1. अपने हिंदी को मान्यता प्रदान के लिए हिंदी में एक सी बूटा (20 वाले 'A' में + 40 वाले 'B' + 60 वाले 'C' में) क्षुद्र विकल्प प्रश्न (MCQ) जिए एवं भज़ाएं। अगर आपके बूट के 'A' में से अधिकतम 15 और बूट 'B': 25 प्रश्नों तथा बूट 'C' में से 20 प्रश्नों के उत्तर देंते हैं, तब अनुप्रस्तुति से अधिक प्रश्नों के उत्तर दिए गए प्रश्न का उत्तर दिए गए। इस तरह से आपके बूट प्रश्न को भी जानें।

2. औपन्यास, उत्तर प्रश्न के अंश में जिए एवं भज़ाएं। अपने रोल नम्बर और केंद्र का नाम लिखें। यदि आपके प्रश्न प्रकार ऐसे हों, तब आपके प्रश्न का उत्तर दिए अथवा उत्तर नहीं दिए हो। यदि ऐसा हो, तब आपके प्रश्न का उत्तर दिए अथवा उत्तर नहीं दिए हो। उत्तर प्रश्न के अनुसार अपना प्रश्न का उत्तर दिए।

3. औपन्यास, उत्तर प्रश्न के अंश 1 में जिए एवं भज़ाएं। अपना रोल नम्बर और नाम लिखें। अपने प्रश्न प्रकार ऐसे हों। यदि आपके प्रश्न प्रकार ऐसे हों, तब आपके प्रश्न का उत्तर दिए।

4. आपके प्रश्न प्रकार ऐसे हों। यदि आपके प्रश्न प्रकार ऐसे हों, तब आपके प्रश्न का उत्तर दिए।

5. भूमि 'A' में प्रश्न का अंश 2 अंक, भूमि 'B' में प्रश्न का अंश 3 अंक, भूमि 'C' में प्रश्न का अंश 4.75 अंक व्यक्त करें।

6. भूमि 'A' तथा भूमि 'B' के प्रश्न का अंश व्यक्त करें।

7. नकल करते हुए या अनुप्रस्तुति तरीकों का प्रयोग करते हुए पाए जाने वाले परीक्षाधिकारियों का इस और अन्य भाषी परीक्षाधिकारियों के लिए अन्य विकल्प हटाने का संस्करण भी शामिल हो सकता है।

8. परीक्षा का उत्तर या रुप प्राप्त विश्लेषण के अनुसार क्रमिकता की ओर उत्तर निर्देशित करें।

9. कंट्रोलर का उपयोग करें।

10. परीक्षा समाप्ति पर दिखा विकल्प प्रश्न स्थान से वेबसाइट पर बूट प्रश्न को भज़ाएं।

11. हिंदी माध्यम संस्करण के प्रश्न में विश्लेषण करें।

12. कंट्रोलर के पूर्ण अवधि तक बूट और परीक्षाधिकारियों को ही परीक्षा पुस्तिका साथ ले जाने की अनुमति ही जारी है।
1. It takes 2 hours for Tiwari and Deo to do a job. Tiwari and Hari take 3 hours to do the same job. Deo and Hari take 6 hours to do the same job. Which of the following statements is incorrect?
1. Tiwari alone can do the job in 3 hours
2. Deo alone can do the job in 6 hours
3. Hari does not work at all
4. Hari is the fastest worker

2. Abdul, Kethrin, and Tijoo travel at speeds of 1/3, 1/2, and 1/3 of Abdul's speed, respectively. If they start at the same time then who reaches first?
1. Abdul
2. Kethrin
3. Tijoo
4. All three together

3. For a certain regular solid: number of faces + number of vertices = number of edges + 2. For three such distinct (not touching each other) objects, what is the total value of faces + vertices - edges?
1. Two
2. Four
3. Six
4. Zero

4. What will be the next figure in the following sequence?

5. A, B, C, D are points on a circle with AB=5 cm, BC=12 cm, AC=13 cm and AD=7 cm. Then, the closest approximation of CD is
1. 9 cm
2. 10 cm
3. 11 cm
4. 14 cm
6. Choose the four digit number, in which the product of the first & fourth digits is 40 and the product of the middle digits is 28. The thousands digit is as much less than the unit digit as the hundreds digit is less than the tens digit.
1. 5478
2. 5748
3. 8745
4. 8475

7. Equilateral triangles are drawn one inside the other as shown. What is the ratio of the two shaded areas?
1. $2 : 1$
2. $\sqrt{3} : 4$
3. $4 : 1$
4. $8 : 1$

8. A frog hops and lands exactly 1 meter away at a time. What is the least number of hops required to reach a point 10 cm away?
1. 1
2. 2
3. 3
4. It cannot travel such a distance

9. A train running at 36 km/h crosses a mark on the platform in 8 sec and takes 20 sec to cross the platform. What is the length of the platform?
1. 120 m
2. 280 m
3. 40 m
4. 160 m

10. When a polynomial $f(x)$ is divided by $x - 5$ or $x - 3$ or $x - 2$ it leaves a remainder of 1. Which of the following would be the polynomial?
1. $x^3 - 10x^2 + 31x + 31$
2. $x^3 - 10x^2 + 31x - 29$
3. $x^3 - 10x^2 + 31x - 31$
4. $x^3 - 10x^2 + 31x + 29$
11. Water is slowly dripping out of a tiny hole at the bottom of a hollow metallic sphere initially full of water. Ignoring the water that has flowed away, the centre of mass of the system
1. remains fixed at the centre of the sphere
2. moves down steadily as the amount of water decreases
3. moves down for some time but eventually returns to the centre of the sphere
4. moves down until half of the water is lost and then moves up

12. The diagram (not to scale) shows the top view and cross section of a pond having a square outline and equal sized steps of 0.5 m width and 0.1m height. What will be the volume of water (in m$^3$) in the pond when it is completely filled?

13. D is a point on AC in the following triangle such that $\angle ADB = \angle ABC$. Then BD (in cm) is

14. The diagram shows the graph of a function $f(x)$ and $x = -1$ is a point where the function changes sign. The function can now be written as

\[ f(x) = \begin{cases} 100 & x < -1 \\
0 & -1 \leq x < 1 \\
1 & x \geq 1 
\end{cases} \]
14. The function \( f(x) \) is plotted against \( x \) as shown. Extrapolate and find the value of the function at \( x = -1 \).

15. A notebook contains only hundred statements as under:
1. This notebook contains 1 false statement.
2. This notebook contains 2 false statements.
3. This notebook contains 99 false statements.
4. This notebook contains 100 false statements.

Which of the statements is correct?
1. 100th
2. 1st
3. 99th
4. 2nd

16. A chocolate bar having \( m \times n \) unit square tiles is given. Calculate the number of cuts needed to break it completely, without stacking, into individual tiles.
1. \( (m \times n) \)
2. \( (m - 1) \times (n - 1) \)
3. \( (m \times n) - 1 \)
4. \( (m \times n) + 1 \)

17. An experiment leads to the following set of observations of the variable \( 't' \) at different times.

<table>
<thead>
<tr>
<th>( t )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v )</td>
<td>5</td>
<td>6.1</td>
<td>9.1</td>
<td>13.7</td>
<td>20.6</td>
<td>30.8</td>
<td>41.4</td>
</tr>
</tbody>
</table>

Which of the statements is correct?
1. \( (m \times n) \)
2. \( (m - 1) \times (n - 1) \)
3. \( (m \times n) - 1 \)
4. \( (m \times n) + 1 \)

18. A person paid income tax at the rate of \( R\% \) for the first Rs 2 lakhs, and at the rate of \( (R+10)\% \) for income exceeding Rs 2 lakhs. If the total tax paid is \( (R+5)\% \) of the annual income, then what is the annual income ?
1. Rs 2.5 lakhs
2. Rs 3.0 lakhs
3. Rs 4.0 lakhs
4. Rs 5.0 lakhs
Allowing for experimental errors, which of the following expressions best describes the relationship between $t$ and $v$?

1. $v \propto t^2$
2. $(v - 5) \propto t^2$
3. $v = 5t + t^2$
4. $(v - 5) = (t + 5)^2$

19. If the father is 899 years old and the son is born 27 years ago, which of the following expressions best describes the relationship between $t$ and $v$?

1. $t = v^2$
2. $v = t^2$
3. $t = v + 27$
4. $v = t + 27$

19. The difference between the squares of the ages (in complete years) of a father and his son is 899. The age of the father when his son was born

1. cannot be ascertained due to inadequate data.
2. is 27 years.
3. is 29 years.
4. is 31 years.

20. A bicycle tube has a mean circumference of 200 cm and a circular cross section of diameter 6 cm. What is the approximate volume of water (in cc) required to completely fill the tube, assuming that it does not expand?

1. $600 \pi$
2. $1200 \pi$
3. $3600 \pi$
4. $1800 \pi$

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24. Consider the function
\[ f(x, y) = \frac{x^2}{y^2}, \quad (x, y) \in [1/2, 3/2] \times [1/2, 3/2] \]

1. 0
2. 1
3. 2
4. -2

24. Consider the function
\[ f(x, y) = \frac{x^2}{y^2}, \quad (x, y) \in [1/2, 3/2] \times [1/2, 3/2] \]

The derivative of the function at (1, 1) along the direction is:
1. 0
2. 1
3. 2
4. -2

25. Consider the improper Riemann integral
\[ \int_{0}^{\infty} y^{-1/2} \, dy. \]

This integral is:
1. continuous in [0, \infty).
2. continuous only in (0, \infty).
3. discontinuous in (0, \infty).
4. discontinuous only in \((\frac{1}{2}, \infty)\).

26. Which one of the following statements is true for the sequence of functions
\[ f_n(x) = \frac{1}{n^{1+x^2}}, \quad n = 1, 2, \ldots, x \in [1/2, 1]? \]
1. The sequence is monotonic and has 0 as the limit for all \(x \in [1/2, 1]\) as \(n \to \infty\).
2. The sequence is not monotonic and has \(f(x) = \frac{1}{x^2}\) as the limit as \(n \to \infty\).
3. The sequence is monotonic and has \(f(x) = \frac{1}{x}\) as the limit as \(n \to \infty\).
4. The sequence is not monotonic but has 0 as the limit.

