2018 (I)  
गणित विज्ञान  
प्रश्न पत्र

1. जब और दो संख्याओं को समान करने पर मल्टीप्ल वाले से दो संख्याओं (20 संवेदन 'A' में 25 संवेदन 'B', 60 संवेदन 'C') का गुणन विषय उपयोग (MCC) दिया गया है। जबकि संवेदन 'A' में समानता 15 और संवेदन 'B' में 25 संवेदन 'C' में से 20 संवेदन के उपर खेला गया है। अर्थात् विद्यार्थी के प्रश्नात्मक कार्य के लिए दिया गया रेखा के खंड में संवेदन 'A' में 15, संवेदन 'B' में 25 और संवेदन 'C' में 20 संवेदन की मात्रा की गई।

2. अपना रास्ता पर जाना है जिसे नियम नहीं है। कारण, रेखा नर्व और सेवा का तात्पर्य नहीं है। संपूर्ण मान जो शिक्षक ने मुख्य आयाम के लिए नहीं दिया गया है। हालांकि, प्रश्न को हल करने के लिए एक तरीका नहीं है। यह योजना है कि उपरोक्त उपचार से पहले अपने रास्ते की पुर्वज प्रश्नों का विवेचना करने की जरूरत है। इसलिए, प्रश्न के चरण को हल करने के लिए अवधारणा पर आधारित है।

3. प्रश्न राशि के अनुसार 1 में दिया गया अभ्यास पर आधारित है। इस राशि द्वारा प्रश्न पत्र का कार्य में लागू होता है। प्रश्न पत्र का इस्तेमाल अपने शताब्दी भी करना चाहिए।

4. जब अपनी अभ्यास काम करने वाला प्रश्न, विशेष उपयोग, पुर्वज प्रश्नों के बीच समान करने का अभ्यास करता है। इस प्रश्न के शास्त्रीय लिखित हैं कि इस अभ्यास के लिए एक राशि दिया गया है। अतः, उपरोक्त राशि के द्वारा प्रश्न पत्र के शताब्दी में लागू है।

5. प्रश्न 'A' में राशि प्रकाश का हिस्सा है। प्रश्न 'B' में विशेष प्रकाश का हिस्सा है। प्रश्न 'C' में प्रकाश का अभ्यास 4.75 अंक का है। प्रश्न 'D' में अपने उपरोक्त प्रकाश का अभ्यास 0.5 अंक का है। प्रश्न 'E' में अपने उपरोक्त प्रकाश का अभ्यास 0.75 अंक का है।

6. प्रश्न 'A' में अपने हिस्से के नीचे प्रकाश का अभ्यास है। प्रश्न 'B' में अपने हिस्से का अभ्यास है। प्रश्न 'C' में अपने हिस्से का अभ्यास है। प्रश्न 'D' में अपने हिस्से का अभ्यास है। प्रश्न 'E' में अपने हिस्से का अभ्यास है।

7. प्रश्न 'A' में अपने उपरोक्त प्रकाश का प्रश्न करने के लिए है और प्रश्न 'B' में अपने उपरोक्त प्रकाश का प्रश्न करने के लिए है।

8. प्रश्न 'C' में अपने उपरोक्त प्रकाश का प्रश्न करने के लिए है।

9. प्रश्न 'D' में अपने उपरोक्त प्रकाश का प्रश्न करने के लिए है।

10. प्रश्न 'E' पर अपने उपरोक्त प्रकाश का प्रश्न करने के लिए है।

11. अपने उपरोक्त प्रकाश का प्रश्न करने के लिए है।

12. प्रश्न 'D' में अपने उपरोक्त प्रकाश का प्रश्न करने के लिए है।
PART A

1. किसी भी (2017, 2017), (2027, 2027) और (2037, 2037) से केवल किसी का क्षेत्रफल है?
   1. 2017 2. 100
   3. $100\sqrt{10}$ 4. $100\sqrt{20}$

2. त्रिभुज का एक कोण की यादृच्छिक माप से दो मानों में तोड़ा गया है? छोटे दुरुस्ती की औसत माप क्या है?
   1. $\frac{L}{6}$ 2. $\frac{L}{4}$
   3. $\frac{L}{3}$ 4. $\frac{L}{2}$

3. एक वर्गाकार मिश्र वर्ग का अन्तरिक्ष और प्रत्येक बेलन की प्रवांक विश्लेषण में दर्शाया था है। वर्गा प्रवांक विवरण किस मात्रा में वर्गाकार प्रवांक का दो गुगी व्यक्ति है?

4. बार ग्रुप में $M_1, M_2, M_3, M_4$ और बार ग्रुप $P_1, P_2, P_3, P_4$ एक गोलाकार गेंद का फिराक और पूर्वतर उत्तर यौगिक की हुई हैं। ज्ञान केवल अपने से तीन कोण प्रवांक प्रवर्तित हैं और फिर एक कोण प्रवांक प्रवर्तित है। एक $P_4$ का बेहतर कितना दिशा में है?

3. Number of times a research paper is viewed and cited is shown in the plot. In which month was the percentage increase in citation more than the double of the percentage increase in view?

1. February 2. April
3. May 4. June

4. Four males $M_1, M_2, M_3, M_4$ and four females $P_1, P_2, P_3$, and $P_4$ are sitting around a round table facing away from the table, as shown in the figure. If each one moves
3. Which of the following options is the best choice for the missing number?
   0.1, 0.25, 0.3, 0.2, 0.3, 0.6, 0.3,___,
   0.9, 0.4, 0.6, 1.2
   1. 1.05
   2. 0.85
   3. 0.75
   4. 0.65

4. How many students are girls? Eight students in the class wear blue shirts. Two are neither girls nor wear blue shirts. Five students who wear blue shirts are girls. How many students are there in the class?
   1. 19
   2. 29
   3. 17
   4. 24

5. If the line segment ABCD is divided into equal parts, how many parts are there?
   1. 5
   2. 6
   3. 7
   4. 4

6. Which of the following options is the best choice for the missing number?
   0.1, 0.25, 0.3, 0.2, 0.3, 0.6, 0.3,___,
   0.9, 0.4, 0.6, 1.2
   1. 1.05
   2. 0.85
   3. 0.75
   4. 0.65

7. In the diagram, what is the ratio of the total shaded area (of the circle and semi-circle) to the total area of the square and the rectangle?
   1. 5
   2. 6
   3. 7
   4. 4

8. Prabhisri is choosing partners for a group project. How many partners can she choose?
   1. 19
   2. 29
   3. 17
   4. 24
for three class periods in a row. Calvin does not want to work with Alice. Who should be assigned to work with Bob?

9. The figure shows a semi-circle inside a big circle as shown in the figure. If the radius of the semicircle is \(2\) cm and the radius of the big circle is \(4\) cm, what proportion of the big circle's area is shaded?

- 1. \(\frac{1}{2}\)  
- 2. \(\frac{1}{4}\)  
- 3. \(\frac{1}{8}\)  
- 4. \(\frac{1}{16}\)

10. A ball is dropped from a height of 100 m. The ball after each bounce rises vertically by half its previous height (This means at the first bounce it rises by 50 m, by 25 m at the second bounce and so on). What is the vertical distance travelled by the ball between the first and the fifth bounces?

- 1. \(50 + 25 + 12.5 + 6.25 + 3.125\) m  
- 2. \(125\) m  
- 3. \(187.5\) m  
- 4. \(250\) m

11. Consider a number 54 expressed in a base different from ten. What is the base of this number system if its equivalent value in the decimal system is 49?

- 1. 5  
- 2. 6  
- 3. 7  
- 4. 8

12. A man borrows \(\text{Rs} 15000\) at the rate of \(10\%\) p.a. in 5 years. What is the amount he has to pay back to the lender?

- 1. \(16500\)  
- 2. \(17550\)  
- 3. \(18500\)  
- 4. \(19500\)
12. A fuel station sold diesel costing ₹15000 to 150 persons on a day. If the lower limit of sale to a person is ₹50, what is the maximum amount in rupees for which one person could have purchased diesel on that day?
1. 7450  2. 7500  3. 7550  4. 7600

13. Which diagram shows the given word combination?

14. यदि संगीता की पुत्री मेरी पुत्री की मां है, तो मेरा छत्रीता से किस दिशा है?
1. केवल पुत्र होना ही सम्भव है।
2. केवल मां होना ही सम्भव है।
3. केवल पुत्र होना ही सम्भव है।
4. मां या पुत्री

14. If Sangeeta’s daughter is my daughter’s mother, then how am I related to Sangeeta?
1. Son is the only possibility
2. Son-in-law is the only possibility
3. Daughter is the only possibility
4. Son-in-law or daughter

15. 44 खिलाड़ियों के समूह में, 26 खिलाड़ियों हाफी, 24 खिलाड़ियों फूटबॉल और 24 खिलाड़ियों क्रिकेट खेलते हैं। उनमें से, 8 हाफी और फूटबॉल दोनों, 12 फूटबॉल और क्रिकेट दोनों, और 5 टीम खेलते हैं। फिर भी कितने खिलाड़ियों हाफी और क्रिकेट दोनों खेलते हैं?
1. 10  2. 16  3. कोई नहीं  4. 7

15. In a group of 44 players, 26 play hockey, 24 play football and 24 play cricket. Eight of them play both hockey and football, 12 play both football and cricket, and 5 play all the three games. How many play both hockey and cricket?
1. 10  2. 15  3. None  4. 7
16. \( (a^n) = a \) if \( a > 0 \) and \( n > 0 \) \( = 1 \) if \( a > 0 \) and \( n < 0 \) \( = 0 \) if \( a < 0 \). What is the value of \( (xy)^n = (x^n)(y^n) \)?

