11.1. The triple points of neon and carbon dioxide are 24.57 K and 216.55 K, respectively. Express these temperatures on the Celsius and Fahrenheit scales.

Solution:

The Kelvin and Celsius's scales are related as:

\[ T_C = T_K - 273.15 \quad \ldots \ldots (i) \]

Celsius and Fahrenheit's scales are related as:

\[ T_F = \frac{9}{5} T_C + 32 \quad \ldots\ldots \quad (ii) \]

For neon:

\[ T_K = 24.57 \text{ K} \]
\[ \therefore T_C = 24.57 - 273.15 = -248.58^\circ\text{C} \]
\[ T_F = \frac{9}{5} T_C + 32 \]
\[ = \frac{9}{5} (-248.58) + 32 \]
\[ = 415.44^\circ\text{F} \]

For carbon dioxide:

\[ T_K = 216.55 \text{ K} \]
\[ \therefore T_C = 216.55 - 273.15 = -56.60^\circ\text{C} \]
\[ T_F = \frac{9}{5} (T_C) + 32 \]
\[ = \frac{9}{5} (-56.60) + 32 \]
\[ = -69.88^\circ\text{C} \]

11.2. Two absolute scales A and B have triple points of water defined to be 200 A and 350 B. What is the relation between \( T_A \) and \( T_B \)?

Solution:

The triple point of water on absolute scale A, \( T_1 = 200 \text{ A} \)

The triple point of water on absolute scale B, \( T_2 = 350 \text{ B} \)
The triple point of water on the Kelvin scale, $T_K = 273.15$ K

The temperature of 273.15 K on the Kelvin scale is equivalent to 200 A on absolute scale A.

$T_1 = T_K$

$200A = 273.15$ K

$\therefore A = \frac{273.15}{200}$

The temperature of 273.15 K on the Kelvin scale is equivalent to 350 B on absolute scale B.

$T_2 = T_K$

$350 B = 273.15$

$\therefore B = \frac{273.15}{350}$

$T_A$ is the triple point temperature of the water on scale A.

$T_B$ is the triple point temperature of the water on scale B.

$\therefore \frac{273.15}{200} \times T_A = \frac{273.15}{350} \times T_B$

$T_A = \frac{200}{350} T_B$

Therefore, the ratio $T_A : T_B$ is given as 4:7.

11.3. The electrical resistance in ohms of a certain thermometer varies with temperature according to the approximate law:

$R = R_0 [1 + \alpha (T - T_0)]$

The resistance is 101.6 Ω at the triple-point of water 273.16 K, and 165.5 Ω at the normal melting point of lead (600.5 K). What is the temperature when the resistance is 123.4 Ω?

Solution:

It is given that:

$R = R_0 [1 + \alpha (T - T_0)]$ ... (i)

Where,

$R_0$ and $T_0$ are the initial resistance and temperature respectively $R$, and $T$ are the final resistance and temperature respectively $\alpha$ is a constant

At the triple point of water, $T_0 = 273.15$ K
The resistance of lead, \( R_0 = 101.6 \, \Omega \)

At normal melting point of lead, \( T = 600.5 \, \text{K} \)

The resistance of lead, \( R = 165.5 \, \Omega \)

Putting these values in equation (i), we get:

\[
R = R_0[1 + \alpha(T - T_0)]
\]

\[
165.5 = 101.6[1 + \alpha(600.5 - 273.15)]
\]

\[
1.629 = 1 + \alpha(327.35)
\]

\[
\therefore \alpha = \frac{0.629}{327.35} = 1.92 \times 10^{-3} \text{K}^{-1}
\]

For resistance, \( R_1 = 123.4 \, \Omega \)

\[
R_1 = R_0[1 + \alpha(T - T_0)]
\]

Where \( T \) is the temperature when the resistance of lead is 123.4 \( \Omega \)

\[
123.4 = 101.6[1 + 1.92 \times 10^{-3}(T - 273.15)]
\]

\[
1.214 = 1 + 1.92 \times 10^{-3}(T - 273.15)
\]

\[
\frac{0.214}{1.92 \times 10^{-3}} = T - 273.15
\]

\[
\therefore T = 384.61 \, \text{K}
\]

11.4. Answer the following:

(a) The triple-point of water is a standard fixed point in modern thermometry. Why? What is wrong in taking the melting point of ice and the boiling point of water as standard fixed points (as was originally done in the Celsius scale)?

(b) There were two fixed points in the original Celsius scale as mentioned above which were assigned the number 0 \( ^\circ \text{C} \) and 100 \( ^\circ \text{C} \) respectively. On the absolute scale, one of the fixed points is the triple-point of water, which on the Kelvin absolute scale is assigned the number 273.16 \( \text{K} \). What is the other fixed point on this (Kelvin) scale?

(c) The absolute temperature (Kelvin scale) \( T \) is related to the temperature \( t_c \) on the Celsius scale by \( t_c = T - 273.15 \)

Why do we have 273.15 in this relation, and not 273.16?

(d) What is the temperature of the triple-point of water on an absolute scale whose unit interval size is equal to that of the Fahrenheit scale?

