नाम: 

कक्षा 12 का भीतरी गणित शाखा में दो पाठ्यक्रम हैं: 

1. गणित (A) 
2. गणित (B)

प्रश्न पत्र: 2018 (I)

नोट: 

1. मात्र सभी प्रश्नों को जवाब दें।
2. प्रश्न 1 से 15 के तुल्य मार्क (M.C.Q.) में जवाब दें।
3. प्रश्न 16 से 20 के तुल्य मार्क (M.C.Q.) में जवाब दें।
4. प्रश्न 21 से 25 के तुल्य सिफारिश बांटें।
5. प्रश्न 26 से 29 के तुल्य लिखित जवाब दें।

प्रश्न 1 (२ मार्क) 

(अंकन के साथ दिए गए विवरणों का अनुसार जवाब दें)
**भाग/PART A**

1. दो विद्यार्थी एक प्रश्न को रचत्रतक हल कर रहे हैं। वह दोनों की प्रश्न हल करने की प्रविष्टिका $\frac{1}{2}$ है और पुरस्कार की $\frac{1}{3}$ है, तो कम से कम एक विद्यार्थी के प्रश्न हल करने की प्रविष्टिका या है?
   1. $\frac{17}{25}$
   2. $\frac{19}{25}$
   3. $\frac{22}{25}$
   4. $\frac{23}{25}$

2. 44 विद्यार्थियों के समूह में, 26 विद्यार्थी ही होंगे, 24 विद्यार्थी पुरुष होंगे और 24 विद्यार्थी किकेट खेलेंगे हैं। उनमें से 8 ही होंगे और पुरुष दोनों, 12 पुरुषों और किकेट दोनों, और 5 ही होंगे खेल खेलेंगे हैं। विद्यार्थी ही होंगे और किकेट दोनों खेलेंगे?
   1. 10
   2. 15
   3. कोई नहीं
   4. 7

3. चार मुक्ति $M_1, M_2, M_3, M_4$ और चार 
   महिलाएं $F_1, F_2, F_3, F_4$ एक मोलियाक रेखा के 
   किनारे एक-दूसरे से चली तरे बैठा हैं। 
   जिन्हें हुए बैठे हैं, जो व्यों को जिन दिशाओं 
   दिशाओं में दर्शाया गया है। यदि प्रत्येक उपनयन से चीन 
   कदम दिशानुसार चलता है और किसे एक कदम 
   वापस भट्टता है, तब $F_4$ का बेहतर किस 
   दिशा में है?

4. त्रिकोण (2017, 2017), (2027, 2027) और (2037, 2019) के शीर्षक हैं।
   1. 2017
   2. 100
   3. $100\sqrt{10}$
   4. $186\sqrt{20}$
4. The area of the triangle formed by joining the points (2017, 2017), (2027, 2027) and 
(2037, 2017) is
1. 2017
2. 100
3. \(100\sqrt{10}\)
4. \(1000\sqrt{10}\)

5. In the diagram, what is the ratio of the total shaded area (of the circle and semi-circle) to the total area of the square and the rectangle?

6. Prof. Munzy likes to let her students choose their partners for more than seven class periods in a row. Alice and Bob have worked together for more than seven class periods in a row. Calvin and Denny have worked together for three class periods in a row. Calvin does not want to work with Alice. Who should be assigned to work with Bob?
1. Calvin
2. Alice
3. Denny
4. None

7. It is given that
\[(x^2) = a \quad (a > 0)\] 
\[(a/a) \quad (a > 0)\]
Suppose for two real numbers \(x\) and \(y\),
\[(xy)^2 = (x^2)(y^2)\]. Then which of the following is necessarily true?
1. \(x > 0 \text{ and } y > 0\)
2. \((x < 0 \text{ and } y < 0) \text{ or } (x > 0 \text{ and } y > 0)\)
3. \((x < 0 \text{ and } y > 0) \text{ or } (x > 0 \text{ and } y < 0)\)
4. \((x < 0) \text{ or } (y < 0) \text{ or } (x > 0 \text{ and } y > 0)\)

8. In the diagram, what is the ratio of the shaded area to the total area?

9. In three-dimensional space, given that \(a\) and \(b\) are two vectors, what is the dot product of \(a\) and \(b\)?
1. \(a \cdot b\)
2. \(a^2 \cdot b^2\)
3. \(a + b\)
4. \(a^2 + b^2\)
8. Three semi-circles are drawn inside a big circle as shown in the figure. If the radius of the two identical smaller semi-circles is \(\frac{1}{4}\)th of that of the big circle and the radius of the bigger semi-circle is twice that of the small semi-circle, what proportion of the big circle's area is shaded?

9. The number 54 is expressed in a base different from ten. What is the base of this number system if its equivalent value in the decimal system is 49?

10. A long-distance runner finds a water station after completing \(\frac{1}{4}\)th of the total distance. After covering another \(\frac{1}{6}\)th of the total distance, he gets medical aid. Another runner joins him 4 km after the medical aid station. The second runner stops 4 km before the completion of run, covering \(\frac{1}{2}\) of the total distance. What is the total distance?

11. The factor of a number 54 is to be found. The factors are 3 and 2. The other factor is to be found.

12. A boy who has 100 books, adds books to his collection. If he adds books equivalent to \(\frac{1}{2}\) of his current collection, how many books does he have now?
12. A ball is dropped from a height of 100 m. The ball after each bounce rises vertically by half its previous height (This means at the first bounce it rises by 50 m, by 25 m at the second bounce and so on). What is the vertical distance travelled by the ball between the first and the fifth bounce?

- 1. \(155 \frac{1}{2}\) m
- 2. \(265 \frac{1}{2}\) m
- 3. \(175 \frac{1}{2}\) m
- 4. \(285 \frac{1}{2}\) m

13. यदि संगीता की पुत्री मेरी पुत्री की नहीं है,
तो मेरा संगीता से क्या रिश्ता है?
1. केवल पुत्र होना ही सम्भवता है।
2. केवल दासदास होना ही सम्भवता है।
3. भेंट पुत्री होना ही सम्भवता है।
4. दासदास या पुत्री

14. रिश्त स्थान में कौन सा विकल्प सबसे सटीक है?

<table>
<thead>
<tr>
<th></th>
<th>0.1</th>
<th>0.25</th>
<th>0.3</th>
<th>0.5</th>
<th>0.6</th>
<th>0.3</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.05</td>
<td>2.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.75</td>
<td>4.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

15. रिश्त स्थान के लिए उपयुक्त विकल्प कौन हैं?

- 1. 
- 2. 
- 3. 
- 4. 

15. Which of the options is appropriate for the blank space?

<table>
<thead>
<tr>
<th></th>
<th>0.1</th>
<th>0.25</th>
<th>0.3</th>
<th>0.5</th>
<th>0.6</th>
<th>0.3</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.05</td>
<td>2.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.75</td>
<td>4.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
16. लम्बाई की एक छटी को लीन, रूप से दो भागों में तोड़ा गया है। तोड़े दुसरे के अंत में लम्बाई कितनी है?
   1. L/6
   2. L/4
   3. L/3
   4. L/2

17. एक कार की दूरी समय के साथ नीचे दर्शानी गयी है।

18. Number of times a research paper is viewed and cited is shown in the plot. In which month was the percentage increase in citation more than the double of the percentage increase in view?

19. एक इंस्टीट्यूट एक दिन में 160 लोगों को ₹15000 का वीजल बेचता है। यदि प्रत्येक वीजल का क्रम से क्रम ₹50 का वीजल वर्षीय अनुमान है, तो कितनी वीजल ने वर्षीय रूप से दिन में वापसी की?
   1. 7450
   2. 7500
   3. 7580
   4. 7600

20. A fuel station sold diesel costing ₹15000 to 150 persons on a day. If the lower limit of sale to a person is ₹50, what is the maximum amount in rupees for which one person could have purchased diesel on that day?
   1. 7450
   2. 7500
   3. 7550
   4. 7600
20. A और B एक साथ एक लग्न के 30 मिनट से दूरी घूमते हैं। A को एक नकार लगाता है और वह एक मिनट से अधिक तक नीचे का दौरा करता है। B को एक नकार लगाता है और वह 2 मिनट से अधिक तक दौरा करता है। कहाँ से उठाए गए तीनों लगे A और B के समय?
1. 30
2. 29
3. 31
4. 28

20. A and B move clockwise around a circle, starting from a common point O. A takes 9 minutes to complete a round but re-starts after a delay of 1 minute. B takes 13 minutes to complete a round but restarts after a delay of 2 minutes. How many minutes after they began would they meet again at O?
1. 30
2. 29
3. 31
4. 28

भाग / PART B

UNIT-1

21. ज्ञात कीजिए $a_n = \left(-\frac{1}{2}\right)^n \left(\left(\frac{1}{2}\right)^n + \frac{1}{n+1}\right)$ जब $n \geq 1$ है।
1. लिम sup $a_n = \sqrt{2}$
2. लिम inf $a_n = -\infty$
3. लिम $a_n = \sqrt{2}$
4. सैप $a_n = \sqrt{2}$

21. Define the sequence $(a_n)$ as follows:

$$a_n = 1 \text{ and for } n \geq 1, a_{n+1} = \left(-\frac{1}{2}\right)^n \left(\left(\frac{1}{2}\right)^n + \frac{1}{n+1}\right)$$

Which of the following is true?
1. लिम sup $a_n = \sqrt{2}$
2. लिम inf $a_n = -\infty$
3. लिम $a_n = \sqrt{2}$
4. सैप $a_n = \sqrt{2}$

22. निम्नलिखित समय (real numbers) का एक अन्तरालिका अनुक्रम (convergent sequence) है?