27. Given a \(n \times n\) matrix \(B\), define \(e^B\) by

\[ e^B = \sum_{j=0}^{\infty} \frac{B^j}{j!}. \]

Let \(p\) be the characteristic polynomial of \(B\). Then the matrix \(e^{p(B)}\) is:
1. \(I_{n \times n}\)
2. \(0_{n \times n}\)
3. \(eI_{n \times n}\)
4. \(\pi I_{n \times n}\)

28. Let \(A\) be a \(n \times n\) real symmetric non-singular matrix. Suppose there exists \(x \in \mathbb{R}^n\) such that

\[ x'Ax < 0. \]

Then we can conclude that
1. \(\det(A) < 0\).
2. \(B = -A\) is positive definite.
3. \(\exists y \in \mathbb{R}^n: y'A^{-1}y < 0\)
4. \(\forall y \in \mathbb{R}^n: y'A^{-1}y < 0\)
29. Let $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. Let $f: \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(v, w) = w^T Av$. Pick the correct statement from below:
1. There exists an eigenvector $v$ of $A$ such that $Av$ is perpendicular to $v$.
2. The set $\{v \in \mathbb{R}^2 | f(v, v) = 0\}$ is a nonzero subspace of $\mathbb{R}^2$.
3. If $v, w \in \mathbb{R}^2$ are nonzero vectors such that $f(v, v) = 0 = f(w, w)$, then $v$ is a scalar multiple of $w$.
4. For every $v \in \mathbb{R}^2$, there exists a nonzero $w \in \mathbb{R}^2$ such that $f(v, w) = 0$.

30. Let $A$ be a $n \times m$ matrix and $b$ be a $n \times 1$ vector (with real entries). Suppose the equation $Ax = b$, $x \in \mathbb{R}^m$ admits a unique solution. Then we can conclude that
1. $m \geq n$ 
2. $n \geq m$ 
3. $n = m$ 
4. $n > m$

31. Let $V$ be the vector space of all real polynomials of degree $\leq 10$. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $Tp(x) = p'(x)$ for $p \in V$. Consider the basis $\{1, x, x^2, \ldots, x^{10}\}$ of $V$. Let $A$ be the matrix of $T$ with respect to this basis. Then
1. $\text{Trace}(A) = 1$ 
2. $\det(A) = 0$ 
3. there is no $m \in \mathbb{N}$ such that $A^m = 0$ 
4. $A$ has a nonzero eigenvalue
33. Let $P(x)$ be a polynomial of degree $d \geq 2$. The radius of convergence of the power series

$$\sum_{n=0}^{\infty} P(n)z^n$$

is:
1. 0
2. 1
3. $\infty$
4. dependent on $d$

34. Let $P(z), Q(z)$ be two complex non-constant polynomials of degree $m, n$ respectively. The number of roots of $P(z) = P(z)Q(z)$ counted with multiplicity is equal to:
1. $\min \{m, n\}$
2. $\max \{m, n\}$
3. $m + n$
4. $m - n$

35. $z = 0$ पर फलन $f(z) = e^{-e^{1/2}}$ का अवशेष है:
1. $1 + e^{-1}$
2. $e^{-1}$
3. $-e^{-1}$
4. $1 - e^{-1}$

36. The residue of the function

$$f(z) = e^{-e^{1/2}}$$

at $z = 0$ is:
1. $1 + e^{-1}$
2. $e^{-1}$
3. $-e^{-1}$
4. $1 - e^{-1}$

37. निम्न कथनों में से कौन-सा गलत है? ऐसे पूर्णांक $x$ का अस्तित्व है ताकि
1. $x \equiv 23 \mod 1000$ and $x \equiv 45 \mod 6789$
2. $x \equiv 23 \mod 1000$ and $x \equiv 54 \mod 6789$
3. $x \equiv 32 \mod 1000$ and $x \equiv 54 \mod 9876$
4. $x \equiv 32 \mod 1000$ and $x \equiv 44 \mod 9876$

38. मानें की $p$ एक अभाज्य संख्या है। क्षेत्र $\mathbb{F}_p$ के,
एक के साथ, $p$ गुणनफलीय के कितने मूल उपवल में हैं?
1. 0
2. 1
3. $p$
4. $p^2$
38. Let \( p \) be a prime number. How many distinct sub-rings (with unity) of cardinality \( p \) does the field \( \mathbb{F}_p \times \mathbb{F}_p \) have?
1. \( 0 \)
2. \( 1 \)
3. \( p \)
4. \( p^2 \)

39. Let \( G = (\mathbb{Z}/25\mathbb{Z})^* \) be the group of units (i.e. the elements that have a multiplicative inverse) in the ring \((\mathbb{Z}/25\mathbb{Z})\). Which of the following is a generator of \( G \)?
1. \( 3 \)
2. \( 4 \)
3. \( 5 \)
4. \( 6 \)

40. Let \( p \geq 5 \) be a prime. Then \( \mathbb{F}_p \times \mathbb{F}_p \) has at least five subgroups of order \( p \).
1. \( \mathbb{F}_p \times \mathbb{F}_p \) has at least five subgroups of order \( p \).
2. Every subgroup of \( \mathbb{F}_p \times \mathbb{F}_p \) of the form \( H_1 \times H_2 \) where \( H_1, H_2 \) are subgroups of \( \mathbb{F}_p \).
3. Every subgroup of \( \mathbb{F}_p \times \mathbb{F}_p \) is an ideal of the ring \( \mathbb{F}_p \times \mathbb{F}_p \).
4. The ring \( \mathbb{F}_p \times \mathbb{F}_p \) is a field.

41. Let \( y_1 \) and \( y_2 \) be two solutions of the problem
\[
\begin{align*}
y''(t) + ay'(t) + by(t) &= 0, t \in \mathbb{R} \\
y(0) &= 0
\end{align*}
\]
where \( a \) and \( b \) are real constants. Let \( w \) be the Wronskian of \( y_1 \) and \( y_2 \). Then
1. \( w(t) = 0, \forall t \in \mathbb{R} \)
2. \( \text{constant} \)
3. \( w \) is a nonconstant positive function
4. There exists \( t_1, t_2 \in \mathbb{R} \) such that \( w(t_1) < 0 < w(t_2) \).

42. Let \( \mathbb{A} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -2 \end{bmatrix} \), \( x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \) and
\[
|x(t)| = (x_1^2(t) + x_2^2(t) + x_3^2(t))^{1/2}
\]
where \( x(t) \) is a solution of the system
\[
\begin{align*}
x'(t) &= Ax(t) \\
x(0) &= x_0
\end{align*}
\]
Then any solution of the first order system of the ordinary differential equation
\[
x'(t) = Ax(t) \\
x(0) = x_0
\]
satisfies
1. \( \lim_{t \to \infty} |x(t)| = 0 \)
2. \( \lim_{t \to \infty} |x(t)| = \infty \)
3. \( \lim_{t \to \infty} |x(t)| = 2 \)
4. \( \lim_{t \to \infty} |x(t)| = 12 \)

43. Let \( a, b, c, d \) be four differentiable functions defined on \( \mathbb{R}^2 \). Then the partial differential equation is

1. always hyperbolic
2. always parabolic
3. never parabolic
4. never elliptic

44. For the Cauchy problem

\[
\begin{align*}
&u_t - uu_x = 0, \quad x \in \mathbb{R}, t > 0 \\
&u(x,0) = x, \quad x \in \mathbb{R},
\end{align*}
\]

the solution \( u \) exists for all \( t > 0 \) and breaks down at \( t = \frac{1}{2} \).

45. The curve of fixed length \( l \), that joins the points \((0, 0)\) and \((1, 0)\), lies above the \( x \)-axis, and encloses the maximum area between itself and the \( x \)-axis, is

1. a straight line
2. a parabola
3. an ellipse
4. a circle

46. Consider the integral equation

\[
y(x) = x^3 + \int_0^x \sin(x - t) y(t) \, dt, \quad x \in [0, \pi]
\]

Then the value of \( y(1) \) is

1. \( \frac{19}{20} \)
2. \( 1 \)
3. \( \frac{17}{20} \)
4. \( \frac{21}{20} \)
48. Consider the equations of motion for a system
\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = 0 , \quad i = 1, 2, 3, \ldots, n
\]
where
\[
L = T - V \quad \text{[with } T(t,q_i,\dot{q}_i) \text{ as kinetic energy]}
\]
and \(V(t,q_i)\) as potential energy, the generalized coordinates, and \(\dot{q}_i\) the generalized velocities. Then the equations of motion in the form as above are

1. necessarily restricted to a conservative system but there is no unique choice of \(L\).
2. not necessarily restricted to a conservative system and there is a unique choice of \(L\).
3. necessarily restricted to a conservative system and there is a unique choice of \(L\).
4. not necessarily restricted to a conservative system and there is no unique choice of \(L\).

49. Hundred (100) tickets are marked 1, 2, ..., 100 and are arranged at random. Four tickets are picked from these tickets and are given to four persons A, B, C and D. What is the probability that A gets the ticket with the largest value (among A, B, C, D) and D gets the ticket with the smallest value (among A, B, C, D)?

1. \(\frac{1}{4}\)
2. \(\frac{1}{6}\)
3. \(\frac{1}{2}\)
4. \(\frac{1}{12}\)

50. Let \(X\) and \(Y\) be independent and identically distributed random variables such that \(P(X = 0) = P(X = 1) = \frac{1}{2}\). Let \(Z = X + Y\) and \(W = |X - Y|\). Then which statement is not correct?

1. \(X\) and \(W\) are independent.
2. \(Y\) and \(W\) are independent.
3. \(Z\) and \(W\) are uncorrelated.
4. \(Z\) and \(W\) are independent.

51. मानें कि \(X_i\) तथा \(Y_i\) दो स्वतंत्र शून्य जनन प्रक्रियायें हैं, क्रमशः जनन गतियों \(\lambda_1\) तथा \(\lambda_2\) के साथ। मानें कि \(Z_i = X_i + Y_i\) है। तो

1. \(Z_i\) एक शून्य जनन प्रक्रिया है।
2. \(Z_i\) एक शून्य जनन प्रक्रिया है, जनन गति \(\lambda_1 + \lambda_2\) के साथ।
3. \(Z_i\) एक शून्य जनन प्रक्रिया है, जनन गति \(\lambda_1 \lambda_2\) के साथ।
4. \(Z_i\) एक शून्य जनन प्रक्रिया है, जनन गति \(\frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}\) के साथ।
51. Let \( \{X_t\} \) and \( \{Y_t\} \) be two independent pure birth processes with birth rates \( \lambda_1 \) and \( \lambda_2 \) respectively. Let \( Z_t = X_t + Y_t \). Then
1. \( \{Z_t\} \) is not a pure birth process.
2. \( \{Z_t\} \) is a pure birth process with birth rate \( \lambda_1 + \lambda_2 \).
3. \( \{Z_t\} \) is a pure birth process with birth rate \( \min(\lambda_1, \lambda_2) \).
4. \( \{Z_t\} \) is a pure birth process with birth rate \( \lambda_1 \lambda_2 \).

