CBSE NCERT Solutions for Class 9 Mathematics Chapter 2

Back of Chapter Questions

Exercise: 2.1

1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 - 3x + 7$

(ii) $y^2 + \sqrt{2}$

(iii) $3\sqrt{t} + t\sqrt{2}$

(iv) $y + \frac{2}{y}$

(v) $x^{10} + y^3 + t^{50}$

Solution:

(i) Given expression is a polynomial

It is of the form $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ where $a_n, a_{n-1}, \ldots a_0$ are constants. Hence given expression $4x^2 - 3x + 7$ is a polynomial.

(ii) Given expression is a polynomial

It is of the form $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ where $a_n, a_{n-1}, \ldots a_0$ are constants. Hence given expression $y^2 + \sqrt{2}$ is a polynomial.

(iii) Given expression is not a polynomial. It is not in the form of

$a_n x^n + a_{n-1}2^{n-1} + \cdots + a_1 x + a_0$

where $a_n, a_{n-1}, \ldots a_0$ all constants.

Hence given expression $3\sqrt{t} + t\sqrt{2}$ is not a polynomial.

(iv) Given expression is not a polynomial

$$y + \frac{2}{y} = y + 2.y^{-1}$$

It is not of form $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$, where $a_n, a_{n-1}, \ldots a_0$ are constants.

Hence given expression $y + \frac{2}{y}$ is not a polynomial.

(v) Given expression is a polynomial in three variables. It has three variables x, y, t.

Hence the given expression $x^{10} + y^3 + t^{50}$ is not a polynomial in one variable.

2. Write the coefficients of x^2 in each of the following:

(i) $2 + x^2 + x$

(ii) $2 - x^2 + x^3$

(iii) $\frac{\pi}{2}x^2 + x$

(iv) $\sqrt{2}x - 1$

Solution:

(i) The constant multiplied with the term x^2 is called the coefficient of the x^2.

Given polynomial is $2 + x^2 + x$.

Hence, the coefficient of x^2 in given polynomial is equal to 1.

(ii) The constant multiplied with the term x^2 is called the coefficient of the x^2.

Given polynomial is $2 - x^2 + x^3$.

Hence, the coefficient of x^2 in given polynomial is equal to -1.

(iii) The constant multiplied with the term x^2 is called the coefficient of the x^2.

Given polynomial is $\frac{\pi}{2}x^2 + x$.

Hence, the coefficient of x^2 in given polynomial is equal to $\frac{\pi}{2}$.

(iv) The constant multiplied with the term x^2 is called the coefficient of the x^2.

Given polynomial is $\sqrt{2}x - 1$.

In the given polynomial, there is no x^2 term.

Hence, the coefficient of x^2 in given polynomial is equal to 0.

3. Give one example each of a binomial of degree 35 and of a monomial of degree 100^o.

Solution:

Degree of polynomial is highest power of variable in the polynomial. And number of terms in monomial and binomial respectively equals to one and two.

A binomial of degree 35 can be $x^{35} + 7$

A monomial of degree 100 can be $2x^{100} + 9$

4. Write the degree of each of the following polynomials

(i) $5x^3 + 4x^2 + 7x$

(ii) $4 - y^2$

(iii) $5t - \sqrt{7}$

(iv) 3

Solution:

(i) Degree of polynomial is highest power of variable in the polynomial.

Given polynomial is $5x^3 + 4x^2 + 7x$

Hence, the degree of given polynomial is equal to 3.

(ii) Degree of polynomial is highest power of variable in the polynomial.

Given polynomial is $4 - y^2$

Hence, the degree of given polynomial is 2.

(iii) Degree of polynomial is highest power of variable in the polynomial

Given polynomial is $5t - \sqrt{7}$

Hence, the degree of given polynomial is 1.

(iv) Degree of polynomial 1, highest power of variable in the polynomial.

Given polynomial is 3.

Hence, the degree of given polynomial is 0.

5. Classify the following as linear, quadratic and cubic polynomials.

(i) $x^2 + x$

(ii) $x - x^3$

(iii) $y + y^2 + 4$

(iv) $1 + x$

(v) $3t$

(vi) r^2

(vii) $7x^3$

Solution:

(i) Linear, quadratic, cubic polynomials have degrees $1, 2, 3$ respectively.

Given polynomial is $x^2 + x$

It is a quadratic polynomial as its degree is 2.

(ii) Linear, quadratic, cubic polynomials have its degree $1, 2, 3$ respectively.

Given polynomial is $x - x^3$.

It is a cubic polynomial as its degree is 3.

(iii) Linear, quadratic, cubic polynomial has its degree $1, 2, 3$ respectively.

Given polynomial is $y + y^2 + 4$.

It is a quadratic polynomial as its degree is 2.

(v) Linear, quadratic, cubic polynomial has its degree $1, 2, 3$ respectively.

Given polynomial is $1 + x$.

It is a linear polynomial as its degree is 1.

(v) Linear, quadratic, cubic polynomial has its degree $1, 2, 3$ respectively.

Given polynomial is $3t$

It is a linear polynomial as its degree is 1.

(vi) Linear, quadratic, cubic polynomial has its degree $1, 2, 3$ respectively.

Given polynomial is r^2.

It is a quadratic polynomial as its degree is 2.

(vii) Linear, quadratic, cubic polynomial has its degree $1, 2, 3$ respectively.

Given polynomial is $7x^3$.

It is a cubic polynomial as its degree is 3.

Exercise: 2.2

1. Find the value of the polynomial $5x - 4x^2 + 3$ at

(i) $x = 0$

(ii) $x = -1$

(iii) $x = 2$

Solution:

(i) Given polynomial is $5x - 4x^2 + 3$

Value of polynomial at $x = 0$ is $5(0) - 4(0)^2 + 3$

$= 0 - 0 + 3$

$= 3$

Therefore, value of polynomial $5x - 4x^2 + 3$ at $x = 0$ is equal to 3.

(ii) Given polynomial is $5x - 4x^2 + 3$

Value of given polynomial at $x = -1$ is $5(-1) - 4(-1)^2 + 3$

$= -5 - 4 + 3$

$= -6$

Therefore, value of polynomial $5x - 4x^2 + 3$ at $x = -1$ is equal to -6.