Solution:
(a) The triple water point has a unique 273.16 K value. The triple point of water is always 273.16 K at specific volume and pressure values. There are no specific values for the melting point of ice and boiling point of water, as these points depend on pressure and temperature.

(b) The absolute zero or 0 K is the other fixed point on the Kelvin absolute scale.

(c) The temperature of 273.16 K is the triple point of water. It is not the melting point of ice. The temperature 0 °C on the Celsius scale is the melting point of ice. Its corresponding value on the Kelvin scale is 273.15 K.

Accordingly, absolute temperature (Kelvin scale) T, on the Celsius scale as:
\[ t_c = T - 273.15 \]

(c) Let \( T_F \) be the temperature on Fahrenheit scale and \( T_K \) be the temperature on the absolute scale. Both the temperatures can be related as:
\[ \frac{T_F - 32}{180} = \frac{T_K - 273.15}{100} \] ........ (ii)

It is given that:
\[ T_K - T_K = 1 \text{ K} \]

Subtracting equation (i) from equation (ii), we get:
\[ T_F - T_F = \frac{T_K - T_K}{100} = \frac{1}{100} \]
\[ T_F - T_F = \frac{1 \times 180}{100} = \frac{9}{5} \]

The triple point of water = 273.16 K

\[ \therefore \text{The triple point of water on absolute scale} = 273.16 \times \frac{9}{5} = 491.69 \]

11.5. Two ideal gas thermometers A and B use oxygen and hydrogen, respectively. The following observations are made:

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Pressure thermometer A</th>
<th>Pressure thermometer B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triple-point of water</td>
<td>1.250 ( \times 10^5 ) Pa</td>
<td>0.200 ( \times 10^5 ) Pa</td>
</tr>
<tr>
<td>Normal melting point of sulphur</td>
<td>1.797 ( \times 10^5 ) Pa</td>
<td>0.287 ( \times 10^5 ) Pa</td>
</tr>
</tbody>
</table>

(a) What is the absolute temperature of the normal melting point of sulphur as read by thermometers A and B?

(b) What do you think is the reason behind the slight difference in answers of thermometers A and B? (The thermometers are not faulty). What further procedure is needed in the experiment to reduce the discrepancy between the two readings?
Solution:
(a) The triple point of water, $T = 273.16$ K.
At this temperature, pressure in thermometer $A$, $P_A = 1.250 \times 10^5$ Pa
Let $T_1$ be the normal melting point of sulphur.
At this temperature, pressure in thermometer $A$, $P_1 = 1.797 \times 10^5$ Pa
According to Charles’ law:
$$\frac{P_A}{T} = \frac{P_1}{T_1}$$
$$\therefore T_1 = \frac{P_1 T}{P_A} = \frac{1.797 \times 10^5 \times 273.16}{1.250 \times 10^5}$$
$$= 392.69 \text{ K}$$
The absolute temperature of the normal sulfur melting point as read by thermometer $A$ is therefore $392.69$ K.
At triple point $273.16$ K, the pressure in thermometer $B$, $P_B = 0.200 \times 10^5$ Pa
At temperature $T_1$, the pressure in thermometer $B$, $P_2 = 0.287 \times 10^5$ Pa
According to Charles’ law:
$$\frac{P_B}{T} = \frac{P_1}{T_1}$$
$$\frac{0.200 \times 10^5}{273.16} = \frac{0.287 \times 10^5}{T_1}$$
$$\therefore T_1 = \frac{0.287 \times 10^5 \times 273.16}{0.200 \times 10^5} = 391.98 \text{ K}$$
The absolute temperature of the normal sulfur melting point as read by thermometer $A$ is therefore $391.98$ K.
(b) The oxygen and hydrogen gas contained in the A and B thermometers are not ideal gases. Therefore, the measurements of thermometers A and B differ slightly. The experiment should be carried out under low-pressure circumstances to decrease the discrepancy between the two measurements. These gasses act as perfect ideal gases at low pressure.

11.6. A steel tape $1$ m long is correctly calibrated for a temperature of $27.0 \degree \text{C}$. The length of a steel rod measured by this tape is found to be $63.0 \text{ cm}$ on a hot day when the temperature is $45.0 \degree \text{C}$. What is the actual length of the steel rod on that day? What is the length of the same steel rod on a day when the temperature is $27.0 \degree \text{C}$? Coefficient of linear expansion of steel = $1.20 \times 10^{-5} \text{ K}^{-1}$. 
Solution:

Length of the steel tape at temperature \( T = 27 \, ^\circ\text{C} \), \( l = 1 \, \text{m} = 100 \, \text{cm} \)

At temperature \( T_1 = 45 \, ^\circ\text{C} \), the length of the steel rod, \( l_1 = 63 \, \text{cm} \)

The Coefficient of linear expansion of steel, \( \alpha = 1.20 \times 10^{-5} \, \text{K}^{-1} \)

Let \( l_2 \) be the actual length of the steel rod and \( l' \) be the length of the steel tape at \( 45 \, ^\circ\text{C} \).