52. Let \( X_1 \sim N(0, 1) \) be such that \( X_2 = \begin{cases} -X_1, & -2 \leq X_1 \leq 2 \\ X_1, & \text{otherwise} \end{cases} \), \( X_2 \), and \( \{X_t\} \) be independent.
1. For each \( (X_1, X_2) \), \( \text{corr}(X_1, X_2) = 1 \).
2. \( X_2 \) does not have \( N(0, 1) \) distribution.
3. \( (X_1, X_2) \) has a bivariate normal distribution.
4. \( (X_1, X_2) \) does not have a bivariate normal distribution.

53. Let \( X_1, \cdots, X_n \) be a random sample from \( N(\theta, 1) \), where \( \theta \in \{1, 2\} \). Let \( \chi^2_{n, a/2} \) be an upper \((a/2)\)th percentile point of a \( \chi^2_n \) distribution. Then a confidence interval for \( \theta \) is given by
1. \( \left( \frac{\sum_i^n X_i - \mu^2}{n \chi^2_{n, a/2}}, \frac{\sum_i^n X_i - \mu^2}{n \chi^2_{n, 1-a/2}} \right) \)
2. \( \left( \frac{\sum_i^n (X_i - \mu)^2}{(n-1) \chi^2_{n-1, a/2}}, \frac{\sum_i^n (X_i - \mu)^2}{(n-1) \chi^2_{n-1, 1-a/2}} \right) \)
3. \( \left( \frac{\sum_i^n (X_i - \bar{X})^2}{n \chi^2_{n, a/2}}, \frac{\sum_i^n (X_i - \bar{X})^2}{n \chi^2_{n, 1-a/2}} \right) \)
4. \( \left( \frac{\sum_i^n (X_i - \bar{X})^2}{n \chi^2_{n, a/2}}, \frac{\sum_i^n (X_i - \bar{X})^2}{n \chi^2_{n, 1-a/2}} \right) \)

54. Let \( X_1, \cdots, X_n \) denote a random sample from \( N(\mu, \sigma^2) \) distribution. Let \( \mu \in \mathbb{R} \) be known and \( \sigma^2 (> 0) \) be unknown. Let \( \chi^2_{n, a/2} \) be an upper \((a/2)\)th percentile point of a \( \chi^2_n \) distribution. Then a 100\((1 - \alpha)\)% confidence interval for \( \sigma^2 \) is given by
1. \( \left( \frac{n \sum_i^n X_i^2 - \mu^2}{\chi^2_{n, a/2}}, \frac{n \sum_i^n X_i^2 - \mu^2}{\chi^2_{n, 1-a/2}} \right) \)
2. \( \left( \frac{n \sum_i^n (X_i - \mu)^2}{(n-1) \chi^2_{n-1, a/2}}, \frac{n \sum_i^n (X_i - \mu)^2}{(n-1) \chi^2_{n-1, 1-a/2}} \right) \)
3. \( \left( \frac{n \sum_i^n (X_i - \bar{X})^2}{n \chi^2_{n, a/2}}, \frac{n \sum_i^n (X_i - \bar{X})^2}{n \chi^2_{n, 1-a/2}} \right) \)
4. \( \left( \frac{n \sum_i^n (X_i - \bar{X})^2}{n \chi^2_{n, a/2}}, \frac{n \sum_i^n (X_i - \bar{X})^2}{n \chi^2_{n, 1-a/2}} \right) \)

55. Let \( X_1, \cdots, X_n \) be a random sample from \( N(\theta, 1) \), where \( \theta \in \{1, 2\} \). Then which of the following statement is true?
1. MLE of \( \theta \) does not exist.
2. MLE of \( \theta \) is \( \bar{X} \).
3. MLE of \( \theta \) exists but it is not \( \bar{X} \).
4. MLE of \( \theta \) is an unbiased estimator of \( \theta \).
3. In the context of testing of statistical hypotheses, which one of the following statements is true?

1. When testing a simple hypothesis \( H_0 \) against an alternative simple hypothesis \( H_1 \), the likelihood ratio principle leads to the most powerful test.

2. When testing a simple hypothesis \( H_0 \) against an alternative simple hypothesis \( H_1 \), the desired level of the power of the test is 1.

3. For testing a simple hypothesis \( H_0 \) against an alternative simple hypothesis \( H_1 \), randomized test is used to achieve the desired level of the power of the test.

4. UMP tests for testing a simple hypothesis \( H_0 \) against an alternative composite \( H_1 \), always exist.

5. Let \( X \sim N_3(\mu, \Sigma) \) where \( \mu = (1, 1, 1) \) and \( \Sigma = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & c \\ 1 & c & 2 \end{pmatrix} \). The value of \( c \) such that \( X_2 \) and \( -X_1 + X_2 - X_3 \) are independent is

   1. \( -2 \)
   2. \( 0 \)
   3. \( 2 \)
   4. \( 1 \)

56. Let \( Y_1, Y_2, Y_3 \) be uncorrelated observations with common variance \( \sigma^2 \) and expectations given by \( \mathbb{E}(Y_1) = \beta_1, \mathbb{E}(Y_2) = \beta_2 \) and \( \mathbb{E}(Y_3) = \beta_1 + \beta_2 \), where \( \beta_1, \beta_2 \) are unknown parameters. The best linear unbiased estimator of \( \beta_1 + \beta_2 \) is

   1. \( Y_3 \)
   2. \( Y_1 + Y_2 \)
   3. \( \frac{1}{3}(Y_1 + Y_2 + 2Y_3) \)
   4. \( \frac{1}{2}(Y_1 + Y_2 + Y_3) \)

58. A sample of size \( n \geq 2 \) is drawn without replacement from a finite population of size \( N \), using an arbitrary sampling scheme. Let \( \pi_i \) denote the inclusion probability of the \( i \)-th unit and \( \pi_{ij} \), the joint inclusion probability of units \( i \) and \( j, 1 \leq i < j \leq N \). Which of the following statements is always true?
59. Consider a series system with two independent components. Let the component lifespan have exponential distribution with density

\[ f(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0, \\ 0, & \text{otherwise.} \end{cases} \]

If \( n \) observations \( X_1, X_2, \ldots, X_n \) on lifespan of this component are available and

\[ \bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i, \]

then the maximum likelihood estimator of the reliability of the system is given by

1. \( (1 - e^{-t/\bar{X}})^2 \)
2. \( 1 - (1 - e^{-t/\bar{X}})^2 \)
3. \( e^{-2t/\bar{X}} \)
4. \( 1 - e^{-2t/\bar{X}} \)

60. Customers arrive at an ice cream parlour according to a Poisson process with rate 2. Service time distribution has density function

\[ f(x) = \begin{cases} 3e^{-3x}, & x > 0, \\ 0, & x \leq 0. \end{cases} \]

Upon being served a customer may rejoin the queue with probability 0.4, independently of new arrivals; also a returning customer’s service time is the same as that of a new arriving customer. Customers behave independently of each other. Let \( X(t) \) = number of customers in the queue at time \( t \). Which among the following is correct?

1. \( X(t) \) grows without bound with probability 1.
2. \( X(t) \) has stationary distribution given by \( \pi_k = \left( \frac{1}{3} \right) (\frac{2}{3})^k, k = 0, 1, 2, \ldots \)
3. \( X(t) \) has stationary distribution given by \( \pi_k = (0.1)(0.9)^k, k = 0, 1, 2, \ldots \)
4. \( X(t) \) has stationary distribution given by \( \pi_k = (0.4)(0.6)^k, k = 0, 1, 2, \ldots \)
17

भाग \textit{PART 'C'}

Unit-1

61. मानें कि \( x_1 = 0, x_2 = 1 \), तथा \( n \geq 3 \) के लिए\par 
परिभाषित करें \( x_n = \frac{x_{n-1} + x_{n-2}}{2} \). निम्न में से कौन सा/से सही है?
1. \{\( x_n \)\} एक एक्सट्रिम अनुक्रम है।
2. \( \lim_{n \to \infty} x_n = \frac{1}{2} \).
3. \{\( x_n \)\} एक कोशी अनुक्रम है।
4. \( \lim_{n \to \infty} x_n = \frac{2}{3} \).

62. मानें कि \{\( x_n \)\} वास्तविक संख्याओं का एक स्वेच्छ अनुक्रम है। तो?
1. कुछ \( 1 < p < \infty \) के लिए \( \sum_{n=1}^{\infty} |x_n|^p < \infty \) का अर्थ है किसी \( q > p \) के लिए
\( \sum_{n=1}^{\infty} |x_n|^q \) \( < \infty \).
2. कुछ \( 1 < p < \infty \) के लिए \( \sum_{n=1}^{\infty} |x_n|^p < \infty \) का अर्थ है किसी \( 1 \leq q < p \) के लिए
\( \sum_{n=1}^{\infty} |x_n|^q \) \( < \infty \).
3. किसी \( 1 < p < q < \infty \) के दिये जाने पर, एक वास्तविक अनुक्रम \{\( x_n \)\} का अस्तित्व है ताकि \( \sum_{n=1}^{\infty} |x_n|^p < \infty \) परस्तु \( \sum_{n=1}^{\infty} |x_n|^q \) \( = \infty \).
4. किसी \( 1 < q < p < \infty \) के दिये जाने पर, एक वास्तविक अनुक्रम \{\( x_n \)\} का अस्तित्व है ताकि \( \sum_{n=1}^{\infty} |x_n|^p < \infty \) परस्तु \( \sum_{n=1}^{\infty} |x_n|^q \) \( = \infty \).