(iii) Given polynomial is $5x - 4x^2 + 3$

Value of given polynomial at $x = 2$ is $5(2) - 4(2)^2 + 3$

$= 10 - 16 + 3$

$= -3$

Therefore, value of polynomial $5x - 4x^2 + 3$ at $x = 2$ is equal to -3

2. Find $P(0), P(1)$ and $P(2)$ for each of the following polynomials.

(i) $P(y) = y^2 - y + 1$

(ii) $P(t) = 2 + t + 2t^2 - t^3$

(iii) $P(x) = x^3$

(iv) $P(x) = (x - 1)(x + 1)$

Solution:

(i) Given polynomial is $P(y) = y^2 - y + 1$

$P(0) = (0)^2 - 0 + 1$

$= 1$

$P(1) = (1)^2 - 1 + 1$

$= 1$

$P(2) = (2)^2 - 2 + 1$

$= 4 - 2 + 1$

$= 3$

(ii) Given polynomial is $P(t) = 2 + t + 2t^2 - t^3$

$P(0) = 2 + 0 + 2.(0)^2 - (0)^3$

$= 2$

$P(1) = 2 + 1 + 2(1)^2 - (1)^3$

$= 4$

$P(2) = 2 + 2 + 2.(2)^2 - (2)^3$

$= 4$

(iii) Given polynomial is $P(x) = x^3$

$P(0) = (0)^3 = 0$

$P(1) = (1)^3 = 1$

$P(2) = (2)^3 = 8$

(iv) Given polynomial is $p(x) = (x - 1)(x + 1)$

$P(0) = (0 - 1)(0 + 1)$

$= (-1)(1)$

$= -1$

$P(1) = (1 - 1)(1 + 1)$

$= (0)(2)$

$= 0$

$P(2) = (2 - 1)(2 + 1)$

$= 3$

3. Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $P(x) = 3x + 1, x = -\frac{1}{3}$

(ii) $P(x) = 5x - \pi, x = \frac{4}{5}$

(iii) $P(x) = x^2 - 1, x = 1, -1$

(iv) $P(x) = (x + 1)(x - 2), x = -1, 2$

(v) $P(x) = x^2, x = 0$

(vi) $P(x) = lx + m, x = -\frac{m}{l}$

(vii) $P(x) = 3x^2 - 1, x = \frac{-1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

(viii) $P(x) = 2x + 1, x = \frac{1}{a}$

Solution:

(i) For a polynomial $P(n)$, if $n = a$ is zero then $P(a)$ must be equal to zero

Given polynomial is $P(x) = 3x + 1$

At $x = -\frac{1}{3}$

$P\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right) + 1$

$= -1 + 1$

$= 0$

Hence $\frac{-1}{3}$ is a zero of polynomial $P(x) = 3x + 1$.

(ii) For a polynomial $P(x)$, if $x = a$ is zero, then $P(a)$ must be equal to zero.

Given polynomial is $P(x) = 5x - \pi$

At $x = \frac{4}{5}$

$P\left(\frac{4}{5}\right) = 5\left(\frac{4}{5}\right) - \pi$

$= 4 - \pi$

$\neq 0$

Hence $x = \frac{4}{5}$ is not a zero of polynomial $5x - \pi$

(iii) For a polynomial $P(x)$, if $x = a$ is zero, then $P(a)$ must be equal to zero.

Given polynomial is $P(x) = x^2 - 1$

At $x = 1$

$P(1) = (1)^2 - 1$

$= 0$

And $x = -1$

$P(-1) = (-1)^2 - 1$

$= 1 - 1$

$= 0$

Hence $x = 1, -1$ are zeroes of polynomial $x^2 - 1$.

(iv) For a polynomial $P(x)$, if $x = a$ is zero, then $P(a)$ must be equal to zero.

Given polynomial is $P(x) = (x + 1)(x - 2)$

At $x = -1$,

$P(-1) = (-1 + 1)(-1 - 2)$

$= (0)(-3)$

$= 0$

And $x = 2$,

$P(2) = (2 + 1)(2 - 2)$

$= (0)(3)$

$= 0$

Hence $x = -1, 2$ are zeroes of polynomial $(x + 1)(x - 2)$

(v) For a polynomial $P(x)$, if $x = a$ is zero, then $P(a)$ must be equal to zero.

Given polynomial is $P(x) = x^2$

$P(0) = (0)^2$

$= 0$

Hence $x = 0$ is zero of polynomial x^2.

(vi) For a polynomial $P(n)$, if $n = a$ is zero, then $P(a)$ must be equal to zero

Given polynomial is $P(x) = lx + m$

At $x = -\frac{m}{l}$,

$P\left(-\frac{m}{l}\right) = l\left(-\frac{m}{l}\right) + m$

$= -m + m$

$= 0$

Hence $x = -\frac{m}{l}$ is zero of polynomial $lx + m$

(vii) For a polynomial $P(x)$, if $x = a$ is zero then $P(x)$ must be equal to zero

Given polynomial is $P(x) = 3x^2 - 1$

At $x = \frac{-1}{\sqrt{3}}$,

$P\left(\frac{-1}{\sqrt{3}}\right) = 3.\left(\frac{-1}{\sqrt{3}}\right)^2 - 1$

$= 3 \times \frac{1}{3} - 1$

$= 1 - 1$

$= 0$

Now at $x = \frac{2}{\sqrt{3}}$,

$$P\left(\frac{2}{\sqrt{3}}\right) = 3.\left(\frac{2}{\sqrt{3}}\right)^2 - 1$$

$$= 3.\frac{4}{3} - 1$$

$$= 3$$

Therefore, $x = \frac{-1}{\sqrt{3}}$ is zero of polynomial $3x^2 - 1$.

And $x = \frac{2}{\sqrt{3}}$ is not a zero of polynomial $3x^2 - 1$

(viii) For a polynomial $P(x)$, if $x = a$ is zero, then $P(a)$ must be equal to zero.

Given polynomial is $P(x) = 2x + 1$

At $x = \frac{1}{2}$,

$$P\left(\frac{1}{2}\right) = 2.\frac{1}{2} + 1$$

$$= 2$$

Hence $x = \frac{1}{2}$ is not a zero of polynomial $2x + 1$

4. Find the zero of the polynomials in each of the following cases.

(i) $P(x) = x + 5$

(ii) $P(x) = x - 5$

(iii) $P(x) = 2x + 5$

(iv) $P(x) = 3x - 2$

(v) $P(x) = 3x$

(vi) $P(x) = ax, a \neq 0$

(vii) $P(x) = cx + d, c \neq 0, c, d$ are real numbers.

Solution:

For a polynomial $P(x)$, if $x = a$ is said to be a zero of the polynomial $p(x)$, then $P(a)$ must be equal to zero.