1. \( x > 0 \) and \( y > 0 \)
2. \( x < 0 \) and \( y < 0 \) or \( x > 0 \) and \( y > 0 \)
3. \( x \leq 0 \) and \( y \leq 0 \) or \( x \geq 0 \) and \( y \geq 0 \)
4. \( x > 0 \) or \( y > 0 \) or \( x < 0 \) and \( y < 0 \)

16. It is given that \( (a^n) = a \) if \( a > 0 \) for any real number \( a \) and \( n > 0 \).

Suppose for two real numbers \( x \) and \( y \), \( (xy)^n = (x^n)(y^n) \). Then which of the following is necessarily true?

1. \( x > 0 \) and \( y > 0 \)
2. \( x < 0 \) and \( y < 0 \) or \( x > 0 \) and \( y > 0 \)
3. \( x \leq 0 \) and \( y \leq 0 \) or \( x \geq 0 \) and \( y \geq 0 \)
4. \( x > 0 \) or \( y > 0 \) or \( x < 0 \) and \( y < 0 \)

17. A long-distance runner finds a water station after completing \( \frac{1}{4} \) of the total distance. After covering another \( \frac{1}{6} \) of the total distance he gets medical aid. Another runner joins him 4 km after the medical aid station. The second runner stops 4 km before the completion of run, covering \( \frac{1}{5} \) of the total distance. What is the total distance?

1. 21 km
2. 30 km
3. 42 km
4. 50 km

18. A and B move clockwise around a circle, starting from a common point O. A takes 9 minutes to complete a round and restarts after a delay of 1 minute. B takes 13 minutes to complete the round and restarts after a delay of 2 minutes. How many minutes after they began would they meet again at O?

1. 30
2. 29
3. 31
4. 28

19. Two students are solving the same problem independently. If the probability that the first one solves the problem is \( \frac{2}{5} \) and the probability that the second solves the problem is \( \frac{3}{5} \), what is the probability that at least one of them solves the problem?

1. 0.2
2. 0.7
3. 0.8
4. 0.9
20. A car’s speed is given by
\[\lambda x^2 + 2xyz + y^2 = (ax + by)^2 + (cx + dy)^2\]
for all \(x, y \in \mathbb{R}\). This implies
1. \(\lambda = -5\)
2. \(\lambda \geq 1\)
3. \(0 < \lambda < 1\)
4. There is no such \(\lambda \in \mathbb{R}\).

22. Let \(R^n, n \geq 2\), be equipped with standard inner product. Let \(\{v_1, v_2, \ldots, v_n\}\) be \(n\) column vectors forming an orthonormal basis of \(R^n\). Let \(A\) be the \(n \times n\) matrix formed by the column vectors \(v_1, v_2, \ldots, v_n\).
Then
1. \(A = A^{-1}\)
2. \(A = A^T\)
3. \(A^{-1} = A^T\)
4. \(\text{Det}(A) = 1\)

23. Given \((a_n), (b_n)\) two monotonically increasing sequences of real numbers, and that \(\sum a_n, b_n\) is convergent, which of the following is true?
1. \(\sum a_n, \sum b_n\) are convergent
2. At least one of \(\sum a_n, \sum b_n\) is convergent
3. \((a_n)\) is bounded and \((b_n)\) is bounded
4. At least one of \((a_n), (b_n)\) is bounded

\(\lambda x^2 + 2xyz + y^2 = (ax + by)^2 + (cx + dy)^2\)
holds for all \(x, y \in \mathbb{R}\). This implies
1. \(\lambda = -5\)
2. \(\lambda \geq 1\)
3. \(0 < \lambda < 1\)
4. There is no such \(\lambda \in \mathbb{R}\).

22. Let \(R^n, n \geq 2\), be equipped with standard inner product. Let \(\{v_1, v_2, \ldots, v_n\}\) be \(n\) column vectors forming an orthonormal basis of \(R^n\). Let \(A\) be the \(n \times n\) matrix formed by the column vectors \(v_1, v_2, \ldots, v_n\). Then
1. \(A = A^{-1}\)
2. \(A = A^T\)
3. \(A^{-1} = A^T\)
4. \(\text{Det}(A) = 1\)

23. Given \((a_n), (b_n)\) two monotonically increasing sequences of real numbers, and that \(\sum a_n, b_n\) is convergent, which of the following is true?
1. \(\sum a_n, \sum b_n\) are convergent
2. At least one of \(\sum a_n, \sum b_n\) is convergent
3. \((a_n)\) is bounded and \((b_n)\) is bounded
4. At least one of \((a_n), (b_n)\) is bounded
24. Let $S = \{(x,y) \mid x^2 + y^2 = \frac{1}{n^2}, \text{where } n \in \mathbb{N} \text{ and either } x \in \mathbb{Q} \text{ or } y \in \mathbb{Q}\}$. Here $\mathbb{N}$ is the set of natural numbers and $\mathbb{Q}$ is the set of rational numbers. Which of the following is true?
1. $S$ is a finite non-empty set
2. $S$ is countable
3. $S$ is uncountable
4. $S$ is empty

25. The sequence $(a_n)$ is defined as follows:
$a_0 = 1, \quad a_{n+1} = (-1)^n \left(1 + \frac{n}{a_n}\right) \quad \text{for } n \geq 1$
Which of the following is true?
1. $\lim \sup a_n = \sqrt{2}$
2. $\lim \inf a_n = -\infty$
3. $\lim a_n = \sqrt{2}$
4. $\sup a_n = \sqrt{2}$

26. Define the sequence $(a_n)$ as follows:
$n \geq 1, \quad a_{n+1} = (-1)^n \left(1 + \frac{n}{a_n}\right)$
Which of the following is true?
1. $\lim \sup a_n = \sqrt{2}$
2. $\lim \inf a_n = -\infty$
3. $\lim a_n = \sqrt{2}$
4. $\sup a_n = \sqrt{2}$

27. The difference
$\log(2) - \sum_{n=1}^{\infty} \frac{1}{2^{n+1}}$ is
1. less than 0
2. greater than 1
3. less than $\frac{1}{2^{n+1}}$
4. greater than $\frac{1}{2^{n+1}}$

28. If $f(x,y) = \log \left(\cos^2(x+y) + \sin(x+y)\right)$, then
$\frac{\partial f}{\partial y}$, $\frac{\partial f}{\partial x}$, $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial y^2}$, $\frac{\partial^2 f}{\partial x \partial y}$ are
1. $\cos(x+y)$
2. 0
3. $\sin(x+y)$
4. $\cos(x+y)$
28. Let \( f(x, y) = \log \left( \frac{\cos \theta (e^{x+y})}{1 + \sin \theta (e^{x+y})} \right) + \sin(x + y) \).

Then \( \frac{\partial}{\partial y} f(x, y) \) is

1. \( \frac{\cos(e^y - 1)}{1 + \sin\theta (e^{x+y})} - \cos(x + y) \)
2. 0
3. -\sin(x + y)
4. \cos(x + y)

29. Let \( A \) be a \((m \times n)\) matrix and \( B \) be a \((n \times m)\) matrix over real numbers with \( m \times n \). Then

1. \( AB \) is always nonsingular
2. \( BA \) is always singular
3. \( AB \) is always nonsingular
4. \( BA \) is always singular

30. Let \( A \) be a \((m \times n)\) matrix over real numbers with \( m \times n \). Then

1. \( \det(A) = 0 \)
2. \( A = 0 \)
3. \( \text{tr}(A) = 0 \)
4. \( A \) is nonsingular

31. If \( A \) is a \((2 \times 2)\) matrix over \( \mathbb{R} \) with

\( \det(A + I) = 1 + \det(A) \)

then we can conclude that

1. \( \det(A) = 0 \)
2. \( A = 0 \)
3. \( \text{tr}(A) = 0 \)
4. \( A \) is nonsingular

32. Consider \( f(x) = x^3 - 5x + 2 \) which is

1. \( f(x) \) has no real root
2. \( f(x) \) has exactly one real root
3. \( f \) has exactly three real roots
4. all roots of \( f \) are real

34. Consider the space \( S = \{(x, y) | x, y \in \mathbb{Q}\} \subset \mathbb{R}^2 \) where \( \mathbb{Q} \) is the set of rational numbers. Then
1. \( S \) is closed in \( \mathbb{R}^2 \)
2. \( S^c \) is connected in \( \mathbb{R}^2 \)
3. \( S \) is closed in \( \mathbb{R}^2 \)
4. \( S^c \) is closed in \( \mathbb{R}^2 \)

35. Suppose \( f \) is a non-constant analytic function defined over \( \mathbb{C} \), then which one of the following is false?
1. \( f \) is unbounded
2. \( f \) sends open sets into open sets
3. There exists an open connected domain \( U \) on which \( f \) is never zero but \( |f| \) attains its minimum at some point of \( U \)
4. The image of \( f \) is dense in \( \mathbb{C} \)

36. The value of the integral
\[
\int_{|z|=1} \frac{g^2}{z^2 - 1} \, dz
\]
1. 0
2. \((n!)(a)
3. \((n!)(a)-(n!)(e^{-1})
4. \((e + e^{-1})

37. Let \( f : \{z | |z| < 1\} \to \mathbb{C} \) be a non-constant analytic function. Which of the following conditions can possibly be satisfied by \( f \)?
1. \( f_1(z) = f_1(z_0) = \frac{1}{n} \quad \forall n \in \mathbb{N} \)
2. \( f_1(z) = f_1(z_0) = \frac{1}{n} \quad \forall n \in \mathbb{N} \)
3. \( f_1(z) < 2^{-n} \forall n \in \mathbb{N} \)
4. \( \frac{1}{n^2} < f_1(z) < \frac{1}{n^2} \quad \forall n \in \mathbb{N} \)

38. Consider the map \( \varphi : \mathbb{C} \to \mathbb{C} \) given by \( \varphi(z) = \frac{z^2 + 1}{z^2 - 1} \). Which of the following is true?
1. \( \varphi(z) \in \mathbb{C} | |z| < 1 \) \( \subset \mathbb{C} \) \( |z| < 1 \)
2. \( \varphi(z) \in \mathbb{C} | Re(z) < 0 \) \( \subset \mathbb{C} \) \( Re(z) < 0 \)
3. \( \varphi \) is onto
4. \( \varphi(\mathbb{C}(1)) = \mathbb{C}(1) \)
39. Let $S_7$ denote the group of permutations of the set $\{1, 2, 3, 4, 5, 6, 7\}$. Which of the following statements about $S_7$ being a group (group) is true?
1. $S_7$ is not a group (element) of order 6.
2. $S_7$ is a group (element) of order 7.
3. $S_7$ is a group (element) of order 8.
4. $S_7$ is a group (element) of order 10.

