CBSE NCERT Solutions for Class 11 Physics Chapter 8

Back of Chapter Questions

8.1. (a) You can shield a charge from electrical forces by putting it inside a hollow conductor. Can you shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means?

(b) An astronaut inside a small space ship orbiting around the earth cannot detect gravity. If the space station orbiting around the earth has a large size, can he hope to detect gravity?

(c) If you compare the gravitational force on the earth due to the sun to that due to the moon, you would find that the Sun’s pull is greater than the moon’s pull.

(You can check this yourself using the data available in the succeeding exercises). However, the tidal effect of the moon’s pull is greater than the tidal effect of sun. Why?

Solution:

(a) No. There is no way to screen the gravitational influence of matter on neighbouring objects. This is because, unlike electrical forces, the gravitational force is independent of the material medium's nature. It is also independent of the status of the object.

(b) Yes. If the size of the space station is sufficiently large, then the astronaut will detect the change in Earth’s gravity (g).

(c) The tidal effect relies inversely upon the cube of the distance while the gravitational force relies inversely on the square of the distance. Since the distance between the Moon and the Earth is less than the distance between the Sun and the Earth, the tidal effect Moon's pull's is greater than the tidal effect Sun's pull.

8.2. Choose the correct alternative:

(a) Acceleration due to gravity increases/decreases with increasing altitude.

(b) Acceleration due to gravity increases/decreases with increasing depth. (assume the earth to be a sphere of uniform density).

(c) Acceleration due to gravity is independent of mass of the earth/mass of the body.

(d) The formula $-GMm(1/r_2 - 1/r_1)$ is more/less accurate than the formula $mg(r_2 - r_1)$ for the difference of potential energy between two points $r_2$ and $r_1$ distance away from the centre of the earth.
Solution:

Explanation:

Decreases

The relation which gives acceleration due to gravity at depth $h$ is given by:

$$(g_h) = \left(1 - \frac{2h}{R_e}\right)g$$

Where,

$R_e =$ Radius of the Earth

$g =$ Acceleration due to gravity on the surface of the Earth

It is clear from the given relation that with an increase in height, acceleration due to gravity decreases.

Decreases

Acceleration due to gravity at depth $d$ is given by the relation:

$$(g_d) = \left(1 - \frac{d}{R_e}\right)g$$

It is clear from the given relation that with an increase in depth, acceleration due to gravity decreases.

Mass of the body.

Acceleration due to the gravity of the body of mass $m$ is given by the relation:

$$g = \frac{GM}{R^2}$$

Where,

$G =$ Universal gravitational constant

$M =$ Mass of the Earth

$R =$ Radius of the Earth

Therefore, acceleration due to gravity can be inferred independent of the body's mass.

More

The gravitational potential energy of two points $r_2$ and $r_1$ distance away from the centre of the Earth is respectively given by:

$$V(r_1) = -\frac{GmM}{r_1}$$
Gravitation

\[ V(r_2) = -\frac{GmM}{r_2} \]

\[ \therefore \text{The difference in potential energy}, \ V = V(r_2) - V(r_1) = -GmM \left( \frac{1}{r_2} - \frac{1}{r_1} \right) \]

Hence, this formula is more precise than the formula \( m g (r_2 - r_1) \).

### 8.3.

Suppose there existed a planet that went around the sun twice as fast as the earth. What would be its orbital size as compared to that of the earth?

**Solution:**

Earth's time to complete a revolution around the Sun,

\[ T_e = 1 \text{ year} \]

The Earth's orbital radius in its orbit, \( R_e = 1 \text{ AU} \)

The Time taken by the planet to complete one revolution around the Sun,

\[ T_p = \frac{1}{2} T_e = \frac{1}{2} \text{ year} \]

The orbital radius of the planet \( = R_p \)

From Kepler’s third law of planetary motion:

\[ \left( \frac{R_p}{R_e} \right)^3 = \left( \frac{T_p}{T_e} \right)^2 \]

\[ \frac{R_p}{R_e} = \left( \frac{T_p}{T_e} \right)^{\frac{2}{3}} = \left( \frac{\frac{1}{2}}{1} \right)^{\frac{2}{3}} = (0.5)^{\frac{2}{3}} = 0.63 \]

Therefore, the planet's orbital radius will be 0.63 times lower than the Earth's.

### 8.4.

\( I_0 \), one of the satellites of Jupiter, has an orbital period of 1.769 days and the radius of the orbit is \( 4.22 \times 10^8 \text{ m} \). Show that the mass of Jupiter is about one-thousandth that of the sun.