(i) Given polynomial is $P(x) = x + 5$

Now, $P(x) = 0$

$\Rightarrow x + 5 = 0$

$\Rightarrow x = -5$

Hence $x = -5$ is zero of polynomial $P(x) = x + 5$

(ii) Given polynomial is $P(x) = x - 5$

Now, $P(x) = 0$

$\Rightarrow x - 5 = 0$

$\Rightarrow x = 5$

Hence $x = 5$ is zero of polynomial $P(x) = x - 5$.

(iii) Given polynomial is $P(x) = 2x + 5$

Now, $P(x) = 0$

$\Rightarrow 2x + 5 = 0$

$\Rightarrow x = \frac{-5}{2}$

Hence $x = -\frac{5}{2}$ is zero of polynomial $P(n) = 2x + 5$.

(iv) Given polynomial is $P(x) = 3x - 2$

Now, $P(x) = 0$

$\Rightarrow 3x - 2 = 0$

$\Rightarrow x = \frac{2}{3}$

Hence $x = \frac{2}{3}$ is zero of polynomial $P(n) = 3x - 2$

(v) Given polynomial is $P(x) = 3x$

Now, $P(n) = 0$

$\Rightarrow 3x = 0$

$\Rightarrow x = 0$

Hence $x = 0$ is zero of polynomial $P(n) = 0$.

(vi) Given polynomial is $P(x) = ax$

Now, $P(x) = 0$

$\Rightarrow ax = 0$

$\Rightarrow a = 0$ or $x = 0$

But given that $a \neq 0$

Hence $x = 0$ is zero of polynomial $P(x) = ax$.

(vii) Given polynomial is $P(x) = cx + d$

$P(x) = 0$

$\Rightarrow cx + d = 0$

$\Rightarrow cx = -d$

$\Rightarrow x = -\frac{d}{c}$

Hence $x = -\frac{d}{c}$ is zero of given polynomial $P(n) = cx + d$

Exercise: 2.3

1. Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by

(i) $x + 1$

(ii) $x - \frac{1}{2}$

(iii) x

(iv) $x + \pi$

(v) $5 + 2x$

Solution:

We know, the remainder of polynomial $P(n)$ when divided by another polynomial $(an + b)$ where a and b are real numbers $a \neq 0$ is equal to $P\left(\frac{-b}{a}\right)$.

(i) Given polynomial is $P(x) = x^3 + 3x^2 + 3x + 1$

When $P(x)$ is divided by $x + 1$, then the remainder is $P(-1)$

Hence, remainder $= P(-1) = (-1)^3 + 3.(-1)^2 + 3(-1) + 1$

$= -1 + 3 - 3 + 1$

$= 0$

Remainder when polynomial $x^3 + 3x^2 + 3x + 1$ is divided by $x + 1$ is equal to 0

(ii) Given polynomial is $P(x) = x^3 + 3x^2 + 3x + 1$

When $P(x)$ is divided by $x - \frac{1}{2}$, then the remainder is $P\left(\frac{1}{2}\right)$

Hence, remainder $= P\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + 3.\left(\frac{1}{2}\right)^2 + 3.\left(\frac{1}{2}\right) + 1$

$= \frac{1}{8} + \frac{3}{4} + \frac{3}{2} + 1$

$= \frac{1}{8} + \frac{9}{4} + 1$

$= \frac{19}{8} + 1 = \frac{27}{8}$

The remainder when polynomial $x^3 + 3x^2 + 3x + 1$ is divided by $x - \frac{1}{2}$ is equal to $\frac{27}{8}$

(iii) Given polynomial is $P(x) = x^3 + 3x^2 + 3x + 1$

When $P(x)$ is divided by x, then the remainder is $P(0)$

Hence, remainder $= P(0) = (0)^3 + 3.(0)^2 + 3(0) + 1$

$= 1$

The remainder when polynomial $x^3 + 3x^2 + 3x + 1$ is divided by x is equal to 1.

(iv) Given polynomial is $P(x) = x^3 + 3x^2 + 3x + 1$

When $P(x)$ is divided by $x + \pi$, then the remainder is $P(-\pi)$

Hence, remainder $= P(-\pi) = (-\pi)^3 + 3.(-\pi)^2 + 3(-\pi) + 1$

$= -\pi^3 + 3\pi^2 - 3\pi + 1$

$= (-\pi + 1)^3$

The remainder when polynomial $P(n) = x^3 + 3x^2 + 3x + 1$ is divided by $x + \pi$ is equal to $(-\pi + 1)^3$.

(v) Given polynomial is $P(x) = x^3 + 3x^2 + 3x + 1$

When $P(x)$ is divided by $5 + 2x$, then the remainder is $P\left(-\frac{5}{2}\right)$

Hence, remainder $= P\left(\frac{-5}{2}\right) = \left(\frac{-5}{2}\right)^3 + 3.\left(\frac{-5}{2}\right)^2 + 3.\left(\frac{-5}{2}\right) + 1$

$= \frac{-125}{8} + 3.\left(\frac{25}{4}\right) - \frac{15}{2} + 1$

$= -\frac{125}{8} + \frac{75}{4} - \frac{15}{2} + 1$

$= \frac{25}{8} - \frac{15}{2} + 1$

$= \frac{-35}{8} + 1$

$$= -\frac{27}{8}$$

The remainder when $x^3 + 3x^2 + 3x + 1$ is divided by $5 + 2x$ is equal to $-\frac{27}{8}$.

2. Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

Solution:

The remainder of polynomial $P(x)$ when divided by another polynomial $(ax + b)$ where a and b are real numbers $a \neq 0$ is equal to $P\left(\frac{-b}{a}\right)$

Given polynomial is $P(x) = x^3 - ax^2 + 6x - a$

When $P(x)$ is divided by $x - a$, then the remainder is $P(a)$

Hence, remainder = $P(a) = a^3 - a.(a)^2 + 6(a) - a$

$= a^3 - a^3 + 6a - a$

$= 5a$

The remainder when polynomial $P(x) = x^3 - ax^2 + 6x - a$ is divided by $x - a$ is equal to $5a$

3. Check whether $7 + 3x$ is factor of $3x^3 + 7x$.

Solution:

Given polynomial is $P(x) = 3x^3 + 7x$

For $7 + 3x$ to be a factor of $3x^3 + 7x$, remainder when polynomial $3x^3 + 7x$ divided by $7 + 3x$ must be zero.