40. Consider the group homomorphism $\varphi: G \to G$ defined by $\varphi(x) = g_1xg_2$, where $G = \mathbb{Z}_{10}$ and $\varphi$ is injective (kernel) $\{0\}$.
1. $\varphi(x)$ is a homomorphism.
2. $\varphi(x)$ is not a homomorphism.
3. $\varphi(x)$ is an isomorphism.
4. $\varphi(x)$ is a bijection.

41. The number of group homomorphisms from $\mathbb{Z}_{10}$ to $\mathbb{Z}_{20}$ is
1. zero
2. one
3. five
4. ten

UNIT 3

42. The Lagrangian for a simple pendulum $L = \frac{1}{2}mL^2\dot{\theta}^2 + mgL\cos \theta$.
1. $H(p, \theta) = \frac{p^2}{2mL} + mgL\cos \theta$
2. $H(p, \theta) = \frac{p^2}{2mL} - mgL\cos \theta$
3. $H(p, \theta) = \frac{p^2}{2mL} - mgL\sin \theta$
4. $H(p, \theta) = \frac{p^2}{2mL} + mgL\sin \theta$

43. The function $y = y(x) = x^2 + x + 1$ is
1. decreasing
2. increasing
3. bounded
4. unbounded

44. Consider the ordinary differential equation $y' + P(x)y' + Q(x)y = 0$.
1. Which of the following statements is true?
   1. $y(0) = 0.5$ and $y$ is decreasing
   2. $y(0) = 1.2$ and $y$ is increasing
   3. $y(0) = 2.5$ and $y$ is unbounded
   4. $y(0) < 0$ and $y$ is bounded

45. The function $y = y(x)$ satisfies the differential equation $y' = f(x)$.
1. $y(x)$ is a smooth function.
2. $y(x)$ is not a smooth function.
3. $y(x)$ is bounded.
4. $y(x)$ is unbounded.
44. Consider the ordinary differential equation
\[ y'' + P(x)y' + Q(x)y = 0 \]
where \( P \) and \( Q \) are smooth functions. Let \( y_1 \) and \( y_2 \) be any two solutions of the ODE. Let \( W(x) \) be the corresponding Wronskian. Then which of the following is always true?
1. If \( y_1 \) and \( y_2 \) are linearly dependent then \( x = x_1, x_2 \) such that \( W(x_1) = 0 \) and \( W(x_2) \neq 0 \)
2. If \( y_1 \) and \( y_2 \) are linearly independent then \( W(x) = 0 \) \( \forall x \)
3. If \( y_1 \) and \( y_2 \) are linearly independent then \( W(x) = 0 \) \( \forall x \)
4. If \( y_1 \) and \( y_2 \) are linearly independent then \( W(x) = 0 \) \( \forall x \)

45. Consider the Cauchy problem
\[ 2u_x + 3u_y = 5 \]
\[ u(x, 1) = y(x, 0) = 0 \] on the line \( 2x - 2y = 0 \) \( \forall x \). How many solutions can you find?
1. exactly one solution
2. exactly two solutions
3. infinitely many solutions
4. no solution

46. Consider the equation
\[ f(x) = 0, x \in [0, 1] \] and \( f(x + 1) = f(x) \) \( \forall x \in \mathbb{R} \).
46. Let \( u \) be the unique solution of
\[ u(x, 0) = f(x), \frac{\partial u}{\partial x}(x, 0) = 0, x \in \mathbb{R}, t > 0 \]
where \( f: \mathbb{R} \to \mathbb{R} \) satisfies the relations
\[ f(x) = x, x \in [0, 1] \] and \( f(x + 1) = f(x) \) \( \forall x \in \mathbb{R} \).
Then \( u(x, t) = \) is
1. \( \frac{1}{6} \)
2. \( \frac{1}{12} \)
3. \( \frac{1}{16} \)
4. \( \frac{1}{16} \)

47. Let
\[ f(x) = \frac{\alpha x^3 + \beta x + c}{x^3 + 1} \] where \( \alpha, \beta, c \) are constants. How many solutions can you find?
1. \( \alpha = 0, \beta = 1, c = -2 \)
2. \( \alpha = 0, \beta = -1, c = -2 \)
3. \( \alpha = 2, \beta = 3, c = 2 \)
4. \( \alpha = 0, \beta = -1, c = -2 \)

48. The Cauchy problem
\[ 2u_x + 3u_y = 5 \]
\[ u = 1 \text{ on the line } 2x - 2y = 0 \]
how many solutions can you find?
1. exactly one solution
2. exactly two solutions
3. infinitely many solutions
4. no solution

49. Consider the equation
\[ f(x) = 0, x \in [0, 1] \] and \( f(x + 1) = f(x) \) \( \forall x \in \mathbb{R} \).
49. The values of \( a, b, c \) such that
\[ \int_0^1 f(x)dx = \int_0^1 (af(x) + bf(x) + cf(x))dx \] is exact for polynomials \( f \) of degree as high as possible are
1. \( a = 0, b = 1, c = -2 \)
2. \( a = 1, b = 1, c = 2 \)
3. \( a = -1, b = 2, c = -2 \)
4. \( a = 0, b = 1, c = -2 \)
48. \[ f(y) = \int_0^1 (y')^2 + 2y' \, dx \]

**Solution:**

1. $2 \frac{23}{12}$
2. $2 \frac{23}{24}$
3. $\frac{18}{25}$
4. does not exist

**UNIT-4**

49. Consider

\[ f(y) = \int_0^1 (y')^2 + 2y' \, dx \]

subject to $y(0) = 0, y(1) = 1$. Then \( \inf f(y) \) does not exist.

1. \( \frac{23}{12} \)
2. \( \frac{23}{24} \)
3. \( \frac{18}{25} \)
4. does not exist

**50.** In a Latin Square Design the error degrees of freedom is 30. The “treatment degrees of freedom” for any treatment is

1. \( 1 \)
2. \( 2 \)
3. \( 3 \)
4. \( 4 \)

**51.** Suppose that \( |3x| + |2y| \leq 1 \) in the region.

1. \( 1 \)
2. \( 2 \)
3. \( 3 \)
4. \( 4 \)

**52.** A standard fair die is rolled until some face other than 5 or 6 turns up. Let \( X \) denote the face value of the last roll, and \( A = \{ X \text{ is even} \} \) and \( B = \{ X \text{ is at most } 2 \} \). Then

1. \( P(A|B) = 0 \)
2. \( P(A|B) = 1/6 \)
3. \( P(A|B) = 1/4 \)
4. \( P(A|B) = 1/3 \)

**53.** Let \( X \) and \( Y \) be i.i.d. uniform (0, 1) random variables. Let \( Z = \max \{X, Y\} \) and \( W = \min \{X, Y\} \). Then \( P(Z - W > 1/2) \) is

1. \( 1/2 \)
2. \( 3/4 \)
3. \( 1/4 \)
4. \( 2/3 \)

**54.** A Markov chain (Markov chain) is the Markov chain. The state space \( S = \{1, 2, 3, 4\} \) is a transition probability matrix.

\[ P = (p_{ij}) \]

where

1. \( 1 \)
2. \( 2 \)
3. \( 3 \)
4. \( 4 \)

\[ \lim_{n \to \infty} p_{22}^{(n)} = 0, \sum_{n=0}^{\infty} p_{22}^{(n)} = \infty \]

\[ \lim_{n \to \infty} p_{32}^{(n)} = 0, \sum_{n=0}^{\infty} p_{32}^{(n)} < \infty \]

\[ \lim_{n \to \infty} p_{11}^{(n)} = 1, \sum_{n=0}^{\infty} p_{11}^{(n)} = \infty \]

**4.G.H**
53. Consider a Markov chain having state space $S = \{1, 2, 3, 4\}$ with transition probability matrix $P = (p_{ij})$ given by

$$
\begin{array}{cccc}
1 & 2 & 3 & 4 \\
1/2 & 0 & 1/2 & 0 \\
1/4 & 1/4 & 1/4 & 1/4 \\
1/3 & 0 & 1/3 & 1/3 \\
1/2 & 0 & 1/2 & 0 \\
\end{array}
$$

Then

1. $\lim_{n \to \infty} p_{2,2}^{(n)} = 0$, $\sum_{n=0}^{\infty} p_{2,2}^{(n)} = \infty$

2. $\lim_{n \to \infty} p_{1,2}^{(n)} = 0$, $\sum_{n=0}^{\infty} p_{1,2}^{(n)} < \infty$

3. $\lim_{n \to \infty} p_{2,2}^{(n)} = 1$, $\sum_{n=0}^{\infty} p_{2,2}^{(n)} = \infty$

4. $\lim_{n \to \infty} p_{2,2}^{(n)} = 1$, $\sum_{n=0}^{\infty} p_{2,2}^{(n)} < \infty$