Therefore, the actual length of the steel rod measured by the steel tape at \( 45 \, ^\circ\text{C} \) can be calculated as:

\[ l' = l + \alpha l (T_1 - T) \]

\[ \therefore l' = 100 + 1.20 \times 10^{-5} \times 100 \times (45 - 27) \]

\[ = 100.0216 \, \text{cm} \]

Therefore, the actual length of the steel rod measured by the steel tape at \( 45 \, ^\circ\text{C} \) can be calculated as:

\[ l_2 = \frac{100.0216}{100} \times 63 = 63.0136 \, \text{cm} \]

Hence, the actual length of the rod at \( 45.0 \, ^\circ\text{C} \) is \( 63.0136 \, \text{cm} \). Its length at \( 27.0 \, ^\circ\text{C} \) is \( 63.0 \, \text{cm} \).

11.7. A large steel wheel is to be fitted on to a shaft of the same material. At \( 27 \, ^\circ\text{C} \), the outer diameter of the shaft is \( 8.70 \, \text{cm} \), and the diameter of the central hole in the wheel is \( 8.69 \, \text{cm} \). The shaft is cooled using ‘dry ice’. At what temperature of the shaft does the wheel slip on the shaft? Assume the coefficient of linear expansion of the steel to be constant over the required temperature range: \( \alpha_{\text{steel}} = 1.20 \times 10^{-5} \, \text{K}^{-1} \).

Solution:

The given temperature, \( T = 27 \, ^\circ\text{C} \) can be written in Kelvin as:

\[ 27 + 273 = 300 \, \text{K} \]

The outer diameter of the steel shaft at \( T \), \( d_1 = 8.70 \, \text{cm} \)

The Diameter of the central hole in the wheel at \( T \), \( d_2 = 8.69 \, \text{cm} \)

Coefficient of linear expansion of steel, \( \alpha_{\text{steel}} = 1.20 \times 10^{-5} \, \text{K}^{-1} \)

After the shaft is cooled using ‘dry ice’, its temperature becomes \( T_1 \).

The wheel will slip on the shaft if the change in diameter, \( \Delta d = 8.69 - 8.70 \)

\[ = -0.01 \, \text{cm} \]

Temperature \( T_1 \), can be calculated from the relation:
\[ \Delta d = d_1 \alpha_{steel} (T_1 - T) \]
\[ = 8.70 \times 1.20 \times 10^{-5} (T_1 - 300) \]
\[ (T_1 - 300) = 95.78 \]
\[ \therefore T_1 = 204.21 \text{ K} \]
\[ = 204.21 - 273.16 \]
\[ = -68.95 \text{ °C} \]

Hence, the wheel will slip on the shaft when the temperature of the shaft is \(-69 \text{ °C}\).

**11.8.** A hole is drilled in a copper sheet. The diameter of the hole is 4.24 cm at 27.0 °C. What is the change in the diameter of the hole when the sheet is heated to 227 °C? Coefficient of linear expansion of copper = \(1.70 \times 10^{-5} \text{ K}^{-1}\).

**Solution:**

Initial temperature, \(T_1 = 27.0 \text{ °C} \)

The diameter of the hole at \(T_1, d_1 = 4.24 \text{ cm} \)

Final temperature, \(T_2 = 227 \text{ °C} \)

The diameter of the hole at \(T_2 = d_2 \)

Co-efficient of linear expansion of copper, \(\alpha_{Cu} = 1.70 \times 10^{-5} \text{ K}^{-1} \)

For the coefficient of superficial expansion \(\beta\), and change in temperature \(\Delta T\), we have the relation:

\[
\frac{\Delta A}{A} = \beta \Delta T
\]

\[
\left( \frac{\pi d_2^2/4 - \pi d_1^2/4}{\pi d_1^2/4} \right) = \frac{\Delta A}{A}
\]

\[
\left( \frac{\pi d_2^2/4 - \pi d_1^2/4}{\pi d_1^2/4} \right) = \frac{\Delta A}{A}
\]

\[
\therefore \frac{\Delta A}{A} = \frac{d_2^2 - d_1^2}{d_1^2}
\]

But \(\beta = 2\alpha\)

\[
\therefore \frac{d_2^2 - d_1^2}{d_1^2} = 2\alpha \Delta T
\]
\[
\frac{d_2^2}{d_1^2} - 1 = 2\alpha(T_2 - T_1)
\]

\[
\frac{d_2^2}{(4.24)^2} = 2 \times 1.7 \times 10^{-5}(227 - 27) + 1
\]

\[
d_2^2 = 17.98 \times 1.0068 = 18.1
\]

\[
\therefore d_2 = 4.2544 \text{ cm}
\]

Change in diameter \(d_2 - d_1 = 4.2544 - 4.24 = 0.0144 \text{ cm}

Hence, the diameter increases by \(1.44 \times 10^{-2} \text{ cm}\).