63. मानें कि \( f: \mathbb{R} \to \mathbb{R} \) एक संतत फलन है तथा सभी \( x \in \mathbb{R} \) के लिए \( f(x+1) = f(x) \) है। तो?
1. \( f \) ऊपर से परिबंध है, परस्तु नीचे से नहीं।
2. \( f \) ऊपर तथा नीचे परिबंध है, परस्तु अपने परिबंध पर शायद नहीं पहुंचता।
3. \( f \) ऊपर तथा नीचे से परिबंध है तथा \( f \) अपने परिबंध पर पहुंचता है।
4. \( f \) एकसमानता: संतत है।

64. संवृत अंतराल \([0,1]\) तथा विवृत अंतराल \((1/3, 2/3)\) को लें। मानें कि \( K = [0,1] \setminus (1/3, 2/3) \). \( x \in [0,1] \) के लिए परिभाषित करें कि \( f(x) = d(x, K) \) जहाँ \( d(x, K) = \inf \{|x-y| : y \in K\} \) है। तो?
1. \( f: [0,1] \to \mathbb{R} \) (0,1) के सभी बिंदुओं पर अवकलनीय है।
2. \( f: [0,1] \to \mathbb{R} \) 1/3 तथा 2/3 पर अवकलनीय नहीं है।
3. \( f: [0,1] \to \mathbb{R} \) 1/2 पर अवकलनीय नहीं है।
4. \( f: [0,1] \to \mathbb{R} \) संतत नहीं है।

65. Take the closed interval \([0,1]\) and open interval \((1/3, 2/3)\). Let \( K = [0,1] \setminus (1/3, 2/3) \). For \( x \in [0,1] \) define \( f(x) = d(x, K) \) where \( d(x, K) = \inf \{|x-y| : y \in K\} \). Then?
1. \( f: [0,1] \to \mathbb{R} \) is differentiable at all points of (0,1)
2. \( f: [0,1] \to \mathbb{R} \) is not differentiable at 1/3 and 2/3

66. Given any \( 1 < p < q < \infty \), there is a real sequence \{\( x_n \)\} such that
\( \sum_{n=1}^{\infty} |x_n|^p < \infty \) but \( \sum_{n=1}^{\infty} |x_n|^q = \infty \).
4. Given any \( 1 < q < p < \infty \), there is a real sequence \{\( x_n \)\} such that
\( \sum_{n=1}^{\infty} |x_n|^p < \infty \) but \( \sum_{n=1}^{\infty} |x_n|^q = \infty \).
3. $f: [0,1] \rightarrow \mathbb{R}$ is not differentiable at 1/2
4. $f: [0,1] \rightarrow \mathbb{R}$ is not continuous

65. "Which of the following is/are true?"
1. (0,1) with the usual topology admits a metric which is complete
2. (0,1) with the usual topology admits a metric which is not complete
3. [0,1] with the usual topology admits a metric which is not complete
4. [0,1] with the usual topology admits a metric which is complete

66. "Let $V$ be the span of (1,1,1) and (0,1,1) $\in \mathbb{R}^3$. Let $u_1 = (0,0,1), u_2 = (1,1,0)$ and $u_3 = (1,0,1)$. Which of the following are correct?"
1. $(\mathbb{R}^3 \setminus V) \cup \{(0,0,0)\}$ is not connected.
2. $(\mathbb{R}^3 \setminus V) \cup \{(t,1-t)u_1: 0 \leq t \leq 1\}$ is connected.
3. $(\mathbb{R}^3 \setminus V) \cup \{(t,1-t)u_2: 0 \leq t \leq 1\}$ is connected.
4. $(\mathbb{R}^3 \setminus V) \cup \{(t,2t,2t): t \in \mathbb{R}\}$ is connected.

67. "Let $A$ be any set. Let $\mathbb{P}(A)$ be the power set of $A$, that is, the set of all subsets of $A$; $\mathbb{P}(A) = \{B: B \subseteq A\}$. Then which of the following is/are true about the set $\mathbb{P}(A)$?"
1. $\mathbb{P}(A) = \emptyset$ for some $A$.
2. $\mathbb{P}(A)$ is a finite set for some $A$.
3. $\mathbb{P}(A)$ is a countable set for some $A$.
4. $\mathbb{P}(A)$ is an uncountable set for some $A$.

68. "Let $f$ be the span of (1,1,1) and (0,1,1) $\in \mathbb{R}^3$. Let $u_1 = (0,0,1), u_2 = (1,1,0)$ and $u_3 = (1,0,1)$. Which of the following are correct?"
1. $(\mathbb{R}^3 \setminus V) \cup \{(0,0,0)\}$ is not connected.
2. $(\mathbb{R}^3 \setminus V) \cup \{(t,1-t)u_1: 0 \leq t \leq 1\}$ is connected.
3. $(\mathbb{R}^3 \setminus V) \cup \{(t,1-t)u_2: 0 \leq t \leq 1\}$ is connected.
4. $(\mathbb{R}^3 \setminus V) \cup \{(t,2t,2t): t \in \mathbb{R}\}$ is connected.

69. "Which of the following functions is/are uniformly continuous on the interval (0,1)?"
1. $\frac{1}{x}$
2. $\sin \frac{1}{x}$
3. $x \sin \frac{1}{x}$
4. $\sin \frac{x}{x}$
3. \( f \) रीमान समाकलनीय तथा \( \int_0^1 f(x)dx = \frac{1}{3} \) है।

4. \( \frac{1}{4} = \int_0^1 f(x)dx < \int_0^1 f(x)dx = \frac{1}{3}, \) जहां
\( \int_0^1 f(x)dx \) तथा \( \int_0^1 f(x)dx \) क्रमशः \( f \) के निचले तथा ऊपरी रीमान समाकल हैं।

69. Define \( f \) on \([0,1]\) by
\[
 f(x) = \begin{cases} 
 x^2 & \text{if } x \text{ is rational} \\
 x^3 & \text{if } x \text{ is irrational} 
\end{cases}
\]
Then
1. \( f \) is not Riemann integrable on \([0,1]\).
2. \( f \) is Riemann integrable and \( \int_0^1 f(x)dx = \frac{1}{4} \).
3. \( f \) is Riemann integrable and \( \int_0^1 f(x)dx = \frac{1}{3} \).
4. \( \frac{1}{4} = \int_0^1 f(x)dx < \int_0^1 f(x)dx = \frac{1}{3}, \)
where \( \int_0^1 f(x)dx \) and \( \int_0^1 f(x)dx \) are the lower and upper Riemann integrals of \( f \).

70. मानें कि \( V \) धात \( p \leq n \) के सभी समिश्र बहुपदों \( p \) की सदिश समस्या है। मानें कि \( T: V \rightarrow V (Tp)(x) = p'(1), x \in \mathbb{C} \) का प्रतिचित्र है। लाम्स में से कौन-से सही हैं?
1. \( \text{विभाजन } Ker T = n \)
2. \( \text{विभाजन } T = 1 \)
3. \( \text{विभाजन } Ker T = 1 \)
4. \( \text{विभाजन } T = n + 1 \)

71. धात \( d \) के समाकलन या कम वाले बहुपदों की वास्तविक सदिश समस्या \( V \) पर विचार। \( p \in V \) के लिए परिभाषित करें कि
\[
 \|p\|_k = \text{उच्चतम} \{ |p(0)|, |p^{(1)}(0)|, \ldots, |p^{(k)}(0)| \},
\]
जहां \( p^{(i)}(0) \), \( p \) का \( i^{th} \) अवकलज है जो 0 पर मूल्याकृत है। तो \( \|p\|_k V \) पर एक मानक परिभाषित करता है यदि तथा केवल यदि
1. \( k \geq d - 1 \) \quad 2. \( k < d \)
3. \( k \geq d \) \quad 4. \( k < d - 1 \)

71. Consider the real vector space \( V \) of polynomials of degree less than or equal to \( d \). For \( p \in V \) define
\[
 \|p\|_k = \max \{ |p(0)|, |p^{(1)}(0)|, \ldots, |p^{(k)}(0)| \},
\]
where \( p^{(i)}(0) \) is the \( i^{th} \) derivative of \( p \) evaluated at 0. Then \( \|p\|_k \) defines a norm on \( V \) if and only if
1. \( k \geq d - 1 \) \quad 2. \( k < d \)
3. \( k \geq d \) \quad 4. \( k < d - 1 \)

72. मानें कि \( A, B \) \( n \times n \) वास्तविक आद्यूर हैं तथापि सारणिक \( A > 0 \) तथा सारणिक \( B < 0 \) हैं।
\( 0 \leq t \leq 1 \) के लिए \( C(t) = t A + (1-t)B \) पर विचार। तो
1. \( \text{हर } t \in [0,1] \) के लिए \( C(t) \) व्युक्तक्रमणीय है।
2. \( \text{ऐसे } t_0 \in (0,1) \) अस्तित्व है तथापि \( C(t_0) \) व्युक्तक्रमणीय नहीं है।
3. \( \text{हर } t \in [0,1] \) के लिए \( C(t) \) व्युक्तक्रमणीय है।
4. \( \text{केवल एक उपरी } t_0 \in [0,1] \) के लिए \( C(t) \) व्युक्तक्रमणीय है।

73. मानें कि \( A \) एक \( n \times n \) वास्तविक आद्यूर है। \( \text{द्वारा } \) से सही \( U, V \in \mathbb{R}^n \) के लिए
\[
 (Av)^T(Av) > 0 \]
3. $A^T A$ का हर अभिलक्षणिक मान एक अक्षण वास्तविक संख्या है।
4. $I + A^T A$ व्युत्क्रमणीय है।

73. Let $A$ be an $n \times n$ real matrix. Pick the correct answer(s) from the following
1. $A$ has at least one real eigenvalue.
2. For all nonzero vectors $v, w \in \mathbb{R}^n$, $(Aw)^T (Av) > 0$.
3. Every eigenvalue of $A^T A$ is a nonnegative real number.
4. $I + A^T A$ is invertible.

