1. Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N}$:

$$1 + 3 + 3^2 + \ldots + 3^{a-1} = \frac{(3^n - 1)}{2}$$

**Solution:**

Step 1:

Considering the given statement as $P(n)$, i.e.,

$$P(n): 1 + 3 + 3^2 + \ldots + 3^{a-1} = \frac{(3^n - 1)}{2}$$

For $n = 1$, we have

$$P(1) = \frac{(3^1 - 1)}{2} = \frac{3 - 1}{2} = \frac{2}{2} = 1\text{. Which is true.}$$

Consider, $P(k)$ be true for some positive integer $k$, i.e.,

$$1 + 3 + 3^2 + \ldots + 3^{k-1} = \frac{(3^k - 1)}{2} \ldots (i)$$

Now to prove that $P(k+1)$ is true.

$$1 + 3 + 3^2 + \ldots + 3^{k-1} + 3^{k+1} - 1$$

$$= (1 + 3 + 3^2 + \ldots + 3^{k-1}) + 3^k$$

$$= \frac{(3^k - 1)}{2} + 3^k \quad [\text{Using } (i)]$$

$$= \frac{(3^k - 1) + 2 \times 3^k}{2}$$

$$= \frac{3^k(1+2) - 1}{2}$$

$$= \frac{3 \times 3^k - 1}{2}$$

$$= \frac{3^{k+1} - 1}{2}$$

Therefore, $P(k + 1)$ is true when ever $P(k)$ is true.
Therefore, $P(k + 1)$ is true when ever $P(k)$ is true.

Thus, by the principle of mathematical induction, statement $P(n)$ is true for all natural numbers i.e., $N$.

OVERALL HINT: Add $3^k$ on both sides

2. Prove the following by using the principle of mathematical induction for all $n \in N$:

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n + 1)}{2}\right)^2$$

Solution:

STEP 1:

Consider the given statement as $P(n)$, i.e.,

$$P(n): 1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n + 1)}{2}\right)^2$$

For $n = 1$, we have

$$P(1): 1^3 = 1 = \left(\frac{1(1+1)}{2}\right)^2 = \left(\frac{2}{2}\right)^2 = 1^2 = 1,$$ which is true.

Let’s $P(k)$ be true for some positive integer $k$, i.e.,

$$1^3 + 2^3 + 3^3 + \cdots + k^3 = \left(\frac{k(k+1)}{2}\right)^2 \quad \text{...(i)}$$

Now to prove that $P(k + 1)$ is true.

STEP 2:

Consider

$$1^3 + 2^3 + 3^3 + \cdots + k^3 + (k + 1)^3$$

$$= \left(1^3 + 2^3 + 3^3 + \cdots + k^3\right) + (k + 1)^3$$

$$= \left(\frac{k(k+1)}{2}\right)^2 + (k + 1)^3 \quad \text{[Using(i)]}$$

$$= \frac{k^2(k + 1)^2}{4} + (k + 1)^3$$
Principle of Mathematical Induction

\[ \frac{k^2(k + 1)^2 + 4(k + 1)^3}{4} \]

\[ = \frac{(k + 1)^2(k^2 + 4k + 4)}{4} \]

\[ = \frac{(k + 1)^2(k + 2)^2}{4} \]

\[ = \frac{(k + 1)^2(k + 1 + 1)^2}{4} \]

\[ = \left( \frac{(k + 1)(k + 1 + 1)}{2} \right)^2 \]

Therefore, \( P(k + 1) \) is true when \( P(k) \) is true.

Thus, by the principle of mathematical induction, statement \( P(n) \) is true for all natural numbers i.e., \( N \).

\[ P(n): 1^3 + 2^3 + 3^3 + \cdots + n^3 = \left( \frac{n(n + 1)}{2} \right)^2 \]

3. Prove the following by using the principle of mathematical induction for all \( n \in N \):

\[ 1 + \frac{1}{(1 + 2)} + \frac{1}{(1 + 2 + 3)} + \cdots + \frac{1}{(1 + 2 + 3 + \cdots + n)} = \frac{2n}{(n + 1)} \]

Solution:

Step 1:

Consider the given statement as \( P(n) \), i.e.,

\[ P(n): 1 + \frac{1}{(1 + 2)} + \frac{1}{(1 + 2 + 3)} + \cdots + \frac{1}{(1 + 2 + 3 + \cdots + n)} = \frac{2n}{(n + 1)} \]

For \( n = 1 \), we have

\[ P(1): 1 + \frac{1}{1+2} = \frac{2}{1+2} = 1 \text{, which is true.} \]

And, \( P(k) \) be true for some positive integer \( k \), i.e.,
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\[ 1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \cdots + \frac{1}{1+2+3+\cdots+k} = \frac{2k}{k+1} \quad \text{(i)} \]

Now to prove that \( P(k+1) \) is true.

