transformation from $\mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$. Then
1. If $x = 0$, then $DF(x, 0) = 0$.
2. If $y = 0$, then $DF(0, y) = 0$.
3. If $(x, y) = (0, 0)$ then $DF(x, y) = 0$.
4. If $x = 0$ or $y = 0$, then $DF(x, y) = 0$.

69. Let $f: \mathbb{R}^2 \to \mathbb{R}^2$ be a function given by 
$f(x, y) = (x^2 + 3xy^2 - 15x - 12y, x + y)$. Let $S = \{(x, y) \in \mathbb{R}^2 | f$ is locally invertible at $(x, y)\}$. Then
1. $S = \mathbb{R}^2 \setminus \{(0, 0)\}$.
2. $S$ is open in $\mathbb{R}^2$.
3. $S$ is dense in $\mathbb{R}^2$.
4. $\mathbb{R}^2 \setminus S$ is countable.

70. Let $X = \mathbb{N}$, the set of positive integers.
Consider the metrics $d_1, d_2$ on $X$ given by 
$d_1(m, n) = |m - n|, m, n \in X$
$d_2(m, n) = \frac{1}{m+n}, m, n \in X$

Let $X_1, X_2$ denote the subspaces of $\mathbb{N}$, $\mathbb{N}$ respectively. Then
1. $X_1$ is complete.
2. $X_2$ is complete.
3. $X_1$ is totally bounded.
4. $X_2$ is totally bounded.

71. Let $\mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation $f$ which
satisfies
$f(x, y) = (x^2 + 3xy^2 - 15x - 12y, x + y)$. Then

1. $f$ is invertible on $\mathbb{R}^2$.
2. $f$ is not invertible on $\mathbb{R}^2$.
3. $f$ is not continuous on $\mathbb{R}^2$.
4. $f$ is not differentiable on $\mathbb{R}^2$.
71. Let \( T : \mathbb{R}^n \to \mathbb{R}^n \) be a linear map that satisfies \( T^2 = T - I_n \). Then which of the following are true?
1. \( T \) is invertible
2. \( T - I_n \) is not invertible
3. \( T \) has a real eigenvalue
4. \( T^3 = -I_n \)

72. Let \( M = \begin{bmatrix} 2 & 0 & 3 & 2 & 0 & -2 \\ 0 & 1 & 0 & 1 & 3 & 4 \\ 0 & 0 & 1 & 0 & 4 & 4 \\ 1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix} \)

\( b_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \) and \( b_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \)

Then which of the following are true?
1. both systems \( MX = b_1 \) and \( MX = b_2 \) are consistent
2. both systems \( MX = b_1 \) and \( MX = b_2 \) are inconsistent
3. the system \( MX = b_1 - b_2 \) is consistent
4. the systems \( MX = b_1 - b_2 \) is inconsistent

73. Let \( M = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 4 \\ -2 & 1 & -4 \end{bmatrix} \) be a matrix. Which of the following statements are true:
1. \( M \) is an invertible matrix (\( X - 1)(X + 4) \) is an eigenvalue of \( M \), then which among the following are correct?
2. The minimal polynomial of \( M \) is \( (X - 1)(X + 4) \)
3. \( M \) is symmetric
4. \( M^{-1} = \frac{1}{2}(M + 3I) \)

74. Let \( M = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ 4 & 1 & 3 \end{bmatrix} \) and \( b_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \) and \( b_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \). Then which of the following are true?
1. both systems \( MX = b_1 \) and \( MX = b_2 \) are consistent
2. both systems \( MX = b_1 \) and \( MX = b_2 \) are inconsistent
3. the system \( MX = b_1 - b_2 \) is consistent
4. the systems \( MX = b_1 - b_2 \) is inconsistent

75. Let \( A \) be a real matrix with characteristic polynomial \( (X - 1)^2 \). Pick the correct statements from below:
1. \( A \) is necessarily diagonalizable
2. If the minimal polynomial of \( A \) is \( (X - 1)^2 \), then \( A \) is diagonalizable
3. Characteristic polynomial of \( A^2 \) is \( (X - 1)^2 \)
4. If \( A \) has exactly two Jordan blocks, then \( (A - I)^3 \) is diagonalizable

76. Given that \( P \) is an invertible \( 3 \times 3 \) matrix and \( \text{det}(P(x)) = p(x) + p(x - 1) \) for \( x \in \mathbb{R} \), which of the following statements are correct?
1. \( \text{det}(P(x)) = p(x) + p(x - 1) \)
2. \( P(x) = P(x - 1) + P(x - 1) \)
3. \( P(x) = P(x - 1) + P(x - 1) \)
4. \( P(x) = P(x - 1) + P(x - 1) \)

77. Let \( A \) be a matrix with characteristic polynomial \( (X - 1)^2 \). Pick the correct statements from below:
1. \( A \) is necessarily diagonalizable
2. If the minimal polynomial of \( A \) is \( (X - 1)^2 \), then \( A \) is diagonalizable
3. Characteristic polynomial of \( A^2 \) is \( (X - 1)^2 \)
4. If \( A \) has exactly two Jordan blocks, then \( (A - I)^3 \) is diagonalizable

78. Let \( M \) be a matrix with characteristic polynomial \( (X - 1)^2 \). Pick the correct statements from below:
1. \( M \) is necessarily diagonalizable
2. If the minimal polynomial of \( M \) is \( (X - 1)^2 \), then \( M \) is diagonalizable
3. Characteristic polynomial of \( M^2 \) is \( (X - 1)^2 \)
4. If \( M \) has exactly two Jordan blocks, then \( (M - I)^3 \) is diagonalizable

79. Let \( A \) be a matrix with characteristic polynomial \( (X - 1)^2 \). Pick the correct statements from below:
1. \( A \) is necessarily diagonalizable
2. If the minimal polynomial of \( A \) is \( (X - 1)^2 \), then \( A \) is diagonalizable
3. Characteristic polynomial of \( A^2 \) is \( (X - 1)^2 \)
4. If \( A \) has exactly two Jordan blocks, then \( (A - I)^3 \) is diagonalizable