74. मानें कि $\{a_1, \ldots, a_n\}$ तथा $\{b_1, \ldots, b_n\} \mathbb{R}^n$ के दो आधार हैं। मानें कि $P$ एक $n \times n$ आव्यूह है, वास्तविक प्रविष्टियों के साथ, ताकि $P a_i = b_i \ i = 1, 2, \ldots, n$ है। मानें कि $P$ का हर अभिलक्षणिक मान $-1$ या $1$ है। मानें कि $Q = I + 2P$ है। तो निम्न कथनों में से कौन से सही हैं?
1. $\{a_i + 2b_i \ | \ i = 1, 2, \ldots, n\}$ भी $V$ का एक आधार है।
2. $Q$ व्युत्क्रमणीय है।
3. $Q$ का हर अभिलक्षणिक मान $3$ या $-1$ है।
4. यदि सारणिक $P > 0$ है तो सारणिक $Q > 0$ है।

75. मानें कि $T$ एक $n \times n$ आव्यूह है, गुणाधर्म $T^n = 0$ के साथ। निम्न में से कौन सा/से सही है/है?
1. $T$ के $n$ मिन्न अभिलक्षणिक मान हैं।
2. $T$ का एक अभिलक्षणिक मान है बहुक्ता $n$ के साथ।
3. $T$ का एक अभिलक्षणिक मान $0$ है।
4. $T$ एक विकर्ण आव्यूह के समान है।

76. मानें कि $A$ एक $n \times n$ आव्यूह है, वास्तविक प्रविष्टियों के साथ। परिभाषित करें कि $\langle x, y \rangle_A = \langle Ax, Ay \rangle$, $x, y \in \mathbb{R}^n$. तो $\langle x, y \rangle_A$ आंतरगुणनफल की परिभाषा करता है यदि तथा केवल यदि
1. $Ker A = \{0\}$।
2. $rank A = n$।
3. $A$ के सभी अभिलक्षणिक मान धनात्मक हैं।
4. $A$ के सभी अभिलक्षणिक मान अक्षणात्मक हैं।

77. मानें कि $\mathbb{R}^n$ में $\{v_1, \ldots, v_n\}$ मात्रक सदिश हैं ताकि
$$\|v\|^2 = \sum_{i=1}^{n} \langle v, v \rangle, \forall v \in \mathbb{R}^n$$
तो निम्न में से सही कथनों का निर्णय कीजिये।
1. $v_1, \ldots, v_n$ आपस में लांब हैं।
2. $\{v_1, \ldots, v_n\}$ $\mathbb{R}^n$ के लिए एक आधार है।
3. $v_1, \ldots, v_n$ आपस में लांब नहीं हैं।
4. समुच्चय $\{v_1, \ldots, v_n\}$ में अधिक से अधिक $n - 1$ अवयव लांबिक हो सकते हैं।
77. Suppose \( \{v_1, \ldots, v_n\} \) are unit vectors in \( \mathbb{R}^n \) such that
\[
\|v\|^2 = \sum_{i=1}^{n} |(v_i, v)|^2, \forall v \in \mathbb{R}^n
\]
Then decide the correct statements in the following
1. \( v_1, \ldots, v_n \) are mutually orthogonal.
2. \( \{v_1, \ldots, v_n\} \) is a basis for \( \mathbb{R}^n \).
3. \( v_1, \ldots, v_n \) are not mutually orthogonal.
4. At most \( n - 1 \) of the elements in the set \( \{v_1, \ldots, v_n\} \) can be orthogonal.

78. Let \( V = \{ f : [0, 1] \to \mathbb{R} \mid f \text{ is a polynomial of degree less than or equal to } n \} \). Let \( f_j(x) = x^j \) for \( 0 \leq j \leq n \) and let \( A \) be the \( (n + 1) \times (n + 1) \) matrix given by
\[
a_{ij} = \int_0^1 f_i(x)f_j(x)dx. \]
Then which of the following are true?
1. \( \dim V = n \).
2. \( \dim V > n \).
3. \( A \) is nonnegative definite, i.e., for all \( v \in \mathbb{R}^n \), \( (Av, v) \geq 0 \).
4. \( \det A > 0 \).

79. Let \( f : \mathbb{C} \to \mathbb{C} \) be an entire function. Suppose that \( f = u + iv \) where \( u, v \) are the real and imaginary parts of \( f \) respectively. Then which of the following are true?
1. \( \{u(x, y): z = x + iy\} \) is bounded.
2. \( \{v(x, y): z = x + iy\} \) is bounded.
3. \( \{u(x, y) + v(x, y): z = x + iy\} \) is bounded.
4. \( \{u^2(x, y) + v^2(x, y): z = x + iy\} \) is bounded.
1. $z = -2i$ for $f$ a simple pole.
2. $f(i)f(-i) = 1$ for $f$ a solution.
3. $z = -2i$ for $f$ an essential singularity.
4. $|f(2 + 2i)| = \frac{1}{\sqrt{5}}$

81. Let $H = \{z = x + iy \in \mathbb{C} : y > 0\}$ be the upper half plane and $D = \{z \in \mathbb{C} : |z| < 1\}$ be the open unit disc. Suppose that $f$ is a Mobius transformation, which maps $H$ conformally onto $D$. Suppose that $f(2i) = 0$. Pick each correct statement from below.
1. $f$ has a simple pole at $z = -2i$.
2. $f$ satisfies $f(i)f(-i) = 1$.
3. $f$ has an essential singularity at $z = -2i$.
4. $|f(2 + 2i)| = \frac{1}{\sqrt{5}}$

82. Consider the function $F(z) = \int_1^z \frac{1}{(x-z)^2} dx$, $\text{Im}(z) > 0$, and

Then there is a meromorphic function $G(z)$ on $\mathbb{C}$ that agrees with $F(z)$ when $\text{Im}(z) > 0$, such that
1. $1, \infty$ are poles of $G(z)$.
2. $0, 1, \infty$ are poles of $G(z)$.
3. $1, 2$ are poles of $G(z)$.
4. $1, 2$ are simple poles of $G(z)$.

83. Consider the integral
$$A = \int_0^1 x^n (1 - x)^n \, dx.$$ Pick each correct statement from below.
1. $A$ is not a rational number.
2. $0 < A \leq 4^{-n}$.
3. $A$ is a natural number.
4. $A^{-1}$ is a natural number.

84. Let $G$ be a finite abelian group of order $n$. Pick each correct statement from below.
1. If $d$ divides $n$, there exists a subgroup of $G$ of order $d$.
2. If $d$ divides $n$, there exists an element of order $d$ in $G$.
3. If every proper subgroup of $G$ is cyclic, then $G$ is cyclic.
4. If $H$ is a subgroup of $G$, there exists a subgroup $N$ of $G$ such that $G/N \cong H$.

85. Consider the integral $A = \int_0^1 x^n (1 - x)^n \, dx$.
85. Consider the symmetric group $S_{20}$ and its subgroup $A_{20}$ consisting of all even permutations. Let $H$ be a 7-Sylow subgroup of $A_{20}$. Pick each correct statement from below:
1. $|H| = 49$.
2. $H$ must be cyclic.
3. $H$ is a normal subgroup of $A_{20}$.
4. Any 7-Sylow subgroup of $S_{20}$ is a subset of $A_{20}$.

86. Let $p$ be a prime. Pick each correct statement from below. Up to isomorphism,
1. there are exactly two abelian groups of order $p^2$.
2. there are exactly two groups of order $p^2$.
3. there are exactly two commutative rings of order $p^2$.
4. there is exactly one integral domain of order $p^2$.

87. Let $R$ be a commutative ring with unity, such that $R[X]$ is a UFD. Denote the ideal $(X)$ of $R[X]$ by $I$. Pick each correct statement from below:
1. $I$ is prime.
2. If $I$ is maximal, then $R[X]$ is a PID.
3. If $R[X]$ is a Euclidean domain, then $I$ is maximal.
4. If $R[X]$ is a PID, then it is a Euclidean domain.

88. Let $f(x) \in \mathbb{Z}[x]$ be a polynomial of degree $\geq 2$. Pick each correct statement from below:
1. If $f(x)$ is irreducible in $\mathbb{Z}[x]$, then it is irreducible in $\mathbb{Q}[x]$.
2. If $f(x)$ is irreducible in $\mathbb{Q}[x]$, then it is irreducible in $\mathbb{Z}[x]$.
3. If $f(x)$ is irreducible in $\mathbb{Z}[x]$, then for all primes $p$ the reduction $\overline{f(x)}$ of $f(x)$ modulo $p$ is irreducible in $\mathbb{F}_p[x]$.
4. If $f(x)$ is irreducible in $\mathbb{Z}[x]$, then it is irreducible in $\mathbb{R}[x]$. 

89. Let $\mathbb{C}$ be a commutative ring with unity, such that $(\mathbb{C}, r)$ is a UFD. Pick each correct statement from below:
1. $(\mathbb{C}, r)$ is a field.
2. $(\mathbb{C}, r)$ is a Dedekind domain.
3. \((\mathbb{C}, \tau)\) संबद्ध है।
4. \((\mathbb{C}, \tau)\) में \(\mathbb{Z}\) सम्बन्ध है।

89. Consider the smallest topology \(\tau\) on \(\mathbb{C}\) in which all the singleton sets are closed. Pick each correct statement from below:
1. \((\mathbb{C}, \tau)\) is Hausdorff.
2. \((\mathbb{C}, \tau)\) is compact.
3. \((\mathbb{C}, \tau)\) is connected.
4. \(\mathbb{Z}\) is dense in \((\mathbb{C}, \tau)\).

90. मानें कि \(\{X_\alpha\}_{\alpha \in I}\) विभिन्न सांस्थितिक समस्तियां हैं तथा मानें कि \(X = \prod_{\alpha \in I} X_\alpha\) है। निम्न दिये गये कथनों से हर उन कथन को चुनें जो अर्थ देता है कि \(X\) पर गुणनफल सांस्थितिकी \(X\) पर विभिन्न सांस्थितिकी के समान है।
1. \(I\) परिमित है।
2. \(I\) गणनीयत: अपरिमित है तथा परिमितत: कई \(\alpha\) को छोड़कर बाकी सभी के लिए \(X_\alpha\) एकल है।
3. \(I\) अगणनीयत: अपरिमित है तथा परिमितत: कई \(\alpha\) को छोड़कर बाकी सभी के लिए \(X_\alpha\) एकल है।
4. \(I\) अपरिमित है तथा सभी \(\alpha\) के लिए \(X_\alpha\) अपरिमित हैं।

90. Let \(\{X_\alpha\}_{\alpha \in I}\) be discrete topological spaces and let \(X = \prod_{\alpha \in I} X_\alpha\). From the statements given below, pick each statement that implies that the product topology on \(X\) equals the discrete topology on \(X\).
1. \(I\) is finite.
2. \(I\) is countably infinite and \(X_\alpha\) are singletons for all but finitely many \(\alpha\).
3. \(I\) is uncountably infinite and \(X_\alpha\) are singletons for all but finitely many \(\alpha\).
4. \(I\) is infinite and \(X_\alpha\) are infinite for all \(\alpha\).

91. मानें कि \(y: \mathbb{R} \to \mathbb{R}\) साधारण अवकल समीकरण
\[2y'' + 3y' + y = e^{-3x}, \quad x \in \mathbb{R}\]
\[\lim_{x \to \infty} e^xy(x) = 0\] का समाधान करते हुए, का हल है। तो
1. \(\lim_{x \to \infty} e^xy(x) = 0\)
2. \(y(0) = \frac{1}{10}\)
3. \(\mathbb{R}\) पर \(y\) एक परिमित फलन है।
4. \(y(1) = 0\)

91. Let \(y: \mathbb{R} \to \mathbb{R}\) be a solution of the ordinary differential equation,
\[2y'' + 3y' + y = e^{-3x}, \quad x \in \mathbb{R}\]
satisfying \(\lim_{x \to \infty} e^xy(x) = 0\). Then
1. \(\lim_{x \to \infty} e^xy(x) = 0\)
2. \(y(0) = \frac{1}{10}\)
3. \(y\) is a bounded function on \(\mathbb{R}\).
4. \(y(1) = 0\).