54. Let $X_1, X_2, X_3$ be i.i.d standard normal variables. Which of the following is true?

1. $\frac{\sqrt{2}x_1}{x_1 + x_2} \sim t_2$

2. $\frac{x_2 - 2x_1 + x_3}{\sqrt{2}(x_1 + x_2 + x_3)} \sim t_3$

3. $\frac{(x_1 - x_2)^2}{(x_1 + x_2)^2} \sim F_{2,2}$

4. $\frac{\sqrt{2}x_2}{x_1 + x_2} \sim F_{1,3}$

55. Suppose that the lifetime of an electric bulb follows an exponential distribution with mean $\theta$ hours. In order to estimate $\theta$, $n$ bulbs are switched on at the same time. After $t$ hours, $n - m > 0$ bulbs are found to be in functioning state. If the lifetimes of the other $m > 0$ bulbs are noted as $X_1, X_2, \ldots, X_m$, respectively, then the maximum likelihood estimate of $\theta$ is given by

1. $\beta = \frac{t}{m(n - m)}$

2. $\beta = \frac{\sum x_i}{m}$

3. $\beta = \frac{\sum x_i}{m(n - m)}$

4. $\beta = \frac{\sum x_i}{m(n - m)}$

56. A random variable is uniformly distributed (i.i.d. random variables) $X_1, X_2, \ldots, X_n$ which follow a normal distribution $(\theta_1, \theta_2)$ with $\theta_1 < \theta_2$ and $\theta_1$ are known, $\theta_2$ is unknown. Which of the following is a correct statement?

1. If $k < n$, the distribution of $X^{(k)}$ is $\chi^2$.

2. If $k < n$, the distribution of $X^{(k)}$ is $\chi^2$.
3. If \( k < n \) then \( \frac{x_{(k)}}{x_{(n)}} \) is distributed as \( \chi^2(k) \).

4. If \( k \), \( 1 < k < n \) then \( \frac{x_{(k)}}{x_{(n)}} \) is distributed as \( F(k, n-k-1) \).

56. Let \( X_1, X_2, \ldots, X_n \) be i.i.d. uniform \((\theta_1, \theta_2)\) variables, where \( \theta_1 < \theta_2 \) are unknown parameters. Which of the following is an ancillary statistic?

1. \( \frac{x_{(k)}}{x_{(n)}} \) for any \( k < n \)

2. \( \frac{x_{(k)}}{x_{(n)}} \) for any \( k < n \)

3. \( \frac{x_{(k)}}{x_{(n)}} \) for any \( k < n \)

4. \( \frac{x_{(k)}}{x_{(n)}} \) for any \( k \) where \( 1 < k < n \)

57. If a random variable \( X \sim N(\theta, 1) \), where \( 0 < \theta < \infty \), how should the standard error \( \sigma \) be estimated? For each value \( \theta \), the squared error (squared error loss) is defined as the expected value of \( (X - \theta)^2 \) over all possible \( \theta \). Which of the following options correctly estimates risk?

1. \( k < 0 \)

2. \( 0 < k < 1 \)

3. \( k > 1 \)

4. \( k \) has at least one value which is positive.

58. Consider the problem of estimation of \( \theta \) on the basis of \( X \), where \( X \sim N(\theta, 1) \) and \( -\infty < \theta < \infty \). Under squared error loss, \( X \) has uniformly smaller risk than that of \( kX \), for

1. \( k < 0 \)

2. \( 0 < k < 1 \)

3. \( k > 1 \)

4. no value of \( k \)

59. The covariance matrix of a four dimensional random vector \( X \) is of the form

\[
\begin{bmatrix}
\rho & \rho & \rho & \rho \\
\rho & 1 & \rho & \rho \\
\rho & \rho & 1 & \rho \\
\rho & \rho & \rho & 1
\end{bmatrix}
\]

If \( \rho \) is the variance of the first principal component, then

1. \( \rho \) cannot exceed 5/4

2. \( \rho \) can exceed 5/4, but cannot exceed 4/3

60. To test the equality of effects of 10 schools against all alternatives, we take a random sample of 5 students from each school and note their marks in a common examination. “Between sum of squares” and “total sum of squares” are found to be 180 and 500 respectively. What is the p-value for the standard F-test?

1. \( F_{10, 40} \approx 1.5 \)

2. \( F_{10, 40} \approx 1.6 \)

3. \( F_{10, 40} \approx 3.6 \)

4. \( F_{10, 40} \approx 2.5 \)
3. \( v \) can exceed \( 4/3 \), but cannot exceed \( 3/2 \).
4. \( v \) can exceed \( 3/2 \).

60. A simple random sample of size \( n \) will be drawn from a class of 125 students, and the mean mathematics score of the sample will be computed. If the standard error of the sample mean for "with replacement sampling" is twice as much as the standard error of the sample mean for "without replacement sampling", the value of \( n \) is
\[ \begin{align*}
1. & \quad 32 \\
2. & \quad 63 \\
3. & \quad 79 \\
4. & \quad 94
\end{align*} \]

\section{PART C}

\underline{Unit 1.1}

61. Let \( A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 1 \end{pmatrix} \) and define for \( x, y, z \in \mathbb{R} \)
\[ Q(x, y, z) = (x y z)A \begin{pmatrix} x \\ y \\ z \end{pmatrix} \]
1. Which of the following statements are true?
   1. The matrix of second order partial derivatives of the quadratic form \( Q \) is \( 2A \).
   2. The rank of the quadratic form \( Q \) is 3.
   3. The signature of the quadratic form \( Q \) is \((+ + 0)\).
   4. The quadratic form \( Q \) takes the value 0 for some non-zero vector \( (x, y, z) \).

62. \( \alpha \in \mathbb{R} \) is defined as \( S_{\alpha} = \{ (x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = \alpha \} \). Let \( E = \bigcup_{\alpha \in \mathbb{R}} S_{\alpha} \). Which of the following are true?
   1. The Lebesgue measure of \( E \) is infinite.
   2. \( E \) contains a non-empty open set.
   3. \( E \) is path connected.
   4. Every open set containing \( E \) has infinite Lebesgue measure.

\[ \text{4.C.H} \]

\[ 9'(11) \text{RISE}\[18.—4CH—3A] \]
63. Let \( C_c(K) = \{ f : K \to \mathbb{R} \mid f \text{ is continuous and there exists a compact set } K' \text{ such that } f(x) = 0 \text{ for all } x \in K' \} \). Let \( g(x) = e^{-x^2} \) for all \( x \in \mathbb{R} \). Which of the following statements are true?

1. There exists a sequence \( (f_n) \) in \( C_c(K) \) such that \( f_n \to f \) uniformly.
2. There exists a sequence \( (f_n) \) in \( C_c(K) \) such that \( f_n \to f \) pointwise.
3. If a sequence in \( C_c(K) \) converges pointwise to \( g \) then it must converge uniformly to \( g \).
4. There does not exist any sequence in \( C_c(K) \) converging pointwise to \( g \).

64. Let \( A = \left\{ \sin \left( \frac{\pi}{n} \right) \mid n \in (0, \frac{\pi}{2}) \right\} \). Which of the following set arguments are true?

1. \( A \) is uncountable.
2. \( A \) has a finite cardinality.
3. \( A \) is not null.
4. \( A \) is measurable.

65. Let \( f : \mathbb{R} \to \mathbb{R} \) be continuous. Which of the following statements are true?

1. \( f \) is bounded on any finite interval.
2. \( f \) is bounded on any compact interval.
3. \( f \) is uniformly continuous on any compact interval.
4. \( f \) is continuous on any compact interval.

66. Given that \( a(n) = \frac{1}{10^n} \) and \( b(n) = 10^n \log(n) \), find \( c(n) \) such that \( a(n) < c(n) < b(n) \) for all \( n \).
3. Suppose $f$ is a function on $\mathbb{R}$ such that $f(x) = x^2 + 1$ for all $x \in \mathbb{R}$. Is $f$ uniformly continuous on $\mathbb{R}$? Justify your answer.

4. Let $f: \mathbb{R} \to \mathbb{R}$ be given by $f(x) = x^2 + 1$. Is $f$ uniformly continuous on $\mathbb{R}$? Justify your answer.

67. Let $f: \mathbb{R} \to \mathbb{R}$ be given by $f(x) = ax^2 + bx + c$. Determine the values of $a$, $b$, and $c$ such that $f$ is uniformly continuous on $\mathbb{R}$.

Which of the following are true?
1. $f$ is uniformly continuous on compact intervals of $\mathbb{R}$ for all values of $a$ and $b$.
2. $f$ is uniformly continuous on $\mathbb{R}$ and is bounded for all values of $a$ and $b$.
3. $f$ is uniformly continuous on $\mathbb{R}$ only if $b = 0$.
4. $f$ is uniformly continuous on $\mathbb{R}$ and unbounded if $a = 0$ and $b \neq 0$.

68. Let $\alpha = \int_{-1}^{1} \frac{1}{1 + t^2} dt$. How does this value depend on $\alpha$?
1. $\frac{\pi}{2}$
2. $\alpha + \frac{\pi}{2}$
3. $\log(\alpha) = 1$
4. $\sin(\alpha) = 1$

69. Let $\alpha = \int_{0}^{1} \frac{1}{1 + t^2} dt$. Which of the following are true?
1. $\frac{\pi}{2}$
2. $\alpha$ is a rational number
3. $\log(\alpha) = 1$
4. $\sin(\alpha) = 1$

70. Let $\alpha = \int_{0}^{1} \frac{1}{1 + t^2} dt$. How does this value depend on $\alpha$?
1. $\frac{\pi}{2}$
2. $\alpha$ is a rational number
3. $\log(\alpha) = 1$
4. $\sin(\alpha) = 1$

71. Which of the following functions are of bounded variation?
1. $x^2 + x + 1$ for $x \in (-1, 1)$
2. $\tan(x)$ for $x \in (-\pi/2, \pi/2)$
3. $\sin(x)$ for $x \in (-\pi, \pi)$
4. $\sqrt{1 - x^2}$ for $x \in (-1, 1)$

72. Let $M_n(\mathbb{R})$ denote the space of all $n \times n$ real matrices. Consider the diagonal matrices $M_n(\mathbb{R})$. Which of the following statements are true?
1. $M_n(\mathbb{R})$ is a linear subspace of $\mathbb{R}^{n^2}$.
2. Every element of $M_n(\mathbb{R})$ is invertible.
3. $M_n(\mathbb{R})$ is a field.
4. $M_n(\mathbb{R})$ is an integral domain.