80. Let \( M \) be a matrix with characteristic polynomial \( (X - 1)^2 \). Pick the correct statements from below:
1. \( M \) is necessarily diagonalizable
2. If the minimal polynomial of \( M \) is \( (X - 1)^2 \), then \( M \) is diagonalizable
3. Characteristic polynomial of \( M^2 \) is \( (X - 1)^2 \)
4. If \( M \) has exactly two Jordan blocks, then \( (M - I)^3 \) is diagonalizable
1. \( \det T = 0 \)
2. \((T - 2I)^2 = 0 \) ओर \((T - 2I)^3 = 0 \)
3. \((T - 2I)^3 = 0 \) ओर \((T - 2I)^2 = 0 \)
4. 2 स्कूल आ का अभिलक्षणित्वाच्य है

75. Let \( P_3 \) be the vector space of polynomials with real coefficients and of degree at most 3. Consider the linear map \( T: P_3 \rightarrow P_3 \) defined by \( T(f(x)) = f(x + 1) + p(x - 1) \). Which of the following properties does the matrix of \( T \) (with respect to the standard basis \( B = \{1, x, x^2, x^3\} \) of \( P_3 \)) satisfy?
1. \( \det T = 0 \)
2. \((T - 2I)^2 = 0 \) but \((T - 2I)^3 = 0 \)
3. \((T - 2I)^3 = 0 \) but \((T - 2I)^2 = 0 \)
4. \( T \) is an eigenvalue with multiplicity 4

76. सामान्य कसरी \( M \) एक \( n \times n \) है इसकी लाइज़ (rang) \( k \times n \) है। यदि \( \lambda \neq 0 \) \( M \) का एक अभिलक्षणित्वाच्य हो, एक ऐसा जस्ता मान्य \( u \) के संगम \( Mu = Au \) हो, तो \( \lambda \) से \( k \) से समान साथ है
1. \( rank(M - \lambda uu^T) = k - 1 \)
2. \( rank(M - \lambda uu^T) = k \)
3. \( rank(M - \lambda uu^T) = k + 1 \)
4. \( (M - \lambda uu^T)^2 = M^2 - 4\lambda uu^T \)

76. Let \( M \) be an \( n \times n \) Hermitian matrix of rank \( k \), \( k \neq n \). If \( \lambda \neq 0 \) is an eigenvalue of \( M \) with corresponding unit column vector \( u \), with \( Mu = Au \), then which of the following are true?
1. \( rank(M - \lambda uu^T) = k - 1 \)
2. \( rank(M - \lambda uu^T) = k \)
3. \( rank(M - \lambda uu^T) = k + 1 \)
4. \( (M - \lambda uu^T)^2 = M^2 - 4\lambda uu^T \)

77. \( R^2 \times R^2 \) पर बास्तवित्वें मान्य प्रकरण \( B \) को निम्न प्रकार परिभाषित करें:

| \( p = (x_1, x_2), w = (y_1, y_2) \) का संबंध \( B^2 \) ने है, \( B(w, v) = x_1y_1 - x_2y_1 + 4x_2y_2 \) है। और यदि \( u_0 = (1, 0) \) तला \( W = \{(v \in R^2 : B(v, u_0) = 0) \} \) से \n1. \( B^2 \) का उपरमानित्वाच्य है
2. \( (0,0) \) के बादतर है
3. \( y \)-अक्ष है
4. \( \{(0,0) \} \) से होकर जाने वाली रेखा है

77. Define a real valued function \( B \) on \( R^2 \times R^2 \) as follows. If \( p = (x_1, x_2), w = (y_1, y_2) \) belong to \( R^2 \) define \( B(p, w) = x_1y_1 - x_2y_1 + 4x_2y_2 \). Let \( u_0 = (1, 0) \) and let \( W = \{p \in R^2 : B(p, u_0) = 0\} \). Then \( W \)
1. is not a subspace of \( R^2 \)
2. equals \( \{(0,0)\} \)
3. is the \( y \)-axis
4. is the line passing through \( (0,0) \) and \( (1,1) \)

78. \( R^2 \) पर इन दृष्टिकोण रूपों
\( Q_1(x, y) = xy \)
\( Q_2(x, y) = x^2 + 2xy + y^2 \)
\( Q_3(x, y) = x^2 + 3xy + 2y^2 \)

वर्णितों विचार के से सही भाषणों को (तीत्त्र):
1. \( Q_1 \) तथा \( Q_2 \) है
2. \( Q_2 \) तथा \( Q_3 \) है
3. \( Q_3 \) तथा \( Q_2 \) है
4. सभी सही है

78. Consider the Quadratic forms
\( Q_1(x, y) = xy \)
\( Q_2(x, y) = x^2 + 2xy + y^2 \)
\( Q_3(x, y) = x^2 + 3xy + 2y^2 \)

on \( R^2 \). Choose the correct statements from below:
1. \( Q_1 \) and \( Q_2 \) are equivalent
2. \( Q_2 \) and \( Q_3 \) are equivalent
3. \( Q_2 \) and \( Q_3 \) are equivalent
4. all are equivalent

Unit-2

79. \( f \) के उपरी \( \Delta z \) तल माने अर्थात
\( H = \{(x - y, y) \} \) \( z \in H \) के लिए, निम्न जो से \( \frac{z}{2} \) से \( z \) सही है?
1. \( \frac{z}{2} \in H \\
2. \( \frac{z}{2} \in H \\
3. \frac{z}{2} \in H \\
4. \frac{z}{2} \in H
79. Let $H$ denote the upper half plane, that is
$H = \{z = x + iy : y > 0\}$
For $z \in H$, which of the following are true?
1. $\frac{z}{z+i} \in H$
2. $\frac{1}{z^2} \in H$
3. $\frac{1}{z+i} \in H$