**Solution:**

Orbital period of \( I_0, T_{I_0} = 1.769 \times 24 \times 60 \times 60 \text{ s} \)

Orbital radius of \( I_0, R_{I_0} = 4.22 \times 10^8 \text{ m} \)

Satellite \( I_0 \) is revolving around the Jupiter

The Mass of the latter is given by the relation:

\[ M_j = \frac{4\pi^2 R_{I_0}^3}{GT_{I_0}^2} \text{ ......(i)} \]
Where,

\[ M_J = \text{Mass of Jupiter} \]

\[ G = \text{Universal gravitational constant} \]

The orbital period of the Earth,

\[ T_e = 365.25 \text{ days} = 365.25 \times 24 \times 60 \times 60 \text{ s} \]

The orbital radius of the Earth,

\[ R_e = 1 \text{ AU} = 1.496 \times 10^{11} \text{ m} \]

Mass of Sun is given as:

\[ M_s = \frac{4\pi^2 R_e^3}{G T_e^2} \tag{ii} \]

\[ \therefore \frac{M_s}{M_J} = \frac{4\pi^2 R_e^3}{G T_e^2} \times \frac{G T_{10}^2}{4\pi^2 R_{10}^3} \times \frac{T_{10}^2}{T_e^2} \]

\[ = \left( \frac{1.769 \times 24 \times 60 \times 60}{365.25 \times 24 \times 60 \times 60} \right)^2 \times \left( \frac{1.496 \times 10^{11}}{4.22 \times 10^8} \right)^3 \]

\[ = 1045.04 \]

\[ \therefore \frac{M_s}{M_J} \sim 1000 \]

\[ M_s \sim 1000 \times M_J \]

Therefore, it can be inferred that Jupiter's mass is about one-thousandth of the Sun's mass.

8.5. Let us assume that our galaxy consists of \(2.5 \times 10^{11}\) stars each of one solar mass. How long will a star at a distance of 50,000 ly from the galactic centre take to complete one revolution? Take the diameter of the Milky Way to be \(10^5\) ly.

**Solution:**

The Mass of our galaxy Milky Way, \( M = 2.5 \times 10^{11}\) solar mass

Solar mass = Mass of Sun = \(2.0 \times 10^{36}\) kg

Mass of our galaxy, \( M = 2.5 \times 10^{11} \times 2 \times 10^{36} = 5 \times 10^{41}\) kg

The diameter of Milky Way, \( d = 10^5\) ly

The radius of Milky Way, \( r = 5 \times 10^4\) ly

\(1\) ly = \(9.46 \times 10^{15}\) m

\[ r = 5 \times 10^4 \times 9.46 \times 10^{15} \]
\[ r = 4.73 \times 10^{20} \text{ m} \]

Since a star revolves around the galactic centre of the Milky Way, its time period is given by the relation:

\[
T = \left( \frac{4\pi^2 r^3}{GM} \right)^{\frac{1}{2}}
\]

\[
= \left( \frac{4 \times (3.14)^2 \times (4.73)^3 \times 10^{60}}{6.67 \times 10^{-11} \times 5 \times 10^{41}} \right)^{\frac{1}{2}}
\]

\[
= \left( 125.27 \times 10^{30} \right)^{\frac{1}{2}} = 1.12 \times 10^{16} \text{ s}
\]

1 year = 365 \times 324 \times 60 \times 60 \text{ s}

\[
1 \text{ s} = \frac{1}{365 \times 24 \times 60 \times 60} \text{ years}
\]

\[
\therefore 1.12 \times 10^{16} \text{ s} = \frac{1.12 \times 10^{16}}{365 \times 24 \times 60 \times 60} \text{ years}
\]

\[
= 3.55 \times 10^{8} \text{ years}
\]

### 8.6. Choose the correct alternative:

(a) If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic/potential energy.

(b) The energy required to launch an orbiting satellite out of earth’s gravitational influence is more/less than the energy required to project a stationary object at the same height (as the satellite) out of earth’s influence.

### Solution:

(a) Kinetic energy

A satellite's total mechanical energy is the sum of its kinetic energy (always positive) and potential energy (may be negative). At infinity, the satellite's gravitational potential is zero. The total energy of the satellite is negative because the Earth-satellite system is a bound system.

Thus, an orbiting satellite's total energy at infinity is equivalent to the negative of its kinetic energy.

(b) Less

An orbiting satellite gains a certain amount of energy that allows it to revolve around the Earth. This energy is provided by its orbit. To move out of the influence of the Earth's gravitational field, it needs relatively
less energy than a stationary object on the Earth’s surface that initially contains no energy.