We know, the remainder of polynomial $P(x)$ when divided by another polynomial $(ax + b)$, where a and b are real numbers $a \neq 0$ is equal to $P\left(\frac{-b}{a}\right)$

Hence, remainder $= P\left(-\frac{7}{3}\right) = 3\left(-\frac{7}{3}\right)^3 + 7.\left(-\frac{7}{3}\right)^2$

$$= 3\left(\frac{343}{27}\right) - \frac{49}{3}$$

$$= -\frac{343}{9} - \frac{49}{3}$$

$$= -\frac{490}{9}$$

As remainder is not equal to zero

Hence $7 + 3x$ is not a factor of $3x^2 + 7x$

Exercise: 2.4

1. Determine which of the following polynomials has $(x + 1)$ as factor:

(i) $x^3 + x^2 + x + 1$

(ii) $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

(iv) $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

Solution:

For polynomials $(x + 1)$ to be a factor of given polynomial, remainder when given polynomials divided by $(x + 1)$ must be equal to zero.

The remainder of polynomial $p(x)$ when divided by $(ax + b)$ where a and b are real numbers, $a \neq 0$ is equal to $p\left(-\frac{b}{a}\right)$

(i) Given polynomial is $p(x) = x^3 + x^2 + x + 1$

Hence, remainder $= p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$

$= -1 + 1 - 1 + 1$

$= 0.$

Hence $x + 1$ is a factor of polynomial $x^3 + x^2 + x + 1$.

(ii) Given polynomial is $p(x) = x^4 + x^3 + x^2 + x + 1$

Hence, remainder $= p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$

$= 1 - 1 + 1 - 1 + 1$

$= 1$

As remainder $\neq 0$,

Hence $x + 1$ is not a factor of polynomial $x^4 + x^3 + x^2 + x + 1$.

(iii) Given polynomial is $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$.

Hence, remainder $= p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$

$= 1 - 3 + 3 - 1 + 1 = 1$

As remainder $\neq 0$.

Hence $(x + 1)$ is not a factor of polynomial $x^4 + 3x^3 + 3x^2 + x + 1$

(iv) Given polynomial is $p(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

$p(-1) = (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2} = 2\sqrt{2}$

As remainder $\neq 0$,

Hence $(x + 1)$ is not a factor of polynomial $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$.

2. Use the factor theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x) = 2x^3 + x^2 - 2x - 1, g(x) = x + 1$

(ii) $p(x) = x^3 + 3x^2 + 3x + 1, g(x) = x + 2$

(iii) $p(x) = x^3 - 4x^2 + x + 6, g(x) = x - 3$

Solution:

For polynomial $g(x)$ to be a factor of polynomial $p(x)$, remainder when polynomial $p(x)$ is divided by polynomial $g(x)$ must be equal to zero.

(i) Given polynomial is $p(x) = 2x^3 + x^2 - 2x - 1$

We know, the remainder of polynomial $p(x)$ when divided by $(ax + b)$ where a and b are real numbers, $a \neq 0$ is equal to $p\left(-\frac{b}{a}\right)$.

Hence, remainder $= p(-1) = 2(-1)^3 + (-1)^3 - 2(-1) - 1$

$= -2 + 1 + 2 - 1$

$= 0.$

As remainder when polynomial $p(x)$ is divided by polynomial $g(x)$ is equal to zero, polynomial $g(x) = x + 1$ is a factor of polynomial $p(x) = 2x^3 + x^2 - 2x - 1$.

(ii) Given polynomial is $p(x) = x^3 + 3x^2 + 3x + 1$

We know, the remainder of polynomial $p(x)$ when divided by $(ax + b)$ where a and b are real numbers, $a \neq 0$ is equal to $p\left(-\frac{b}{a}\right)$.

Hence, remainder $= p(-2) = (-2)^3 + 3(2)^2 + 3(-2) + 1$

$= -8 + 12 - 6 + 1$

$= -1$

Since remainder $\neq 0$, the polynomial $g(x) = x + 2$ is not a factor of polynomial $p(x) = x^3 + 3x^2 + 3x + 1$.

(iii) Given polynomial is $p(x) = x^3 - 4x^2 + x + 6$

We know, the remainder of polynomial $p(x)$ when divided by $(ax + b)$ where a and b are real numbers, $a \neq 0$ is equal to $p\left(-\frac{b}{a}\right)$.

Hence, remainder $= p(3) = (3)^3 - 4(3)^2 + 3 + 6$

$= 27 - 36 + 9$

$= 0$

Since reminder $= 0$, the polynomial $g(x) = x - 3$ is factor of polynomial $p(x) = x^3 - 4x^2 + x + 6$.

3. Find the value of k if $x - 1$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x) = x^2 + x + k$

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

(iv) $p(x) = kx^2 - 3x + k$

Solution:

For polynomial $(x - 1)$ to be a factor of polynomial $p(x)$ then the remainder when polynomial $p(x)$ is divided by polynomial $(x - 1)$ must be equal to zero.

(i) Given polynomial is $p(x) = x^2 + x + k$

We know, the remainder of polynomial $p(x)$ when divided by $(ax + b)$ where a and b are real numbers, $a \neq 0$ is equal to $p\left(-\frac{b}{a}\right)$.

Hence, remainder $= p(1) = (1)^2 + (1) + k$

$= k + 2.$

Remainder should be equal to zero.

$\Rightarrow k + 2 = 0$

$\Rightarrow k = -2$

For $k = -2, x - 1$ is a factor of polynomial $p(x) = x^2 + x + k$.

(ii) Given polynomial is $p(x) = kx^2 - \sqrt{2}x + 1$

We know, the remainder of polynomial $p(x)$ when divided by $(ax + b)$ where a and b are real numbers, $a \neq 0$ is equal to $p\left(-\frac{b}{a}\right)$.

Hence, remainder $= p(1) = 2(1)^2 + K(1) + \sqrt{2}$

$= 2 + \sqrt{2} + k$

Now, $p(1) = 0$

$\Rightarrow 2 + \sqrt{2} + k = 0$

$\Rightarrow K = -(2 + \sqrt{2})$

For $k = -(2 + \sqrt{2})$, $x - 1$ is a factor of polynomial $p(x) = 2x^2 + kx + \sqrt{2}$

(iii) Given polynomial is $p(x) = kx^2 - \sqrt{2}x + 1$

We know, the remainder of polynomial $p(x)$ when divided by $(ax + b)$ where a and b are real numbers, $a \neq 0$ is equal to $p\left(-\frac{b}{a}\right)$.

Hence, remainder $= p(1) = k(1)^2 - \sqrt{2}(1) + 1$

$= k - \sqrt{2} + 1$

Now, $p(1) = 0$

$\Rightarrow k + 1 - \sqrt{2} = 0$

For $k = (\sqrt{2} - 1)$, $x - 1$ as factor of polynomial $p(x) = kx^2 - 3x + k$

(iv) Given polynomial is $p(x) = kx^2 - 3x + k$

We know, the remainder of polynomial $p(x)$ when divided by $(ax + b)$ where a and b are real numbers, $a \neq 0$ is equal to $p\left(-\frac{b}{a}\right)$.