**STEP 2**

Consider

\[ 1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \cdots + \frac{1}{1+2+3+\cdots+k} + \frac{1}{1+2+3+\cdots+k+(k+1)} \]

\[ = \left( 1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \cdots + \frac{1}{1+2+3+\cdots+k} \right) + \frac{1}{1+2+3+\cdots+k+k+1} \]

\[ = \frac{2k}{k+1} + \frac{1}{1+2+3+\cdots+k+(k+1)} \quad \text{[Using (i)]} \]

\[ = \frac{2k}{k+1} + \frac{1}{\left( \frac{(k+1)(k+1+1)}{2} \right)} \]

\[ = \frac{2k}{(k+1)(k+2)} \]

\[ = \frac{2}{(k+1)} \left( k + \frac{1}{k+2} \right) \]

\[ = \frac{2}{(k+1)} \left( \frac{k^2 + 2k + 1}{k+2} \right) \]

\[ = \frac{2}{(k+1)} \left( \frac{(k+1)^2}{k+2} \right) \]

\[ = \frac{2(k+1)}{k+2} \]

Therefore, \( P(k+1) \) is true whenever \( P(k) \) is true.

Thus, by the principle of mathematical induction, statement \( P(n) \) is true for all natural numbers i.e., \( N \).

OVER ALL HINT: TO FIND \( P(n) \); where \( n=1 \).

\[ P(n): 1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \cdots + \frac{1}{1+2+3+\cdots+n} = \frac{2n}{(n+1)} \]

4. Prove the following by using the principle of mathematical induction for all \( n \in N \):

\[ 123 + 2.3.4 + \cdots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4} \]
Solution:

STEP 1:
Consider the given statement be \( P(n), \) i.e.,

\[
P(n): 1.2.3 + 2.3.4 + \cdots + n(n + 1)(n + 2) = \frac{n(n + 1)(n + 2)(n + 3)}{4}
\]

For \( n = 1, \) we have

\[
P(1): 1.2.3 = 6 = \frac{1(1+1)(1+2)(1+3)}{4} = \frac{1.2.3.4}{4} = 6, \text{ which is true.}
\]

Consider, \( P(k) \) be true for some positive integer \( k, \) i.e.,

\[
1.2.3 + 2.3.4 + \cdots + k(k + 1)(k + 2) = \frac{k(k+1)(k+2)(k+3)}{4} \quad \text{(i)}
\]

Now to prove that \( P(k + 1) \) is true.

STEP 2:
Consider

\[
1.2.3 + 2.3.4 + \cdots + k(k + 1)(k + 2) + (k + 1)(k + 2)(k + 3)
\]

\[
= \{1.2.3 + 23.4 + \cdots + k(k + 1)(k + 2)\} + (k + 1)(k + 2)(k + 3)
\]

\[
= \frac{k(k+1)(k+2)(k+3)}{4} + (k + 1)(k + 2)(k + 3)
\]

\[
= (k + 1)(k + 2)(k + 3) \left[ \frac{k}{4} + 1 \right]
\]

\[
= \frac{(k + 1)(k + 2)(k + 3)(k + 4)}{4}
\]

\[
= \frac{(k + 1)(k + 1 + 1)(k + 1 + 2)(k + 1 + 3)}{4}
\]

Therefore, \( P(k + 1) \) is true when \( P(k) \) is true.