92. \(\lambda \in \mathbb{R}\) के लिए निम्न अवकल समीकरण पर विचारें।
\[y'(x) = \lambda \sin(x + y(x)), \quad y(0) = 1\]
\(\lambda\) का किसी भी सामीप में कोई हल नहीं है। तो इस प्रारंभिक मान समस्या का:
1. \(0\) के किसी भी सामीप में कोई हल नहीं है।
2. \(\lambda\) की \(1 < \lambda\) है तो \(\mathbb{R}\) में एक हल है।
3. \(0\) के सामीप में एक हल है।
4. मान यदि \(\lambda > 1\) है तो \(\mathbb{R}\) में एक हल है।

92. For \(\lambda \in \mathbb{R}\), consider the differential equation
\[y'(x) = \lambda \sin(x + y(x)), \quad y(0) = 1\]
Then this initial value problem has:
1. no solution in any neighbourhood of 0.
2. a solution in \(\mathbb{R}\) if \(\lambda < 1\).
3. a solution in a neighbourhood of 0.
4. a solution in \(\mathbb{R}\) only if \(\lambda > 1\).
Let $u: \mathbb{R} \times [0, \infty) \to \mathbb{R}$ be a solution of the initial value problem

\begin{align*}
  u_{tt} - u_{xx} &= 0, \quad \text{for } (x,t) \in \mathbb{R} \times (0, \infty) \\
  u(x,0) &= f(x), \quad x \in \mathbb{R} \\
  u_t(x,0) &= g(x), \quad x \in \mathbb{R}
\end{align*}

Suppose $f(x) = g(x) = 0$ for $x \not\in [0,1]$, then we always have

1. $u(x,t) = 0$ for all $(x,t) \in \{ (-\infty, 0) \times (0, \infty) \}$.
2. $u(x,t) = 0$ for all $(x,t) \in \{ (1, \infty) \times (0, \infty) \}$.
3. $u(x,t) = 0$ for all $(x,t)$ satisfying $x + t < 0$.
4. $u(x,t) = 0$ for all $(x,t)$ satisfying $x - t > 1$.

Consider the Cauchy problem for the Eikonal equation

\[ p^2 + q^2 = 1; \quad p \equiv \frac{\partial u}{\partial x}, \quad q \equiv \frac{\partial u}{\partial y} \]

$u(x,y) = 0$ on $x + y = 1$, $(x,y) \in \mathbb{R}^2$.

Then

1. The Charpit’s equations for the differential equation are

\[ \frac{dx}{dt} = 2p; \quad \frac{dy}{dt} = 2q; \quad \frac{du}{dt} = 2; \quad \frac{dp}{dt} = -p; \quad \frac{dq}{dt} = -q. \]

2. The Charpit’s equations for the differential equation are

\[ \frac{dx}{dt} = 2p; \quad \frac{dy}{dt} = 2q; \quad \frac{du}{dt} = 2; \quad \frac{dp}{dt} = 0; \quad \frac{dq}{dt} = 0. \]

3. $u \left(1, \sqrt{2}\right) = \sqrt{2}$.

4. $u \left(1, \sqrt{2}\right) = 1$. 

95. Consider the Cauchy problem for the Eikonal equation

\[ p^2 + q^2 = 1; \quad p \equiv \frac{\partial u}{\partial x}, \quad q \equiv \frac{\partial u}{\partial y} \]

$u(x,y) = 0$ on $x + y = 1$, $(x,y) \in \mathbb{R}^2$.

Then

1. The Charpit’s equations for the differential equation are

\[ \frac{dx}{dt} = 2p; \quad \frac{dy}{dt} = 2q; \quad \frac{du}{dt} = 2; \quad \frac{dp}{dt} = -p; \quad \frac{dq}{dt} = -q. \]

2. The Charpit’s equations for the differential equation are

\[ \frac{dx}{dt} = 2p; \quad \frac{dy}{dt} = 2q; \quad \frac{du}{dt} = 2; \quad \frac{dp}{dt} = 0; \quad \frac{dq}{dt} = 0. \]

3. $u \left(1, \sqrt{2}\right) = \sqrt{2}$.

4. $u \left(1, \sqrt{2}\right) = 1$. 

96. Suppose $f(x) = g(x) = 0$ for all $x \not\in [0,1]$.

Then we always have

1. $u(x,t) = 0$ for all $(x,t) \in \{ (-\infty, 0) \times (0, \infty) \}$.
2. $u(x,t) = 0$ for all $(x,t) \in \{ (1, \infty) \times (0, \infty) \}$.
3. $u(x,t) = 0$ for all $(x,t)$ satisfying $x + t < 0$.
4. $u(x,t) = 0$ for all $(x,t)$ satisfying $x - t > 1$.
1. उच्च \[ x \rightarrow |f(x) - H(x)| = \frac{1}{16} \text{ है।} \]
2. \[ |f(x) - H(x)| \text{ का उच्चतम } x = \frac{1}{2} \text{ पर पाया जाता है।} \]
3. उच्च \[ x \rightarrow |f(x) - H(x)| = \frac{1}{2^4} \text{ है।} \]
4. \[ |f(x) - H(x)| \text{ का उच्चतम } x = \frac{1}{4} \text{ पर पाया जाता है।} \]

96. Let \( H(x) \) be the cubic Hermite interpolation of \( f(x) = x^4 + 1 \) on the interval \( I = [0,1] \) interpolating at \( x = 0 \) and \( x = 1 \). Then
   1. \[ \max_{x \in I} |f(x) - H(x)| = \frac{1}{16}. \]
   2. The maximum of \( |f(x) - H(x)| \) is attained at \( x = \frac{1}{2} \).
   3. \[ \max_{x \in I} |f(x) - H(x)| = \frac{1}{2^4}. \]
   4. The maximum of \( |f(x) - H(x)| \) is attained at \( x = \frac{1}{4} \).

97. मानें कि \( f : [0,3] \rightarrow \mathbb{R} \) दिया गया हो।
f(x) = |1 - |x - 2|| \text{ से प्राप्त} |f| \text{ निर्देशांक को निर्दिष्ट करता है। तो } \int_0^3 f(x) \, dx, \text{ के संच्यात्मक समीकरण के लिए निम्नलिखित कथनों में से कौन-से सही हैं?
1. संयुक्त समस्या नियम, तीन समान उपांतरों के साथ, यथायथ है।
2. संयुक्त अभिविद्या नियम, तीन समान उपांतरों के साथ, यथायथ है।
3. संयुक्त समस्या नियम, चार समान उपांतरों के साथ, यथायथ है।
4. संयुक्त अभिविद्या नियम, चार समान उपांतरों के साथ, यथायथ है।

98. मानें कि \( u \) सीमा मान समस्या
\[ u_{xx} + u_{yy} = 0, \quad 0 < x, y < \pi \text{ के लिए} \]
\[ u(x, 0) = 0 = u(x, \pi), \quad 0 \leq x \leq \pi \text{ के लिए} \]
\[ u(0, y) = 0, \quad u(\pi, y) = \sin y + \sin 2y, \quad 0 \leq y \leq \pi \text{ के लिए} \]
का हल है। तो
1. \[ u \left(1, \frac{\pi}{2}\right) = (\sinh(\pi))^{-1} \sinh(1). \]
2. \[ u \left(1, \frac{\pi}{2}\right) = (\sinh(1))^{-1} \sinh(\pi). \]
3. \[ u \left(1, \frac{\pi}{4}\right) = (\sinh(\pi))^{-1} (\sinh(1)) \frac{1}{\sqrt{2}} + (\sinh(2\pi))^{-1} \sinh(2). \]
4. \[ u \left(1, \frac{\pi}{4}\right) = (\sinh(1))^{-1} (\sinh(\pi)) \frac{1}{\sqrt{2}} + (\sinh(2))^{-1} \sinh(2\pi). \]

99. प्रारंभिक मान समस्या \( y'(x) = f(x, y(x)), \)
\( y(x_0) = y_0 \) के संच्यात्मक के लिए निम्न रूप के संग-कुछ विधि पर विचारिएः
\[ y_{n+1} = y_n + ak_1 + bk_2 \]
\[ k_1 = hf(x_n, y_n) \]
\[ k_2 = hf(x_n + ah, y_n + \beta k_1) \]
\[ a, b, \alpha, \beta \text{ के निम्न वर्णों में से कौन-से एक द्वितीय कोटि विधि प्रदान करते हैं?} \]
1. $a = \frac{1}{2}, \ b = \frac{1}{2}, \ a = 1, \ \beta = 1$
2. $a = 1, \ b = 1, \ a = \frac{1}{2}, \ \beta = \frac{1}{2}$
3. $a = \frac{1}{4}, \ b = \frac{3}{4}, \ a = \frac{2}{3}, \ \beta = \frac{2}{3}$
4. $a = \frac{3}{4}, \ b = \frac{1}{4}, \ a = 1, \ \beta = 1$

99. Consider the Runge-Kutta method of the form
\[ y_{n+1} = y_n + ak_1 + bk_2 \]
\[ k_1 = hf(x_n, y_n) \]
\[ k_2 = hf(x_n + \alpha h, y_n + \beta k_1) \]
to approximate the solution of the initial value problem
\[ y'(x) = f(x, y(x)), \ y(x_0) = y_0. \]
Which of the following choices of $a, b, \alpha$ and $\beta$ yield a second order method?
1. $a = \frac{1}{2}, \ b = \frac{1}{2}, \ a = 1, \ \beta = 1$
2. $a = 1, \ b = 1, \ a = \frac{1}{2}, \ \beta = \frac{1}{2}$
3. $a = \frac{1}{4}, \ b = \frac{3}{4}, \ a = \frac{2}{3}, \ \beta = \frac{2}{3}$
4. $a = \frac{3}{4}, \ b = \frac{1}{4}, \ a = 1, \ \beta = 1$

100. The curve $y = y(x)$, which is defined by the following property (Volterta integral equation of the first kind)
\[ \int_{x_0}^{x_2} f(y) \, dy \bigg|_{y = y_0} = 4\sqrt{y}. \]
\[ \text{where } f(y) = \sqrt{1 + \frac{1}{y^2}}, \text{ is the part of a} \]
1. straight line.
2. circle.
3. parabola.
4. cycloid.