8.7. Does the escape speed of a body from the earth depend on
(a) the mass of the body
(b) the location from where it is projected
(c) the direction of projection
(d) the height of the location from where the body is launched?

Solution:
(a) No
(b) No
(c) No
(d) Yes

The escape velocity of a body from the Earth is given by the relation:

\[ v_{esc} = \sqrt{2gR} \quad \ldots \ldots \text{(i)} \]

\[ g \] = Acceleration due to gravity
\[ R \] = Radius of the Earth

It is clear from equation (i) that escape velocity \( v_{esc} \) is independent of the mass of the body and the direction of its projection. However, at the point where the body is launched, it depends on the gravitational potential. Because this potential depends marginally on the height of the point, escape velocity also marginally depends on these factors.

8.8. A comet orbits the Sun in a highly elliptical orbit. Does the comet have a constant
(a) linear speed
(b) angular speed
(c) angular momentum
(d) kinetic energy
(e) potential energy
(f) total energy throughout its orbit?

Neglect any mass loss of the comet when it comes very close to the Sun.

Solution:
(a) No
At all points of a comet's orbit moving in a highly elliptical orbit around the Sun, angular momentum and total energy are constant. Its linear speed, angular speed, kinetic and potential energy vary in the orbit from point to point.

8.9. Which of the following symptoms is likely to afflict an astronaut in space

(a) swollen feet
(b) swollen face
(c) headache
(d) orientational problem?

Solution: (b), (c), and (d)

(a) Due to gravitational pull, the legs hold the entire mass of a body in standing position. In space, due to the absence of gravity, an astronaut feels weightlessness. Hence, an astronaut's swollen feet do not affect him/her in space.

(b) In general, a swollen face is caused by apparent weightlessness in space. Sense organs like eyes, ears, nose, and mouth constitute a person's face. This symptom can affect an astronaut in space.

(c) Due to mental strain, headaches are triggered. It may influence an astronaut's work in space.

(d) Space has different orientations. Therefore, the orientational problem can affect an astronaut in space.

8.10. Choose the correct answer from among the given ones:

The gravitational intensity at the centre of a hemispherical shell of uniform mass density has the direction indicated by the arrow (see Figure) (i) a, (ii) b, (iii) c, (iv) O.
Solution: (iii)

The gravitational potential \( V \) is constant at all points in a spherical shell. Hence, the gravitational potential gradient \( \frac{dV}{dr} \) is zero everywhere inside the spherical shell. The gravitational potential gradient is equal to the negative of gravitational intensity. Hence, the intensity is also zero at all points inside the spherical shell. This indicates that gravitational forces acting at a point in a spherical shell are symmetric.

If the upper half of a spherical shell is cut out (as shown in the figure in question), then the net gravitational force acting on a particle in the centre O will be in the downward direction.

Because at that stage, the gravitational intensity is described as the gravitational force per unit mass, it will also act in the downward direction. The gravitational intensity of the given hemispheric shell at centre O, therefore, has the direction indicated by arrow c.

8.11. Choose the correct answer from among the given ones:

For the problem 8.10, the direction of the gravitational intensity at an arbitrary point P is indicated by the arrow (i) d, (ii) e, (iii) f, (iv) g.

Solution: (ii)

The gravitational potential \( V \) is constant at all points in a spherical shell. Hence, the gravitational potential gradient \( \frac{dV}{dr} \) is zero everywhere inside the spherical shell. The gravitational potential gradient is equal to the negative of gravitational intensity.
intensity. Therefore, the intensity within the spherical shell is also zero at all points. This indicates symmetric gravitational forces at a point in a spherical shell.

If the upper half of a spherical shell is cut out (as shown in the figure), then the net gravitational force at an arbitrary point P will be in the downward direction.

Because at that point, the gravitational intensity is defined as the gravitational force per unit mass, it will also act in the downward direction. Consequently, the gravitational intensity of the hemispheric shell at an arbitrary point P has the direction indicated by arrow e.

8.12. A rocket is fired from the earth towards the sun. At what distance from the earth’s centre is the gravitational force on the rocket zero?

Mass of the sun = \(2 \times 10^{30}\) kg, mass of the earth = \(6 \times 10^{24}\) kg. Neglect the effect of other planets etc. (orbital radius = \(1.5 \times 10^{11}\) m).