(i) $(a_n)$ अन्तरालिका समय का एक अनंतिक अनुक्रम (bounded sequence) है।
2. $\{x_n + y_n\}$ परिपथ है।
3. $\{x_n - y_n\}$ का कोई भी अन्तरालिका अनुक्रम (convergent subsequence) नहीं है।
4. $\{x_n - y_n\}$ का कोई भी परिपथ अनुक्रम (bounded subsequence) नहीं है।

22. If $(x_n)$ is a convergent sequence in $\mathbb{R}$ and $(y_n)$ is a bounded sequence in $\mathbb{R}$, then we can conclude that:
1. $(x_n + y_n)$ is convergent
2. $(x_n - y_n)$ is bounded
3. $(x_n - y_n)$ has no convergent subsequence
4. $(x_n + y_n)$ has no bounded subsequence

23. समय $\log(2) = \sum_{n=1}^{\infty} \frac{1}{2^n - n}$ है?
1. सैप से केवल
2. $1$ से समांतर
3. $\frac{1}{2n+1}$ से समांतर
4. $\frac{1}{2n+1}$ से अधिक

23. The difference

$$\log(2) = \sum_{n=1}^{100} \frac{1}{2^n - n}$$

1. less than $0$
2. greater than $1$
3. less than $\frac{1}{100}$
4. greater than $\frac{1}{100}$

24. निम्नलिखित

$$f(x, y) = \log\left(\cos^2(x + y)\right) + \sin(x + y)$$

$\frac{\partial}{\partial x} f(x, y) = (1)$
$\frac{\partial}{\partial y} f(x, y) = (2)$

1. $\cos(x + y) - \cos(x + y)$
2. $0$
3. $-\sin(x + y)$
4. $\cos(x + y)$
24. Let \( f(x, y) = \log \left( \cos^2(e^{xy}) \right) + \sin(x + y). \)

Then \( \frac{\partial}{\partial y} f(x, y) \) is

1. \( \frac{\cos(e^{xy})}{\sin(e^{xy})} \cdot e^{xy} \cdot \cos(x + y) \)
2. 0
3. \(-\sin(x + y)\)
4. \(\cos(x + y)\)

25. Let \( A \) be a \( m \times n \) matrix and \( B \) be a \( n \times m \) matrix over real numbers with \( m < n \). Then

1. \( AB \) is nonsingular
2. \( BA \) is singular
3. \( BA \) is nonsingular
4. \( BA \) is nonsingular

26. Let \( A \) be a \( (2 \times 2) \) matrix over \( R \) with \( \det(A + I) = 1 + \det(A) \), then we can conclude that

1. \( \det(A) = 0 \)
2. \( A = 0 \)
3. \( \text{Tr}(A) = 0 \)
4. \( A \) is nonsingular

27. The system of equations:

1. \( x + 2x^2 + 3xy + 0 \cdot y = 6 \)
2. \( 2x + 1x^2 + 3xy + 1y = 5 \)
3. \( 1x = 1x + 0 \cdot x + 1y = 7 \)

28. The system of equations:

1. \( x + 2 \cdot x^2 + 3 \cdot xy + 0 \cdot y = 6 \)
2. \( 2x + 1 \cdot x^2 + 3 \cdot xy + 1y = 5 \)
3. \( 1x = 1x^2 + 0 \cdot xy + 1y = 7 \)

29. Given that there are real constants \( a, b, c, d \) such that the identity

30. The trace of the matrix

\[
\begin{pmatrix}
0 & 2 & 0 \\
0 & 0 & 3
\end{pmatrix}
\]

1. \( 1 + 2 \cdot 3^2 \)
2. \( 3 \cdot 2 + 3 \cdot 3^2 \)
3. \( 2 + 2^2 + 3^2 \)
4. \( 2^2 + 3^2 + 1 \)
\[ ax^2 + 2cy + y^2 = (ax + by)^2 + (cx + dy)^2 \]
holds for all \(x, y \in \mathbb{R} \). This implies
1. \( \lambda = -5 \)
2. \( \lambda \geq 1 \)
3. \( 0 < \lambda < 1 \)
4. there is no such \( \lambda \in \mathbb{R} \).

30. Consider the set of matrices \( \{A_1, A_2, ..., A_n \} \) of order \( n \times n \) such that \( A_i A_j = A_j A_i \) for all \( i, j \). Let \( \det(A) \neq 0 \). Then
1. \( A = A^{-1} \)
2. \( A = A^T \)
3. \( A^{-1} = A^T \)
4. \( \det(A) = 1 \).

32. Let \( \mathbb{R}^n, n \geq 2 \), be equipped with standard inner product. Let \( \{v_1, v_2, ..., v_n \} \) be \( n \) column vectors forming an orthonormal basis of \( \mathbb{R}^n \). Let \( A \) be the \( n \times n \) matrix formed by the column vectors \( v_1, ..., v_n \). Then
1. \( A = A^{-1} \)
2. \( A = A^T \)
3. \( A^{-1} = A^T \)
4. \( \det(A) = 1 \).

33. Given \( \{a_n\}, \{b_n\} \) two monotone sequences of real numbers and that \( \sum a_n, \sum b_n \) are convergent, which of the following is true?
1. \( \sum a_n \) is convergent and \( \sum b_n \) is convergent.
2. At least one of \( \sum a_n, \sum b_n \) is convergent.
3. \( \{a_n\} \) is bounded and \( \{b_n\} \) is bounded.
4. At least one of \( \{a_n\}, \{b_n\} \) is bounded.

34. Consider the set \( \mathbb{Q} \) of rational numbers. Let \( S \) be a non-empty set. Then
1. \( S \) is a set (finite or empty).
2. \( S \) is countable.
3. \( S \) is uncountable.
4. \( S \) is empty.

UNIT-2

33. Let \( f: \mathbb{R} \rightarrow \mathbb{R} \) be a function. Which of the following statements is true?
1. \( f \) is a constant function.
2. \( f \) is a linear function.
3. \( f \) is a polynomial function.
4. \( f \) is an exponential function.

34. Let \( f: \mathbb{R} \rightarrow \mathbb{R} \) be a function. Which of the following conditions can possibly be satisfied by \( f \)?
1. \( f \) is a constant function.
2. \( f \) is a linear function.
3. \( f \) is a polynomial function.
4. \( f \) is an exponential function.
34. Consider the map \( \varphi : \mathbb{C}(1) \to \mathbb{C} \) given by \( \varphi(z) = \frac{1}{z} \).
Which of the following is true?
1. \( \varphi([z \in \mathbb{C} | |z| < 1]) \subseteq [x \in \mathbb{C} | |x| < 1] \)
2. \( \varphi([z \in \mathbb{C} | Re(z) < 0]) \subseteq [x \in \mathbb{C} | Re(x) < 0] \)
3. \( \varphi \) is onto
4. \( \varphi(\mathbb{C}(1)) = \mathbb{C}(\{0\}) \)

35. Let \( S_7 \) be the symmetric group of permutations of the set \( \{1, 2, 3, 4, 5, 6, 7\} \). Which of the following is true?
1. \( S_7 \) is a group of order 6.
2. \( S_7 \) is a group of order 7.
3. \( S_7 \) is a group of order 5.
4. \( S_7 \) is a group of order 0.

36. Let \( G \) be a group. Which of the following is true?
1. \( G = \{e\} \)
2. \( G \) is the trivial group.
3. \( G \) is the cyclic group.
4. \( G \) is the symmetric group.

37. Let \( f(x) = x^5 - 5x + 2 \) be a continuous function. Which of the following is true?
1. \( f \) has no real root.
2. \( f \) is exactly one real root.
3. \( f \) is exactly three real roots.
4. \( f \) is exactly five real roots.

38. Suppose that \( f \) is a non-constant analytic function defined on \( \mathbb{C} \). Which of the following is false?
1. \( f \) is unbounded.
2. \( f \) sends open sets into open sets.
3. \( f \) is continuous on \( \mathbb{C} \).
4. The image of \( f \) is dense in \( \mathbb{C} \).

39. Evaluate the integral \( \int_{\gamma} e^{z^2} \, dz \) where \( \gamma \) is the unit circle in the complex plane.

40. Calculate the value of the integral \( \int_{\gamma} e^{z^2} \, dz \) for \( \gamma \) the unit circle.
\[ \int_{1-x}^{x} \frac{e^x}{x^2 - 1} \, dx \] is

1. 0 2. \( (\pi) e \)
3. \((\pi) e - (\pi) e^{-1} \)
4. \((e + e^{-1}) \)

UNIT 3

41. सोपी प्रण (Cauchy problem)

\[ 2u + 3y = 3 \]
\[ 3x - 2y = 0 \text{ when } x = 1 \]

यह सीधे नैसर्गिक रूप से नहीं है?
1. इसके लिए एक हल है?
2. इसके दो हल है?
3. इसके अनग्रन्थ है?
4. इसका कोई हल नहीं है?