11.9. A brass wire 1.8 m long at 27 °C is held taut with little tension between two rigid supports. If the wire is cooled to a temperature of −39 °C, what is the tension developed in the wire, if its diameter is 2.0 mm? Co-efficient of linear expansion of brass

\(= 2.0 \times 10^{-5} \text{ K}^{-1}\); Young’s modulus of brass = \(0.91 \times 10 \text{ Pa}\)

**Solution:**

Initial temperature, \(T_1 = 27 \text{ °C}\)

Length of the brass wire at \(T_1, l = 1.8 \text{ m}\)

Final temperature, \(T_2 = −39 \text{ °C}\)

The diameter of the wire, \(d = 2.0 \text{ mm} = 2 \times 10^{-3} \text{ m}\)

Tension developed in the wire = \(F\)

The Coefficient of linear expansion of brass, \(\alpha = 2.0 \times 10^{-5} \text{ K}^{-1}\)

Young’s modulus of brass, \(Y = 0.91 \times 10^{11} \text{ Pa}\)

Young’s modulus is given by the relation:

\[
Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F}{A} \frac{\Delta L}{L}
\]

\[
\Delta L = \frac{FL}{AY} \ldots (i)
\]

Where,

\(F = \text{Tension developed in the wire}\)

\(A = \text{Area of a cross-section of the wire}\).

\(\Delta L = \text{Change in the length}\),

given by the relation:
\[ \Delta L = \alpha L (T_2 - T_1) \ldots \text{(ii)} \]

Equating equations (i) and (ii), we get:

\[ \alpha L (T_2 - T_1) = \frac{FL}{\pi \left(\frac{d}{2}\right)^2 \times Y} \]

\[ F = \alpha (T_2 - T_1) \pi Y \left(\frac{d}{2}\right)^2 \]

\[ F = 2 \times 10^{-5} \times (-39 - 27) \times 3.14 \times 0.91 \times 10^{11} \times \left(\frac{2 \times 10^{-3}}{2}\right)^2 \]

\[ = -3.8 \times 10^2 \text{N} \]

(The negative sign shows that the tension is inside)

Hence, the tension developed in the wire is \(3.8 \times 10^2\) N.

11.10. A brass rod of length 50 cm and diameter 3.0 mm is joined to a steel rod of the same length and diameter. What is the change in length of the combined rod at 250 °C, if the original lengths are at 40.0 °C? Is there a ‘thermal stress’ developed at the junction? The ends of the rod are free to expand (Co-efficient of linear expansion of brass \(= 2.0 \times 10^{-5} \text{ K}^{-1}\), steel \(= 1.2 \times 10^{-5} \text{ K}^{-1}\)).

Solution:

Initial temperature, \(T_1 = 40 \degree \text{C}\)

Final temperature, \(T_2 = 250 \degree \text{C}\)

Change in temperature, \(\Delta T = T_2 - T_1 = 210 \degree \text{C}\)

Length of the brass rod at \(T_1, l_1 = 50 \text{ cm}\)

The diameter of the brass rod at \(T_1, d_1 = 3.0 \text{ mm}\)

Length of the steel rod at \(T_1, l_2 = 50 \text{ cm}\)

The diameter of the steel rod at \(T_2, d_2 = 3.0 \text{ mm}\)

Coefficient of linear expansion of brass, \(\alpha_1 = 2.0 \times 10^{-5} \text{ K}^{-1}\)

Coefficient of linear expansion of steel, \(\alpha_2 = 1.2 \times 10^{-5} \text{ K}^{-1}\)

For the expansion in the brass rod, we have the given formulae:

\[ \frac{\text{Change in length}(\Delta l_1)}{\text{Original length}(l_1)} = \alpha_1 \Delta T \]

\[ \therefore \Delta l_1 = 50 \times (2.1 \times 10^{-5}) \times 210 \]

\[ = 0.2205 \text{ cm} \]
For the expansion in the steel rod, we have the given formulae:

\[
\frac{\text{Change in length}(\Delta l_2)}{\text{Original length}(l_2)} = \alpha_2 \Delta T
\]

\[
\therefore \Delta l_2 = 50 \times (1.2 \times 10^{-5}) \times 210
\]

\[= 0.126 \text{ cm}\]

Total change in the lengths of brass and steel is given by,

\[\Delta l = \Delta l_1 + \Delta l_2\]

\[= 0.2205 + 0.126\]

\[= 0.346 \text{ cm}\]

Total change in the length of the combined rod = 0.346 cm

No thermal stress is developed at the junction when the rod expands freely from both ends.

11.11. The coefficient of volume expansion of glycerin is \(49 \times 10^{-5} \text{ K}^{-1}\). What is the fractional change in its density for a 30\(^\circ\)C rise in temperature?

**Solution:**

Coefficient of volume expansion of glycerin, \(\alpha_v = 49 \times 10^{-5} \text{ K}^{-1}\)

Rise in temperature, \(\Delta T = 30^\circ\text{C}\)

Fractional change in its volume = \(\frac{\Delta V}{V}\)

This change is related to the change in temperature as:

\[
\frac{\Delta V}{V} = \alpha_v \Delta T
\]

\[
V_{T_2} - V_{T_1} = V_{T_1} \alpha_v \Delta T
\]

\[
\frac{m}{\rho_{T_2}} - \frac{m}{\rho_{T_1}} = \frac{m}{\rho_{T_1}} \alpha_v \Delta T
\]

Where, \(m = \text{Mass of glycerine}\)

\(\rho_{T_1} = \text{Initial density at } T_1\)

\(\rho_{T_2} = \text{Final density at } T_2\)

\[
\frac{\rho_{T_1} - \rho_{T_2}}{\rho_{T_2}} = \alpha_v \Delta T
\]

Where,
\[ \frac{\rho_1 - \rho_2}{\rho_2} = \text{Fractional change in density} \]

\[ \therefore \text{Fractional change in the density of glycerin} = 49 \times 10^{-5} \times 30 = 1.47 \times 10^{-2} \]

11.12. A 10 kW drilling machine is used to drill a bore in a small aluminium block of mass 8.0 kg. How much is the rise in temperature of the block in 2.5 minutes, assuming 50% of power is used up in heating the machine itself or lost to the surroundings? Specific heat of aluminium = 0.91 J g\(^{-1}\) K\(^{-1}\).