**Solution:**

Mass of the Sun, \(M_s = 2 \times 10^{30}\) kg

Mass of the Earth, \(M_e = 6 \times 10^{24}\) kg

Orbital radius, \(r = 1.5 \times 10^{11}\) m

Mass of the rocket = \(m\)
Let x be the distance from the centre of the Earth where the gravitational force acting on satellite P becomes zero.

From Newton’s gravitational law, under the influence of the Sun and Earth, we can equate gravitational forces acting on satellite P:

\[
\frac{G m M_s}{(r - x)^2} = \frac{G m M_e}{x^2}
\]

\[
\left(\frac{r - x}{x}\right)^2 = \frac{M_s}{M_e}
\]

\[
r - x = \left(\frac{2 \times 10^{30}}{60 \times 10^{24}}\right)^{\frac{1}{2}} = 577.35
\]

\[
1.5 \times 10^{11} - x = 577.35 x
\]

\[
578.35x = 1.5 \times 10^{11}
\]

\[
x = \frac{1.5 \times 10^{11}}{578.35} = 2.59 \times 10^8 \text{ m}
\]

8.13. How will you ‘weigh the sun’, that is estimate its mass? The mean orbital radius of the earth around the sun is $1.5 \times 10^8 \text{ km}$.

**Solution:**

The orbital radius of the Earth around the Sun, $r = 1.5 \times 10^{11} \text{ m}$

Time taken by the Earth to complete a single revolution around the Sun,

\[
T = 1 \text{ year} = 365.25 \text{ days}
\]

\[
= 365.25 \times 24 \times 60 \times 60 \text{ s}
\]

Universal gravitational constant, \( G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2} \)

Thus, the mass of the Sun can be calculated using the relation,

\[
M = \frac{4 \pi^2 r^3}{GT^2}
\]

\[
= \frac{4 \times (3.14)^2 \times (1.5 \times 10^{11})^3}{6.67 \times (365.25 \times 24 \times 60 \times 60)^2}
\]

\[
= \frac{133.24 \times 10}{6.64 \times 10^{4}} = 2.0 \times 10^{30} \text{ kg}
\]

Hence, the mass of the Sun is $2 \times 10^{30} \text{ kg}$

8.14. A Saturn year is 29.5 times the earth year. How far is the Saturn from the sun if the earth is $1.50 \times 10^8 \text{ km}$ away from the sun?
Solution:

Given:
The distance of the Earth from the Sun, \( r_e = 1.5 \times 10^8 \text{ km} = 1.5 \times 10^{11} \text{ m} \)
The time period of the Earth = \( T_e \)
The time period of Saturn, \( T_s = 29.5 \ T_e \)
The distance of Saturn from the Sun = \( r_s \)

From Kepler’s third law of planetary motion:

\[
T = \left( \frac{4\pi^2 r^3}{GM} \right)^{\frac{1}{2}}
\]

For Saturn and Sun, we can write

\[
\frac{r_s^3}{r_e^3} = \left( \frac{T_s}{T_e} \right)^2
\]

\[
r_s = r_e \left( \frac{T_s}{T_e} \right)^{\frac{2}{3}}
\]

\[
= 1.5 \times 10^{11} \left( \frac{29.5T_e}{T_e} \right)^{\frac{2}{3}}
\]

\[
= 1.5 \times 10^{11} \left( 29.5 \right)^{\frac{2}{3}}
\]

\[
= 1.5 \times 10^{11} \times 9.55
\]

\[
= 14.32 \times 10^{11} \text{ m}
\]

Hence, the distance between Saturn and the Sun is \( 1.43 \times 10^{12} \text{ m} \).

8.15. A body weighs 63 N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth?

Solution:

Weight of the body, \( W = 63 \text{ N} \)

The relation for acceleration due to gravity at height \( h \) from the Earth’s surface:

\[
g' = \frac{g}{\left( \frac{1 + h}{R_e} \right)^2}
\]

Where,

\( g \) = Acceleration due to gravity on the Earth’s surface
Gravitation

\( R_e = \) Radius of the Earth

For \( h = \frac{R_e}{2} \)

\[
g' = \frac{g}{\left(1 + \frac{R_e}{2} \times \frac{R_e}{R_e}\right)^2} = \frac{g}{\left(1 + \frac{1}{2}\right)^2} = \frac{4}{9}g
\]

The weight of a body of mass \( m \) at height \( h \) is given as:

\[
W' = mg'
\]

\[
= m \times \frac{4}{9}g = \frac{4}{9} \times mg
\]

\[
= \frac{4}{9}W
\]

\[
= \frac{4}{9} \times 63 = 28 \text{ N}
\]

8.16. Assuming the earth to be a sphere of uniform mass density, how much would a body weigh halfway down to the centre of the earth if it weighed 250 N on the surface?