80. Which of the following statements are true?
1. $\tan z$ is an entire function
2. $\tan z$ is a meromorphic function on $\mathbb{C}$
3. $\tan z$ has an isolated singularity at $\infty$
4. $\tan z$ has a non-isolated singularity at $\infty$

81. If $f: \mathbb{C} \to \mathbb{C}$ is an analytic function. Then
which of the following statements are true?
1. If $|f'(z)| \leq 1$ for all $z \in \mathbb{C}$, then $f'$ has
infinitely many zeros in $\mathbb{C}$
2. If $f$ is onto, then the function $f(\cos z)$
is onto
3. If $f$ is onto, then the function $f(e^z)$ is
onto
4. If $f$ is one-one, then the function $f(e^z + z + 2)$ is
one-one

82. Which of the following statements are true?
1. $\tan z$ is an entire function
2. $\tan z$ is a meromorphic function on $\mathbb{C}$
3. $\tan z$ has an isolated singularity at $\infty$
4. $\tan z$ has a non-isolated singularity at $\infty$

83. Let $a_0 < a_1 < \cdots < a_n$ be given distinct
natural numbers such that $1 \leq a_i \leq 100$
for all $i = 1, 2, \ldots, 50$. Then which of the
following are correct?
1. There exist $t$ and $j$ with $1 \leq t < j \leq 50$
satisfying $a_t$ divides $a_j$
2. There exist $t$ with $1 \leq t \leq 51$ such that
$a_t$ is an odd integer.
3. There exists $j$ with $1 \leq j \leq 51$ such that
$a_j$ is an even integer.
4. There exist $t$ such that $|a_t - a_j| > 51$.

84. Consider the entire functions $f(z) = 1 + z + x^2$ and $g(z) = e^{x^2} \in \mathbb{C}$. Which of
the following statements are true?
1. $\lim_{z \to \infty} f(z) = \infty$
2. $\lim_{z \to \infty} g(z) = \infty$
3. $f^{-1}(\{z \in \mathbb{C}: |z| \leq R\})$ is bounded for
every $R > 0$
4. $g^{-1}(\{z \in \mathbb{C}: |z| \leq R\})$ is bounded for
every $R > 0$
3. Let $G$ be a group. Show that $Aut(G)$ is also a group.

4. Let $G$ be a group of order $p^2$, where $p$ is a prime number. Show that the group $G$ is cyclic.

5. Let $G$ be a group of order $p^3$, where $p$ is a prime number. Show that the group $G$ is abelian.

6. Let $R$ be the ring $\mathbb{Z}[x]/(x^2 + 1)$. Pick the correct statements from below:
   1. $\dim R = 2$
   2. $R$ has exactly two prime ideals
   3. $R$ is a UFD
   4. $(x)$ is a maximal ideal of $R$

7. Let $f(x) = x^3 - 105x + 12$. Find the roots of $f(x)$ in $\mathbb{Z}_5$.

8. Let $f(x) = x^2 - 105x + 12$. Then which of the following are correct?
   1. $f(x)$ is reducible over $\mathbb{Q}$
   2. There exists an integer $m$ such that $f(m) = 105$
   3. There exists an integer $m$ such that $f(m) = 2$
   4. $f(m)$ is not a prime number for any integer $m$.

9. Let $G$ be a group with the following property: Given any positive integers $m$, $n$ and $r$ there exist elements $g$ and $h$ in $G$ such that $|g| = m$, $|h| = n$ and $|gh| = r$. Then which of the following are necessarily true?
   1. $G$ has to be of infinite order
   2. $G$ cannot be a cyclic group
   3. $G$ has finitely many cyclic subgroups
   4. $G$ has to be a non-abelian group.

10. Let $G$ be a group with the following property: There exists an element $g$ in $G$ such that $|g| = 3$.

11. Let $\sigma = \frac{\sqrt{2}}{2} \in \mathbb{R}$ and $\xi = \exp\left(\frac{2\pi i}{3}\right)$. Let $K = \mathbb{Q}(\xi)$. Pick the correct statements from below:
   1. There exists a field automorphism of $\mathbb{C}$ such that $\sigma(K) = K$ and $\sigma \neq id$
   2. There exists a field automorphism of $\mathbb{C}$ such that $\sigma(K) \neq K$.
3. There exists a finite extension $E$ of $Q$ such that $K \subseteq E$ and $\sigma(K) \subseteq E$ for every field automorphism $\sigma$ of $E$.

4. For all field automorphisms $\sigma$ of $K$, $\sigma(\alpha) = a\alpha$.

98. Let $\mathbf{X} = \{(x_i)_{i=1}^n : x_i \in (0,1) \text{ for all } i \geq 1\}$ with the metric $d((x_i), (y_i)) = \sum_{i=1}^{\infty} |x_i - y_i|^{2^{-i}}$.

99. Let $X = \{(x_i)_{i=\infty} : x_i \in [0,1] \text{ for all } i \geq 1\}$ with the metric $d((x_i), (y_i)) = \sum_{i=1}^{\infty} |x_i - y_i|^{2^{-i}}$.

100. Let $X = \{(x_i)_{i=1}^n : x_i \in (0,1) \text{ for all } i \geq 1\}$ with the metric $d((x_i), (y_i)) = \sum_{i=1}^{\infty} |x_i - y_i|^{2^{-i}}$.

101. Let $f: X \to [0,1]$ be the function defined by $f((x_i)_{i=1}^n) = \sum_{i=1}^{n} x_i^{2^{-i}}$.

102. Choose the correct statements from below:

1. $f$ is continuous.
2. $f$ is onto.
3. $f$ is one-to-one.
4. $f$ is open.

103. If $A$ is a subgroup of a topological group $H$, then $A$ is a subgroup of $H$.

104. Let $A$ be a subset of $\mathbb{R}$ satisfying $A = \bigcup_{n=1}^{\infty} A_n$, where each $A_n$ is an open dense subset of $\mathbb{R}$. Which of the following are correct?