**Solution:**

Power of the drilling machine, \( P = 10 \text{ kW} = 10 \times 10^3 \text{ W} \)

Mass of the aluminium block, \( m = 8.0 \text{ kg} = 8 \times 10^3 \text{ g} \)

Time for which the machine is used, \( t = 2.5 \text{ min} = 2.5 \times 60 = 150 \text{ s} \)

Specific heat of aluminium, \( c = 0.91 \text{ J g}^{-1} \text{ K}^{-1} \)

The Rise in the temperature of the block after drilling = \( \delta T \)

The total energy of the drilling machine = \( Pt \)
\[ = 10 \times 10^3 \times 150 \]
\[ = 1.5 \times 10^6 \text{ J} \]

It is given that only 50% of the power is useful.

Hence, Useful energy, \( \Delta Q = \frac{50}{100} \times 1.5 \times 10^6 = 7.5 \times 10^5 \text{ J} \)

But \( \Delta Q = mc\Delta T \)

\[ \therefore \Delta T = \frac{\Delta Q}{mc} \]
\[ = \frac{7.5 \times 10^5}{8 \times 10^3 \times 0.91} \]
\[ = 103 \text{ } ^\circ \text{C} \]

Hence, in 2.5 minutes of drilling, the rise in the temperature of the block is 103 \( ^\circ \text{C} \).

11.13. A copper block of mass 2.5 kg is heated in a furnace to a temperature of 500 \( ^\circ \text{C} \) and then placed on a large ice block. What is the maximum amount of ice that can melt? (Specific heat of copper = 0.39 J g\(^{-1}\) K\(^{-1}\); the heat of fusion of water = 335 J g\(^{-1}\)).

**Solution:**

Mass of the copper block, \( m = 2.5 \text{ kg} = 2500 \text{ g} \)

The Rise in the temperature of the copper block, \( \Delta \theta = 500^\circ \text{C} \)
Specific heat of copper, \( C = 0.39 \text{ J g}^{-1} \text{C}^{-1} \)

The heat of fusion of water, \( L = 335 \text{ J g}^{-1} \)

Maximum heat the copper block can lose, \( Q = mC\Delta \theta \)

\[
= 2500 \times 0.39 \times 500 \\
= 487500 \text{ J}
\]

Let \( m_1 \) g be the amount of ice that melts when the copper block is placed on the ice block.

Hence, The heat gained by the melted ice, \( Q = m_1L \)

\[
\therefore m_1 = \frac{Q}{L} = \frac{487500}{335} = 1455.22 \text{ g}
\]

Hence, the maximum amount of ice that can melt is 1.45 kg.

**11.14.** In an experiment on the specific heat of a metal, a 0.20 kg block of the metal at 150 °C is dropped in a copper calorimeter (of water equivalent 0.025 kg) containing 150 cm³ of water at 27 °C. The final temperature is 40 °C. Compute the specific heat of the metal. If heat losses to the surroundings are not negligible, is your solution greater or smaller than the actual value for the specific heat of the metal?

**Solution:**

Mass of the metal, \( m = 0.20 \text{ kg} = 200 \text{ g} \)

The initial temperature of the metal, \( T_1 = 150 \text{ °C} \)

The final temperature of the metal, \( T_2 = 40 \text{ °C} \)

The calorimeter has water equivalent of mass, \( m' = 0.025 \text{ kg} = 25 \text{ g} \)

The volume of water, \( V = 150 \text{ cm}^3 \)

Mass (M) of water at temperature \( T = 27 \text{ °C} \):

\[
150 \times 1 = 150 \text{ g}
\]

Fall in the temperature of the metal:

\[
\Delta T = T_1 - T_2 = 150 - 40 = 110 \text{ °C}
\]

Specific heat of water, \( C_w = 4.186 \text{ J/g/°K} \)

Specific heat of the metal = \( C \)

Heat lost by the metal, \( \theta = mC\Delta T \) ……… (i)

The rise in the temperature of the water and calorimeter system:

\[
\Delta T' = 40 - 27 = 13 \text{ °C}
\]
The Heat gained by the water and calorimeter system:
\[ \Delta \theta^\prime = m_1 C_w \Delta T' \]

\[ = (M + m') C_w \Delta T' \quad \text{…… (ii)} \]

Heat lost by the metal = Heat gained by the calorimeter system and water
\[ m C \Delta T = (M + m') C_w \Delta T' \]

\[ 200 \times C \times 110 = (150 + 25) \times 4.186 \times 13 \]

\[ \therefore C = \frac{175 \times 4.186 \times 13}{110 \times 200} = 0.43 \text{ Jg}^{-1}\text{K}^{-1} \]

If some heat is lost to the surroundings, then the value of \( C \) will be smaller than the actual value.