73. Let $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$ denote the Euclidean space of all $2 \times 2$ matrices. Consider the column vector $x = (x_1, x_2)$ in $\mathbb{R}^2$. Define $f: M_2(\mathbb{R}) \to \mathbb{R}$ by $f(A) = A^2 x$. Which of the following statements are true?
1. $f$ is linear.
2. $f$ is differentiable.
3. $f$ is continuous but not differentiable.
4. $f$ is unbounded.

74. Let $y = x^3$. How does $y$ depend on $x$?
1. $y$ is a polynomial function of degree 3.
2. $y$ is a rational function.
3. $y$ is a trigonometric function.
4. $y$ is an exponential function.

75. For any $y \in \mathbb{R}$, let $[y]$ denote the greatest integer less than or equal to $y$. Define $f: \mathbb{R} \to \mathbb{R}$ by $f(x) = x^{[y]}$. Which of the following statements are true?
1. $f$ is continuous on $\mathbb{R}$.
2. For every $y \in \mathbb{R}$, $x \mapsto f(x, y)$ is continuous on $\mathbb{R} \setminus \{0\}$.
3. For every $x \in \mathbb{R}$, $y \mapsto f(x, y)$ is continuous on $\mathbb{R}$.
4. $f$ is continuous at all points of $\mathbb{R}$.

4-C-H
72. Let $V$ denote the vector space of all sequences $x = (x_1, x_2, ...)$ of real numbers such that $\sum |x_n|^2$ converges.

Define $\|x\| = \sum |x_n|$. Which of the following are true?

1. $V$ contains only the sequence $(0, 0, ...)$
2. $V$ is finite dimensional
3. $V$ has a countable basis
4. $V$ is a complete normed space

73. Let $V$ be a vector space of all sequences $x = (x_1, x_2, ...)$ of real numbers such that $\sum |x_n|^2$ converges.

Define $\|x\| = \sum |x_n|$. Which of the following are true?

1. $\|x\| = 0$
2. $\|x - y\| = 0$
3. $\|x\| = 0$
4. $\|x - y\| = 0$

74. Let $A$ be a $5 \times 5$ matrix with one eigenvalue $-5$ and one eigenvalue $0$. Which of the following are true?

1. $\text{Rank}(A) \leq 3$
2. $\text{Rank}(A) \geq 3$
3. $\text{Rank}(A) = 3$
4. $\text{Det}(A) = 0$

75. If $A$ is a $5 \times 5$ matrix and the condition of the solution space of $Ax = 0$ is at least two, then

1. $\text{Rank}(A^2) \leq 3$
2. $\text{Rank}(A^2) \geq 3$
3. $\text{Rank}(A^2) = 3$
4. $\text{Det}(A^2) = 0$

76. Let $A$ be an $n \times n$ matrix (with $n > 1$) satisfying $A^2 - 7A + 12I_{n \times n} = 0$, where $I_{n \times n}$ is the identity matrix. Which of the following statements are true?

1. $A$ is invertible
2. $A^2 - 7A + 12I_{n \times n} = 0$
3. $A^2 - 7A + 12I_{n \times n} = 0$
4. $A^2 - 7A + 12I_{n \times n} = 0$
77. Let \( A \) be a (6 x 6) matrix with characteristic polynomial \((x - 3)^2(x - 2)^4\) and minimal polynomial \((x - 3)(x - 2)^2\). Then the Jordan canonical form of \( A \) can be

\[
\begin{bmatrix}
3 & 0 & 0 & 0 & 0 & 0 \\
0 & 3 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 1 & 0 & 0 \\
0 & 0 & 0 & 2 & 1 & 0 \\
0 & 0 & 0 & 0 & 2 & 1 \\
0 & 0 & 0 & 0 & 0 & 2 \\
\end{bmatrix}
\]

78. Let \( V \) be an inner product space and \( S \) be a non-empty subset of \( V \). Then \( S^1 = S \) if and only if \( S = \{0\} \) or \( S = V \). If \( S \) is closed, then \( S^1 = \overline{S} \). If \( S \) is open, then \( S^1 = \mathbb{R} \backslash S \).

79. Let \( G \) be a group. Then \( G \) is cyclic if there exists a generator \( a \in G \) such that every element of \( G \) can be written as \( a^n \) for some integer \( n \). If \( G \) is cyclic, then \( G \) is abelian. If \( G \) is abelian, then \( G \) is isomorphic to \( \mathbb{Z}_n \) for some integer \( n \). If \( G \) is non-abelian, then \( G \) is not cyclic.
79. Let $G = S_3$ be the permutation group of 3 symbols. Then:
1. $G$ is isomorphic to a subgroup of a cyclic group
2. there exists a cyclic group $H$ such that $G$ maps homomorphically onto $H$
3. $G$ is a product of cyclic groups
4. there exists a nontrivial group homomorphism from $G$ to the additive group $(\mathbb{Q}, +)$ of rational numbers

80. Let $f(x) = 1 \mod (x - 1)$ and $f(x) = 0 \mod (x - 2)$. Which of the following statements are true?
1. $f(x)$ is empty
2. $f(x)$ is a singleton
3. $f(x)$ is a finite non-empty set
4. $f(x)$ is countably infinite

82. Let $S$ be the set of polynomials $f(x)$ with integer coefficients satisfying:

- $f(x) \equiv 1 \mod (x - 1)$
- $f(x) \equiv 0 \mod (x - 2)$

Which of the following statements are true?
1. $S$ is empty
2. $S$ is a singleton
3. $S$ is a finite non-empty set
4. $S$ is countably infinite

83. Let $E = \{a_1, a_2, \ldots, a_n\} \subseteq \Omega$. Suppose that $f: \Omega \to \mathbb{C}$ is a function such that $f(a_i)$ is analytic. Then $f$ is analytic on $\Omega$ if

1. $f$ is continuous on $\Omega$
2. $f$ is bounded on $\Omega$
3. for every $j$, if $\sum_{m=0}^{\infty} a_m (x - a_j)^m$ is a Laurent series expansion of $f$ at $a_j$; then $a_m = 0$ for $m = -1, -2, -3, \ldots$
4. for every $j$, if $\sum_{m=0}^{\infty} a_m (x - a_j)^m$ is a Laurent series expansion of $f$ at $a_j$; then $a_{-1} = 0$.
83. Let \( D \) be the open unit disk centered at 0 in \( \mathbb{C} \) and \( f: \mathbb{D} \to \mathbb{C} \) be an analytic function. Let \( f = u + iv \), where \( u, v \) are the real and imaginary parts of \( f \). If \( f(z) = \sum a_n z^n \) is the power series of \( f \), then \( f \) is constant if
1. \( f \) is analytic
2. \( u(1/2) = u(z) \quad \forall z \in \mathbb{D} \)
3. The set \( \{ n \in \mathbb{N} \mid a_n = 0 \} \) is infinite
4. For any 
\[ f \left( \frac{1}{n} \right) = 0 \quad \forall a \in D \text{ with } |a| \geq 1/2 \]

84. निरूपित \( n \) से \( n \) तक तकनीकी कलान वाला है?
1. यदि \( (a_n) \) गतिरिष्ट (bounded) है तो \( \sum a_n z^n \) एक शैक्षिक कलान को \( p \)-रिक्षित बनाता है।
2. यदि \( \sum a_n z^n \) हाइक्रिट्स विद्युतीय (open unit disk) या \( \mathbb{C} \) एक शैक्षिक कलान को परिनिर्देशित कर 
3. यदि तत्त्व तीन वाला कलान (power series functions)
\[ f(z) = \sum_{n=0}^{\infty} a_n z^n \]
\[ g(z) = \sum_{n=0}^{\infty} b_n z^n \]
(\( a_n \) और \( b_n \) के अलग-अलग वैकल्पिक विनिर्देशित फलन) \( f: g \) हाइक्रिट्स विद्युतीय (open unit disk) पर \( \mathbb{C} \) एक अलग-अलग (power series)
\[ \sum_{n=0}^{\infty} c_n z^n \text{ द्वारा परिनिर्देशित किया जाना है।} \]
4. \( f(z) = \sum_{n=0}^{\infty} a_n z^n \) की जरूरत विद्युतीय (radius of convergence) \( \Omega \) है तो जरूरत विद्युतीय \( |z| \leq 1 \) \( f \) कायम होगा।

85. Which of the following statements are true?
1. Every compact metric space is separable
2. If a metric space \( (X, d) \) is separable, then the metric \( d \) is not the discrete metric
3. Every separable metric space is second countable
4. Every first countable topological space is separable

86. एक गणितीय गणित (topological space) \( X \) के एक अलग अलग \( A \) के जीवन को \( \phi \) राजा है?
1. \( \phi \) वह \( X \) का \( X \) की दशा (nowhere dense) है तो \( X \) का \( A \) सबसे (dense) राजा।
2. \( \phi \) या \( \emptyset \) नहीं है तो \( X \) एक \( \emptyset \) का सबसे नहीं राजा।
3. \( \phi \) वह \( X \) का अंतर (interior) है तो \( X \) का \( A \) होगा।
4. \( \phi \) या \( \emptyset \) नहीं सबसे है तो \( X \) का \( A \) का अंतर (interior) है।

86. Let \( X \) be a topological space and \( A \) be a non-empty subset of \( X \). Then one can conclude that
1. \( A \) is dense in \( X \) if \( (X \setminus A) \) is nowhere dense in \( X \)
2. \( (X \setminus A) \) is nowhere dense in \( X \) if \( A \) is dense in \( X \)
3. \( A \) is dense in \( X \), if the interior of \( (X \setminus A) \) is empty
4. the interior of \( (X \setminus A) \) is empty, if \( A \) is dense in \( X \)
87. Which of the following statements are true?
1. The multiplicative group of a finite field is always cyclic.
2. The additive group of a finite field is always cyclic.
3. There exists a finite field of any given order.
4. There exists at most one finite field (up to isomorphism) of any given order.