41. The Cauchy problem

\[ 2u + 5uy = 5 \] \( u = 1 \) on the line \( 3x - 2y = 0 \)

1. exactly one solution
2. exactly two solutions
3. infinitely many solutions
4. no solution

42. यदि \( \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0, x \in \mathbb{R}, \ y > 0 \); \( u(x, 0) = f(x), \frac{\partial u}{\partial y}(x, 0) = 0, x \in \mathbb{R} \)

इसके लिए एक हल है?
1. \( u(x, t) = f(x) \)
2. \( u(x, t) = f(x) + g(x) \)
3. \( u(x, t) = f(x) - g(x) \)
4. \( u(x, t) = f(x) \cdot g(x) \)

43. यदि \( I = \int_{0}^{h} f(x) \, dx = h \left[ af(0) + bf \left( \frac{h}{2} \right) + cf(h) \right] \)

इसके लिए एक हल है?
1. \( a = 0, b = \frac{h}{2}, c = \frac{1}{2} \)
2. \( a = \frac{h}{2}, b = \frac{1}{2}, c = \frac{1}{4} \)
3. \( a = \frac{h}{4}, b = \frac{1}{4}, c = \frac{1}{8} \)
4. \( a = 0, b = \frac{1}{4}, c = \frac{1}{8} \)

44. The values of \( a, b, c \) such that

\[ \int_{0}^{h} f(x) \, dx = h \left[ af(0) + bf \left( \frac{h}{2} \right) + cf(h) \right] \]

is exact for polynomials \( f \) of degree at least as high as possible are?
1. \( a = 0, b = \frac{h}{2}, c = \frac{1}{4} \)
2. \( a = \frac{h}{2}, b = \frac{1}{4}, c = \frac{1}{8} \)
3. \( a = 0, b = \frac{1}{4}, c = \frac{1}{8} \)
4. \( a = 0, b = \frac{1}{8}, c = \frac{1}{16} \)

44. जब \( f(x) = f_{0}(y'x)^{2} + 2y \) तो उक्तकी

\[ y(0) = 0, y(1) = 1 \]

इसके लिए \( y \) का हल है?
1. \( \frac{23}{12} \)
2. \( \frac{21}{12} \)
3. \( \frac{10}{24} \)
4. अभिल प्रतिक नहीं है?

45. Consider \( f(x) = f_{0}(y'x)^{2} + 2y \) dx

subject to \( y(0) = 0, y(1) = 1 \). Then \( \ln f(y) \) is

1. \( \frac{23}{12} \)
2. \( \frac{21}{12} \)
3. \( \frac{18}{25} \)
4. does not exist.
\[ \Phi(x) = x^2 + \int_0^t e^{t-x} \phi(t) \, dt \]

45. The resolvent kernel for the integral equation

\[ \Phi(x) = x^2 + \int_0^x e^{t-x} \phi(t) \, dt \]

1. \( e^{x-x} \)
2. 1
3. \( e^{x-t} \)
4. \( x^2 + e^{x-t} \)

46. A simple pendulum (simple pendulum) is given by the Lagrangian:

\[ L = \frac{1}{2} m \dot{\theta}^2 + mgl \cos \theta \]

where \( m \) is the mass of the pendulum bob, \( g \) is the acceleration due to gravity, and \( \theta \) is the angle the pendulum makes with its vertical position. The equation of motion is:

\[ \ddot{\theta} + \frac{g}{l} \sin \theta = 0 \]

1. \( H(p, \theta) = \frac{p^2}{2m} + mgl \cos \theta \)
2. \( H(p, \theta) = \frac{p^2}{2m} - mgl \cos \theta \)
3. \( H(p, \theta) = \frac{p^2}{2m} - mgl \cos \theta \)
4. \( H(p, \theta) = \frac{p^2}{2m} + mgl \cos \theta \)

47. Consider the ordinary differential equation:

\[ y'' + P(x)y' + Q(x)y = 0 \]

1. \( y(0) = 1.2 \) is a solution.
2. \( y'(0) = 2.5 \) is a solution.
3. \( y(0) < 0 \) is bounded below.
4. \( y(0) < 0 \) is unbounded.

48. Consider the ordinary differential equation:

\[ y'' + P(x)y' + Q(x)y = 0 \]

1. If \( y(0) = 0 \) then \( y \) is not decreasing.
2. If \( y(0) = 1.2 \) then \( y \) is increasing.
3. If \( y(0) = 2.5 \) then \( y \) is unbounded.
4. If \( y(0) < 0 \) then \( y \) is bounded below.
49. A Markov chain (Markov chain) is a sequence of states (state space) $S = \{1, 2, 3, 4\}$ with transition probability matrix $P = (p_{ij})$ given by:

\[
\begin{bmatrix}
1 & 2 & 3 & 4 \\
1/2 & 0 & 1/2 & 0 \\
1/4 & 1/4 & 1/4 & 1/4 \\
3/4 & 0 & 1/3 & 1/3 \\
4/5 & 0 & 1/2 & 0
\end{bmatrix}
\]

1. $\lim_{n \to \infty} p_{1,2}^{(n)} = 0$, $\sum_{n=0}^{\infty} p_{1,2}^{(n)} = \infty$
2. $\lim_{n \to \infty} p_{2,1}^{(n)} = 0$, $\sum_{n=0}^{\infty} p_{2,1}^{(n)} < \infty$
3. $\lim_{n \to \infty} p_{1,2}^{(n)} = 1$, $\sum_{n=0}^{\infty} p_{1,2}^{(n)} = \infty$
4. $\lim_{n \to \infty} p_{2,1}^{(n)} = 1$, $\sum_{n=0}^{\infty} p_{2,1}^{(n)} < \infty$

50. Let $X_1, X_2, X_3$ be i.i.d. standard normal variables. Which of the following is true?

\[\begin{align*}
1. & \quad \frac{X_1^2 + X_2 + X_3}{\sqrt{2}} \sim \chi_2 \\
2. & \quad \frac{X_1^2 - 2X_3}{\sqrt{2}} \sim \chi_5 \\
3. & \quad \frac{(X_1 - X_2)^2}{X_1^2 + X_2^2} \sim F_{2,2} \\
4. & \quad \frac{X_1^2}{X_1^2 + X_2^2} \sim F_{1,3}
\end{align*}\]

51. Consider a Markov chain having state space $S = \{1, 2, 3, 4\}$ with transition probability matrix $P = (p_{ij})$ given by:

\[
\begin{bmatrix}
1 & 2 & 3 & 4 \\
1/2 & 0 & 1/2 & 0 \\
1/4 & 1/4 & 1/4 & 1/4 \\
3/4 & 0 & 1/3 & 1/3 \\
4/5 & 0 & 1/2 & 0
\end{bmatrix}
\]

Then $\lim_{n \to \infty} p_{1,2}^{(n)} = 0$, $\sum_{n=0}^{\infty} p_{1,2}^{(n)} = \infty$

2. $\lim_{n \to \infty} p_{2,1}^{(n)} = 0$, $\sum_{n=0}^{\infty} p_{2,1}^{(n)} < \infty$
3. $\lim_{n \to \infty} p_{1,2}^{(n)} = 1$, $\sum_{n=0}^{\infty} p_{1,2}^{(n)} = \infty$
4. $\lim_{n \to \infty} p_{2,1}^{(n)} = 1$, $\sum_{n=0}^{\infty} p_{2,1}^{(n)} < \infty$

51. Suppose that the lifetime of an electric bulb follows an exponential distribution with mean $\theta$ hours. In order to estimate $\theta$, $n$ bulbs are switched on at the same time. After $T$ hours, $m$ ($m > 0$) bulbs are found to be in functioning state. If the lifetimes of the other $n-m$ ($> 0$) bulbs are noted as $x_1, x_2, \ldots, x_{n-m}$, then $\theta$ can be estimated using the maximum likelihood estimate (MLE) $\hat{\theta}$.

\[\hat{\theta} = \frac{1}{m} \sum_{i=1}^{m} \ln x_i - \frac{1}{n-m} \sum_{i=m+1}^{n} \ln x_i\]
maximum likelihood estimate of $\theta$ is given by
1. $\hat{\theta} = \frac{1}{\ln(10^{-\theta})}$
2. $\hat{\theta} = \frac{\sum x_i}{n}$
3. $\hat{\theta} = \frac{\sum x_i (x_i - n\theta)}{n}$
4. $\hat{\theta} = \frac{\sum x_i (x_i - n\theta)}{n}$

52. Suppose random variables $X_1, X_2, \ldots, X_n$ are independent and identically distributed (i.i.d.) with distribution $F_1$ and $F_2$. Suppose $F_1 < F_2$ on the parameter $\theta$. What are the likelihood ratios of $F_1$ and $F_2$? (The likelihood ratio is an auxiliary statistic, which is always non-decreasing.)

1. If $k < n$, then $\frac{L_{F_2}}{L_{F_1}}$ is a valid likelihood ratio.
2. If $k < n$, then $\frac{L_{F_2}}{L_{F_1}}$ is a valid likelihood ratio.
3. If $k < n$, then $\frac{L_{F_2}}{L_{F_1}}$ is a valid likelihood ratio.
4. If $k > 1$, then $\frac{L_{F_2}}{L_{F_1}}$ is a valid likelihood ratio.