Hence, remainder $= p(1) = k(1)^2 - 3(1) + k$

$= 2k - 3$

Now, $p(1) = 0$

$\Rightarrow 2k - 3 = 0$

$\Rightarrow k = \frac{3}{2}$

For $k = \frac{3}{2}$, $x - 1$ is a factor of polynomial $p(x) = kx^2 - 3x + k$

4. Factorise;

(i) $12x^2 - 7x + 1$

(ii) $6x^2 + 5x - 6$

(iii) $6x^2 + 5x - 6$

(iv) $3x^2 - x - 4$

Solution:

(i) Given polynomial is $12x^2 - 7x + 1$

$12x^2 - 7x + 1$

$= 12x^2 - 4x - 3x + 1$

$= 4x(3x - 1) - 1(3x - 1)$

$= (4x - 1)(3x - 1)$

$12x^2 - 7x + 1 = (3x - 1)(4x - 1)$

(ii) Given polynomial is $2x^2 + 7x + 3$

$2x^2 + 7x + 3$

$= 2x^2 + 6x + x + 3$

$= 2x(x + 3) + 1(x + 3)$

$= (2x + 1)(x + 3)$

$2x^2 + 7x + 3 = (2x + 1)(x + 3)$

(iii) Given polynomial is $6x^2 + 5x - 6$

$6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6$

$= 3x(2x + 3) - 2(2x + 3)$

$6x^2 + 5x - 6 = (3x - 2)(2x + 3)$

(iv) Given polynomial is $3x^2 - x - 4$

On splitting middle term

$= 3x^2 - 4x + 3x - 4$

$= x(3x - 4) + 1(3x - 4)$

$= (x + 1)(3x - 4)$

$3x^2 - x - 4 = (x + 1)(3x - 4)$

5. Factorise:

(i) $x^3 - 2x^2 - x + 2$

(ii) $x^3 - 3x^2 - 9x - 5$

(iii) $x^3 + 13x^2 + 32x + 20$

(iv) $2y^3 + y^2 - 2y - 1$

Solution:

(i) Given polynomial is $x^3 - 2x^2 - x + 2$

Put x = 1

$(1)^3 - 2.(1)^2 - 1 + 2$

$= 1 - 2 - 1 + 2$

$= 0$

By trial and error method, we got $(x - 1)$ is a factor of given polynomial $x^3 - 2x^2 - x + 2$.

We can find other factors by long division method.

$$
\begin{array}{r|l|l}
x-1 & x^3-2x^2-x+2 & x^2-x-2 \\
 & \underline{\overset{-}{x^3}\ \overset{+}{-x^2}} & \\
 & \quad -x^2-x & \\
 & \quad \underline{\overset{+}{-}x^2\ \overset{-}{+x}} & \\
 & \qquad -2x+2 & \\
 & \qquad \underline{\overset{+}{-2x}\ \overset{-}{+2}} & \\
 & \qquad\quad \underline{0} &
\end{array}
$$

Quotient $= x^2 - x - 2$

$= x^2 - 2x + x - 2$

$= x(x - 2) + 1(x - 2)$

$(x + 1)(x - 1)$

Hence on factorization,

$x^3 - 2x^2 - x + 2 = (x - 1)(x + 1)(x - 2)$

(ii) Given polynomial is $x^3 - 3x^2 - 9x - 5$

Put $x = -1$ in given polynomial,

$(-1)^3 - 3(-1)^2 - 9(-1) - 5$

$= -1 - 3 + 9 - 5$

$= 0$

By trial and error method, we got $(x + 1)$ is a factor of given polynomial $x^3 - 2x^2 - 9x - 5$.

We can find other factor by long division method.

$$
\begin{array}{r|l|l}
x+1 & x^3-3x^2-9x-5 & x^2-4x-5 \\
 & \underline{\overset{-}{x^3}+\overset{-}{x^2}} & \\
 & -4x^2-9x & \\
 & \underline{\overset{+}{-4x^2}\ \overset{+}{-4x}} & \\
 & -5x-5 & \\
 & \underline{\overset{+}{-5x}\ \overset{+}{-5}} & \\
 & 0 &
\end{array}
$$

Quotient $= x^2 - 4x - 5$

$= x^2 - 5x + x - 5$

$= x(x - 5) + 1(x - 5)$

$= (x + 1)(x - 5)$

Hence, $x^3 - 3x^2 - 9x - 5 = (x + 1)^2(x - 5)$

(iii) Given polynomial is $x^3 + 13x^2 + 32x + 20$

Put $x = -1$ in given polynomial,

$(-1)^3 + 13(-1)^2 + 32(-1) + 20$

$= -1 + 13 - 32 + 20$

$= 12 - 12$

$= 0$

By trial and error method, we got $(x + 1)$ is factor of given polynomial.

The remaining factors can be found by long division method

$$
\begin{array}{r|l}
x+1 & x^3+13x^2+32x+20 \;\big(\; x^2+12x+20 \\
 & \underline{\underset{-}{x^3}+\underset{-}{x^2}} \\
 & \quad 12x^2+32x \\
 & \quad \underline{\underset{-}{12x^2}+\underset{-}{12x}} \\
 & \qquad 20x+20 \\
 & \qquad \underline{\underset{-}{20x}+\underset{-}{20}} \\
 & \qquad \underline{\quad 0 \quad}
\end{array}
$$

Quotient $= x^2 + 12 + 20$

$= x^2 + 10x + 2x + 20$

$= x(x + 10) + 2(x + 10)$

$= (x + 2)(x + 10)$

$= (x + 2)(x + 10)$

Hence, $x^3 + 13x^2 + 32x + 20 = (x + 1)(x + 2)(x + 10)$

(iv) Given polynomial is $2y^3 + y^2 - 2y - 1$

Put $y = 1$ in given polynomial

$2.(1)^3 + (1_{=})^2 - 2(1) - 1$

$= 2 + 1 - 2 - 1$

$= 0$

By trial and error method, we got $(y - 1)$ is factor of given polynomial.