1. $A$ is a non-empty set.
2. $A$ is countable.
3. $A$ is uncountable.
4. $A$ is dense in $\mathbb{R}$.

91. Three solutions of a certain second order non-homogeneous linear differential equation are

\[ y_1(x) = 1 + x e^x, \quad y_2(x) = (1 + x) e^x - 3, \quad y_3(x) = 1 + e^x. \]

Which of the following is (are) general solution(s) of the differential equation?

1. $y_1(x) = 1 + x e^x - 3$, $y_2(x) = (1 + x) e^x$, $y_3(x) = 1 + e^x$.

92. Let $x \in \mathbb{R}$ and $\phi(x), \psi(x), \phi(x)$ are continuous and $\phi(x) \phi(x) + \psi(x) = 0$.

1. Find $\phi(x)$ and $\psi(x)$ such that $\phi(x) \phi(x) + \psi(x) = 0$.

93. Let $\mathbf{X}$ be a compact metric space.

1. Show that $\mathbf{X}$ is complete.

4-AH

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1. Consider the eigenvalue problem

\[ y'' + 2x = 0 \quad \text{for} \quad x \in (-1, 1) \]

\[ y(-1) = y(1) \]

\[ y'(-1) = y'(1) \]

Which of the following statements are true?

1. All eigenvalues are strictly positive.
2. All eigenvalues are non-negative.
3. Distinct eigenvalues are orthogonal in \( L^2[-1, 1] \).
4. The sequence of eigenvalues is bounded above.

93. The method of variation of parameters to solve the differential equation

\[ y'' + p(x)y' + q(x)y = r(x) \]

where \( x \in I \) and \( p(x), q(x), r(x) \) are non-zero continuous functions on an interval \( I \), seeks a particular solution of the form

\[ y_p(x) = v_1(x)v_2(x) \]

where \( y_1 \) and \( y_2 \) are linearly independent solutions of

\[ y'' + p(x)y' + q(x)y = 0 \]

and \( y_1(x) \) and \( y_2(x) \) are functions to be determined. Which of the following statements are necessarily true?

1. The Wronskian of \( y_1 \) and \( y_2 \) is never zero in \( I \).
2. \( v_1, v_2, u_1, u_2 \) are twice differentiable.
3. \( v_2, y_2 \) are twice differentiable in \( I \).
4. The solution set of \( y'' + p(x)y' + q(x)y = r(x) \) consists of functions of the form \( u_1(x)v_1 + u_2(x)v_2 \) where \( a, b, c, d \in \mathbb{R} \) are arbitrary constants.

94. Consider the IVP:

\[ u_{xx} + u_{tt} = u + 1 \quad x \in \mathbb{R}, t \geq 0 \]

\[ u(x, 0) = x^2, \quad t = x^2 \quad \text{initial conditions} \]

1. The solution is singular at \((0, 0)\).
2. The given space curve \((x, t, u) = (x^2, t^2, x^2)\) is not a characteristic curve at \((0, 0)\).
3. There is no characteristic curve in the \((x, t)\) plane passing through \((0, 0)\).
4. A necessary condition for the IVP to have a unique solution at \((0, 0)\) does not hold.

95. To check if a function \( u(x, t) \) is an admissible solution of

\[ u_t + u u_x = 1, \quad x, t \in \mathbb{R}, t > 0 \]


\[ u \left( \frac{\partial u}{\partial t} \right) = \frac{1}{2} \]

is to follow these steps:

1. Check if \( u \) is twice differentiable.
2. Check if \( u \) satisfies the boundary conditions of the corresponding partial differential equation (PDE).
3. Check if \( u \) is a solution of the IVP.
Let \( u(x, t) \) be a function that satisfies the PDE

\[ u_{tt} + uu_x = 0, \quad x \in \mathbb{R}, \quad t > 0, \]

and the initial condition \( u \left( \frac{t}{1+t} \right) = \frac{1}{2} \). Then the IVP has

1. only one solution
2. two solutions
3. an infinite number of solutions
4. solutions none of which is
differentiable on the characteristic curve.

Let \( f: [0, 1] \to [0, 1] \) be the function defined by

\[ f(x) = x, \]

and consider the differential equation

\[ f'(x) = x, \quad x \in [0, 1]. \]

If \( L = \max_{x \in [0, 1]} |f'(x)| \), then which of the following are true?

1. \( L < 1 \) implies \( x_n \to x_0 \), where \( x_0 \in [0, 1] \).
2. \( L > 1 \) implies \( x_n \to x_\infty \), where \( x_\infty \in \{1, -1\} \).
3. \( L = 1 \) implies \( |x_n| \leq |x_0| \) for all \( n \geq 0 \).
4. \( L > 1 \) implies \( |x_n| \leq |x_0| \) for all \( n \geq 0 \).

Let \( f: [0, 1] \to [0, 1] \) be twice continuously differentiable with a unique fixed point \( f(x_0) = x_0 \). For a given \( x_0 \in (0, 1) \) consider the iteration \( x_{n+1} = f(x_n) \) for \( n \geq 0 \).

If \( L = \max_{x \in [0, 1]} |f'(x)| \), then which of the following are true?

1. \( L < 1 \) implies \( x_n \to x_0 \), where \( x_0 \in (0, 1) \).
2. \( L > 1 \) implies \( x_n \to x_\infty \), where \( x_\infty \in \{1, -1\} \).
3. \( L = 1 \) implies \( |x_n| \leq |x_0| \) for all \( n \geq 0 \).
4. \( L > 1 \) implies \( |x_n| \leq |x_0| \) for all \( n \geq 0 \).