53. Consider the problem of estimating a parameter $\theta$ on the basis of $X$, where $X \sim N(\theta, 1)$ and $-\infty < \theta < \infty$. Under squared error loss, $X$ has uniformly smaller risk than that of $kX$, for
1. $k > 0$
2. $0 < k < 1$
3. $k > 1$
4. no value of $k$

54. Testing against all alternatives: Suppose $F_1$ and $F_2$ are two different distributions. Let $X$ be a random variable generated according to $F_1$ or $F_2$. Consider the following test statistic:
1. $P[F_{4.55} \geq 1.5]$
2. $P[F_{4.45} \geq 1.6]$
3. $P[F_{4.55} \geq 3.6]$
4. $P[F_{4.48} \geq 2.5]$

55. For a $n \times n$ random normal vector $X$ with covariance matrix $\Sigma$,
$$\begin{pmatrix} 1 & \rho & \rho \\ \rho & 1 & \rho \\ \rho & \rho & 1 \end{pmatrix} : \rho < 0$$
55. The covariance matrix of a four dimensional random vector \( \mathbf{X} \) is of the form
\[
\begin{pmatrix}
\rho & \rho & \rho & \rho \\
\rho & 1 & \rho & \rho \\
\rho & \rho & 1 & \rho \\
\rho & \rho & \rho & 1
\end{pmatrix}, \text{ where } \rho < 0.
\]
If \( \rho \) is the variance of the first principal component, then
1. \( \rho \) cannot exceed 5/4.
2. \( \rho \) can exceed 5/4, but cannot exceed 4/3.
3. \( \rho \) can exceed 4/3, but cannot exceed 3/2.
4. \( \rho \) can exceed 3/2.

56. In a Latin Square Design the “error degrees of freedom” is 30. The “treatment degrees of freedom” for any treatment is
\[
\begin{align*}
1 & : 4 \\
2 & : 5 \\
3 & : 6 \\
4 & : 7
\end{align*}
\]
57. Suppose that \( |3x| + |2y| \leq 1 \) in a particular \( 9x + 4y \) of the maximum mean is
\[
\begin{align*}
1 & : 1 \\
2 & : 2 \\
3 & : 3 \\
4 & : 4
\end{align*}
\]
58. A simple random sample of size \( n \) will be drawn from a class of 25 students, and the mean and variance of the sample will be computed. If the standard error of the sample mean for “with replacement sampling” is twice as much as the standard error of the sample mean for “without replacement sampling,” the value of \( n \) is
\[
\begin{align*}
1 & : 32 \\
2 & : 39 \\
3 & : 79 \\
4 & : 94
\end{align*}
\]
60. Let $X$ and $Y$ be i.i.d. uniform $(0, 1)$ random variables. Let $Z = \max (X, Y)$ and $W = \min (X, Y)$.
Then $P(\{Z > W > 1/2\})$ is
1. $1/2$
2. $3/4$
3. $1/4$
4. $2/3$

**PART C**

**Unit 1**

61. Given $C_r(R) = \{ f: \mathbb{R} \rightarrow \mathbb{R} | f \text{ is compact} \}$ and $K$ is a compact set, show that $f(x) = 0$ for all $x \in K$. Let $g(x) = e^{-x^2} \forall x \in \mathbb{R}$ to show $\int g(x) dx$ is finite.
1. Let $C_r(R)$ be given by the formula $\left( e^{-x^2} \right)$ uniformly for all $x \in R$. Which of the following statements are true?
   - $\int g(x) dx$ is finite.
   - $e^{-x^2}$ is uniformly continuous on $R$.
   - $e^{-x^2}$ is bounded on $R$.
   - $\forall x \in R, e^{-x^2}$ is bounded.

62. Given $a(n) = \frac{1}{\log n}$.
   - $b(n) = 10^{\log n} \log n$.
   - $c(n) = \frac{1}{n^2} n^2$.
   - Which of the following statements are true?
   1. $a(n) > b(n)$ for all sufficiently large $n$.
   2. $b(n) > c(n)$ for all sufficiently large $n$.
   3. $c(n) > a(n)$ for all sufficiently large $n$.

63. Given $f: \mathbb{R} \rightarrow \mathbb{R}$ and $f(x) = \frac{a}{x+b}, a, b \in \mathbb{R}, b \neq 0$.

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = \frac{a}{x+b}, a, b \in \mathbb{R}, b \neq 0$.

Which of the following statements are true?
1. $f$ is uniformly continuous on $\mathbb{R}$ for all values of $a$ and $b$.
2. $f$ is uniformly continuous on $\mathbb{R}$ and is bounded for all values of $a$ and $b$.
3. $f$ is uniformly continuous on $\mathbb{R}$ only if $b = 0$.
4. $f$ is uniformly continuous on $\mathbb{R}$ and unbounded if $a \neq 0, b \neq 0$.
64. Let:
\[ a = \int_{0}^{\infty} \frac{1}{1 + e^x} \, dx. \]
Which of the following are true?
1. \[ \frac{a}{2} = \frac{1}{e^2}. \]
2. \(a\) is a rational number
3. \(\log(a) = 1\)
4. \(\sin(a) = 1\)

65. Which of the following functions are of bounded variation?
1. \(x \in (-1, 1)\) for \(x^2 + x + 1\)
2. \(x \in (-1, 1)\) for \(\tan\left(\frac{x}{2}\right)\)
3. \(x \in (-\pi, \pi)\) for \(\sin\left(\frac{x}{2}\right)\)
4. \(x \in (-1, 1)\) for \(\sqrt{1-x^2}\)

66. Let \(M_n(R)\) denote the space of all \(n \times n\) real matrices. Fix a column vector \(x \neq 0\) in \(R^n\). Define \(f: M_n(R) \to R\) by \(f(A) = (A^2, x)\). Then
1. \(f\) is linear
2. \(f\) is differentiable
3. \(f\) is continuous but not differentiable
4. \(f\) is unbounded

67. Which of the following sets \(y\) are in \([x]\) such that \(x > 0\) and \(y > 0\)?
1. \(y \in R^2\)
2. \(y \in R\) for \(x = n\)
3. \(y \in R\) for \(x = f(x, y)\)
4. \(y \in R\) for \(x = f(x, y)\)

68. Let \(f: R^2 \to R\) be defined by \(f(x, y) = x^2 + y^2\). Then
1. \(f\) is continuous on \(R^2\)
2. \(f\) is continuous on \(R\{0\}\)
3. \(f\) is continuous on \(R\)
4. \(f\) is continuous at no point of \(R^2\)

69. Let \(M_n(R)\) denote the space of all \(n \times n\) real matrices. Fix a column vector \(x \neq 0\) in \(R^n\). Define \(f: M_n(R) \to R\) by \(f(A) = (A^2, x)\). Then
1. \(f\) is linear
2. \(f\) is differentiable
3. \(f\) is continuous but not differentiable
4. \(f\) is unbounded

68. Let \(V\) denote the vector space of all sequences \(\alpha = (\alpha_1, \alpha_2, ...)\) of real numbers such that \(\sum |\alpha_i|\) converges. Define \(\|\cdot\|: V \to R\) by \(\|\alpha\| = \sum |\alpha_i|\). Which of the following are true?
1. \(V\) contains only the sequence \((0, 0, ...)\)
2. \(V\) is finite dimensional
3. \(V\) has a countable linear basis
4. \(V\) is a complete normalized space

69. Let \(V\) denote the vector space of all sequences \(\mathbf{a} = (\alpha_1, \alpha_2, ...)\) of real numbers such that \(\sum |\alpha_i|\) converges. Define \(\|\cdot\|: V \to R\) by \(\|\alpha\| = \sum |\alpha_i|\). Which of the following are true?
1. \(V\) contains only the sequence \((0, 0, ...)\)
2. \(V\) is finite dimensional
3. \(V\) has a countable linear basis
4. \(V\) is a complete normalized space
(linear transformation) की विकल्प उपलब्ध 
अवकलज्ञता रूप (eigenvalue) 1 हो तो इनके से 
कौन के विकल्प रूप (eigenvalue) 2 हो सकते हैं?
1. \( T - I = 0 \)
2. \( (T - I)^2 = 0 \)
3. \( (T - I)^3 = 0 \)
4. \( (T - I)^4 = 0 \)

69. दिने \( V \) एक वektor space over \( C \) इके शिर्ठता \( n \) हो। \( T: V \rightarrow V \) एक लाइनेर एक्सट्रीमल के अभिलाख अवलंबित रूप (linear transformation) जहाँ इनके शिर्धता \( n \) हो सकते हैं?
1. \( T = I \)
2. \( (T - I)^2 = 0 \)
3. \( (T - I)^3 = 0 \)
4. \( (T - I)^4 = 0 \)

70. निम्न \( A \) एक \( (5 \times 5) \) matrix हो जिसके लिए 
संस्थान का \( Ax = 0 \) के लिये \( A \) की उपकरण (vector space) की विकल्प (dimension) \( n \) हो 
1. \( \text{Rank}(A^3) \leq 3 \)
2. \( \text{Rank}(A^3) = 3 \)
3. \( \text{Rank}(A^3) = 3 \)
4. \( \text{Det}(A^3) = 0 \)

71. निम्न के लिए \( A \in M_5(R) \) के लिए \( A^5 = I_{5 \times 5} \) हो सकते हैं?
1. \( A \) के अवकलज्ञता \( A \) अवकलज्ञता (minimal polynomial) 
2. \( A \) के अवकलज्ञता \( A \) अवकलज्ञता (minimal polynomial) 
3. \( A \) के अवकलज्ञता \( A \) अवकलज्ञता (minimal polynomial) 
4. \( A \) के अवकलज्ञता \( A \) अवकलज्ञता (minimal polynomial) 