88. Prove that if \( f(x) \in \mathbb{Z}[x] \) is a monic polynomial, then \( f(x) \) has a root in \( \mathbb{Z} \).

89. Prove that if \( f(x) \in \mathbb{Z}[x] \) is a monic polynomial, then \( f(x) \) has a root in \( \mathbb{Z} \).

90. Let \( f(x) \in \mathbb{Z}[x] \) be a monic polynomial. Then the roots of \( f(x) \)
1. can belong to \( \mathbb{Z} \).
2. always belong to \( (\mathbb{Z}(\mathbb{Q})) \cup \mathbb{Z} \).
3. always belong to \( \mathbb{Z} \).
4. can belong to \( (\mathbb{Z}(\mathbb{Q})) \).

91. Which of the following statements are true?
1. A subring of an integral domain is an integral domain.
2. A subring of an unique factorization domain (UFD) is a UFD.
3. A subring of a principal ideal domain (PID) is a PID.
4. A subring of an Euclidean domain is an Euclidean domain.

92. Prove that if \( G \) is a group of order \( 2 \), then \( G \) is isomorphic to \( \mathbb{Z}/2\mathbb{Z} \).

93. Prove that if \( G \) is a group of order \( 4 \), then \( G \) is isomorphic to \( \mathbb{Z}/4\mathbb{Z} \).

94. Prove that if \( G \) is a group of order \( 8 \), then \( G \) is isomorphic to \( \mathbb{Z}/8\mathbb{Z} \).

95. Prove that if \( G \) is a group of order \( 16 \), then \( G \) is isomorphic to \( \mathbb{Z}/16\mathbb{Z} \).

96. Prove that if \( G \) is a group of order \( 32 \), then \( G \) is isomorphic to \( \mathbb{Z}/32\mathbb{Z} \).

97. Prove that if \( G \) is a group of order \( 64 \), then \( G \) is isomorphic to \( \mathbb{Z}/64\mathbb{Z} \).

98. Prove that if \( G \) is a group of order \( 128 \), then \( G \) is isomorphic to \( \mathbb{Z}/128\mathbb{Z} \).

99. Prove that if \( G \) is a group of order \( 256 \), then \( G \) is isomorphic to \( \mathbb{Z}/256\mathbb{Z} \).

UNIT 3

90. Prove that if \( G \) is a group of order \( 2 \), then \( G \) is isomorphic to \( \mathbb{Z}/2\mathbb{Z} \).

91. Prove that if \( G \) is a group of order \( 4 \), then \( G \) is isomorphic to \( \mathbb{Z}/4\mathbb{Z} \).

92. Prove that if \( G \) is a group of order \( 8 \), then \( G \) is isomorphic to \( \mathbb{Z}/8\mathbb{Z} \).

93. Prove that if \( G \) is a group of order \( 16 \), then \( G \) is isomorphic to \( \mathbb{Z}/16\mathbb{Z} \).

94. Prove that if \( G \) is a group of order \( 32 \), then \( G \) is isomorphic to \( \mathbb{Z}/32\mathbb{Z} \).

95. Prove that if \( G \) is a group of order \( 64 \), then \( G \) is isomorphic to \( \mathbb{Z}/64\mathbb{Z} \).

96. Prove that if \( G \) is a group of order \( 128 \), then \( G \) is isomorphic to \( \mathbb{Z}/128\mathbb{Z} \).

97. Prove that if \( G \) is a group of order \( 256 \), then \( G \) is isomorphic to \( \mathbb{Z}/256\mathbb{Z} \).

98. Prove that if \( G \) is a group of order \( 512 \), then \( G \) is isomorphic to \( \mathbb{Z}/512\mathbb{Z} \).

99. Prove that if \( G \) is a group of order \( 1024 \), then \( G \) is isomorphic to \( \mathbb{Z}/1024\mathbb{Z} \).
91. Assume that a non-singular matrix \( A = L + D + U \) where \( L \) and \( U \) are lower and upper triangular matrices respectively with all diagonal entries are zero, and \( D \) is a diagonal matrix. Let \( x^* \) be the solution of \( Ax = b \). Then the Gauss-Seidel iteration method \( x^{(k+1)} = Hx^{(k)} + c \), \( k = 0, 1, 2, \ldots \) with \( ||H|| < 1 \) converges to \( x^* \) provided \( H \) is equal to:
1. \(-D^{-1}(L + U)\)
2. \(-(D + L)^{-1}U\)
3. \(-D^{-1}(L + U)^{-1}\)
4. \(-L^{-1}(D - L)^{-1}\)

92. Let \( x \in \mathbb{R}^n \). If \( x = x_1 + x_2 \) where \( x_1, x_2 \in \mathbb{R}^n \), then the element-wise multiplication is given by \( x_1 \cdot x_2 \). The Hadamard product is defined as \( x_1 \odot x_2 = (x_1)_i (x_2)_i \). The element-wise division is then \( x_1 \oslash x_2 = (x_1)_i / (x_2)_i \), provided \((x_2)_i \neq 0\). The element-wise composition is defined as \( x_1 \circ x_2 = (x_1)_i \odot (x_2)_i \).

93. If \( u(x, t) \) is the solution of
\[
\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0, \quad 0 < x < 1, \quad t > 0
\]
and the initial condition \( u(x, 0) = f(x) \), \( x \in (0, 1) \). Then the solution is given by
\[
u(x, t) = f(x - ct) + f(x + ct)
\]

94. Assume that \( a: [0, \infty) \rightarrow \mathbb{R} \) is a continuous function. Consider the ordinary differential equation
\[y'(x) = a(x)y(x), \quad x > 0, \quad y(0) = y_0 \neq 0\]
Which of the following statements are true?

1. If \( \int_{a}^{b} |a(x)|dx < \infty \), then \( y \) is bounded.
2. If \( \int_{a}^{b} |v(x)|dx < \infty \), then \( \lim_{x \to \infty} y(x) \) exists.
3. If \( \lim_{x \to \infty} u(x) = 1 \), then \( \lim_{x \to \infty} |y(x)| = \infty \).
4. If \( \lim_{x \to \infty} u(x) = 1 \), then \( y \) is monotone.

95. What is the critical point of \((x, 0)\) of the system

\[
\begin{align*}
\frac{dx}{dt} &= 2x - 7y \\
\frac{dy}{dt} &= 8x - 6y
\end{align*}
\]

Then the critical point \((0, 0)\) of the system is an

1. asymptotically stable node
2. unstable node
3. asymptotically stable spiral
4. unstable spiral

96. Consider the system of differential equations

\[
\begin{align*}
\frac{dy}{dx} &= 0, \quad y(0) = 0 \quad \text{and} \quad y(\pi) = 0
\end{align*}
\]

Which of the following statements are true?

1. If \( \int_{0}^{\pi} |a(x)|dx < \infty \), then \( y \) is bounded.
2. If \( \int_{0}^{\pi} |v(x)|dx < \infty \), then \( \lim_{x \to \infty} y(x) \) exists.
3. If \( \lim_{x \to \infty} u(x) = 1 \), then \( \lim_{x \to \infty} |y(x)| = \infty \).
4. If \( \lim_{x \to \infty} u(x) = 1 \), then \( y \) is monotone.

97. Consider the Sturm-Liouville problem:

\[
y'' + \lambda y = 0, \quad y(0) = 0 \quad \text{and} \quad y(\pi) = 0.
\]

Which of the following statements are true?

1. There exist only countably many characteristic values.
2. There exist uncountably many characteristic values.
3. Each characteristic function corresponding to the characteristic value \( \lambda \) has exactly \( \sqrt{\lambda} \) zeros in \((0, \pi)\).
4. Each characteristic function corresponding to the characteristic value \( \lambda \) has exactly \( \sqrt{\lambda} \) zeros in \((0, \pi)\).