Thus, by the principle of mathematical induction, statement \( P(n) \) is true for all natural numbers i.e., \( N. \)

\[
P(n): 1.2.3 + 2.3.4 + \cdots + n(n + 1)(n + 2) = \frac{n(n + 1)(n + 2)(n + 3)}{4}
\]

5. Prove the following by using the principle of mathematical induction for all \( n \in N: 1.3 + 2.3^2 + 3.3^3 + \cdots + n.3^n = \frac{(2n-1)3^{n+1}+3}{4} \)
Solution:

STEP 1:
Consider the given statement be $P(n)$, i.e.,

$$P(n): 1.3 + 2.3^2 + 3.3^3 + \cdots + n.3^n = \frac{(2n - 1)3^{n+1} + 3}{4}$$

For $n = 1$, we have

$$P(1): 1.3 = \frac{(2(1) - 1)3^{1+1} + 3}{4} = \frac{3^2 + 3}{4} = \frac{12}{4} = 3$$, which is true.

Consider $P(k)$ be true for some positive integer $k$, i.e.,

$$1.3 + 2.3^2 + 3.3^3 + \cdots + k3^k = \frac{(2k - 1)3^{k+1} + 3}{4}$$

Now to prove that $P(k + 1)$ is true.

STEP 2:
Consider

$$1.3 + 2.3^2 + 3.3^3 + \cdots + k3^k + (k + 1) \cdot 3^{k+1}$$

$$= (1.3 + 2.3^2 + 3.3^3 + \cdots + k.3^k) + (k + 1) \cdot 3^{k+1}$$

$$= \frac{(2k - 1)3^{k+1} + 3}{4} + (k + 1)3^{k+1}$$

$$= \frac{(2k - 1)3^{k+1} + 3 + 4(k + 1)3^{k+1}}{4}$$

$$= \frac{3^{k+1}[2k - 1 + 4(k + 1)] + 3}{4}$$

$$= \frac{3^{k+1}[6k + 3] + 3}{4}$$

$$= \frac{3^{k+1} \cdot 3[2k + 1] + 3}{4}$$

$$= \frac{3^{(k+1)+1}[2k + 1] + 3}{4}$$

$$= \frac{[2(k + 1) - 1]3^{(k+1)+1} + 3}{4}$$

Therefore, $P(k + 1)$ is true whenever $P(k)$ is true.
Thus, by the principle of mathematical induction statement $P(n)$ is true for all natural numbers i.e., $N$.

OVERALL HINT:

$$P(n): 1.3 + 2.3^2 + 3.3^3 + \cdots + n.3^n = \frac{(2n - 1)3^{n+1} + 3}{4}$$

6. Prove the following by using the principle of mathematical induction for all $n \in N$:

$$1.2 + 2.3 + 3.4 + \cdots + n(n+1) = \left[\frac{n(n+1)(n+2)}{3}\right]$$

Solution:

STEP1:

Consider the given statement be $P(n)$, i.e.,

$$1.2 + 2.3 + 3.4 + \cdots + k(k+1) = \left[\frac{k(k+1)(k+2)}{3}\right]\ldots(i)$$

Now to prove that $P(k+1)$ is true.

MARKS:1

DL1:L

STEP2:

Consider

$$1.2 + 2.3 + 3.4 + \cdots + k. (k + 1) + (k + 1). (k + 2)$$

$$= [1.2 + 2.3 + 3.4 + \cdots + k. (k + 1)] + (k + 1). (k + 2)$$

$$= \frac{k(k+1)(k+2)}{3} + (k + 1)(k + 2)\ [\text{using}(i)]$$

$$= (k + 1)(k + 2)\left(\frac{k}{3} + 1\right)$$

$$= \frac{(k + 1)(k + 2)(k + 3)}{3}$$

$$= \frac{(k + 1)(k + 1 + 1)(k + 1 + 2)}{3}$$

Thus, $P(k+1)$ is true whenever $P(k)$ is true.

Thus, by the principle of mathematical induction, statement $P(n)$ is true for all natural numbers i.e., $N$. 


OVERALL HINT: $1.2+2.3+3.4+\cdots+k(k+1) = \left[\frac{k(k+1)(k+2)}{3}\right]$ AND P(K+1)

7. Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N}$: $1.3+3.5+5.7+\cdots+(2n-1)(2n+1) = \frac{n(4n^2+6n-1)}{3}$

Solution:

STEP1:

Consider the given statement be $P(n)$, i.e.,

$P(n): 1.3+3.5+5.7+\cdots+(2n-1)(2n+1) = \frac{n(4n^2+6n-1)}{3}$

Form $n=1$, we have

$P(1): 1.3 = 3 = \frac{1(4(1)^2+6(1)-1)}{3} = \frac{1+6-1}{3} = \frac{6}{3} = 2$, which is true.