72. अवकलज्ञता \( A \) एक \( n \times n \) अवकलज्ञता \( n \) हो (\( n > 1 \)). \( A^2 \) \( 7(A + 12I) = 0 \) \( \text{Det}(A) \) \( A \) \( A \) के अवकलज्ञता \( A \) अवकलज्ञता (minimal polynomial) \( 0 \) \( \text{Det}(A) \) \( A \) के अवकलज्ञता \( A \) अवकलज्ञता (minimal polynomial) 
1. \( A \) के अवकलज्ञता \( A \) अवकलज्ञता (minimal polynomial) 
2. \( A^2 - 7A + 12I = 0 \) \( \text{Det}(A) \) \( A \) के अवकलज्ञता \( A \) अवकलज्ञता (minimal polynomial) 
3. \( A^2 - 7A + 12I = 0 \) \( \text{Det}(A) \) \( A \) के अवकलज्ञता \( A \) अवकलज्ञता (minimal polynomial)
4. \( A^2 - 7A + 12I = 0 \) \( \text{Det}(A) \) \( A \) के अवकलज्ञता \( A \) अवकलज्ञता (minimal polynomial) 
5. \( A^2 - 7A + 12I = 0 \) \( \text{Det}(A) \) \( A \) के अवकलज्ञता \( A \) अवकलज्ञता (minimal polynomial) 

73. \( \text{Det}(A) \) \( A \) एक \( 6 \times 6 \) अवकलज्ञता \( A \) के अवकलज्ञता \( A \) अवकलज्ञता (characteristic polynomial) \( x - 3 \) \( x - 2 \) \( A \) के अवकलज्ञता \( A \) के अवकलज्ञता (minimal polynomial) \( x - 3 \) \( x - 2 \) \( A \) के अवकलज्ञता \( A \) के अवकलज्ञता (minimal polynomial) 
1. \( A \) के अवकलज्ञता \( A \) के अवकलज्ञता (minimal polynomial) 
2. \( A \) के अवकलज्ञता \( A \) के अवकलज्ञता (minimal polynomial) 
3. \( A \) के अवकलज्ञता \( A \) के अवकलज्ञता (minimal polynomial) 
4. \( A \) के अवकलज्ञता \( A \) के अवकलज्ञता (minimal polynomial) 

74. अवकलज्ञता \( A \) के \( 3 \times 3 \) अवकलज्ञता \( A \) के अवकलज्ञता (minimal polynomial) \( (x - 3)(x - 2) \) \( A \) के अवकलज्ञता \( A \) के अवकलज्ञता (minimal polynomial) 
1. \( A \) के अवकलज्ञता \( A \) के अवकलज्ञता (minimal polynomial) 
2. \( A \) के अवकलज्ञता \( A \) के अवकलज्ञता (minimal polynomial) 
3. \( A \) के अवकलज्ञता \( A \) के अवकलज्ञता (minimal polynomial) 
4. \( A \) के अवकलज्ञता \( A \) के अवकलज्ञता (minimal polynomial) 

75. अवकलज्ञता \( A \) के \( 3 \times 3 \) अवकलज्ञता \( A \) के अवकलज्ञता (minimal polynomial) \( (x - 3)(x - 2) \) \( A \) के अवकलज्ञता \( A \) के अवकलज्ञता (minimal polynomial) 
1. \( A \) के अवकलज्ञता \( A \) के अवकलज्ञता (minimal polynomial) 
2. \( A \) के अवकलज्ञता \( A \) के अवकलज्ञता (minimal polynomial) 
3. \( A \) के अवकलज्ञता \( A \) के अवकलज्ञता (minimal polynomial) 
4. \( A \) के अवकलज्ञता \( A \) के अवकलज्ञता (minimal polynomial) 

76. अवकलज्ञता \( A \) के \( 3 \times 3 \) अवकलज्ञता \( A \) के अवकलज्ञता (minimal polynomial) \( (x - 3)(x - 2) \) \( A \) के अवकलज्ञता \( A \) के अवकलज्ञता (minimal polynomial) 
1. \( A \) के अवकलज्ञता \( A \) के अवकलज्ञता (minimal polynomial) 
2. \( A \) के अवकलज्ञता \( A \) के अवकलज्ञता (minimal polynomial) 
3. \( A \) के अवकलज्ञता \( A \) के अवकलज्ञता (minimal polynomial) 
4. \( A \) के अवकलज्ञता \( A \) के अवकलज्ञता (minimal polynomial)
74. Let $V$ be an inner product space and $S$ be a subset of $V$. Let $\bar{S}$ denote the closure of $S$ in $V$ with respect to the topology induced by the metric given by the inner product. Which of the following statements are true?
1. $S = (S^+)\overline{1}$
2. $\bar{S} = (S^+)\overline{1}$
3. span$(S) = (S^+)\overline{1}$
4. $S^+ = (S^+)\overline{1}$

75. Let $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & -2 \end{pmatrix}$.

1. $Q(x,y,z) = (x,y,z)A \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

76. Which of the following statements are true?
1. The matrix of second order partial derivatives of the quadratic form $Q$ is $2A$.
2. The rank of the quadratic form $Q$ is 2.
3. The signature of the quadratic form $Q$ is $\{+, +\}$.
4. The quadratic form $Q$ takes the value $0$ for some non-zero vector $(x, y, z)$.

76. Let $A = \{t \sin \left(\frac{\pi}{2}\right) \mid t \in \left(0, \frac{\pi}{2}\right)\}$.
1. The Lebesgue measure of $A$ is $0$.
2. $A$ contains a non-empty open set.
3. $A$ is path connected.
4. Every open set containing $A$ has infinite Lebesgue measure.

77. Which of the following sets are uncountable?
1. The set of all functions from $\mathbb{R}$ to $[0, 1]$
2. The set of all functions from $[0, 1]$ to $\mathbb{N}$
3. The set of all finite subsets of $\mathbb{N}$
4. The set of all subsets of $\mathbb{N}$
80. Which of the following statements are true?
1. Every compact metric space is separable.
2. If a metric space \((X, d)\) is separable, then the metric of \(X\) is not the discrete metric.
3. Every separable metric space is second countable.
4. Every first countable topological space is separable.

81. Which of the following statements are true?
1. A principal ideal domain (integral domain) is a unique factorization domain.
2. A unique factorization domain is a principal ideal domain.
3. A principal ideal domain is a Euclidean domain.
4. A Euclidean domain is an integral domain.

82. Let \(f(x) \in \mathbb{Z}[x]\) be a monic polynomial.
1. \(f(0) = 0\) implies \(\deg f = 1\).
2. \(f(0)\) is a prime number.
3. \(f(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0\) for some integers \(a_i\).
4. \(\gcd(a_0, a_1, \ldots, a_n) = 1\).

83. Which of the following statements are true?
1. A finite group is cyclic.
2. Every finite group is abelian.
3. Every finite group is a product of cyclic groups.
4. Every finite group is isomorphic to a cyclic group.

84. Suppose that \(f : \mathbb{C} \to \mathbb{C}\) is an analytic function. Then \(f\) is a polynomial if
1. \(f(0) = 0\).
2. \(f(z) = \sum a_n z^n\) has only finitely many terms.
3. \(f(1/n) \to 0\) as \(n \to \infty\).
4. \(|f(z)| \leq M|z|^n\) for some \(M\) and for all \(z\) with \(|z| > R\).
85. Let $\Omega$ be an open connected subset of $\mathbb{C}$. Let $E = \{z_1, z_2, \ldots, z_n\} \subseteq \Omega$. Suppose that $f : \Omega \to \mathbb{C}$ is a function such that $f(z)_{(z_1)}$ is analytic. Then $f$ is analytic on $\Omega$ if:
1. $f$ is continuous on $\Omega$.
2. $f$ is bounded on $\Omega$.
3. For every $f$, if $\sum_{m=\infty}^\infty e_m (z - z_j)^m$ is analytic at $z_j$, then $a_m = 0$ for $m = -1, -2, -3, \ldots$.
4. For every $f$, if $\sum_{m=\infty}^\infty e_m (z - z_j)^m$ is analytic at $z_j$, then $a_m = 0$ for $m = -1, -2, -3, \ldots$.

86. Let $S$ be the set of polynomials $f(x)$ with integer coefficients satisfying $f(x) \equiv 1 \pmod{x - 1}$ or $f(x) \equiv 0 \pmod{x - 3}$. Which of the following statements are true?
1. $S$ is empty.
2. $S$ is a singleton.
3. $S$ is a finite non-empty set.
4. $S$ is countably infinite.

87. Let $G = S_n$ be the permutation group of $n$ symbols. Then:
1. $G$ is isomorphic to a subgroup of a cyclic group.
2. There exists a nontrivial group $H$ such that $G$ maps homomorphically onto $H$.
3. $G$ is a product of cyclic groups.
4. There exists a nontrivial group homomorphism from $G$ to the additive group $\mathbb{Q}^+$ of rational numbers.

88. Suppose $K$ is a subgroup of $G$ with $|G| = 96$. Then:
1. $H \cap K = \{e\}$.
2. $H \cap K \neq \{e\}$.
3. $H \cap K$ is Abelian (Abelian).
4. $H \cap K$ is not Abelian (Non-Abelian).