The remaining factors can be found by long division method

$$
\begin{array}{r|l|l}
y-1 & 2y^3 + y^2 - 2y - 1 & 2y^2 + 3y + 1 \\
 & \underline{\overset{-}{}2y^3 \overset{+}{-} 2y^2} & \\
 & \quad 3y^2 - 2y & \\
 & \quad \underline{\overset{-}{}3y^2 \overset{+}{-} 3y} & \\
 & \qquad\quad y - 1 & \\
 & \qquad\quad \underline{\overset{-}{}y \overset{+}{-} 1} & \\
 & \qquad\quad \underline{0} &
\end{array}
$$

Quotient $= 2y^2 + 3y + 1$

$= 2y^2 + 2y + y + 1$

$= 2y(y + 1) + 1(y + 1)$

$= (2y + 1)(y + 1)$

Hence, $2y^3 + y^2 - 2y - 1 = (y - 1)(2y + 1)(y + 1)$

Exercise: 2.5

1. Use suitable identities to find the following products:

(i) $(x + 4)(x + 10)$

(ii) $(x + 8)(x - 10)$

(iii) $(3x + 4)(3x - 5)$

(iv) $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$

(v) $(3 - 2x)(3 + 2x)$

Solution:

(i) We know that

$(x + a)(x + b) = x^2 + (a + b)x + ab$

Given polynomial is $(x + 4)(x + 10)$

Here, $a = 4, b = 10$

$(x + 4)(x + 10) = x^2 + (4 + 10)x + 40$

$= x^2 + 14x + 40$

(ii) We know that

$(x+a)(x+b) = x^2 + (a+b)x + ab$

Given polynomial is $(x+8)(x-10)$

Here $a = 8, b = -10$

$(x+8)(x-10) = x^2 + (8-10)x - 80$

$= x^2 - 2x - 80.$

(iii) We know that

$(x+a)(x+b) = x^2 + (a+b)x + ab$

Given polynomial is $(3x+4)(3x-5) = 3\left(x+\frac{4}{3}\right) 3\left(x-\frac{5}{3}\right)$

$= 9\left(x+\frac{4}{3}\right)\left(x-\frac{5}{3}\right)$

Here $a = \frac{4}{3}, b = \frac{-5}{3}$.

$(3x+4)(3x-5) \ = 9\left(x+\frac{4}{3}\right)\left(x-\frac{5}{3}\right)$

$= 9\left(x^2 - \frac{x}{3} - \frac{20}{9}\right)$

$(3x+4)(3x-5) = 9x^2 - 3x - 20.$

(iv) We know that

$(x+a)(x-a) = x^2 - a^2.$

Given polynomial is $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$

Here $x = y^2, a = \frac{3}{2}$

$\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right) = (y^2)^2 - \left(\frac{3}{2}\right)^2$

$= y^4 - \frac{9}{4}.$

(v) We know that $(x+a)(x-a) = x^2 - a^2$

Given Polynomial is $(3-2x)(3+2x)$

Here, $x = 3, a = \frac{3}{2}$

$(3-2x)(3+2x) = -2\left(x - \frac{3}{2}\right) \cdot 2\left(x + \frac{3}{2}\right)$

$$= -4\left(x + \frac{3}{2}\right)\left(x - \frac{3}{2}\right)$$

$$= -4\left(x^2 - \frac{9}{4}\right)$$

$$= -4x^2 + 9.$$

2. Evaluate the following products without multiplying directly:

(i) 103×107

(ii) 95×96

(iii) 104×96

Solution:

(i) $103 \times 107 = (100 + 3) \times (100 + 7)$

We know that $(x + a)(x + b) = x^2 + (a + b)x + ab$

Here $x = 100, a = 3, b = 7$

$(100 + 3)(100 + 7) = (100)^2 + 10 \times 100 + 3 \times 7$

$= 10000 + 1000 + 21$

$= 11021$

(ii) $95 \times 96 = (100 - 5)(100 - 4)$

We know that $(x + a)(x + b) = x^2 + (a + b)x + ab$

Here $x = 100, a = -5, b = -4$

$95 \times 96 = (100 - 5)(100 - 4)$

$= (100)^2 + \big(-5 + (-4)\big)100 + (-5)(-4)$

$= 10000 - 900 + 20$

$= 9120.$

(iii) $104 \times 96 = (100 + 4)(100 - 4)$

We know that $(x + a)(x - a) = x^2 - a^2$

Here $x = 100, a = 4$

$(100 + 4)(100 - 4) = (100\,)^2 - (4)^2$

$= 10000 - 16$

$= 9984$

3. Factorise the following using appropriate identities:

(i) $9x^2 + 6xy + y^2$

(ii) $4y^2 - 4y + 1$

(iii) $x^2 - \frac{y^2}{100}$

Solution:

(i) $9x^2 + 6xy + y^2 = (3x)^2 + 2 \cdot 3x \cdot y + (y)^2$

We know that $x^2 + 2xy + y^2 = (x + y)^2$

Comparing obtained expression with above identity

$9x^2 + 6xy + y^2 = (3x^2) + 2 \cdot 3x \cdot y + (y)^2$

$= (3x + y)^2$

$= (3x + y)(3x + y)$

(ii) $4y^2 - 4y + 1 = (2y)^2 - 2 \cdot 2y + 1$

We know that $x^2 - 2xy + y^2 = (x - y)^2$

Comparing obtained expression with above identity

$(2y)^2 - 2 \cdot 2y + 1 = (2y - 1)^2$

$= (2y - 1)(2y - 1)$

(iii) $x^2 - \frac{y^2}{100} = x^2 - \left(\frac{y}{10}\right)^2$

We know that $a^2 - b^2 = (a - b)(a + b)$

$x^2 - \left(\frac{y}{10}\right)^2 = \left(x - \frac{y}{10}\right)\left(x + \frac{y}{10}\right)$

4. Expand each of the following, using suitable Identities

(i) $(x + 2y + 4z)^2$

(ii) $(2x - y + z)^2$

(iii) $(-2x + 3y + 2z)^2$

(iv) $(3a - 7b - c)^2$

(v) $(-2x + 5y - 3z)^2$

(vi) $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$

Solution:

We know that

$(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz$

(i) $(x+2y+4z)^2 = x^2 + (2y)^2 + (4z)^2 + 2 \cdot x \cdot 2y + 2 \cdot 2y \cdot 4z + 2 \cdot x \cdot 4z$

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8xz$

(ii) $(2x-y+z)^2 = (2x)^2 + (-y)^2 + (z)^2 + 2 \cdot 2x(-y) + 2(-y)(z) + 2 \cdot 2x \cdot z$

$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$

(iii) $(-2x+3y+2z)^2 = (-2x)^2 + (3y)^2 + (2z)^2 + 2 \cdot (-2x) \cdot 3y + 2 \cdot 3y \cdot 2z + 2 \cdot (-2x) \cdot 2z$

$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8xz$

(iv) $(3a-7b-c)^2 = (3a)^2 + (-7b)^2 + (-c)^2 + 2 \cdot 3a \cdot (-7b) + 2 \cdot (-7b)(-c) + 2.(3a).(-c)$