Solution:

Weight of a body of mass \( m \) at the Earth’s surface, \( W = mg = 250 \text{ N} \)

Body of mass \( m \) is located at depth, \( d = \frac{1}{2}R_e \)

Where,

\( R_e = \) Radius of the Earth

Acceleration due to gravity at depth \( g(d) \) is given by the relation:

\[
g' = \left(1 - \frac{d}{R_e}\right)g
\]

\[
= \left(1 - \frac{R_e}{2 \times R_e}\right)g = \frac{1}{2}g
\]

Weight of the body at depth \( d \),

\[
W' = mg'
\]

\[
= m \times \frac{1}{2}g = \frac{1}{2}mg = \frac{1}{2}W
\]

\[
= \frac{1}{2} \times 250 = 125 \text{ N}
\]
8.17. A rocket is fired vertically with a speed of 5 km s\(^{-1}\) from the earth’s surface. How far from the earth does the rocket go before returning to the earth? Mass of the earth = \(6.0 \times 10^{24}\) kg; mean radius of the earth = \(6.4 \times 10^{6}\) m; \(G = 6.67 \times 10^{-11}\) N m\(^2\) kg\(^{-2}\).

Solution:

The velocity of the rocket, \(v = 5\) km/s = \(5 \times 10^3\) m/s

Mass of the Earth, \(M_e = 6.0 \times 10^{24}\) kg

The radius of the Earth, \(R_e = 6.4 \times 10^6\) m

The height reached by rocket mass, \(m = h\)

At the surface of the Earth,

The total energy of the rocket = Kinetic energy + Potential energy

\[
\frac{1}{2}mv^2 + \left( -\frac{GM_e m}{R_e} \right)
\]

At highest point \(h\),

\(v = 0\)

And, Potential energy = \(-\frac{GM_e m}{R_e + h}\)

The total energy of the rocket = 0 + \(\left( -\frac{GM_e m}{R_e + h} \right) = \frac{GM_e m}{R_e + h}\)

From the law of conservation of energy:

The total energy of the rocket at the Earth’s surface = Total energy at height \(h\)

\[
\frac{1}{2}mv^2 + \left( -\frac{GM_e m}{R_e} \right) = \frac{GM_e m}{R_e + h}
\]

\[
\frac{1}{2}v^2 = GM_e \left( \frac{1}{R_e} - \frac{1}{R_e + h} \right)
\]

\[
\frac{1}{2}v^2 = GM_e \left( \frac{R_e + h - R_e}{R_e (R_e + h)} \right)
\]

\[
\frac{1}{2}v^2 = \frac{GM_e h}{R_e (R_e + h)} \times \frac{R_e}{R_e}
\]

\[
\frac{1}{2} \times v^2 = \frac{gR_e h}{R_e + h}^2
\]

Where \(g = \frac{GM}{R_e^2} = 9.8\) m/s\(^2\) (Acceleration due to gravity on the Earth’s surface)

\(\therefore \ 2gR_e h = \frac{1}{2}mv^2(R_e + h)\)
\[ v^2 R_e = h = (2gR_e - v^2) \]

\[ h = \frac{R_e v^2}{2gR_e - v^2} \]

\[ = \frac{6.4 \times 10^6 \times (5 \times 10^3)^2}{2 \times 9.8 \times 6.4 \times 10^6 - (5 \times 10^3)^2} \]

\[ h = \frac{6.4 \times 25 \times 10^{12}}{100.44 \times 10^6} = 1.6 \times 10^6 \text{m} \]

The height of the rocket in relation to the Earth's center

\[ = R_e + h \]

\[ = 6.4 \times 10^6 + 1.6 \times 10^6 \]

\[ = 8.0 \times 10^6 \text{m} \]

8.18. The escape speed of a projectile on the earth’s surface is 11.2 km s\(^{-1}\). A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets.

**Solution:**

The escape velocity of a projectile from the Earth, \( v_{esc} = 11.2 \text{ km/s} \)

Projection velocity of the projectile, \( v_p = 3 \ v_{esc} \)

Mass of the projectile = \( m \)

The velocity of the projectile far away from the Earth = \( v_f \)

The total energy of the projectile on the Earth = \( \frac{1}{2} mv_p^2 - \frac{1}{2} mv_{esc}^2 \)

The gravitational potential energy of the projectile far away from the Earth is zero.