11.15. Given below are observations on molar specific heats at room temperature of some common gases.

<table>
<thead>
<tr>
<th>Gas</th>
<th>Molar specific heat (Cv) (cal mol(^{-1}) K(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen</td>
<td>4.87</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>4.97</td>
</tr>
<tr>
<td>Oxygen</td>
<td>5.02</td>
</tr>
<tr>
<td>Nitric oxide</td>
<td>4.99</td>
</tr>
<tr>
<td>Carbon monoxide</td>
<td>5.01</td>
</tr>
<tr>
<td>Chlorine</td>
<td>6.17</td>
</tr>
</tbody>
</table>

The measured molar specific heats of these gases are markedly different from those for monatomic gases. Typically, molar specific heat of a monatomic gas is 2.92 cal/mol K. Explain this difference. What can you infer from the somewhat larger (than the rest) value for chlorine?

**Solution:**

The gasses in the table shown are diatomic. They have other degrees of freedom (modes of motion) in addition to the translational degree of freedom. To raise the temperature of these gases, heat must be provided. This increases the average energy of all the modes of motion.

\[ gas = \frac{5}{2} R \]

\[ = \frac{5}{2} \times 1.98 = 4.95 \text{ cal mol}^{-1}\text{K}^{-1} \]
All the observations in the table given, with the exception of chlorine, agree with \( \left( \frac{5}{2} R \right) \).

This is because, in addition to rotational and translational modes of motion, chlorine also has vibrational mode of motion at room temperature.

11.16. The following questions based on the \( P - T \) phase diagram of carbon dioxide:

(a) At what temperature and pressure can the solid, liquid and vapour phases of \( CO_2 \) co-exist in equilibrium?

(b) What is the effect of the decrease of pressure on the fusion and boiling point of \( CO_2 \)?

(c) What are the critical temperature and pressure for \( CO_2 \)? What is their significance?

(d) Is \( CO_2 \) solid, liquid or gas at (a) \(-70^\circ C\) under 1 atm, (b) \(-60^\circ C\) under 10 atm, (c) \(15 ^\circ C\) under 56 atm?

**Solution:**

(a) The P-T phase diagram for \( CO_2 \) is shown in the following figure.

\[ P \text{ (atm)} \]

\[ T^\circ (C) \]

\( C \) is the triple point of the \( CO_2 \) phase diagram. This means that at the pressure and temperature corresponding to this point (i.e., at \(-56.6 ^\circ C\) and 5.11 atm), the solid phase, liquid phase, and vaporous phases of \( CO_2 \) co-exist in equilibrium.

(b) The fusion and boiling points of \( CO_2 \) decrease with a decrease in pressure.

(c) The critical temperature and critical pressure of \( CO_2 \) are 31.1 \( ^\circ C \) and 73 atm respectively.

Even if it is compressed to a pressure greater than 73 atm, \( CO_2 \) will not liquefy above the critical temperature.
It can be concluded from the P-T phase diagram of \( CO_2 \) that:

(a) \( CO_2 \) is gaseous at \(-70 \, ^\circ C\), under 1 atm pressure
(b) \( CO_2 \) is solid at \(-60 \, ^\circ C\), under 10 atm pressure
(c) \( CO_2 \) is liquid at \(15 \, ^\circ C\), under 56 atm pressure

11.17. Answer the following questions based on the P-T phase diagram of \( CO_2 \):

(a) \( CO_2 \) at 1 atm pressure and temperature \(-60 \, ^\circ C\) is compressed isothermally. Does it go through a liquid phase?
(b) What happens when \( CO_2 \) at 4 atm pressure is cooled from room temperature at constant pressure?
(c) Describe qualitatively the changes in a given mass of solid \( CO_2 \) at 10 atm pressure and temperature \(-65 \, ^\circ C\) as it is heated up to room temperature at constant pressure.
(d) \( CO_2 \) is heated to a temperature \(70 \, ^\circ C\) and compressed isothermally. What changes in its properties do you expect to observe?

Solution:

(a) No
(b) It condenses to solid directly.
(c) The fusion and boiling points are given by the intersection point where this parallel line cuts the fusion and vaporisation curves.
(d) It departs from ideal gas behaviour as pressure increases.

Explanation:

(a) The P-T phase diagram for \( CO_2 \) is shown in the following figure.
At 1 atm pressure and at $-60 ^\circ C$, $CO_2$ lies to the left of $-56.6 ^\circ C$ (triple point $C$). Hence, it lies in the region of solid and vaporous phases.

Thus, $CO_2$ condenses into the solid state directly, without going through the liquid state.

(b) At 4 atm pressure, $CO_2$ lies below 5.11 atm (triple point $C$). Therefore, It's in the vaporous and solid phase region. It therefore condenses straight into the solid state, without going through the liquid state.

(c) When the temperature of a mass of solid $CO_2$ (at 10 atm pressure and at $-65 ^\circ C$) is increased; it changes to the liquid phase and then to the vaporous phase. It forms a line parallel to the temperature as at 10 atm. The fusion and boiling points are provided by the intersection point where the fusion and vaporization curves are cut off by this parallel line.