Let \( u(x) \) satisfy the boundary value problem

\[ u'' + u = 0, \quad u(0) = 0, \quad u(1) = 1. \]

Consider the finite difference approximation to this problem.

\[ u'' + u = 0, \quad u(0) = 0, \quad u(1) = 1. \]
1. There exists a solution to \((\Delta Y^2)\), of the form \(u_j = ax^2 + b\) for some \(a, b \in \mathbb{R}\) with \(r = 1\) and \(r\) satisfying \((2 + a)\) \[r^2 - 4r - (2 - h) = 0\]
2. \(u_j = (r^2 - 1)/(r^n - 1)\) where \(r\) satisfies \((2 + h)\) \[r^2 - 4r - (2 - h) = 0\] and \(r \neq 1\)
3. \(u\) is monotonic in \(y\)
4. \(u\) is monotonic in \(f\)

98. \(y(0) = 0, y(1) = 0\) तथा \(f\) का पालन करते हुए फलनक \(f(y) = \int (y')^2 - (y)^2\) dx

पर किसी के द्वितीय चरण एक वांछित शर्त \(y(0) = 0, y(1) = 0\) है जिसको अवकलन से परिवर्तित रीती में सिद्ध करने का प्रयास किया है। इस किसी से किसी से क्या सही है?

1. कई दूहों दिल्ली दिन नहीं है तथा \(y = 0\) एक वर्ग है
2. कई वर्तमान अद्वितीय दिन होता है
3. प्रथम के तत्त्व अप्रवृत्त दिन अद्वितीय दिन है
4. अन्य दुहों दिल्ली दिन है

98. Consider the functional \(f(y) = \int (y')^2 - (y)^2\) dx subject to \(y(0) = 0, y(1) = 0\). A broken extremal is a continuous extremal whose derivative has jump discontinuities at a finite number of points. Then which of the following statements are true?

1. There are no broken extremals and \(y = 0\) is an extremal
2. There is a unique broken extremal
3. There exist more than one and finitely many broken extremals
4. There exist infinitely many broken extremals

99. \(y(x) = y(0) = y(1) = 0, y'(1) = 6\) का पालन करते हुए फलनक \(f(y) = \int (720x^2y - (y)^2)\) dx के क्रम हैं:

1. \(x^2 + 2x^2 - 3x^2\)
2. \(x^3 + 4x^2 - 5x^2\)
3. \(x^3 + 2x^2 - 4x^2\)
4. \(x^3 + 4x^2 - 6x^2\)

99. The extremals of the functional \(f[y] = \int (720x^2y - (y)^2)\) dx, subject to \(y(0) = y'(0) = y(1) = 0, y'(1) = 6\) are

1. \(x^2 + 2x^2 - 3x^2\)
2. \(x^3 + 4x^2 - 5x^2\)
3. \(x^3 + 2x^2 - 4x^2\)
4. \(x^3 + 4x^2 - 6x^2\)

100. Consider the solution \(p(x) = 1 - 2x - 4x^2 + \int \phi(x - t)\) dx, where \(\phi(x)\) is equal to

1. 2
2. 4
3. 6
4. 8

100. If \(p(x)\) is the solution of \(p(x) = 1 - 2x - 4x^2 + \int \phi(x - t)\) dx, then \(\phi(0) = 2\) is equal to

1. 2
2. 4
3. 6
4. 8

101. A characteristic number and the corresponding eigenfunction of the homogeneous Fredholm integral equation with kernel \(K(x, t) = \{(x - t, 0) \leq x \leq t\}

1. \(x = \pi^2, y(x) = \sin x\)
2. \(x = -\pi^2, y(x) = \sin 2x\)
3. \(x = -3\pi^2, y(x) = \sin 3x\)
4. \(x = -4\pi^2, y(x) = \sin 2x\)
102. Consider a point mass of mass $m$ which is attached to a massless rigid rod of length $a$. The other end of the rod is made to move vertically such that its downward displacement from the origin at time $t$ is given by $z(t) = x_0 \cos \theta(t)$. The mass is moving in a fixed plane and its position vector at time $t$ is given by

$$\mathbf{r}(t) = (a \sin \theta(t), x(t) + z_0 \cos \theta(t)).$$

Then the equation of motion of the point mass is

1. $a \ddot{\theta} + (g + z_0 \omega^2 \cos (\omega t)) \sin \theta = 0$
2. $a \ddot{x} + (g - z_0 \omega^2 \cos (\omega t)) \sin \theta = 0$
3. $a \ddot{z} + (g + z_0 \omega^2 \cos (\omega t)) \cos \theta = 0$
4. $a \ddot{\omega} + (g - z_0 \omega^2 \cos (\omega t)) \cos \theta = 0$

**Unit 4**

103. Suppose $X_1, X_2, \ldots, X_n$ is a random sample from the uniform distribution on $(0, 2)$ and $M_n = \max \{X_1, X_2, \ldots, X_n\}$ for every positive integer $n$. Then which of the following statements are true?

1. $M_n \rightarrow 2$ in probability.
2. $M_n \rightarrow 2$ in distribution.
3. $M_n \rightarrow 2$ in distribution.
4. $\frac{M_n^2}{n}$ converges in distribution to
5. normal distribution.

104. Let $X_1, X_2, \ldots$ be a sequence of i.i.d. $N(0, 1)$ random variables. Let $S_n = X_1^2 + X_2^2 + \cdots + X_n^2$.

\[ S_n \rightarrow N(n, 0) \text{ for all } n \geq 1. \]

Which of the following statements are correct?

1. $\frac{S_n}{n} \rightarrow N(0, 1)$ for all $n \geq 1$.
2. For all $\varepsilon > 0$, $P\left(\left|\frac{S_n}{n} - 2\right| > \varepsilon\right) \rightarrow 0$ as $n \rightarrow \infty$.
3. $\frac{S_n}{n} \rightarrow 1$ in probability.
4. $P(S_n \leq n + \sqrt{n}) \rightarrow P(Y \leq x)$ for all $x \in \mathbb{R}$, where $Y \sim N(0, 2)$. 