The total energy of the projectile far away from the Earth = \( \frac{1}{2} mv_f^2 \)

From the law of conservation of energy,

\[ \frac{1}{2} mv_p^2 - \frac{1}{2} mv_{esc}^2 = \frac{1}{2} mv_f^2 \]

\[ v_f = \sqrt{v_p^2 - v_{esc}^2} \]

\[ = \sqrt{(3v_{esc})^2 - (v_{esc})^2} \]

\[ = \sqrt{8} \ v_{esc} \]

\[ = \sqrt{8} \times 11.2 = 31.68 \text{ km/s} \]
8.19. A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expended to rocket the satellite out of the earth’s gravitational influence? Mass of the satellite = 200 kg; mass of the earth = $6.0 \times 10^{24}$ kg; radius of the earth = $6.4 \times 10^6$ m; $G = 6.67 \times 10^{-11}$ N m$^2$ kg$^{-2}$.

**Solution:**

Mass of the Earth, $M = 6.0 \times 10^{24}$ kg

Mass of the satellite, $m = 200$ kg

The radius of the Earth, $Re = 6.4 \times 10^6$ m

Universal gravitational constant, $G = 6.67 \times 10^{-11}$ Nm$^2$ kg$^{-2}$

Height of the satellite, $h = 400$ km = $4 \times 10^6$ m = $0.4 \times 10^6$ m

The total energy of the satellite at height $h = \frac{1}{2} mv^2 + \left(-\frac{GMm}{Re+h}\right)$

Orbital velocity of the satellite, $v = \sqrt{\frac{GM}{Re+h}}$

The total energy of height, $h = \frac{1}{2} m \left(\frac{GM}{Re+h}\right) - \frac{GMm}{Re+h} = -\frac{1}{2} \left(\frac{GM}{Re+h}\right)$

The negative sign indicates that the satellite is bound to the Earth. This is called bound energy of the satellite.

The energy required to send the satellite out of its orbit = $- (\text{Bound energy})$

\[
= \frac{1}{2} \frac{GM_{e}m}{(Re+h)}
= \frac{1}{2} \times \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 200}{(6.4 \times 10^6 + 0.4 \times 10^6)}
= \frac{1}{2} \times \frac{6.67 \times 6 \times 2 \times 10}{6.8 \times 10^6} = 5.9 \times 10^9 J
\]

8.20. Two stars each of one solar mass ($= 2 \times 10^{30}$ kg) are approaching each other for a head-on collision. When they are a distance 109 km, their speeds are negligible. What is the speed with which they collide? The radius of each star is 104 km. Assume the stars to remain undistorted until they collide.

(Use the known value of $G$).

**Solution:**

Mass of each star, $M = 2 \times 10^{30}$ kg

The radius of each star, $R = 10^4$ km = $10^7$ m

Distance between the stars, $r = 10^9$ km = $10^{12}$ m
For negligible speeds, \( v = 0 \) total energy of two stars separated at distance \( r \)

\[
\begin{align*}
\text{Total Energy} &= -\frac{GMM}{r} + \frac{1}{2} m v^2 \\
&= -\frac{GMM}{r} + 0 \quad \text{(i)}
\end{align*}
\]

Now, consider the case when the stars are about to collide:

The velocity of the stars = \( v \)

Distance between the centres of the stars = \( 2R \)

The total kinetic energy of both stars = \( \frac{1}{2} M v^2 + \frac{1}{2} M v^2 = M v^2 \)

The total potential energy of both stars = \( \frac{-GMM}{2R} \)

The total energy of the two stars = \( M v^2 - \frac{GMM}{2R} \) \quad \text{(ii)}

Using the law of conservation of energy:

\[
\begin{align*}
M v^2 - \frac{GMM}{2R} &= -\frac{GMM}{r} \\
v^2 &= \frac{G M}{r} + \frac{G M}{2R} = GM \left( \frac{1}{r} + \frac{1}{2R} \right) \\
&= 6.67 \times 10^{-11} \times 2 \times 10^3 \left[ -\frac{1}{10^{12}} + \frac{1}{2 \times 10^7} \right] \\
&= 13.34 \times 10^{19} \left[ -10^{-12} + 5 \times 10^{-8} \right] \\
&\approx 13.34 \times 10^{19} \times 5 \times 10^{-8} \\
&\approx 6.67 \times 10^{12}
\end{align*}
\]

\[
v = \sqrt{6.67 \times 10^{12}} = 2.58 \times 10^6 \text{ m/s}
\]

**8.21.** Two heavy spheres each of mass 100 kg and radius 0.10 m are placed 1.0 m apart on a horizontal table. What is the gravitational force and potential at the midpoint of the line joining the centres of the spheres? Is an object placed at that point in equilibrium? If so, is the equilibrium stable or unstable?