89. If $|G| = 96$, then $G$ is isomorphic to $S_9$. Suppose $H$ and $K$ are subgroups of $G$ with $|H| = 12$ and $|K| = 16$. Then:
1. $H \cap K = \{e\}$.
2. $H \cap K \neq \{e\}$.
3. $H \cap K$ is Abelian.
4. $H \cap K$ is not Abelian.
3. If \( f(x) = \sum a_n x^n \) is a power series function, then \( f(x) = \sum b_n x^n \) is another power series function with the same radius of convergence. The condition for convergence is given by the ratio test:

\[
\lim_{n \to \infty} \left| \frac{b_{n+1}}{b_n} \right| = L
\]

4. The radius of convergence is defined as the largest value of \( r \) such that the series converges for \( |x| < r \). If \( r = \infty \), the series converges for all \( x \) in \( \mathbb{C} \).

UNIT-3

91. The values of \( \lambda \) for which the following equation has a non-trivial solution:

\[
\phi(x) = \lambda \int_0^x K(x,t) \phi(t) dt,
\]

where \( K(x,t) = \frac{\sin x \cos t}{x^2 - t^2} \) and \( 0 \leq x \leq \pi \) are

1. \( \left( \frac{n+1}{2} \right)^2 - 1, n \in \mathbb{N} \)
2. \( \frac{\pi^2}{4} - 1, n \in \mathbb{N} \)
3. \( \frac{\pi^2}{4} - 1, n \in \mathbb{N} \)
4. \( \frac{\pi^2}{4} - 1, n \in \mathbb{N} \)

92. The Hamiltonian (Hamiltonian) for the simple harmonic oscillator (simple harmonic oscillator) is given by:

\[
H(p,q) = \frac{p^2}{2m} + \frac{kq^2}{2}
\]

where \( H \) is the total energy, \( p \) is momentum, \( q \) is position, \( m \) is mass, and \( k \) is spring constant.
1. \[ L = \frac{1}{2}mq^2 - \frac{\beta}{2} q^2 \]
2. \[ L = \frac{1}{2}mq^2 - \frac{\beta}{2} (q^2 + 3q^2) \]
3. \[ L = \frac{1}{2}mq^2 + \frac{\beta}{2} q^2 \]
4. \[ L = \frac{1}{2}mq^2 + \frac{\beta}{2} (q^2 + 3q^2) q \]

93. The extremal for the functional
\[ J[y] = \int_0^1 y^2(x)dx \]
subject to \( y(0) = 0 \), \( y(1) = 1 \) and 
\[ \int_0^1 y(x)dx = 0 \] is 
1. \( 3x^3 - 2x \)
2. \( 8x^3 - 9x^2 + 2x \)
3. \( \frac{x^2}{2} - \frac{x}{2} \)
4. \( \frac{-21}{2} x^3 + 10x^4 + 4x^3 + \frac{5}{2} x \)

94. Consider the integral equation
\[ \phi(x) = \lambda \int_0^1 (\cos x \cos t - 2\sin x \sin t) \phi(t) dt + \cos 7x, \quad 0 \leq x \leq \pi \]
Which of the following statements are true?
1. For every \( \lambda \in \mathbb{R} \), a solution exists
2. There exists \( \lambda \in \mathbb{R} \) such that solution does not exist
3. There exists \( \lambda \in \mathbb{R} \) such that there are more than one but finitely many solutions
4. There exists \( \lambda \in \mathbb{R} \) such that there are infinitely many solutions.
1. Since \( \int_{a}^{b} |a(x)| \, dx < \infty \), hence \( y = \phi(x) \) is bounded (bounded function).

2. Since \( \int_{a}^{b} |a(x)| \, dx < \infty \), hence \( x(0) \) is bounded

\[ \lim_{x \to 0} x(0) = \phi(0) = 0. \]

3. If \( \lim_{x \to \pm \infty} a(x) = 0 \), then \( \lim_{x \to \pm \infty} a(y) = 0 \) holds.

4. Hence \( \lim_{x \to \infty} a(x) = 1 \) exists, hence \( y \) is monotone (monotone function).

96. Assume that \( a: [0, \infty) \to \mathbb{R} \) is a continuous

function. Consider the ordinary differential equation

\[ y''(x) + a(x)y(x), \quad x > 0, \quad y(0) = y_0 = 0. \]

Which of the following statements are true?

1. If \( \int_{0}^{\infty} |a(x)| \, dx < \infty \), then \( y \) is bounded

2. If \( \int_{0}^{\infty} |a(x)| \, dx < \infty \), then \( \lim_{x \to \infty} y(x) \) exists

3. If \( \lim_{x \to \infty} a(x) = 1 \), then \( \lim_{x \to \infty} y(x) = \infty \)

4. If \( \lim_{x \to \infty} a(x) = 1 \), then \( y \) is monotone

97. Given \( u(x,t) \) satisfies

\[ \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, \quad t > 0 \]

\[ u(x,0) = 1 + x + \sin(x) \cos(x), \quad u(0,t) = 1, \quad u(1,t) = 2. \]

Then verify that

1. \( u \left( \frac{1}{2}, t \right) = \frac{3}{2} \)

2. \( u \left( \frac{1}{2}, t \right) = \frac{3}{2} \)

3. \( u \left( \frac{3}{4}, t \right) = \frac{3}{4} + \frac{1}{4} e^{-3t} \)

4. \( u \left( \frac{1}{4}, t \right) = \frac{5}{8} + \frac{1}{4} e^{-3t} \)

98. Let \( \alpha \) be a fixed real constant. Consider the first order partial differential equation

\[ \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0, \quad x \in \mathbb{R}, \quad t > 0 \]

with the initial data \( u(x,0) = u_0(x), \quad x \in \mathbb{R} \) where

\[ u_0 \] is a continuously differentiable function.

Consider the following two statements:

1. There exists a bounded function \( u_0 \) for which the solution \( u \) is unbounded.

2. If \( u_0 \) vanishes outside a compact set for each fixed \( T > 0 \) there exists a compact set \( K_T \subset \mathbb{R} \) such that

\[ u(x, T) \] vanishes for \( x \in K_T \).

Which of the following are true?

1. \( u_0 \) is true and \( S_2 \) is false

2. \( S_2 \) is true and \( S_2 \) is also true

3. \( S_2 \) is false and \( S_2 \) is true

4. \( S_2 \) is false and \( S_2 \) is also false

99. If \( u(x,t) \) is the solution of

\[ \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, \quad t > 0 \]

\[ u(x,0) = 1 + x + \sin(x) \cos(x), \quad u(0,t) = 1, \quad u(1,t) = 2. \]

Then verify that

1. \( u \left( \frac{1}{2}, t \right) = \frac{3}{2} \)

2. \( u \left( \frac{1}{2}, t \right) = \frac{3}{2} \)

3. \( u \left( \frac{3}{4}, t \right) = \frac{3}{4} + \frac{1}{4} e^{-3t} \)

4. \( u \left( \frac{1}{4}, t \right) = \frac{5}{8} + \frac{1}{4} e^{-3t} \)

The matrices

\[ A = I + D + U \]

are known as

1. \( A \) is a special type of matrix called singular matrices.

2. \( A \) is an upper triangular matrix.

3. \( A \) is a lower triangular matrix.

The Gauss-Seidel iterative method
\[ x^{(k+1)} = Hx^{(k)} + c, \quad k = 0, 1, 2, \ldots \] where \( x \) is an asymptotically convergent (converge) upon the solution \( x \) and \( H \) is a diagonal matrix. Let \( x^* \) be the solution of \( Ax = b \). Then the Gauss-Seidel iteration method \[ x^{(k+1)} = Hx^{(k)} + c, \quad k = 0, 1, 2, \ldots \] with \( ||H|| < 1 \) converges to \( x^* \) provided \( H \) is equal to

1. \[ -D^{-1}(L+U) \]
2. \[ -(L+D)U^{-1} \]
3. \[ -D(U+D)^{-1} \]
4. \[ -U(D-L)^{-1} \]

99. Assume that a non-singular matrix \( A = L + D + U \) where \( L \) and \( U \) are lower and upper triangular matrices respectively with all diagonal entries are zero, and \( D \) is a diagonal matrix. Let \( x^* \) be the solution of \( Ax = b \). Then the Gauss-Seidel iteration method \[ x^{(k+1)} = Hx^{(k)} + c, \quad k = 0, 1, 2, \ldots \] with \( ||H|| < 1 \) converges to \( x^* \) provided \( H \) is equal to

1. \[ -D^{-1}(L+U) \]
2. \[ -(L+D)U^{-1} \]
3. \[ -D(U+D)^{-1} \]
4. \[ -U(D-L)^{-1} \]

100. The forward difference operator is defined as \( U_{n+1} = U_n + \Delta U_n \) (difference equation) which is a linear equation having an unbounded solution.