98. For the simple harmonic oscillator (simple harmonic oscillator) of the Hamiltonian Hamiltonian:

\[
H(p, q) = \frac{p^2}{2m} + \frac{1}{2} m \ddot{q}^2.
\]

Then a possible Lagrangian corresponding to \( H \) can be

\[
\begin{align*}
1. & L = \frac{1}{2} m q^2 - \frac{1}{2} m \ddot{q}^2 \\
2. & L = \frac{1}{2} m q^2 - \frac{1}{2} m \ddot{q}^2 + \frac{1}{4} (q^2 + 3q^4) \\
3. & L = \frac{1}{2} m q^2 + \frac{1}{2} m \ddot{q}^2 \\
4. & L = \frac{1}{2} m q^2 + \frac{1}{2} m \ddot{q}^2 + \frac{1}{4} (q^2 + 3q^4)
\end{align*}
\]

99. For a simple harmonic oscillator in \( H(p, q) = \frac{p^2}{2m} + \frac{1}{2} m \ddot{q}^2 \), then a possible Lagrangian corresponding to \( H \) can be

\[
\begin{align*}
1. & L = \frac{1}{2} m q^2 - \frac{1}{2} m \ddot{q}^2 \\
2. & L = \frac{1}{2} m q^2 - \frac{1}{2} m \ddot{q}^2 + \frac{1}{4} (q^2 + 3q^4) \\
3. & L = \frac{1}{2} m q^2 + \frac{1}{2} m \ddot{q}^2 \\
4. & L = \frac{1}{2} m q^2 + \frac{1}{2} m \ddot{q}^2 + \frac{1}{4} (q^2 + 3q^4)
\end{align*}
\]

100. For the simple harmonic oscillator (simple harmonic oscillator) of the Hamiltonian Hamiltonian:

\[
H(p, q) = \frac{p^2}{2m} + \frac{1}{2} m \ddot{q}^2.
\]

Then a possible Lagrangian corresponding to \( H \) can be

\[
\begin{align*}
1. & L = \frac{1}{2} m q^2 - \frac{1}{2} m \ddot{q}^2 \\
2. & L = \frac{1}{2} m q^2 - \frac{1}{2} m \ddot{q}^2 + \frac{1}{4} (q^2 + 3q^4) \\
3. & L = \frac{1}{2} m q^2 + \frac{1}{2} m \ddot{q}^2 \\
4. & L = \frac{1}{2} m q^2 + \frac{1}{2} m \ddot{q}^2 + \frac{1}{4} (q^2 + 3q^4)
\end{align*}
\]

4-C-H
98. The values of $\lambda$ for which the following equation has a non-trivial solution

$$\phi(x) = \lambda \int_0^\pi [\cos x \cos t - 2\sin x \sin t] \phi(t) \, dt$$

where $K(x, t) = \begin{cases} \sin x \cos t, & 0 \leq x \leq t \\ \cos x \sin t, & t \leq x \leq \pi \end{cases}$

are

1. $(n + \frac{1}{2})^2 - 1, n \in \mathbb{N}$
2. $n^2 - 1, n \in \mathbb{N}$
3. $\frac{1}{2} (n + 1)^2 - 1, n \in \mathbb{N}$
4. $\frac{1}{2} (2n + 1)^2 - 1, n \in \mathbb{N}$

99. Consider the integral equation

$$\phi(x) = \lambda \int_0^\pi \left[ \cos x \cos t - 2\sin x \sin t \right] \phi(t) \, dt$$

$$+ \cos x, \quad 0 \leq x \leq \pi$$

Which of the following statements are true?"
3. \[ \Delta^2 U_n = 2 \Delta U_n + 2U_n = 0 \]
4. \[ \Delta^2 U_{n+1} - \frac{3}{2} \Delta^2 U_n = 0 \]

102. The forward difference operator is defined as \( \Delta U_n = U_{n+1} - U_n \). Then which of the following difference equations has an unbounded general solution?
1. \[ \Delta^2 U_n - 3 \Delta U_n + 2U_n = 0 \]
2. \[ \Delta^2 U_n + 4 \Delta U_n + 3U_n = 0 \]
3. \[ \Delta^2 U_n - 2 \Delta U_n + 2U_n = 0 \]
4. \[ \Delta^3 U_{n+1} - \frac{3}{2} \Delta^2 U_n = 0 \]

Unit - 4

103. Two random variables \( X \) and \( Y \) are two independent exponential random variables with means \( \theta \) and \( 2\theta \) respectively, where \( \theta \) is unknown. Which of the following statements are true?
1. \( X + 2Y \) is sufficient for \( \theta \).
2. Right-tailed test based on \( X + 2Y \) is UMP for testing \( H_0: \theta < 1 \) against \( H_1: \theta > 1 \).
3. Left-tailed test based on \( 2X + Y \) is UMP for testing \( H_0: \theta = 1 \) against \( H_1: \theta < 1 \).
4. UMP test does not exist for testing \( H_0: \theta = 1 \) against \( H_0: \theta = 0 \).

104. Suppose \( X \) and \( Y \) are two independent normal random variables with means \( \mu \) and \( \sigma^2 \) respectively, where \( \sigma^2 \) is unknown. Which of the following statements are true?
1. \( X + 2Y \) is sufficient for \( \theta \).
2. Right-tailed test based on \( X + 2Y \) is UMP for testing \( H_0: \theta < 1 \) against \( H_1: \theta > 1 \).
3. Left-tailed test based on \( 2X + Y \) is UMP for testing \( H_0: \theta = 1 \) against \( H_1: \theta < 1 \).
4. UMP test does not exist for testing \( H_0: \theta = 1 \) against \( H_0: \theta = 0 \).

105. Suppose \( X_1, X_2, \ldots, X_n \) are independent and identically distributed (i.i.d.) normal random variables with means \( \mu \) and \( \sigma^2 \) respectively. Which of the following statements are true?
1. \( X + 2Y \) is sufficient for \( \theta \).
2. Right-tailed test based on \( X + 2Y \) is UMP for testing \( H_0: \theta < 1 \) against \( H_1: \theta > 1 \).
3. Left-tailed test based on \( 2X + Y \) is UMP for testing \( H_0: \theta = 1 \) against \( H_1: \theta < 1 \).
4. UMP test does not exist for testing \( H_0: \theta = 1 \) against \( H_0: \theta = 0 \).
Which of the following statements are true?
1. \( X_1 + 2X_2 \) is a sufficient statistic
2. \( X_1 - X_2 \) is a sufficient statistic
3. \( X_1^2 + X_2^2 \) is a sufficient statistic
4. \( X_1^2 + X_2 \) is a sufficient statistic

106. Let \( W \) be a random variable defined as \( W = X + Y \) where \( X \) and \( Y \) are independent exponential random variables with parameter 1. Define \( U = X/(X+Y) \). Which of the following are true?
1. \( E(U) = 1/2 \)
2. \( U \) is uniform on \((0,1)\)
3. \( W, U \) are independent
4. \( W, U \) are uncorrelated (uncorrelated)

107. Let \( \{X_n\}_{n=1}^{\infty} \) be a sequence of i.i.d. random variables with \( E(X_i) = 0 \) and \( V(X_i) = 1 \). Which of the following are true?
1. \( \frac{1}{n} \sum_{i=1}^{n} X_i^2 \to 0 \) (सत्य)
2. \( \frac{1}{n^{3/2}} \sum_{i=1}^{n} X_i \to 0 \) (सत्य)
3. \( \frac{1}{n^{1/2}} \sum_{i=1}^{n} X_i \to 0 \) (सत्य)
4. \( \frac{1}{n} \sum_{i=1}^{n} X_i^2 \to 1 \) (सत्य)

108. Let \( S = \{1, 2, 3\} \) be a Markov chain on state space \( S \) with transition probability matrix \( P \) given by
\[
P = \begin{pmatrix}
0 & 1/2 & 1/2 \\
1/2 & 0 & 1/2 \\
1/2 & 1/2 & 0
\end{pmatrix}
\]
Let \( \pi = (\pi_1, \pi_2, \pi_3) \) be a stationary distribution of the Markov chain. Which of the following statements are correct?
1. \( d(1) = 1 \)
2. \( d(1) = 2 \)
3. \( \pi_1 = 1/2 \)
4. \( \pi_3 = 1/3 \)

109. Let \( X, Y \) be two random variables such that \( X \geq 0, Y \geq 0 \), \( E(X) = 3, V(X) = 9 \), \( E(Y) = 2, V(Y) = 4 \). Which of the following are true?
1. \( 0 \leq \text{Cov}(X,Y) \leq 4 \)
2. \( E(XY) \leq 6 \)
3. \( V(X+Y) \leq 25 \)
4. \( E(X+Y)^2 \leq 25 \)
109. Let $X$ and $Y$ be two random variables satisfying:

\[ X \geq 0, Y \geq 0, \mathbb{E}(X) = 3, \mathbb{V}(X) = 9, \mathbb{E}(Y) = 2 \]

and $\mathbb{V}(Y) = 4$.

Which of the following statements are correct?

1. $0 \leq \text{Cov}(X, Y) \leq 4$
2. $\mathbb{E}(XY) \leq 6$
3. $\mathbb{V}(X + Y) \leq 25$
4. $\mathbb{E}(X + Y)^2 \geq 25$

110. A random variable $X$ and $Y$ are jointly defined as $f(x, y) = \begin{cases} \frac{1}{2}, & \text{if } 0 \leq x^2 + y^2 \leq 1 \\ 0, & \text{otherwise} \end{cases}$

Which of the following statements are correct?

1. $X$ and $Y$ are independent
2. $P(X > 0) = 1/2$
3. $E(Y) = 0$
4. $\text{Cov}(X, Y) = 0$

111. Let $X_1, X_2, \ldots, X_{20}$ be 20 observations in the interval $[0, 1]$. Let $x$ and $\bar{x}$ be the mean and the median of these observations, and let $s^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2$.

1. If 15 observations are smaller than 0.3, then $x$ cannot exceed 0.5
2. $s^2$ will be maximum if 10 of these observations are 1 and the rest are 0
3. If all observations except one are smaller than 0.5, then $\bar{x}$ cannot be smaller than $x$
4. $s^2 \leq \mathbb{V}(X - \bar{x})$

112. Consider a single server $M/M/1$ queue with arrival rate $\lambda$ and service rate $\mu$. Further assume that $\lambda < \mu$. Then, which of the following statements are true?