- 4 A-H
- 27 CIS/18-4AH-5A
105. Consider a Markov chain with transition probability matrix \( P \) given by
\[
P = \begin{pmatrix}
1/2 & 1/2 & 0 \\
1/3 & 1/3 & 1/3 \\
1/3 & 1/3 & 1/3
\end{pmatrix}
\] For any two states \( i \) and \( j \), let \( p_{ij}^{(n)} \) denote the \( n \)-step transition probability of going from \( i \) to \( j \). Identify correct statements.
1. \( \lim_{n \to \infty} p_{ij}^{(n)} = 2/9 \)
2. \( \lim_{n \to \infty} p_{ij}^{(n)} = 0 \)
3. \( \lim_{n \to \infty} p_{ij}^{(n)} = 1/3 \)
4. \( \lim_{n \to \infty} p_{ij}^{(n)} = 1/3 \)

106. Consider a Markov chain with transition probability matrix \( P \) given by
\[
P = \begin{pmatrix}
1/2 & 1/2 & 0 \\
1/3 & 1/3 & 1/3 \\
1/3 & 1/3 & 1/3
\end{pmatrix}
\] For any two states \( i \) and \( j \), let \( p_{ij}^{(n)} \) denote the \( n \)-step transition probability of going from \( i \) to \( j \). Let \( d(i) \) denote the period of state \( i \) (\( i \in S \)). Which of the following statements are correct?
1. If \( d(i) = d(j) \) then \( \lim_{n \to \infty} p_{ij}^{(n)} = 0 \)
2. If \( d(i) = d(j) \) then \( p_{ij}^{(n)} = 0 \) and \( p_{ii}^{(n)} = 0 \) for some \( n, m \geq 1 \)
3. If \( p_{ij}^{(n)} > 0 \) and \( p_{ii}^{(n)} > 0 \) for some \( n, m \geq 1 \), then \( d(i) = d(j) \)
4. \( \lim_{n \to \infty} p_{ij}^{(n)} = 0 \) implies \( d(i) = d(j) \)

107. Suppose that \( (X_n, Y_n) \) follows a bivariate distribution with common marginal distribution \( F \) and \( \text{Corr}(X_1, X_2) = 0 \). Then which of the following statements are correct?
1. \( F = \text{Uniform}(0, 1) \Rightarrow X_1 \) and \( X_2 \) are independent.
2. \( F = \text{Bernoulli}(\theta) \Rightarrow X_1 \) and \( X_2 \) are independent.
3. \( F = \text{Discrete Uniform} \{-1, 0, 1\} \Rightarrow X_1 \) and \( X_2 \) are independent.
4. \( F = \text{Normal}(0, 1) \Rightarrow X_1 \) and \( X_2 \) are independent.

108. Let \( X_1, X_2, \ldots, X_n \) be \( n \) real numbers such that \( X_1 \geq X_2 \geq \ldots \geq X_n \). Which of the following statements are correct?
1. \( X_n \) is the maximum of the \( X_i \).
2. \( X_1 \) is the minimum of the \( X_i \).
3. The median of the \( X_i \) is \( X_{n/2} \).
4. The mean of the \( X_i \) is \( \frac{1}{n} \sum_{i=1}^{n} X_i \).

109. Consider a Markov chain with transition probability matrix \( P \) given by
\[
P = \begin{pmatrix}
1/2 & 1/2 & 0 \\
1/3 & 1/3 & 1/3 \\
1/3 & 1/3 & 1/3
\end{pmatrix}
\] For any two states \( i \) and \( j \), let \( p_{ij}^{(n)} \) denote the \( n \)-step transition probability of going from \( i \) to \( j \). Let \( d(i) \) denote the period of state \( i \) (\( i \in S \)). Which of the following statements are correct?
1. If \( d(i) = d(j) \) then \( \lim_{n \to \infty} p_{ij}^{(n)} = 0 \)
2. If \( d(i) = d(j) \) then \( p_{ij}^{(n)} = 0 \) and \( p_{ii}^{(n)} = 0 \) for some \( n, m \geq 1 \)
3. If \( p_{ij}^{(n)} > 0 \) and \( p_{ii}^{(n)} > 0 \) for some \( n, m \geq 1 \), then \( d(i) = d(j) \)
4. \( \lim_{n \to \infty} p_{ij}^{(n)} = 0 \) implies \( d(i) = d(j) \)

110. Suppose that \( (X_n, Y_n) \) follows a bivariate distribution with common marginal distribution \( F \) and \( \text{Corr}(X_1, X_2) = 0 \). Then which of the following statements are correct?
1. \( F = \text{Uniform}(0, 1) \Rightarrow X_1 \) and \( X_2 \) are independent.
2. \( F = \text{Bernoulli}(\theta) \Rightarrow X_1 \) and \( X_2 \) are independent.
3. \( F = \text{Discrete Uniform} \{-1, 0, 1\} \Rightarrow X_1 \) and \( X_2 \) are independent.
4. \( F = \text{Normal}(0, 1) \Rightarrow X_1 \) and \( X_2 \) are independent.
1. \( \prod_{i=1}^{n} X_i, \theta \) के स्वरूप परिभाषित है।
2. \( \sum_{i=1}^{n} \ln X_i, \theta \) के स्वरूप परिभाषित है।
3. \( \prod_{i=1}^{n} X_i, \theta \) के स्वरूप अभिव्यक्तिवादिता आकस्मिक है।
4. \( -\frac{1}{n} \sum_{i=1}^{n} \ln X_i, \theta \) के स्वरूप अभिव्यक्तिवादिता आकस्मिक है।

108. यदि \( X_1, X_2, \ldots, X_n \) एक साधारण समूह से निकाला गया तो किसी दिए गए क्रम के \( x_1, x_2, \ldots, x_n \) का प्रायिकता वितरण 
\[ f_{\theta}(x) = \begin{cases} \frac{x^{\theta-1}}{\Gamma(\theta)}, & 0 < x < 1, \\ 0, & \\end{cases} \]
where \( \theta > 0 \). तो निम्नलिखित कोण आवश्यक है?
1. \( \prod_{i=1}^{n} X_i \) सुवर्धन for \( \theta \).
2. \( -\frac{1}{n} \sum_{i=1}^{n} \ln X_i \) सुवर्धन for \( \theta \).
3. \( \prod_{i=1}^{n} X_i \) मानक अभिव्यक्तिवादिता for \( \theta \).
4. \( -\frac{1}{n} \sum_{i=1}^{n} \ln X_i \) मानक अभिव्यक्तिवादिता for \( \theta \).