$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ac.$

(v) $(-2x+5y-3z)^2 = (-2x)^2 + (5y)^2 + (-3z)^2 + 2 \cdot (-2x)(5y) + 2 \cdot (5y)(-3z) + 2 \cdot (-2x)(-3z)$

$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12xz$

(vi) $\left(\frac{1}{4}a - \frac{1}{2}b + 1\right)^2$

$$= \left(\frac{1}{4}a\right)^2 + \left(\frac{-1}{2}b\right)^2 + (1)^2 + 2 \cdot \left(\frac{1}{4}a\right)\left(\frac{-1}{2}b\right) + 2 \cdot \left(\frac{-1}{2}b\right)(1) + 2 \cdot \left(\frac{1}{4}a\right)(1)$$

$$= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{a}{2}.$$

5. Factorise

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

Solution:

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$= (2x)^2 + (3y)^2 + (-4z)^2 + 2 \cdot 2x \cdot 3y + 2 \cdot (3y)(-4z) + 2 \cdot (2x)(-4z)$

We know that $x^2 + y^2 + z^2 + 2xy + 2yz + 2xz = (x+y+z)^2$

$= (2x)^2 + (3y)^2 + (-4z)^2 + 2 \cdot 2x \cdot 3y + 2 \cdot (3y)(-4z) + 2 \cdot (2x)(-4z)$

$= (2x + 3y - 4z)^2$

$= (2x + 3y - 4z)(2x + 3y - 4z)$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

$= \left(-\sqrt{2}x\right)^2 + (y)^2 + \left(2\sqrt{2}z\right)^2 + 2 \cdot \left(-\sqrt{2}x\right)(y) + 2 \cdot (y)\left(2\sqrt{2}z\right) + 2 \cdot \left(-\sqrt{2}x\right)\left(2\sqrt{2}z\right)$

We know that $x^2 + y^2 + z^2 + 2xy + 2yz + 2xz = (x + y + z)^2$

$2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xy = \left(-\sqrt{2}x + y + 2\sqrt{2}z\right)^2$

$= \left(-\sqrt{2}x + y + 2\sqrt{2}z\right)\left(-\sqrt{2}x + y + 2\sqrt{2}z\right)$

6. Write the following cubes in expanded form:

(i) $(2x + 1)^3$

(ii) $(2a - 3b)^3$

(iii) $\left(\frac{3}{2}x + 1\right)^3$

(iv) $\left(x - \frac{2}{3}y\right)^3$

Solution:

(i) We know that $(a + b)^3 = a^3 + b^3 + 3ab\,(a + b)$

Given polynomial is $(2x + 1)^3$

$a = 2x, b = 1$

$(2x + 1)^3 = (2x)^3 + (1)^3 + 3 \cdot (2x) \cdot (1)(2x + 1)$

$= 8x^3 + 1 + 6x(2x + 1)$

$= 8x^3 + 1 + 12x^2 + 6x$

$= 8x^3 + 12x^2 + 6x + 1$

(ii) We know that $(a - b)^3 = a^3 - b^3 - 3ab\,(a - b)$

$(2a - 3b)^3 = (2a)^3 - (3b)^3 - 3(2a)(3b)(2a - 3b)$

$= 8a^3 - 27b^3 - 18ab(2a - 3b)$

$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$

$= 8a^3 - 36a^2b + 54ab^2 - 27b^3$

(iii) We know that $(a+b)^3 = a^3 + b^3 + 3ab\,(a+b)$

$$\left[\frac{3}{2}x+1\right]^3 = \left(\frac{3}{2}x\right)^3 + (1)^3 + 3\cdot\frac{3x}{2}\cdot 1\left(\frac{3}{2}x+1\right)$$

$$= \frac{27x^3}{8} + 1 + \frac{9x}{2}\left(\frac{3x}{2}+1\right)$$

$$= \frac{27}{8}x^3 + 1 + \frac{27x^2}{4} + \frac{9x}{2}$$

$$= \frac{27}{8}x^3 + \frac{27x^2}{4} + \frac{9x}{2} + 1$$

(iv) We know that $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

$$\left(x-\frac{2}{3}y\right)^3 = x^3 - \left(\frac{2y}{3}\right)^3 - 3\cdot x\cdot\frac{2}{3}y\left(x-\frac{2}{3}y\right)$$

$$= x^3 - \frac{8}{27}y^3 - 2xy\left(x-\frac{2}{3}y\right)$$

$$= x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$$

$$= x^3 - 2x^2y + \frac{4}{3}xy^2 - \frac{8x}{27}y^3$$

7. Evaluate the following using suitable identities

(i) $(99)^3$

(ii) $(102)^3$

(iii) $(998)^3$

Solution:

(i) $(99)^3 = (100-1)^3$

We know that $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

$a = 100, b = 1$

$(99)^3 = (100-1)^3 = (100)^3 - (1)^3 - 3(100)(1)(99)$

$= 1000000 - 1 - 29,700$

$= 9\ 70\ 299.$

(ii) $(102)^3 = (100+2)^3$

We know that $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$a = 100, b = 2$

$(102)^3 = (100 + 2)^3 = (100)^3 + (2)^3 + 3 \cdot 100 \cdot 2(100 + 2)$

$= 1000000 + 8 + 600 \times 102$

$= 1000008 + 61{,}200$

$= 1061208.$

(iii) $(998)^3 = (1000 - 2)^3$

We know that $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

Here $a = 1000, b = 2$

$(998)^3 = (1000 - 2)^3 = (1000)^3 - 8 - 3(1000)(2)(998)$

$= 1000000000 - 8 - 6000 \times 998$

$= 994011992$

8. Factorise each of the following:

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

(iii) $27 - 125a^3 - 135a + 225a^2$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Solution:

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

$= (2a)^3 + (b)^3 + 3 \cdot (2a)(b)(2a + b)$

We know that $a^3 + b^3 + 3ab(a + b) = (a + b)^3$

$8a^3 + b^3 + 3(2a) \cdot b(2a + b) = (2a + b)^3$

$= (2a + b)(2a + b)(2a + b)$

(ii) We know that $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$8a^3 - b^3 - 12a^2b + 6ab^2 = (2a)^3 - (b)^3 - 3 \cdot (2a)(b)(2a - b)$

$= (2a - b)^3$

$= (2a - b)(2a - b)(2a - b)$

(iii) we know that $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$27 - 125a^3 - 135a + 225a^2 = -(125a^3 - 27 - 225a^2 + 135a)$