1. \[ \Delta U_n = 3U_n + 2U_n = 0 \]
2. \[ \Delta U_n + \Delta U_n + 2U_n = 0 \]
3. \[ \Delta U_n = 3U_n + 2U_n = 0 \]
4. \[ \Delta U_n + \frac{1}{2} \Delta^2 U_n = 0 \]

101. The system of differential equations

\[ \frac{dx}{dt} = 2x - 7y \]
\[ \frac{dy}{dt} = 3x - 8y \]

Then the critical point \((0,0)\) of the system is

1. asymptotically stable node
2. unstable node
3. asymptotically stable spiral
4. unstable spiral

102. Consider the system of differential equations

\[ \frac{dx}{dt} = 2x - 7y \]
\[ \frac{dy}{dt} = 3x - 8y \]

Then the critical point \((0,0)\) of the system is

1. asymptotically stable node
2. unstable node
3. asymptotically stable spiral
4. unstable spiral

103. The system of differential equations

\[ y'' + Ay = 0, \quad y(0) = 0, \quad y'(0) = 0 \]

is asymptotically stable (asymptotically stable node)

1. \( A \) is a countably many eigenvalues
2. \( A \) is an asymptotically stable node
3. \( A \) is an asymptotically stable matrix
4. \( A \) is a countably many eigenvalues

104. The system of differential equations

\[ y'' + Ay = 0, \quad y(0) = 0, \quad y'(0) = 0 \]

is asymptotically stable (asymptotically stable node)

1. \( A \) is a countably many eigenvalues
2. \( A \) is an asymptotically stable node
3. \( A \) is an asymptotically stable matrix
4. \( A \) is a countably many eigenvalues
103. Consider a single server $M/M/1$ queue with arrival rate $\lambda$ and service rate $\mu$. Further assume that $\lambda < \mu$. Then, which of the following statements are true? 
1. Queue length becomes 0 in infinitely many time intervals with probability 1.
2. Queue length becomes 0 in at most finitely many time intervals with probability 1.
3. Steady state exists for the queue.
4. $\lim_{t \to \infty} P(L(t) > 0) = \frac{1}{\mu}$ where $L(t)$ is the number of customers in the system at time $t$.

104. Let $x_1, x_2, \ldots, x_N$ be 20 observations in the interval $[0, 1]$. Let $\overline{x}$ and $\mu$ be the mean and the median of these observations, and let $s^2 = \frac{1}{N} \sum (x_i - \overline{x})^2$.
1. If $15$ observations are smaller than $0.3$, then $\overline{x}$ cannot be greater than $0.3$.
2. $s^2$ will be maximum if $10$ of these observations are $1$ and the rest are $0$.
3. If all observations except one are smaller than $0.5$, then $\overline{x}$ cannot be smaller than $x$.
4. $s^2 \leq \bar{x} (1 - \bar{x})$.

105. A statistician has drawn a simple random sample of size 2 with replacement from 4 boys with distinct heights. Let $\overline{x}$ be the sample mean of their heights. Then, another statistician has drawn a simple random sample of size 2 without replacement from those 4 boys. Let $\overline{x}'$ be the sample mean of their heights. Which of the following statements are correct? 
1. $\overline{x}'$ has larger variance than that of $(2x_1 + 3x_2)/5$.
2. $(\overline{x} + \overline{x}')/2$ has larger variance than that of $(2x_1 + 3x_2)/5$.
3. $(\overline{x} + \overline{x}')/2$ has smaller variance than that of $(2x_1 + 3x_2)/5$.
4. $(\overline{x} + \overline{x}')/3$ has smaller variance than that of $(2x_1 + 3x_2)/5$.

106. In a data set with mean 2.5 and standard deviation 0.5, the median must be bigger than 2.5.
3. the median must be smaller than 3
4. the median must be bigger than 2

107. उस पृथ्वी, h और m से विभिन्न {0,∞} में विभिन्न व्यक्ति अनसुगति का क्रम (life distribution function) तथा पृथ्वी (hazard function) है है जो विभिन्न (mean residual lifetime function) को निर्धारित करता है। यदि पृथ्वी असुगति (absolutely continuous) हो तो निम्न में से हो सकता है?
1. \( \int_0^\infty h(t)\,dt = 1 \)
2. \( m(t) = \frac{h(t)}{1 - F(t)} \), for \( t > 0 \)
3. यदि अवधारणा है कि व्यक्ति 0 > 0 असुगति तथा अनसुगति क्रम तो \( t > 0 \) से समय \( m(t) \) का मान कोई बिंदु है।
4. यदि अवधारणा है कि व्यक्ति 0 > 0 असुगति तथा अनसुगति क्रम तो \( t > 0 \) के लिए \( h(t)m(t) = 1 \) होगा।

107. Let \( F \), \( h \) and \( m \) be the lifetime distribution function, the hazard function and the mean residual lifetime function respectively, defined on \([0,\infty)\). Assume that \( F \) is absolutely continuous. Which of the following statements are true?
1. \( \int_0^\infty h(t)\,dt = 1 \)
2. \( m(t) = \frac{h(t)}{1 - F(t)} \), for \( t > 0 \)
3. \( m(t) \) is strictly increasing in \( t \) if the lifetime distribution is exponential with mean \( \lambda > 0 \)
4. \( h(t)m(t) = 1 \) for all \( t > 0 \) if the lifetime distribution is exponential with mean \( \lambda > 0 \)

108. अनसुगति क्रम (state space) \( S = \{1, 2, 3\} \) पर एक यांत्रिक सुस्कार का अवधारणा अनुसार (transition matrix)
\[
P = \begin{pmatrix}
0 & 1/2 & 1/2 \\
1/2 & 0 & 1/2 \\
1/2 & 1/2 & 0
\end{pmatrix}
\]
है। यदि \( S \) पर यांत्रिक सुस्कार का अवधारणा अनुसार (stationary distribution) \( \pi = (\pi_1, \pi_2, \pi_3) \) हो तो \( d(1) \) अवधारणा अनुसार \( S \) के अवधारणा अनुसार \( S \) के निम्न में से कौन सा सही है?
1. \( d(1) = 1 \)
2. \( d(1) = 2 \)
3. \( \pi_1 = 1/2 \)
4. \( \pi_3 = 1/3 \)

109. Consider a Markov chain on state space
\( S = \{1, 2, 3\} \) with transition probability matrix \( P \) given by
\[
P = \begin{pmatrix}
0 & 1/2 & 1/2 \\
1/2 & 0 & 1/2 \\
1/2 & 1/2 & 0
\end{pmatrix}
\]
Let \( \pi = (\pi_1, \pi_2, \pi_3) \) be a stationary distribution of the Markov chain and \( d(1) \) denote the period of state 1. Which of the following statements are correct?
1. \( d(1) = 1 \)
2. \( d(1) = 2 \)
3. \( \pi_1 = 1/2 \)
4. \( \pi_3 = 1/3 \)

109. अवधारणा \( S = \{1, 2, 3\} \) का अनुसार व्यक्ति बिंदु (identical independently distributed) जननिक चर का अवधारणा अनुसार \( E(X_i) = 0 \) और \( V(X_i) = 1 \) हो तो \( \bar{X} \) निम्न में से कौन सा सही है?
1. \[ \frac{1}{n} \sum_{i=1}^{n} X_i^2 \to 0 \] (आवश्यक)
2. \[ \frac{1}{n^{1/2}} \sum_{i=1}^{n} X_i \to 0 \] (आवश्यक)
3. \[ \frac{1}{n^{1/2}} \sum_{i=1}^{n} X_i \to 0 \] (आवश्यक)
4. \[ \frac{1}{n} \sum_{i=1}^{n} X_i^2 \to 1 \] (आवश्यक)

109. Let \( X_1, X_2, \ldots \) be a sequence of i.i.d. random variables with \( E(X_1) = 0 \) and \( V(X_1) = 1 \). Which of the following are true?
1. \[ \frac{1}{n} \sum_{i=1}^{n} X_i^2 \to 0 \] in probability
2. \[ \frac{1}{n^{1/2}} \sum_{i=1}^{n} X_i \to 0 \] in probability
3. \[ \frac{1}{n^{1/2}} \sum_{i=1}^{n} X_i \to 0 \] in probability
4. \[ \frac{1}{n} \sum_{i=1}^{n} X_i^2 \to 1 \] in probability
110. Let $X$ and $Y$ be i.i.d. exponential random variables with parameter $1$. Define, $W = X + Y$ and $U = X/(X + Y)$. Which of the following are true?
1. $E(U) = 1/2$
2. $U$ is uniform on $(0, 1)$
3. $W, U$ are independent
4. $W, U$ are uncorrelated (uncorrelated)

111. Let $X_1, X_2, ..., X_n$ be i.i.d. random variables (i.i.d.) and $Y_1, Y_2, ..., Y_n$ be independent. Let $\theta$ be a parameter of interest (probability mass function)
$$f(x, y) = \theta^x(1 - \theta)^{(n - 2)x} ; \theta \in (0, 1)$$
Find the following:
1. $X_i + 2X_j$ is a sufficient statistic (sufficient statistic)
2. $X_i - X_j$ is a sufficient statistic (sufficient statistic)
3. $X_i^2 + X_j^2$ is a sufficient statistic (sufficient statistic)
4. $X_i^2 + X_j^2$ is a sufficient statistic (sufficient statistic)

112. Let $X_1, X_2, ..., X_n, \eta \geq 3$ be independent random variables (i.i.d.) $N(\mu, \sigma^2)$ (correlation coefficient) $\rho$ such that
$$F_{X_i, X_j} = \left(1 - \rho^2\right) \frac{1}{2}\left(1 + \rho\right)$$
and $F_{X_i, Y_j} = \left(1 - \rho^2\right) \frac{1}{2}\left(1 + \rho\right)$

113. Suppose that for $n \geq 3, X_1, X_2, ..., X_n$ are i.i.d. $\sim N(\mu_1, \sigma_1^2)$ and $Y_1, Y_2, ..., Y_n$ are i.i.d. $\sim N(\mu_2, \sigma_2^2)$. Assume further that the $X_i$'s and the $Y_i$'s are independent. Let $\tau$ be the correlation coefficient computed from the bivariate data
$$(X_1, Y_1), (X_2, Y_2), ..., (X_n, Y_n)$$
Then
$$\tau = \frac{\text{cov}(X, Y)}{\sqrt{\text{var}(X)\text{var}(Y)}}$$
113. Suppose \( X \) and \( Y \) are two independent exponential random variables with means 0 and 1 respectively, where \( \theta \) is unknown. Which of the following statements are true?