109. \( (-2, -1, 1, 2) \) पर \( X \) एक विशिष्ट वास्तविक स्वरूप यह दिखाया है कि किसी वास्तविक स्वरूप \( P_X(x, \theta), \theta \in (0, 2) \) हैं।

<table>
<thead>
<tr>
<th>( X )</th>
<th>(-2)</th>
<th>(-1)</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta = 0 )</td>
<td>0.05</td>
<td>0.6</td>
<td>0.3</td>
<td>0.05</td>
</tr>
<tr>
<td>( \theta = 1 )</td>
<td>0.2</td>
<td>0.4</td>
<td>0.2</td>
<td>0.2</td>
</tr>
</tbody>
</table>

उद्देश्य \( H_0: \theta = 0 \), \( H_1: \theta = \theta_1 \) का परिभाषण करता है। निम्नलिखित कोण आवश्यक है?
1. \( \prod_{i=1}^{n} X_i \) मानक अभिव्यक्तिवादिता 0.05 अंतर का एक सबसे शक्तिशाली परिभाषण है।
2. \( -\frac{1}{n} \sum_{i=1}^{n} \ln X_i \) मानक अभिव्यक्तिवादिता 0.05 अंतर का एक सबसे शक्तिशाली परिभाषण है।
3. \( -\frac{1}{n} \sum_{i=1}^{n} X_i \) मानक अभिव्यक्तिवादिता 0.05 अंतर का एक सबसे शक्तिशाली परिभाषण है।
4. \( -\frac{1}{n} \sum_{i=1}^{n} X_i \) मानक अभिव्यक्तिवादिता 0.05 अंतर का एक सबसे शक्तिशाली परिभाषण है।

110. \( X \) is a discrete random variable on \((-2, -1, 1, 2)\) with probability mass functions \( f_\theta(x) \), \( \theta \in (0, 2) \) given below

<table>
<thead>
<tr>
<th>( X )</th>
<th>(-2)</th>
<th>(-1)</th>
<th>1</th>
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</tr>
<tr>
<td>( \theta = 1 )</td>
<td>0.2</td>
<td>0.4</td>
<td>0.2</td>
<td>0.2</td>
</tr>
</tbody>
</table>

The aim is to test \( H_0: \theta = \theta_0 \) against \( H_1: \theta = \theta_1 \). Which of the following statements are correct?
1. The test procedure with critical region \( x = -2 \) is the most powerful test of size 0.05.
2. The test procedure with critical region \( x = -2 \) is the most powerful test of size 0.05.
3. The test procedure with critical region \( x = -1 \) is the most powerful test of size 0.05.
4. The test procedure with critical region \( x = -1 \) is not a most powerful test of size 0.05.

110. Suppose \( X_1, X_2, \ldots, X_n \) is a random sample from uniform distribution on \((0, \theta + 1)\), where \( \theta \in \mathbb{R} \) is an unknown parameter. Let \( X_{(1)} < X_{(2)} < \cdots < X_{(n)} \) be the corresponding order statistics. Which of the following are 100(1 - \( \alpha \))% confidence intervals for \( \theta \)?
1. \((-\infty, X_0 - a^{2/n})
2. \((X_0 + a^{2/n}, +\infty)\)
3. \(X_0 + \frac{2 \sigma}{\sqrt{n}}
4. \(-\infty, X_0 - \sigma\)

112. \(\beta > 0 \) तथा \(\theta > 0 \) तथा \(\theta \) का पूर्व ज्ञान त्र के प्रकार दिया जाता है:
\(\phi(\theta) \propto \theta^\beta - 1 \), जहाँ \(\alpha > 0 \) तथा \(\beta > 0 \)
है. जिसमें से कौन से कौन से काम करता है?
1. \(Y \) का पूर्व बंटन हाँ, यह ज्ञात है.
2. \(\theta \) के पूर्व बंटन \(Y = y \) के गामा से दिया गया है.
3. \(\alpha \) का संपूर्ण अवधारणा है.
4. \(Y = \frac{1}{\theta} \) है. यह लाइन के लिए वेट रूप से \(\theta \) का उपयोग \(e^{\alpha + \beta} \)

113. \(\lambda \) एक निर्धारित मानदंड \(Y_{\text{ex}} = X_{\text{ex}} + \beta_{\text{ex}} + e_{\text{ex}} \)
पर निर्धारी किया जाता है. जब \(\text{Disp}(e) = o^2, \text{यदि} o^2 > 0 \)
के लिए.
\(e_{\text{ex}} \) का अर्थ तह ि से अभिनवता है।
\(\beta_{\text{ex}} \) की तेरा समायोजन \([-1, 1] \) में से है।
\(X_0 \) के लिए 3 संभाव अवधारणाएँ पर निर्धार किया जाता है.

\(X_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \)
\(X_2 = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \)
\(X_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} \)

4-A-1
113. Consider a linear model

\( Y_{ac1} = X_{ac} \alpha + \varepsilon_{ac1} \),

where

\( \text{Disp}(\varepsilon) = \sigma^2 I_4 \) for some \( \sigma^2 > 0 \). One needs to choose the design matrix \( X \) such that its elements take values in the set \( \{-1, 0, 1\} \). Now, consider the following three choices of \( X \):

\[
X_1 = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[
X_2 = \begin{bmatrix}
1 & -1 & 0 & 0 \\
1 & 1 & 1 & 1 \\
0 & 0 & 1 & -1 \\
0 & 0 & 1 & 1 \\
\end{bmatrix}
\]

\[
X_3 = \begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & -1 & -1 & -1 \\
1 & -1 & -1 & 1 \\
1 & 1 & 1 & -1 \\
\end{bmatrix}
\]

Which of the following statements are true?