(d) If $CO_2$ is heated to 70 $^\circ C$ and compressed isothermally, then it will not exhibit any transition to the liquid state. This is because 70 $^\circ C$ is higher than the critical temperature of $CO_2$. It will remain in the vapour state but as pressure increases, it will depart from its ideal state.

11.18. A child running a temperature of 101$^\circ$F is given an antipyrin (i.e. medicine that lowers fever) which causes an increase in the rate of evaporation of sweat from his body. If the fever is brought down to 98 $^\circ$F in 20 min, what is the average rate of extra evaporation caused, by the drug? Assume the evaporation mechanism to be the only way by which heat is lost. The mass of the child is 30 kg. The specific heat of the human body is approximately the same as that of water, and latent heat of evaporation of water at that temperature is about 580 cal g$^{-1}$.

**Solution:**

The initial temperature of the body of the child, $T_1 = 101 ^\circ F$

The final temperature of the body of the child $T_2 = 98 ^\circ F$

Change in temperature, $\Delta T = [(101 - 98) \times \frac{5}{9}] _{\circ C}$

Time taken to reduce the temperature, $t = 20$ min

Mass of the child, $m = 30 \text{ kg} = 30 \times 10^3 \text{ g}$

The Specific heat of the human body = Specific heat of water = $c = 1000 \text{ cal/kg} / ^\circ C$

Latent heat of evaporation of water, $L = 580 \text{ cal g}^{-1}$

The heat lost by the child is given as:

$\Delta \theta = mc \Delta T$
\[= 30 \times 1000 \times (101 - 98) \times \frac{5}{9}\]

\[= 50000 \text{ cal}\]

Let \(m_1\) be the mass of the water evaporated from the child’s body in 20 min.

Loss of heat through water is given by:

\[\Delta \theta = m_1L\]

\[\therefore m_1 = \frac{\Delta \theta}{L}\]

\[= \frac{50000}{580} = 86.2 \text{ g}\]

\[\therefore \text{ The average rate of extra evaporation caused by the drug } = \frac{m_1}{t}\]

\[= \frac{86.2}{200} = 4.3 \text{ g/min}\]

11.19. A ‘thermacole’ icebox is a cheap and efficient method for storing small quantities of cooked food in summer in particular. A cubical icebox of side 30 cm has a thickness of 5.0 cm. If 4.0 kg of ice is put in the box, estimate the amount of ice remaining after 6 h. The outside temperature is 45 \(^\circ\)C, and co-efficient of thermal conductivity of thermacole is 0.01 \(\text{J s}^{-1} \text{ m}^{-1} \text{ K}^{-1}\). \([\text{Heat of fusion of water } = 335 \times 10^3 \text{ J kg}^{-1}\)]

**Solution:**

Side of the given cubical ice box, \(s = 30 \text{ cm} = 0.3 \text{ m}\)

The thickness of the icebox, \(l = 5.0 \text{ cm} = 0.05 \text{ m}\)

Mass of ice kept in the icebox, \(m = 4 \text{ kg}\)

Time gap, \(t = 6 \text{ h} = 6 \times 60 \times 60 \text{ s}\)

Outside temperature, \(T = 45 \text{ }^\circ\text{C}\)

Coefficient of thermal conductivity of thermacole, \(K = 0.01 \text{ J s}^{-1} \text{ m}^{-1} \text{ K}^{-1}\)

The heat of fusion of water, \(L = 335 \times 10^3 \text{ J kg}^{-1}\) Let

Let \(m'\) be the total amount of ice that melts in 6 h.

The amount of heat lost by the food is given by:

\[\theta = \frac{KA(T - 0)t}{l}\]

Where,
A = Surface area of the box = 6s² = 6 × (0.3)² = 0.54 m³

\[ \theta = \frac{0.01 \times 0.54 \times (45) \times 6 \times 60 \times 60}{0.05} = 104976 \, J \]

But \( \theta = m'L \)

\[ \because m' = \frac{\theta}{L} \]

\[ = \frac{104976}{335 \times 10^3} = 0.313 \, kg \]

Mass of ice left = 4 − 0.313 = 3.687 kg

Hence, the amount of ice remaining after 6 h is 3.687 kg.

11.20. A brass boiler has a base area of 0.15 m² and thickness of 1.0 cm. It boils water at the rate of 6.0 kg/min when placed on a gas stove. Estimate the temperature of the part of the flame in contact with the boiler. Thermal conductivity of brass = 109 J s⁻¹ m⁻¹ K⁻¹; Heat of vaporisation of water = 2256 × 10³ J kg⁻¹.