101. Let $y = y(x)$ be the extremal of the functional
\[ I[y(x)] = \int_{x_1}^{x_2} \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx, \]
subject to the condition that the left end of the extremal moves along $y = x^2$, while the right end moves along $x - y = 5$. Then the
1. shortest distance between the parabola and the straight line is $\left( \frac{19\sqrt{2}}{8} \right)$.
2. slope of the extremal at $(x, y)$ is $\left( -\frac{3}{2} \right)$.
3. point $\left( \frac{3}{4}, 0 \right)$ lies on the extremal.
4. extremal is orthogonal to the curve $y = \frac{x}{2}$.

102. A particle $\alpha$ an angle $x$-axis the direction of the $L = \frac{1}{12} x^4 + \frac{1}{2} x^2 \lambda x^2 - x^2$.
Mane ki $Q = x^2 \lambda$ by the state of the $\lambda$ (the change in the $\lambda$) the principle on the basis of which, $x = x(0) + 1$ then $x(0) = 1$ then $\dot{x}$ is the mean $\dot{x} - 0 = 1$ and $x = 1$ on 1
102. A particle of unit mass moves in the direction of $x$-axis such that it has the Lagrangian

$$L = \frac{1}{12} \dot{x}^4 + \frac{1}{2} x \ddot{x}^2 - x^2.$$  

Let $Q = x^2 \ddot{x}$ represent a force (not arising from a potential) acting on the particle in the $x$-direction. If $x(0) = 1$ and $\dot{x}(0) = 1$, then the value of $\dot{x}$ is

1. some non-zero finite value at $x = 0$.
2. $1$ at $x = 1$.
3. $\sqrt{5}$ at $x = \frac{1}{2}$.
4. $0$ at $x = \frac{3}{\sqrt{2}}$.

103. Let $(\Omega, \mathcal{F}, P)$ be a probability space and let $A$ be an event with $P(A) > 0$. In which of the following cases does $Q$ define a probability measure on $(\Omega, \mathcal{F})$?

1. $Q(D) = P(A \cup D)$ $\forall$ $D \in \mathcal{F}$
2. $Q(D) = P(A \cap D)$ $\forall$ $D \in \mathcal{F}$
3. $Q(D) = P(A | D)$, if $D \in \mathcal{F}$ and $P(D) > 0$, then $P(D) = 0$
4. $Q(D) = P(D | A)$ $\forall$ $D \in \mathcal{F}$

104. The joint probability density function of $(X, Y)$ is

$$f(x, y) = \begin{cases} 6(1-x), & 0 < y < x, 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Which among the following are correct?

1. $X$ and $Y$ are independent
2. $f_Y(y) = \begin{cases} 3(y - \frac{1}{2})^2, & 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$
3. $X$ and $Y$ are dependent
4. $f_Y(y) = \begin{cases} 3(y - \frac{1}{2})^2, & 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$

105. Let $X_n$ be the result of the $n$-th roll of a fair die, $n \geq 1$. Let $S_n = \sum_{i=1}^{n} X_i$ and $Y_n = \text{last digit of } X_i \text{ for } i = 1, \ldots, n$. Then, which of the following statements are correct?

1. $\{Y_n : n \geq 0\}$ is an irreducible Markov chain.
2. $\{Y_n : n \geq 0\}$ is an aperiodic Markov chain.
3. \( P(Y_n = 0) \to \frac{1}{6} \) as \( n \to \infty \).
4. \( P(Y_n = 5) \to \frac{1}{10} \) as \( n \to \infty \).

106. \( \{X_i\} \) is a sequence of independent and identically distributed random variables with common density function
\[
f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}
\]
\( \{Y_i\} \) is a sequence of independent identically distributed random variables with common density function
\[
g(y) = \begin{cases} 4e^{-4y}, & y > 0 \\ 0, & \text{otherwise} \end{cases}
\]

Also \( \{X_i\}, \{Y_j\} \) are independent families.

Let \( Z_k = Y_k - 3X_k, k = 1, 2, \ldots \) Which among the following are correct?
1. \( P(Z_k > 0) > 0 \)
2. \( \sum_{k=1}^{n} Z_k \to +\infty \) with probability 1
3. \( \sum_{k=1}^{n} Z_k \to -\infty \) with probability 1
4. \( P(Z_k < 0) > 0 \).

107. Suppose \( X \) and \( Y \) are independent and identically distributed random variables and let \( Z = X + Y \). Then the distribution of \( Z \) is in the same family as that of \( X \) and \( Y \) if \( X \) is
1. Normal.
2. Exponential.
3. Uniform.
4. Binomial.

108. Suppose \( X_1, X_2, \ldots, X_n \) are independent random variables with common density function
\[
f(x; \mu, \alpha) = \begin{cases} \frac{1}{\Gamma(\alpha)} (x - \mu)^{\alpha-1} e^{-(x-\mu)}, & x \geq \mu \\ 0, & \text{otherwise} \end{cases}
\]

Also \( \{X_i\}, \{Y_j\} \) are independent families.

Let \( Z_k = Y_k - 3X_k, k = 1, 2, \ldots \) Which among the following are correct?
1. \( P(Z_k > 0) > 0 \)
2. \( \alpha \) is the number of independent \( Y_k \) in the probability function and \( \mu \) is the mean of \( X_k \)
3. \( z_k = Y_k - 3X_k, k = 1, 2, \ldots \) Which among the following are correct?
1. \( P(Z_k > 0) > 0 \)
2. \( \sum_{k=1}^{n} Z_k \to +\infty \) with probability 1
3. \( \sum_{k=1}^{n} Z_k \to -\infty \) with probability 1
4. \( P(Z_k < 0) > 0 \).
108. Let \( X_1, \ldots, X_n \) be a random sample from the following probability density function
\[
f(x; \mu, \alpha) = \begin{cases} \frac{1}{f(\alpha)} (x - \mu)^{\alpha-1} e^{-(x-\mu)}, & x \geq \mu, \\ 0, & \text{otherwise}. \end{cases}
\]
Here \(-\infty < \mu < \infty \) and \( \alpha > 0 \). Then which of the following statements are correct?
1. The method of moment estimators of neither \( \alpha \) nor \( \mu \) exist.
2. The method of moment estimator of \( \mu \) exists and it is a consistent estimator of \( \alpha \).
3. The method of moment estimator of \( \mu \) exists and it is a consistent estimator of \( \mu \).
4. The method of moment estimators of both \( \alpha \) and \( \mu \) exist, but they are not consistent.

109. मानें कि \( X \) एक यादृच्छिक घर है, निम्न
प्रारंभिक के साथ
\[
f(x) = \begin{cases} p e^{-x} + 2(1-p) e^{-2x}, & x > 0, \\ 0, & \text{अन्यथा}, \end{cases}
\]
तथा \( 0 \leq p \leq 1 \) है। तो \( X \) का जोखिम फलन है
\begin{enumerate}
1. अचर फलन, \( p = 0 \) तथा \( p = 1 \) के लिए।
2. अचर फलन, सभी \( 0 \leq p \leq 1 \) के लिए।
3. हासमान फलन, तथा \( 0 < p < 1 \) के लिए।
4. अनैकाक्षेपित फलन, सभी \( 0 < p < 1 \) के लिए।
\end{enumerate}

109. Suppose \( X \) is a random variable with following pdf
\[
f(x) = \begin{cases} p e^{-x} + 2(1-p) e^{-2x}, & x > 0, \\ 0, & \text{otherwise}, \end{cases}
\]
and \( 0 \leq p \leq 1 \). Then the hazard function of \( X \) is a
\begin{enumerate}
1. constant function for \( p = 0 \) and \( p = 1 \)
2. constant function for all \( 0 \leq p \leq 1 \)
3. decreasing function for all \( 0 < p < 1 \)
4. non-monotone function for all \( 0 < p < 1 \)
\end{enumerate}

110. मानें कि \( X_1, \ldots, X_n \)
\[
f(x; \lambda) = \begin{cases} 2 \lambda x e^{-\lambda x^2}, & x > 0 \\ 0, & \text{अन्यथा} \end{cases}
\]

Here \( \lambda > 0 \) is an unknown parameter. It is desired to test the following hypothesis at level \( \alpha > 0 \). We want to test
\[
H_0: \lambda \leq 1 \quad \text{vs} \quad H_1: \lambda > 1.
\]
Then which of the following are true?
\begin{enumerate}
1. UMP test is of the form \( \sum_{i=1}^{n} x_i < c_n \), with \( c_n < c_{n+1} \) for all \( n \).
2. UMP test is of the form \( \sum_{i=1}^{n} x_i^2 < d_n \), with \( d_n < d_{n+1} \) for all \( n \).
3. UMP test is of the form \( \sum_{i=1}^{n} x_i < c_n \), with \( c_{n+1} < c_n \) for all \( n \).
4. UMP test is of the form \( \sum_{i=1}^{n} x_i^2 < d_n \), with \( d_{n+1} < d_n \) for all \( n \).
111. Let $X_1, \ldots, X_n$ be i.i.d. $N(\mu, 1)$. It is proposed to test $H_0: \mu = 0$ versus $H_1: \mu > 0$. Let $p_n(\mu, \alpha)$ denote the power of the UMP test at $\mu$ of size $\alpha$ based on sample size $n$.

Then which of the following statements are correct?
1. $\lim_{n \to \infty} p_n(\mu, \alpha) = 1 \ orall \mu > 0, \forall \alpha > 0$.
2. $\lim_{n \to \alpha} p_n(\mu, \alpha) = \alpha \ \forall \ n \geq 1, \forall \alpha > 0$.
3. $\lim_{\alpha \to 0} p_n(\mu, \alpha) = 0 \ \forall \ n \geq 1, \forall \mu > 0$.
4. $\lim_{\alpha \to 0} p_n(\mu, \alpha) = 0 \ \forall \ n \geq 1, \forall \mu > 0$.

112. Let $X$ be a random sample from a Poisson distribution with parameter $\lambda$. The parameter $\lambda$ has a prior distribution $f(z)$; where

$$f(z) = \begin{cases} e^{-z}; & z > 0 \\ 0; & \text{otherwise} \end{cases}$$

Under the squared error loss function, which of the following statements are correct?
1. The Bayes’ estimator of $e^\lambda$ is $2^{X+1}$.
2. The posterior mean of $\lambda$ is $\frac{X+1}{2}$.
3. The posterior distribution of $\lambda$ is gamma.
4. The Bayes’ estimator of $e^{2\lambda}$ is $2^{(X+1)}$.