1. For all three choices of \( X \),
2. For all three choices of \( X \), \( \beta_1 \) and \( \beta_2 \), the least squared estimates of \( \beta_1 \) and \( \beta_2 \) are uncorrelated for all \( i \neq j \).
3. \( X_2 \) is a better choice than \( X_1 \).
4. \( X_4 \) is a better choice than \( X_3 \).

114. Suppose that \( X_1, X_2, \ldots, X_{10} \) are

i.i.d. \( N(0, 1) \). Which of the following statements are correct?

1. \( P(X_1 > X_2 + X_3 + \ldots + X_{10}) = \frac{3}{2} \)
2. \( P(X_1 > X_2 X_3 \ldots X_{10}) = \frac{1}{2} \)
3. \( P(\text{sin}(X_1) > \text{sin}(X_2) + \text{sin}(X_3) + \ldots + \text{sin}(X_{10})) = \frac{2}{3} \)
4. \( P(\text{sin}(X_1) > \text{sin}(X_2 + X_3 + \ldots + X_{10})) = \frac{1}{2} \)

115. Consider a classification problem between two uniform distributions \( U(0, 2) \) and \( U(1, 5) \).

1. For \( \pi = 0 \), the Bayes' classifier is unique.
2. For \( \pi = 1 \), the Bayes' classifier is unique.
3. For \( \pi = 1 \), the Bayes' classifier is unique.
4. For all choices of \( \pi \), the Bayes' classifier is unique.
1.2\ldots,N\text{ जिन प्रत्येक } n_k = 0 \text{ तथा } n_k = 1\text{ हैं, } k = 2,3,\ldots, N \text{ के लिए ऐसे } (k-1) \text{ इकाइयों से संयुक्त होंगे। तब ढूँढ़ी इकाई के समुह में समरूपी होने की प्रासंगिकता } \frac{k}{2} \text{ है।}

2. पहले तथा अन्तिम इकाई के समुह में समरूपी होने के लिए प्रासंगिकता \frac{N-2}{N-1} \text{ है।}

3. पहली इकाई के समरूपी होने के लिए प्रासंगिकता \frac{2}{N-1} \text{ है।}

4. पहली इकाई के समरूपी होने के लिए प्रासंगिकता \frac{2}{N-1} \text{ है।}

116. Suppose \( n \geq 2 \) units are drawn from a population of \( N \geq n \) units sequentially as follows. A random sample \( U_1, U_2, \ldots, U_n \) of size \( N \) is drawn from \( U(0,1) \). The \( k \)-th population unit is selected if \( U_k < \frac{k}{N} \), \( k = 1,2,\ldots, N \), where \( n_1 = 0 \) and \( n_2 = n \) number of units selected out of first \( k = 1 \) units for each \( k = 2,3,\ldots, N \). Then,

1. The probability of inclusion of the 2nd unit in the sample is \( \frac{2}{N} \).

2. The probability of inclusion of the 1st and 2nd unit in the sample is \( \frac{2}{N(N-1)} \).

3. The probability of not including the 1st unit and including the 2nd unit in the sample is \( \frac{2}{N(N-1)} \).

4. The probability of including the 1st unit but not including the 2nd unit in the sample is \( \frac{2}{N(N-1)} \).

117. तीन पदार्थों का तूफान A, B, C तथा D कोई एक साथ विस्फोटित नहीं प्रतिक्रिया होता जो के \( \gamma \) A तथा B को \( \gamma \) के लिए, क्षेत्र A, B तथा D को \( \delta \) के लिए, क्षेत्र C को \( \gamma \) के लिए रहता था।

तब \( \gamma \) वांछित \( \delta \) क्रियाकलाप

118. Suppose \( X \) is a positive random variable with the following probability density function

\[
 f(x) = (ax^{\alpha-1} + \beta x^{\beta-1})e^{-x^{\alpha-\beta}}, \quad x > 0
\]

for \( \alpha > 0 \) and \( \beta > 0 \). Then the hazard function \( h(x) \) for some choices of \( \alpha \) and \( \beta \) can be

1. an increasing function
2. a decreasing function
3. a constant function
4. a non-monotonic function

119. एक समान्तर \( n \) \( n(2 \bar{1}) \) सर्वयुक्त अवधि \( \text{मी.} \) के प्रति का अधिक क्षेत्र के लिए, सर्वसमान किंची चर ग्रहणी वातावरण क्षेत्र \( h \) जिसका ठोस \( \text{लक्ष्य} \) है। यदि \( X \) तो \( \text{फ्लिप} \) में से कॉट लेने की संभावना \( \text{सर्वभौम} \) है, तो \( X \) ने लिखा दिया है कि वो \( \text{काम} \) से कितना तारीक़ है?
1. The mean of $X$ is greater than equal to 1 for all $n$.
2. The median of $X$ is greater than 100 for some $n$.
3. The mode of $X$ is 0 for some $n$.
4. The mode of $X$ is less than or equal to $n$ for all $n$. 

119. A parallel system has $n$ (≥ 1) identical components. The lifetimes of the $n$ components are independent identically distributed exponential random variables with mean 1. If the lifetime of the system is denoted by $X$, then which of the following statements are true?
1. The mode of $X$ is 0 for some $n$.
2. The mode of $X$ is less than or equal to $n$ for all $n$. 

120. Suppose $ABC$ is a triangle on the $x$-$y$-plane with centroid $D$. Which of the following points can NEVER be a minimizer of the function $7x - 10y + 1$ as $(x, y)$ runs over the triangle $ABC$?
1. A
2. B
3. C
4. D