**Solution:**

0;

\(-2.7 \times 10^{-8}\) J/kg;

Yes;

Unstable

Explanation:
The situation is represented in the given figure:

![Diagram of two spheres](image)

Mass of each sphere, \( M = 100 \) kg
The separation between the spheres, \( r = 1 \) m

\( X \) is the midpoint between the spheres. Gravitational force at point \( X \) will be zero. This is because the gravitational force exerted by each sphere will act in opposite directions.

Gravitational potential at point \( X \):

\[
\begin{align*}
&= -\frac{GM}{\left(\frac{r}{2}\right)} - \frac{GM}{\left(\frac{r}{2}\right)} = -4 \frac{GM}{r} \\
&= \frac{4 \times 6.67 \times 10^{-11} \times 100}{1} \\
&= -2.67 \times 10^{-8} \text{ J/kg}
\end{align*}
\]

Any object placed at point \( X \) will be in an equilibrium state, but the equilibrium is unstable. This is because any change in the position of the object will change the effective force in that direction.

8.22. As you have learnt in the text, a geostationary satellite orbits the earth at a height of nearly 36,000 km from the surface of the earth. What is the potential due to earth’s gravity at the site of this satellite? (Take the potential energy at infinity to be zero).

Mass of the earth = \( 6.0 \times 10^{24} \) kg,
radius = 6400 km.

**Solution:**

Mass of the Earth, \( M = 6.0 \times 10^{24} \) kg

The radius of the Earth, \( R = 6400 \) km = \( 6.4 \times 10^6 \) m

A geostationary satellite’s height from the earth’s surface,

\( h = 36000 \) km = \( 3.6 \times 10^7 \) m
The gravitational potential energy due to Earth’s gravity at height \( h \),

\[
\frac{-GM}{(R + h)}
\]

\[
= \frac{-6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{3.6 \times 10^7 + 0.64 \times 10^7}
\]

\[
= \frac{-6.67 \times 6}{4.24} \times 10^{13-7}
\]

\[
= -9.4 \times 10^6 \text{ J/kg}
\]

8.23. A star 2.5 times the mass of the sun and collapsed to a size of 12 km rotates with a speed of 1.2 rev. per second. (Extremely compact stars of this kind are known as neutron stars. Certain stellar objects called pulsars belong to this category). Will an object placed on its equator remain stuck to its surface due to gravity? (Mass of the sun = \( 2 \times 10^{30} \) kg).

**Solution:**

Yes

A body gets stuck to the surface of a star if the inward gravitational force is greater than the outward centrifugal force caused by the rotation of the star.

Gravitational force, \( f_g = \frac{GMm}{R^2} \)

Where,

\( M \) = Mass of the star = \( 2.5 \times 2 \times 10^{30} = 5 \times 10^{30} \) kg

\( m \) = Mass of the body

\( R \) = Radius of the star = 12 km = \( 1.2 \times 10^4 \) m

\[
\therefore f_g = \frac{6.67 \times 10^{-11} \times 5 \times 10^{30} \times m}{(1.2 \times 10^4)^2} = 2.31 \times 10^{11} mN
\]

Centrifugal force, \( f_c = mr\omega^2 \)

where,

\( \omega \) = Angular speed = \( 2\pi v \)

\( v \) = Angular frequency = 1.2 rev s\(^{-1}\)

\[
f_c = mR(2\pi v)^2
\]

\[
= m \times (1.2 \times 10^4) \times 4 \times (3.14)^2 \times (1.2)^2 = 1.7 \times 10^5 mN
\]

Since \( f_g > f_c \), the body will remain stuck to the surface of the star.
8.24. A spaceship is stationed on Mars. How much energy must be expended on the spaceship to launch it out of the solar system? Mass of the spaceship = 1000 kg; mass of the Sun = $2 \times 10^{30}$ kg; mass of Mars = $6.4 \times 10^{23}$ kg; radius of Mars = 3395 km; radius of the orbit of Mars = $2.28 \times 10^{8}$ km; $G = 6.67 \times 10^{-11}$ m$^2$kg$^{-2}$.