113. Let $Y_1, Y_2, Y_3, Y_4$ be uncorrelated observations such that $E(Y_1) = \beta_1 + \beta_2 + \beta_3 = E(Y_2)$, $E(Y_3) = \beta_1 - \beta_2 = E(Y_4)$ and $Var(Y_i) = \sigma^2$ for $i = 1,2,3,4$. Then, which of the following statements are true?
1. $p_1\beta_1 + p_2\beta_2 + p_3\beta_3$ is estimable if and only if $p_1 + p_2 = 2p_3$.
2. $\sigma^2$ is an unbiased estimator of $\sigma^2$ is $[(Y_1 - Y_2)^2 + (Y_3 - Y_4)^2]/4$.
3. $[Y_1 - Y_2, Y_3, Y_4]$ is a Bayesian estimator of $\lambda$ is $\frac{1}{2}(Y_3 + Y_4)$.
4. $\beta_1 + \beta_2 + \beta_3$ is an unbiased estimator of $\lambda$ is $\frac{1}{2}(Y_3 + Y_4)$.
5. $\beta_1 + \beta_2 + \beta_3$ is the best linear unbiased estimator of $\beta_1 - \beta_2$ is $\frac{1}{2}(Y_3 + Y_4)$.
6. The variance of the best linear unbiased estimator of $\beta_1 - \beta_2$ is $\frac{1}{2}(Y_3 + Y_4)$.
7. The best linear estimator of $\lambda$ is estimable.

114. Let $Y = X \beta + \xi$; where $X$ is an $n \times p$ matrix, $\beta$ is a vector of parameters, $\xi$ is a vector of random errors and $p < n$. Then, which of the following statements are correct?
1. $E(\beta) = \beta$ is the best linear unbiased estimator of $\beta$.
2. $\hat{\lambda} = \frac{1}{n} \sum_{i=1}^{n} X_i \xi_i$ is the best linear unbiased estimator of $\beta$.
3. The variance of the best linear unbiased estimator of $\beta$ is $\frac{1}{n} \sum_{i=1}^{n} X_i \xi_i$.
4. The variance of the best linear unbiased estimator of $\beta$ is $\frac{1}{n} \sum_{i=1}^{n} X_i \xi_i$.
114. Consider a linear regression model \( \mathbf{Y} = X\mathbf{\beta} + \mathbf{\epsilon} \), where \( X \) is an \( n \times p \) matrix, \( \text{rank}(X) = p \), \( \mathbb{E}(\mathbf{\epsilon}) = \mathbf{0} \), \( \mathbb{D}(\mathbf{\epsilon}) = \sigma^2 \mathbf{I} \), \( \mathbb{E}(\cdot) \) stands for expectation, \( \mathbb{D}(\cdot) \) denotes the variance-covariance matrix and \( I \) is the \( n \)-th order identity matrix. Define the \( n \times n \) matrix \( H = \left( (h_{ij}) \right) = X(X'X)^{-1}X' \). Then, which of the following are correct?

1. \( 0 \leq h_{ii} \leq 1, \quad 1 \leq i \leq n \).
2. If \( h_{ii} = 0 \) or 1 for some \( i \), then \( h_{ij} = 0 \) for all \( j \neq i \).
3. The variance-covariance matrix of the vector of the predicted values \( \tilde{Y}(\text{of } Y) \) is \( \sigma^2 H \).
4. For \( 1 \leq i \leq n \), if \( e_i \) is the residual corresponding to \( Y_i \), i.e., \( e_i = Y_i - \tilde{Y}_i \), \( \bar{Y}_i \) being the predicted value of \( Y_i \), then the variance of \( e_i \) equals \( \sigma^2 (1 - h_{ii}) \). (Here, \( \tilde{Y}_i \) is the \( i \)-th component of \( \tilde{Y} \)).

115. Let \( (X, Y) \) follow a bivariate normal distribution with mean vector \((0,0)\), and dispersion matrix \( \Sigma = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \), \( \rho \neq 0 \).

Let \( \mathbf{Z} = \begin{bmatrix} 1 + p \\ X'Y \end{bmatrix} \). Then which of the following statements are correct?

1. \( \frac{1 + p}{1 - p} \mathbf{Z} \) has a student-t distribution.
2. \( \frac{1 - p}{1 + p} \mathbf{Z} \) has a student-t distribution.
3. \( \mathbf{Z} \) is symmetric about 0.
4. \( \mathbb{E}(\mathbf{Z}) \) exists and equals zero.

116. Let \( i \)-वीं इकाई की एक समस्थष्टि से आमाप दो का एक प्रतिदिन निकाला जाता है, आमाप के अनुपात में प्रविष्टका एवं प्रतिदिन शरीरस्थापन के साथ। समस्थष्टि में इकाईयों 1,2,3 तथा 4 के लिए क्रमशः वरण प्रायकता\( p_1 = 0.2, p_2 = 0.3, p_3 = 0.1 \) तथा \( p_4 = 0.4 \). माना कि \( i \)-वीं इकाई के लिए अघ्ययिका चर का मान है \( y_i \), \( i = 1,2,3,4 \). माना कि \( i \)-वीं इकाई की अंतर्वेशन प्रायकता को \( \pi_i \) निर्दिष्ट करता है, तथा इकाईयों \( i \) तथा \( j \) की संयुक्त अंतर्वेशन प्रायकता है \( \pi_{ij}, \quad i < j, i, j = 1,2,3,4 \)। तो निम्न कथनों में से कौन-से सही हैं?

1. समस्थष्टि योग का एक अन्वितित आकलन है \( T = \left( \frac{1}{2} \right) \sum \frac{1}{\pi_i} \)। जहाँ योग प्रतिदिन के इकाईयों के उपर है।
2. \( \pi_1 = 0.36, \pi_2 = 0.51 \).
3. \( \pi_{12} = 0.12 \).
4. \( \pi_1 + \pi_2 + \pi_3 + \pi_4 = 4 \).
116. A sample of size two is drawn from a population of 4 units using probability proportional to size, sampling with replacement. The selection probabilities are \( p_1 = 0.2, p_2 = 0.3, p_3 = 0.1 \) and \( p_4 = 0.4 \) for units 1, 2, 3 and 4 in the population, respectively. Let the value of a study variable for the \( i \)-th unit be \( y_i \). Let \( \pi_i \) denote the inclusion probability of the \( i \)-th unit and \( \pi_{ij} \) the joint inclusion probability of units \( i \) and \( j \), \( i < j, i, j = 1, 2, 3, 4 \). Then, which of the following statements are correct?

1. \( T = \left( \frac{1}{2} \right) \sum \frac{y_i}{p_i} \) is an unbiased estimator of the population total, where the sum is over the units in the sample
2. \( \pi_1 = 0.36, \pi_2 = 0.51 \)
3. \( \pi_{12} = 0.12 \)
4. \( \pi_1 + \pi_2 + \pi_3 + \pi_4 = 2 \)

117. \( v \) उपचार, \( b \) खंड, प्रतिकृति \( r \), खंड आमाप \( k \), तथा युगल: संगमन प्राप्त ल युक्त एक संतुलित अपूर्ण खंड अभिक्षण \( d \) पर विचारें। \( d \) दृष्टा पाये गये आंकों के लिए मानक नीति विनियमित प्रतिनिधित्व माने। निम्न कथनों में से कौन सा (वे) सही है(हैं)?
1. यदि \( k \geq 2 \) है तो अभिक्षण संबंध है।
2. \( d \) के लिए \( \text{असमानता} b \geq v \) पाम है।
3. एक प्रसामान्यैयूक्त उपचार विषमता के \( \text{श्रेष्ठसम्म} \) \( \text{विनियमित} \) \( \text{आकलन} \) (\( \text{से.रे.आम} \)) का \( \text{परामर्श} \) \( \text{गर्भ} \) है।
4. दो लांबिक उपचार विषमताओं के \( \text{से.रे.आम} \) के \( \text{बीच} \) का \( \text{सहस्पर्शन} \) शून्य है।

117. Consider a balanced incomplete block design \( d \) with \( v \) treatments, \( b \) blocks, replication \( r \), block size \( k \) and pairwise concurrence parameter \( \lambda \). Assume the standard fixed effects model for the data obtained through \( d \). Which of the following statement is(are) true?
1. The design is connected if \( k \geq 2 \).
2. The inequality \( b \geq v \) holds for \( d \).
3. The variance of the best linear unbiased estimator (BLUE) of a normalized treatment contrast is a constant.
4. The covariance between the BLUEs of two orthogonal treatment contrasts is zero.

118. विद्युत परिष्यों 1, 2, 3 में तीन प्रकार के घटक उपयोग में लाई जाते हैं जैसे निम्न चित्र में दर्शाया गया है।

![Circuit 1](image1)
![Circuit 2](image2)

माने कि तीनों घटकों में से हर एक प्रायिकता \( p \) तथा एक दूसरे से स्वतंत्र: विफल होते हैं। मानें कि \( q_i = \text{प्रायिकता} \) (परिष्य \( i \) विफल नहीं होता): \( i = 1, 2, 3 \). \( 0 < p < 1 \) के लिए हम पाते हैं:
1. \( q_3 > q_1 \)
2. \( q_1 = q_2 \)
3. \( q_2 > q_1 \)
4. \( q_2 > q_3 \)

118. Three types of components are used in electrical circuits 1, 2, 3 as shown below
Suppose that each of the three components fail with probability \( p \) and independently of each other. Let \( q_i = \text{Prob} (\text{Circuit } i \text{ does not fail}); \) \( i = 1, 2, 3 \). For \( 0 < p < 1 \), we have

1. \( q_3 > q_1 \)
2. \( q_1 = q_2 \)
3. \( q_2 > q_1 \)
4. \( q_2 > q_3 \)

119. Maximize \( 3x + 4y \) subject to

\[
\begin{align*}
    x &\geq 0, \quad y \geq 0, \quad x \leq 3, \\
    \frac{1}{2}x + y &\leq 4, \quad x + y \leq 5.
\end{align*}
\]

Which among the following are correct?

1. The optimal value is 19.
2. The optimal value is 18.
3. \((3, 2)\) is an extreme point of the feasible region.
4. \((3, \frac{5}{2})\) is an extreme point of the feasible region.

120. Let \( X_1, X_2, \ldots, X_{2n+1} \) be a random sample from a uniform distribution on the interval \((\theta - 1, \theta + 1)\). Let

\[
    T_1 = \bar{X}, \quad T_2 = \tilde{X}, \quad T_3 = \frac{T_1 + T_2}{2},
\]

be three estimators of \( \theta \). Then, which of the following statements are correct?

1. \( T_1 \) is consistent for \( \theta \).
2. \( T_1 \) and \( T_2 \) are more efficient than \( T_3 \).
3. All the three estimators are unbiased for \( \theta \).
4. \( T_2 \) is a sufficient statistic for \( \theta \).
FOR ROUGH WORK