Consider $P(k)$ be true for some positive integer $k$, i.e.,

$1.3+3.5+5.7+\cdots+(2k-1)(2k+1) = \frac{k(4k^2+6k-1)}{3}$ ...(i)

Now to prove that $P(k+1)$ is true.

STEP2:

Consider

$(1.3+3.5+5.7+\cdots+(2k-1)(2k+1)+\{2(k+1)−1\}(2(k+1)+1))$

$= \frac{k(4k^2+6k-1)}{3} + (2k+2-1)(2k+2+1) \quad \text{[Using (i)]}$

$= \frac{k(4k^2+6k-1)}{3} + (2k+1)(2k+3)$

$= \frac{k(4k^2+6k-1)}{3} + (4k^2+8k+3)$

$= \frac{k(4k^2+6k-1) + 3(4k^2+8k+3)}{3}$

$= \frac{4k^2+6k^2−k+12k^2+24k+9}{3}$
Principle of Mathematical Induction

\[
\begin{align*}
&\quad = k(4k^2 + 14k + 9) + 1(4k^2 + 14k + 9) \\
&\quad = \frac{(k + 1)(4k^2 + 14k + 9)}{3} \\
&\quad = \frac{(k + 1)(4k^2 + 8k + 4 + 6k + 6 - 1)}{3} \\
&\quad = \frac{(k + 1)(4(k + 1)^2 + 6(k + 1) - 1)}{3} \\
&\text{Therefore, } P(k+1) \text{ is true when } P(k) \text{ is true.}
\end{align*}
\]

Thus, by the principle of mathematical induction, statement \( P(n) \) is true for all natural numbers i.e., \( \mathbb{N} \).

OVERALL HINT:
\[
P(n): 1.3 + 3.5 + 5.7 + \cdots + (2n - 1)(2n + 1) = \frac{n(4n^2 + 6n - 1)}{3}
\]

8. Prove the following by using the principle of mathematical induction for all \( n \in \mathbb{N} \): \( 1.2 + 2.2^2 + 3.2^2 + \ldots + n.2^n = (n - 1)2^{n+1} + 2 \)

Solution:

STEP1:

Considering the given principle as \( p(n): 1.2 + 2.2^3 + 3.2^2 + \ldots + n.2^n = (n - 1)2^{n+1} + 2 \)

For \( n = 1 \), we have

\[
P(1): 1.2 = 2 = (1-1)2^{1+1} + 2 = 0 + 2 = 2, \text{ which is true.}
\]

Consider \( P(k) \) be true for some positive integer \( k \), i.e.,

\[
1.2 + 2.2^2 + 3.2^2 + \ldots + k.2^k = (k - 1)2^{k+1} + 2 \ldots (i)
\]

We shall now prove that \( P(k+1) \) is true.

STEP2:

Consider

\[
\{1.2 + 2.2^2 + 3.2^2 + \ldots + k.2^k\} + (k + 1).2^{k+1}
\]

\[
= (k - 1)2^{k+1} + 2 + (k + 1)2^{k+1} \quad \text{[from } (i)\text{]}
\]


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\[ = 2^{k+1}\{(k - 1) + (k + 1)\} + 2 \]
\[ = 2^{k+1}, 2k + 2 \]
\[ = k, 2^{(k+1)+1} + 2 \]
\[ = \{(k + 1) - 1\}2^{(k+1)+1} + 2 \]

Therefore, \( P(k + 1) \) is true whenever \( P(k) \) is true.

Thus, by the principle of mathematical induction statement \( P(n) \) is true for all natural numbers i.e., \( N \).

OVERALL HINT: as \( p(n): 1.2 + 2.2^2 + 3.2^2 + \ldots + n.2^n = (n - 1)2^{n+1} + 2 \)

9. Prove the following by using the principle of mathematical induction for all \( n \in N \):
\[
\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^n} = 1 - \frac{1}{2^n}
\]

Solution:

STEP:1

Consider the given statement be \( P(n) \), i.e.,

\[ P(n): \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^n} = 1 - \frac{1}{2^n} \]

Form = 1, we have

\[ P(1): \frac{1}{2} = 1 \Rightarrow -\frac{1}{2^1} = \frac{1}{2} \] which is true.