1. Queue length becomes 0 in infinitely many time intervals with probability 1
2. Queue length becomes 0 in at most finitely many time intervals with probability 1
3. Steady state exists for the queue
4. $\lim_{t \to \infty} P(L_t > 0) = \frac{1}{\rho}$, where $L_t$ is the number of customers in the system at time $t$

113. Let $F$, $h$, $r$, $m$, $h$, and $s$ be functions. $\{0, \infty\}$ is a time interval, $\lambda$ is a constant, and $t$ is a variable. The function $h(t)$ is called the lifetime distribution function. The function $h(t)$ is called the hazard function.$h(t)$ is called the mean residual lifetime function. If $h(t)$ is a constant, then $h(t)$ is called an absolutely continuous distribution function.
113. Let $F$, $h$ and $m$ be the lifetime distribution function, the hazard function and the mean residual lifetime function respectively, defined on $(0,\infty)$. Assume that $F$ is absolutely continuous. Which of the following statements are true?
1. $\int_0^\infty h(t)\,dt = 1$
2. $m(t) = \frac{\int_0^t h(u)\,du}{1 - F(t)}$, for $t > 0$
3. $m(t)$ is strictly increasing in $t$ if the lifetime distribution is exponential with mean $\lambda > 0$
4. $h(t)m(t) = 1$ for all $t > 0$ if the lifetime distribution is exponential with mean $\lambda > 0$

114. गोली आकार की सदृशता का तथ्य 2.5 दरम्यान
विचलन 0.5 है तो
1. गोली 2.5 से दरी होनी चाहिए।
2. गोली 2.5 से लायी होनी चाहिए।
3. गोली 3 से लायी होनी चाहिए।
4. गोली 2 से दरी होनी चाहिए।

115. एक समूहकोष का अन्तर-अन्तर कैंसर में 4
लकड़ी में तथा जोड़ीमें (with replacement) 2
नवरी में करने के लाल कुर्सियाँ प्रदर्शित (simple
random sample) किया। इस प्रदर्शन की जोड़ी
को $X, Y$ कहा जाता। इस प्रदर्शन की जोड़ी
को $X, Y$ कहा जाता। इस प्रदर्शन की जोड़ी
को $X, Y$ कहा जाता। इस प्रदर्शन की जोड़ी
को $X, Y$ कहा जाता। इस प्रदर्शन की जोड़ी
को $X, Y$ कहा जाता। इस प्रदर्शन की जोड़ी
को $X, Y$ कहा जाता। इस प्रदर्शन की जोड़ी
को $X, Y$ कहा जाता। इस प्रदर्शन की जोड़ी
को $X, Y$ कहा जाता। इस प्रदर्शन की जोड़ी
को $X, Y$ कहा जाता। इस प्रदर्शन की जोड़ी
को $X, Y$ कहा जाता। इस प्रदर्शन की जोड़ी
को $X, Y$ कहा जाता। इस प्रदर्शन की जोड़ी
को $X, Y$ कहा जाता। इस प्रदर्शन की जोड़ी
को $X, Y$ कहा जाता। इस प्रदर्शन की जोड़ी
को $X, Y$ कहा जाता। इस प्रदर्शन की जोड़ी
को $X, Y$ कहा जाता। इस प्रदर्शन की जोड़ी
को $X, Y$ कहा जाता। इस प्रदर्शन की जोड़ी
को $X, Y$ कहा जाता। इस प्रदर्शन की जोड़ी
को $X, Y$ कहा जाता। इस प्रदर्शन की जोड़ी
को $X, Y$ कहा जाता। इस प्रदर्शन की जोड़ी
को $X, Y$ कहा जाता। इस प्रदर्शन की जोड़ी
को $X, Y$ कहा जाता।
1. $(X_1 + X_2)/2$ का माध्यम (variance)
$(2X_1 + 3X_2)/5$ के माध्यम से अधिक है।
2. $(X_1 + X_2)/3$ का माध्यम
$(2X_1 + 3X_2)/5$ के माध्यम से अधिक है।

116. A statistician has drawn a simple random sample
of size $2$ with replacement from 4
boys with distinct heights. Let $X_1$ be the
sample mean of their heights. Then,
another statistician has drawn a simple
random sample of size $2$ without
replacement from those 4 boys. Let $X_2$ be
the sample mean of their heights. Which of
the following statements are correct?
1. $(X_1 + X_2)/2$ has larger variance
than that of $(2X_1 + 3X_2)/5$
2. $(X_1 + 2X_2)/3$ has larger variance
than that of $(2X_1 + 3X_2)/5$
3. $(X_1 + 2X_2)/2$ has smaller variance
than that of $(2X_1 + 3X_2)/5$
4. $(X_1 + X_2)/3$ has smaller variance
than that of $(2X_1 + 3X_2)/5$

116. टिकिया योगदान सिख (two-class classification
problem) का निष्पादन किया। इसके बाद दो प्रकार
की दो वर्गों मिश्रित प्रकाश से निष्पादित हैं
$f_1(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$
$f_2(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$
इन दोनों की पूर्व आवश्यकता को स्वरूप $\pi_1$ और $\pi_2$ के
विनिथित किया। एक क्रिकेटर (classifier) $\theta$, जो
eके एक आकृति को आदर्श नामकरण करता है।
$x < 1/2$ है, व लिखित वर्ण में प्रतिलोम अनुपर नहीं
$\theta > 1/2$ है, व लिखित वर्ण में प्रतिलोम अनुपर नहीं
1. यदि $\pi_1 = \pi_2$, तो $\theta$ एक आदर्श वर्गीय
(Bayes classifier) है।
2. यदि $\pi_1 > \pi_2$, तो $\theta$ एक आदर्श वर्गीय
है।
3. यदि $\pi_1 < \pi_2$, तो $\theta$ एक आदर्श वर्गीय
है।
4. यदि $\pi_1 = \pi_2$, तो $\theta$ एक आदर्श वर्गीय
है।

116. Consider a two-class classification problem,
where the densities of the two competing
classes are given by
$f_1(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$
and
$f_2(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$
Let $p_1$ and $p_2$ be the prior probabilities of these two classes. Now consider a classifier $\delta$, which classifies an observation $x$ to class 1 if $x < 1/2$ and to class 2 if $x \geq 1/2$.

1. If $p_1 = p_2$, then $\delta$ is the Bayes classifier.
2. If $p_1 > p_2$, then $\delta$ is the Bayes classifier.
3. If $p_1 < p_2$, then $\delta$ is the Bayes classifier.
4. If $p_1 = p_2$, then the average probability of misclassification for $\delta$ is $1/2$.

117. Let $N(\mu, \sigma^2)$ be the normal distribution with mean $\mu$ and variance $\sigma^2$. Then $\sigma^2$ is an unbiased estimator of $\sigma^2$ for the random variable $X$. The distribution function of $X$ is $\phi(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$.

1. The distribution is not normal if $X$ is not $N(\mu, \sigma^2)$.
2. If $X$ is $N(\mu, \sigma^2)$, then the distribution is normal.
3. If $X$ is not $N(\mu, \sigma^2)$, then the distribution is not normal.
4. If $X$ is $N(\mu, \sigma^2)$, then the distribution is normal.

118. In the linear model:

\[ Y_i = \beta_0 x_{i1} + \beta_1 x_{i2} + \epsilon_i \]

\[ Y_i = \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \epsilon_i \]

\[ Y_i = \beta_3 x_{i1} + \beta_4 x_{i2} + \beta_5 x_{i3} + \epsilon_i \]

\[ Y_i = \beta_4 x_{i1} + \beta_5 x_{i2} + \beta_6 x_{i3} + \epsilon_i \]

\[ Y_i = \beta_5 x_{i1} + \beta_6 x_{i2} + \beta_7 x_{i3} + \epsilon_i \]

1. If $\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5$ are i.i.d. $N(0, \sigma^2)$, then by the central limit theorem, $\frac{\sqrt{n}}{\sigma} \left( \sum_{i=1}^{n} \epsilon_i \right)$ is approximately $N(0, 1)$.
2. If $\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5$ are i.i.d. $N(0, \sigma^2)$, then by the central limit theorem, $\frac{\sqrt{n}}{\sigma} \left( \sum_{i=1}^{n} \epsilon_i \right)$ is approximately $N(0, 1)$.
3. $\Sigma_{i=1}^{n} e_{i} \bar{e}_{i}$ is the uniformly minimum variance unbiased estimate (UMVUE) of $\Sigma_{i=1}^{n} e_{i} \beta_{i}$.

4. $\Sigma_{i=1}^{n} e_{i} \beta_{i}$ is BLUE but not UMVUE of $\Sigma_{i=1}^{n} e_{i} \beta_{i}$.

119. If $X \sim \text{Binomial} (n, p)$, then $P(0 < p < 1)$ is true. Let $n \in \{0, 1, 2, \ldots\}$ assume $p = 0$. Then $X \sim \text{Poisson} (\lambda > 0)$ is (Degenerate) true. Hence $n$ is the prior distribution (prior distribution) of a parameter $X \sim $ Poisson (Poisson) distribution. It is not uniformly distributed as $X = 0$.

1. The parameter $X$ is a random variable, but $X \sim $ Poisson (Poisson) distribution.

2. $X = 0$ follows a Poisson distribution $X \sim $ Poisson (Poisson).

3. $X = 1$ follows a hypergeometric distribution $X \sim $ Hypergeometric (Hypergeometric).

4. $X = 0$ follows a distribution $X \sim $ Poisson (Poisson).

120. Consider a two-sample location problem with $n$ observations. Suppose that the distribution of the $i^{th}$ observation is $X_i \sim \text{Normal} (\mu, \sigma^2)$. The following statements are true:

1. $\bar{X}$ is an estimator of $\mu$.
2. $\sum_{i=1}^{n} (X_i - \mu)^2$ is a variance unbiased estimator of the variance of $\mu$.
3. $\sum_{i=1}^{n} (X_i - \mu)^2$ is a variance unbiased estimator of $\sum_{i=1}^{n} (X_i - \mu)^2$.

FOR ROUGH WORK