**Solution:**

The base area of the boiler, \( A = 0.15 \, m^2 \)

The thickness of the boiler, \( l = 1.0 \, \text{cm} = 0.01 \, m \)

Boiling rate of water, \( R = 6.0 \, \text{kg/min} \)

Mass, \( m = 6 \, kg \)

Time, \( t = 1 \, min = 60 \, s \)

Thermal conductivity of brass, \( K = 109 \, J \, s^{-1} \, m^{-1} \, K^{-1} \)

The heat of vaporisation, \( L = 2256 \times 10^3 \, J \, kg^{-1} \)

The amount of heat flowing into the water through the brass base of the boiler is given by:

\[ \theta = \frac{KA(T_1-T_2)}{l} \tag{i} \]

Where,

\( T_1 = \) Temperature of the flame in contact with the boiler
\( T_2 = \) Boiling point of water = 100 °C

The heat required for boiling the water:

\[ \theta = mL \tag{ii} \]

Equating equations (i) and (ii), we get:
\[ mL = \frac{KA(T_1 - T_2)t}{l} \]

\[ T_1 - T_2 = \frac{mLl}{KA}\t \]

\[ = \frac{6 \times 2256 \times 10^3 \times 0.01}{109 \times 0.15 \times 60} \]

\[ = 137.98 \, ^\circ C \]

Therefore, the temperature of the part of the flame in contact with the boiler is 237.98 \, ^\circ C.

11.21. Explain why:

(a) a body with large reflectivity is a poor emitter

(b) a brass tumbler feels much colder than a wooden tray on a chilly day

(c) an optical pyrometer (for measuring high temperatures) calibrated for an ideal black body radiation gives too low a value for the temperature of a red hot iron piece in the open, but gives a correct value for the temperature when the same piece is in the furnace

(d) the earth without its atmosphere would be inhospitably cold

(e) heating systems based on circulation of steam are more efficient in warming a building than those based on circulation of hot water

Solution:

(a) A body with a large reflectivity is a poor absorber of light radiations. In turn, a poor absorber will be a poor radiation emitter. A body with a large reflectivity is, therefore, a poor emitter.

(b) Brass is an excellent heat conductor. When a brass tumbler is touched, heat is easily driven from the body to the brass tumbler. As a result, the body's temperature decreases to a lesser value and one feels cooler. Wood is a bad heat conductor. When a wooden tray is touched, very little heat is carried from the body to the wooden tray. Therefore, the temperature drop of the body is only negligible and one does not feel cool. Thus, a brass tumbler feels colder than a wooden tray on a chilly day.

(c) An optical pyrometer calibrated for ideal black body radiation gives too low a value for the temperature of a red hot iron piece kept in the open.

Black body radiation equation is given by:

\[ E = \sigma(T^4 - T_0^4) \]

Where,
\[ E = \text{Energy radiation} \]
\[ T = \text{Temperature of optical pyrometer} \]
\[ T_0 = \text{Temperature of open space} \]
\[ \sigma = \text{Constant} \]

Hence, an increase in the temperature of open space reduces the radiation energy.

When the same piece of iron is placed in a furnace, the radiation energy,
\[ E = \sigma T^4 \]

The earth would be inhospitably cold without its atmosphere. No additional heat will be trapped in the absence of atmospheric gases. It would radiate all the heat back from the surface of the earth.

(e) A steam-based heating system is more efficient in heating a building than it is based on hot water circulation. This is because steam in the form of latent heat (540 cal/g) contains surplus heat.

11.22. A body cools from 80 °C to 50 °C in 5 minutes. Calculate the time it takes to cool from 60 °C to 30 °C. The temperature of the surroundings is 20 °C.

**Solution:**

According to Newton’s law of cooling:

\[-\frac{dT}{dt} = K(T - T_0)\]

\[ \frac{dT}{K(T - T_0)} = -Kdt \quad \ldots \ (i) \]

Where,

- The temperature of the body = \( T \)
- The temperature of the surroundings = \( T_0 = 20 \) °C
- \( K \) is a constant
- The temperature of the body falls from 80 °C to 50 °C in time, \( t = 5 \text{ min} = 300 \text{ s} \)

Integrating equation (i), we get:

\[ \int_{50}^{80} \frac{dT}{K(T - T_0)} = -\int_{0}^{300} Kdt \]

\[ \frac{\log_e (T - T_0)}{\log_e (50 - 20)} = -K[t]^{300} \]

\[ 2.3026 \frac{80 - 20}{50 - 20} = -300 \]

\[ \frac{80 - 20}{50 - 20} = \frac{60}{30} = 2 \]

\[ 2.3026 \times 2 = 4.6052 \]

\[ -300 \times 4.6052 = -13815.6 \]

Therefore, the time it takes to cool from 60 °C to 30 °C in the given scenario is approximately 13815.6 seconds.
\[
\frac{2.3026}{K} \log_{10} 2 = -300
\]
\[
\frac{-2.3026}{300} \log_{10} 2 = K \quad \text{…… (ii)}
\]

The temperature of the body falls from 60 °C to 30 °C in time = \( t' \)

Hence, we get:
\[
\frac{2.3026}{K} \log_{10} \frac{60 - 20}{30 - 20} = -t'
\]
\[
\frac{-2.3026}{t} \log_{10} 4 = K \quad \text{…… (iii)}
\]

Equating equations (ii) and (iii), we get:
\[
\frac{-2.3026}{t} \log_{10} 4 = \frac{-2.3026}{300} \log_{10} 2
\]

\[
\therefore t = 300 \times 2 = 600 \text{ s} = 10 \text{ min}
\]

Hence, the time taken to cool the body from 60 °C to 30 °C is 10 minutes.