1. \( X + 2Y \) is sufficient for \( \theta \)
2. Right-tailed test based on \( X + 2Y \) is UMP for testing \( H_0: \theta = 1 \) against \( H_1: \theta < 1 \)
3. Left-tailed test based on \( 2X + Y \) is UMP for testing \( H_0: \theta = 1 \) against \( H_1: \theta < 1 \)
4. UMP test does not exist for testing \( H_0: \theta = 1 \) against \( H_1: \theta > 1 \)

114. \( \frac{X}{\theta} \) is distributed as Gamma \((1, \theta)\). If \( X \) and \( Y \) are \( \text{i.i.d.} \), which of the following statements about \( X \) being distributed as Gamma \((1, \theta)\) are true?

1. \( X \) is distributed as Gamma \((1, \theta)\) if \( X \) is distributed as Gamma \((1, \theta)\) for each \( \theta \)
2. \( X \) and \( Y \) are independent Gamma \((1, \theta)\)
3. \( X \) is distributed as Gamma \((1, \theta)\) if \( X \) is distributed as Gamma \((1, \theta)\) for each \( \theta \)

115. If \( X \) is distributed as Gamma \((n, \lambda)\), which of the following statements are true?

1. \( \lambda > 0 \) is the mean of \( X \)
2. \( \lambda < 0 \) is the median of \( X \)
3. \( \lambda = 0 \) is the mode of \( X \)
4. \( \lambda = 1 \) is the variance of \( X \)

116. Suppose \( X \) and \( Y \) are two independent exponential random variables with means 0 and 1 respectively. Suppose that the distribution of \( X \) is Gamma \((1, \theta)\) and the distribution of \( Y \) is Bivariate Normal \((\mu, \Sigma)\). Which of the following statements are true?

1. \( X \) is distributed as Gamma \((1, \theta)\) if \( X \) is distributed as Gamma \((1, \theta)\) for each \( \theta \)
2. \( X \) and \( Y \) are independent Gamma \((1, \theta)\)
3. \( X \) is distributed as Gamma \((1, \theta)\) if \( X \) is distributed as Gamma \((1, \theta)\) for each \( \theta \)

117. If \( X \) is distributed as Gamma \((n, \lambda)\), which of the following statements are true?

1. \( \lambda > 0 \) is the mean of \( X \)
2. \( \lambda < 0 \) is the median of \( X \)
3. \( \lambda = 0 \) is the mode of \( X \)
4. \( \lambda = 1 \) is the variance of \( X \)
1. \((\beta_1, \beta_2, \beta_3)\) are identified (unique) if.
2. \(\sum_{i=1}^{n} \epsilon_i \beta_j\) is consistent and unbiased estimate of \(\beta_j\) (BLUE).
3. \(\sum_{i=1}^{n} \epsilon_i \beta_j\) is a consistent and unbiased estimate of \(\sum_{i=1}^{n} \epsilon_i \beta_j\) (RMVUE).
4. \(\sum_{i=1}^{n} \epsilon_i \beta_j\) is BLUE and consistent.

116. In the linear model:
\[ Y = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \epsilon, \]
where \(\epsilon\) is i.i.d. \(N(0, \sigma^2)\) and \(n \geq 3\).

Let \(X_{21}, X_{22}, X_{23}\) be independent variables, and \(X_{11}, X_{12}, X_{13}\) be independent variables.

Let \((\beta_1, \beta_2, \beta_3)\) be the least squares estimate of \((\beta_1, \beta_2, \beta_3)\). Let \(\epsilon_1, \epsilon_2, \epsilon_3 \in \mathbb{R}\).

1. \((\beta_1, \beta_2, \beta_3)\) is unique.
2. \(\sum_{i=1}^{n} \epsilon_i \beta_j\) is the best linear unbiased estimate (BLUE) of \(\sum_{i=1}^{n} \epsilon_i \beta_j\).
3. \(\sum_{i=1}^{n} \epsilon_i \beta_j\) is the uniformly minimum variance unbiased estimate (UMVUE) of \(\sum_{i=1}^{n} \epsilon_i \beta_j\).
4. \(\sum_{i=1}^{n} \epsilon_i \beta_j\) is BLUE but not UMVUE of \(\sum_{i=1}^{n} \epsilon_i \beta_j\).

117. Let \(D(\mu, \sigma^2)\) be the density function of \(X_1, X_2, \ldots, X_n\), where \(\mu, \sigma^2\) are unknown parameters. Consider a confidence interval for \(\sigma^2\), which is of the form
\[ \sigma^2 \in \left( \frac{X_{(n-1)} f_{n-1, \alpha} \sigma^2}{f_{n-1, 1-\alpha}}, \frac{X_{(n-1)} f_{n-1, \alpha}}{f_{n-1, 1-\alpha}} \right), \]
where \(n > 1\).

Let \(G_{n-1}\) be the cumulative distribution function of a chi-square random variable with \(n\) degrees of freedom. Which of the following statements are true?
1. \(G_{n-1}(1) = 1 - G_{n-1}(0) = 0.95\) if \(X_{(n-1)} f_{n-1, \alpha} \sigma^2 = 1\).
2. \(G_{n-1}(0) = 1 - G_{n-1}(1) = 0.025\) if \(X_{(n-1)} f_{n-1, \alpha} \sigma^2 = 0\).
3. \(G_{n-1}(a) = G_{n-1}(b) = 0.95\) if \(X_{(n-1)} f_{n-1, \alpha} \sigma^2 = 1\).
4. \(G_{n-1}(a) = G_{n-1}(b) = 0.025\) if \(X_{(n-1)} f_{n-1, \alpha} \sigma^2 = 0\).

118. \(f(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}\)

\(f(x) = \begin{cases} \frac{1}{2x} & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}\)

1. If \(X \sim N(\mu, \sigma^2)\) and \(X > 0\), the expected value of \(X^2\) is \(E(X^2) = \mu^2 + \sigma^2\). If \(X \sim \text{Exponential}(\lambda)\), then \(E(X^2) = \frac{2}{\lambda^2}\).
2. If \(X \sim \text{Gamma}(k, \lambda)\), then \(E(X^2) = k^2 \lambda^2 + 2k \lambda^2\).
3. If \(X \sim \text{Beta}(a, b)\), then \(E(X^2) = \frac{a(a + b + 1)}{(a + b)(a + b + 1)}\).
4. If \(X \sim \text{Normal}(\mu, \sigma^2)\), then \(E(X^2) = \mu^2 + \sigma^2\).
5. If \(X \sim \text{Uniform}(a, b)\), then \(E(X^2) = \frac{(a + b)^2}{3}\).

\(f(x) = \begin{cases} e^{-x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}\)

\(f(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}\)

118. Consider a two-class classification problem, where the densities of the two competing classes are given by

\[ f_1(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases} \]

and

\[ f_2(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases} \]

Let \( \pi_1 \) and \( \pi_2 \) be the prior probabilities of these two classes. Now consider a classifier \( \delta \), which classifies an observation \( x \) to class 1 if \( x < 1/2 \) and to class 2 if \( x \geq 1/2 \).

1. If \( \pi_1 = \pi_2 \), then \( \delta \) is the Bayes classifier.
2. If \( \pi_1 > \pi_2 \), then \( \delta \) is the Bayes classifier.
3. If \( \pi_1 < \pi_2 \), then \( \delta \) is the Bayes classifier.
4. If \( \pi_1 = \pi_2 \), then the average probability of misclassification for \( \delta \) is 3/8.

119. Let \( X \) and \( Y \) be two random variables with joint probability density function

\[ f(x, y) = \begin{cases} 2 & \text{if } 0 \leq x^2 + y^2 \leq 1 \\ 0 & \text{otherwise} \end{cases} \]

Which of the following statements are correct?

1. \( X \) and \( Y \) are independent
2. \( P(X > 0) = 1/2 \)
3. \( E(Y) = 0 \)
4. \( \text{Cov}(X, Y) = 0 \)

120. Let \( X \) and \( Y \) be two random variables satisfying

\[ X \geq 0, \quad Y \geq 0, \quad E(X) = 3, \quad V(X) = 9, \quad E(Y) = 2, \quad V(Y) = 4 \]

Which of the following statements are correct?

1. \( 0 \leq \text{Cov}(X, Y) \leq 4 \)
2. \( E(XY) \leq 3 \)
3. \( V(X + Y) \leq 25 \)
4. \( E(X + Y)^2 \geq 25 \)

120. Let \( X \) and \( Y \) be two random variables satisfying

\[ X \geq 0, \quad Y \geq 0, \quad E(X) = 3, \quad V(X) = 9, \quad E(Y) = 2, \quad V(Y) = 4 \]

Which of the following statements are correct?

1. \( 0 \leq \text{Cov}(X, Y) \leq 4 \)
2. \( E(XY) \leq 6 \)
3. \( V(X + Y) \leq 25 \)
4. \( E(X + Y)^2 \geq 25 \)