**Solution:**

Mass of the spaceship, $m_s = 1000 \text{ kg}$

Mass of the Sun, $M = 2 \times 10^{30} \text{ kg}$

Mass of Mars, $m_m = 6.4 \times 10^{23} \text{ kg}$

The orbital radius of Mars, $R = 2.28 \times 10^{8} \text{ km} = 2.28 \times 10^{11} \text{ m}$

The radius of Mars, $r = 3395 \text{ km} = 3.395 \times 10^{6} \text{ m}$

Universal gravitational constant, $G = 6.67 \times 10^{-11} \text{ m}^2\text{kg}^{-2}$

The potential energy of the spaceship due to the gravitational attraction of the Sun

$= -\frac{GMm_s}{R}$

The potential energy of the spaceship due to the gravitational attraction of Mars

$= -\frac{GMm_m}{r}$

Since the spaceship is stationed on Mars, its velocity and therefore its kinetic energy will be zero.

The total energy of the spaceship

$= -\frac{GMm_s}{R} - \frac{GMs m_m}{r}$

$= -Gm_s \left( \frac{M}{R} + \frac{m_m}{r} \right)$

The negative sign indicates that the system is in bound state.

The energy required for launching the spaceship out of the solar system

$= - \text{(Total energy of the spaceship)}$

$= Gm_s \left( \frac{M}{R} + \frac{m_m}{r} \right)$

$= 6.67 \times 10^{-11} \times 10^3 \times \left( \frac{2 \times 10^{30}}{2.28 \times 10^{11}} + \frac{6.4 \times 10^{23}}{3.395 \times 10^6} \right)$

$= 6.67 \times 10^{-8} \times (87.72 \times 10^{17} + 1.88 \times 10^{17})$

$= 6.67 \times 10^{-8} \times 9.50 \times 10^{17}$

$= 596.97 \times 10^9$
8.25. A rocket is fired ‘vertically’ from the surface of Mars with a speed of 2 km s\(^{-1}\). If 20\% of its initial energy is lost due to Martian atmospheric resistance, how far will the rocket go from the surface of Mars before returning to it? Mass of mars = 6.4 \times 10^{23} \text{ kg}; radius of mars = 3395 \text{ km}; G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}.

**Solution:**

Initial velocity of the rocket, v = 2 km/s = 2 \times 10^3 \text{ m/s}

Mass of Mars, M = 6.4 \times 10^{23} \text{ kg}

The radius of Mars, R = 3395 \text{ km} = 3.395 \times 10^6 \text{ m}

Universal gravitational constant, G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}

Mass of the rocket = m

The initial kinetic energy of the rocket = \frac{1}{2}mv^2

The initial potential energy of the rocket = \frac{-GMm}{R}

Total initial energy = \frac{1}{2}mv^2 - \frac{GMm}{R}

If 20\% of the initial kinetic energy is lost because of Martian atmospheric resistance, then only 80\% of its kinetic energy will help to reach a height.

Total initial energy available = \frac{80}{100} \times \frac{1}{2}mv^2 - \frac{GMm}{R} = 0.4mv^2 - \frac{GMm}{R}

The maximum height reached by rocket = h

At this height, the velocity and therefore the rocket's kinetic energy will be zero.

The total energy of the rocket at height h = \frac{GMm}{(R+h)}

By applying the law of conservation of energy for the rocket, we can write:

\[0.4mv^2 - \frac{GMm}{R} = -\frac{GMm}{(R+h)}\]

\[0.4v^2 = \frac{GM}{R} - \frac{GM}{R + h}\]

\[0.4v^2 = GM \left(\frac{1}{R} - \frac{1}{R + h}\right)\]

\[0.4v^2 = GM \left(\frac{R + h - R}{R(R + h)}\right)\]

\[0.4v^2 = \frac{GMh}{R(R + h)}\]
\[
\frac{R + h}{h} = \frac{GM}{0.4v^2R} \\
\frac{R}{h} + 1 = \frac{GM}{0.4v^2R} \\
\frac{R}{h} = \frac{GM}{0.4v^2R} - 1 \\
h = \frac{R}{\frac{GM}{0.4v^2R} - 1} \\
h = \frac{0.4R^2v^2}{GM - 0.4v^2R} \\
\]
\[
h = \frac{0.4 \times (3.395 \times 10^6)^2 \times (2 \times 10^3)^2}{6.67 \times 10^{-11} \times 6.4 \times 10^{23} - 0.4 \times (2 \times 10^3)^2 \times (3.395 \times 10^6)} \\
h = \frac{18.442 \times 10^{18}}{42.688 \times 10^{-12} - 5.432 \times 10^{12}} = \frac{18.442}{37.256} \times 10^6 \\
h = 495 \times 10^3 m = 495 km
\]