$= -[(5a)^3 - (3)^3 - 3 \cdot (5a)(3)(5a - 3)]$

$= -[5a - 3]^3$

$= (3 - 5a)^3$

$= (3 - 5a)(3 - 5a)(3 - 5a)$

(iv) we know that $a^3 - b^3 - 3ab(a - b) = (a - b)^3$

$64a^3 - 27b^3 - 144a^2b + 108ab^2$

$= (4a)^3 - (3b)^3 - 3 \cdot (4a) \cdot (3b)(4a - 3b)$

$= (4a - 3b)^3$

$= (4a - 3b)(4a - 3b)(4a - 3b)$

(v) we know that $a^3 - b^3 - 3ab(a - b) = (a - b)^3$

$27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3 \cdot (3p) \cdot \frac{1}{6}\left(3p - \frac{1}{6}\right)$

$= \left(3p - \frac{1}{6}\right)^3$

$= \left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$

9. Verify

(i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

(ii) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Solution:

(i) We know that $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$\Rightarrow x^3 + y^3 = (x + y)^3 - 3xy(x + y)$

$= (x + y)((x + y)^2 - 3xy)$

We know that $(x + y)^2 = x^2 + y^2 + 2xy$

Now, $x^3 + y^3 = (x + y)(x^2 + y^2 + 2xy - 3xy)$

$= (x + y)(x^2 + y^2 - xy)$

Hence verified.

(ii) We know that

$(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$\Rightarrow x^3 - y^3 = (x - y)^3 + 3xy(x - y)$

$= (x - y)((x - y)^2 + 3xy)$

We know that $(x - y)^2 = x^2 + y^2 - 2xy$

$x^3 - y^3 = (x - y)(x^2 + y^2 - 2xy + 3xy)$

$= (x - y)(x^2 + y^2 + xy)$

Hence verified.

10. Factorise each of the following

(i) $27y^3 + 125z^3$

(ii) $64m^3 - 343n^3$

Solution:

(i) $27y^3 + 125z^3 = (3y)^3 + (5z)^3$

We know that $x^3 + y^3 = (x + y)(x^2 + y^2 - xy)$

$(3y)^3 + (5z)^3 = (3y + 5z)\left((3y)^2 + (5z)^2 - (3y)(5z)\right)$

$27y^3 + 125z^3 = (3y + 5z)(9y^2 + 25z^2 - 15yz)$

(ii) $64m^3 - 343n^3 = (4m)^3 - (7n)^3$

We know that $(x)^3 - (y)^3 = (x - y)(x^2 + xy + y^2)$

$(4m)^3 - (7n)^3 = (4m - 7n)((4m)^2 + (4m)7n + (7n)^2)$

$= (4m - 7n)(16m^2 + 28mn + 49n^2)$

11. Factorise $27x^2 + y^3 + z^3 - 9xyz$

Solution:

$27x^2 + y^3 + z^3 - 9xyz$

$= (3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z)$

We know that

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$

Now, $27x^3 + y^3 + z^3 - 9xyz = (3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z)$

$= (3x + y + z)((3x)^2 + (y)^2 + (z)^2 - 3xy - yz - 3xz)$

$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3xz)$

12. Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (x - z)^2]$

Solution:

We know that

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$

$= (x + y + z)\frac{1}{2}(2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2xz)$

$= (x + y + z)\frac{1}{2}(x^2 + y^2 - 2xy + y^2 + z^2 - 2yz + x^2 + z^2 - 2xz)$

We know that $a^2 + b^2 - 2ab = (a - b)^2$

$= \frac{1}{2}(x + y + z)((x + y)^2 + (y - z)^2 + (x - z)^2)$

$x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)((x - y)^2 + (y - z)^2 + (x - z)^2)$

Hence verified.

13. If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$

Solution:

We know that,

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$

Given that $x + y + z = 0$

$x^3 + y^3 + z^3 - 3xyz = (0)(x^2 + y^2 + z^2 - xy - yz - xz)$

$\Rightarrow x^3 + y^3 + z^3 - 3xyz = 0$

$\Rightarrow x^3 + y^3 + z^3 = 3xyz$

Hence proved

14. Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3 + (7)^3 + (5)^3$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Solution:

(i) $(-12)^3 + (7)^3 + (5)^3$

Let $x = -12, y = 7, z = 5$

$x + y + z = -12 + 7 + 5 = 0$

We know that $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$

But here $x + y + z = 0$

Hence, $x^3 + y^3 + z^3 = 3xyz$

Therefore, $(-12)^3 + (7)^3 + (5)^3 = 3(12)(7)(5)$

$= -1260$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Let $x = -28, y = -15, z = -13$

$x + y + z = 28 - 15 - 13$

$= 0$

We know that $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$

But here $x + y + z = 0$

Hence, $x^3 + y^3 + z^3 = 3xyz$

Therefore, $(28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13)$

$= 16380$

15. Give possible expressions for the length and breadth of each of the following rectangles, in which the areas are given

(i) Area: $25a^3 - 35a + 12$

(ii) Area: $35y^3 - 13y - 12$

Solution:

(i) Given area $= 25a^2 - 35a + 12$

$= 25a^2 - 15a - 20a + 12$

$= 5a(5a - 3) - 4(5a - 3)$

$= (5a - 3)(5a - 4)$

We know that area = length × breadth

So possible expression for breadth $= 5a - 3$

possible expression for breadth $= 5a - 4$.

(ii) Given area $= 35^2 + 13y^2 - 12$

$= 35y^2 + 28y - 15y - 12$

$= 7y(5y + 4) - 3(5y + 4)$

$= (5y + 4)(7y - 3)$

We know that area = length × breadth

So possible expression for breadth $= 5y + 4$

possible expression for breadth $= 7y - 3$.

16. What are the possible expressions for the dimension of the cuboids whose volume are given below?

(i) Volume= $3x^2 - 12x$

(ii) Volume= $12ky^2 + 8ky - 20k$

Solution:

(i) Given volume $= 3x^2 - 12x$

$= 3(x^2 - 4x)$

$= 3x(x - 4)$

We know that Volume of cuboid = length × breadth × height

Possible value of length of cuboid= 3

Possible expression for breadth $= x$

Possible expression for height $= x - 4$.

(ii) Given Volume $= 12ky^2 + 8ky - 20k$

$= 4k(3y^2 + 2y - 5)$

$= 4k(3y^2 + 5y - 3y - 5)$

$= 4k(y(3y + 5) - 1(3y + 5)$

$= 4k(3y + 5)(y - 1)$

Possible value of length of cuboid= $4k$

Possible expression for breadth $= 3y + 5$

Possible expression for breadth $= y - 1$.