Consider \( P(k) \) be true for some positive integer \( k \), i.e.,

\[ \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^k} = 1 - \frac{1}{2^k} \ldots \text{(i)} \]

Now to prove that \( P(k + 1) \) is true.

Consider

\[
\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^k}\right) + \frac{1}{2^{k+1}}
\]
\[ = \left(1 - \frac{1}{2^k}\right) + \frac{1}{2^{k+1}} \quad \text{[Using(i)]} \]
\[ = 1 - \frac{1}{2^k} \left(1 - \frac{1}{2}\right) \]
Principle of Mathematical Induction

\[= 1 - \frac{1}{2^k} \left(\frac{1}{2}\right)\]

\[= 1 - \frac{1}{2^{k+1}}\]

Therefore, \(P(k + 1)\) is true when \(P(k)\) is true.

Thus, by the principle of mathematical induction, statement \(P(n)\) is true for all natural numbers i.e., \(N\).

OVERALL HINT:

\[P(n): \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^n} = 1 - \frac{1}{2^n}\]

10. Prove the following by using the principle of mathematical induction for all \(n \in N\):

\[\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \ldots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{(6n+4)}\]

Solution:

STEP: 1

Consider the given statement be \(P(n)\), i.e.,

\[P(n): \frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \ldots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{(6n+4)}\]

For \(n = 1\), we have

\[P(1) = \frac{1}{2.5} = \frac{1}{10} = \frac{1}{6(1)+4} = \frac{1}{10}\] which is true

Consider \(P(k)\) be true for some positive integer \(k\), i.e.,

\[\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \ldots + \frac{1}{(3k-1)(3k+2)} = \frac{k}{6k+4} \ldots (i)\]

We shall now prove that \(P(k + 1)\) is true.

STEP: 2

Consider

\[\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \ldots + \frac{1}{(3k-1)(3k+2)} + \frac{1}{(3k+1)(3k+3)} + \frac{1}{(3k+1)(3k+2)}\]

\[= \frac{k}{6k+4} + \frac{1}{(3k+3-1)(3k+3+2)} \quad [\text{Using} (i)]\]
Principle of Mathematical Induction

11. Prove the following by using the principle of mathematical induction for all \( n \in \mathbb{N} \):
\[
\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{3 \cdot 4 \cdot 5} + \cdots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}
\]

**Solution:**

Consider the given statement be \( P(n) \), i.e.,
\[
P(n) = \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \cdots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}
\]

For \( n = 1 \), we have
\[
P(1): \frac{1}{1 \cdot 2 \cdot 3} = \frac{1 \cdot (1+3)}{4(1+1)(1+2)} = \frac{4}{4 \cdot 2 \cdot 3} = \frac{1}{1 \cdot 2 \cdot 3}, \text{ which is true.}
\]

Let \( P(k) \) be true for some positive integer \( k \), i.e.,
We shall now prove that $P(k + 1)$ is true.

STEP:2

Consider

\[
\left[ \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \cdots + \frac{1}{k(k + 1)(k + 2)} \right] + \frac{1}{(k + 1)(k + 2)(k + 3)} = \frac{k(k + 3)}{4(k + 1)(k + 2)} \quad [\text{Using (i)}]
\]

\[
= \frac{1}{(k + 1)(k + 2)} \left( \frac{k(k + 3)}{4} + \frac{1}{k + 3} \right)
\]

\[
= \frac{1}{(k + 1)(k + 2)} \left( \frac{k^3 + 6k^2 + 9k + 4}{4(k + 3)} \right)
\]

\[
= \frac{1}{(k + 1)(k + 2)} \left( \frac{k^3 + 2k^2 + k + 4k^2 + 8k + 4}{4(k + 3)} \right)
\]

\[
= \frac{1}{(k + 1)(k + 2)} \left( \frac{k^3 + 2k^2 + 2k + 1 + 4(k^2 + 2k + 1)}{4(k + 3)} \right)
\]

\[
= \frac{1}{(k + 1)(k + 2)} \left( \frac{k(k + 1)^2 + 4(k + 1)^2}{4(k + 3)} \right)
\]

\[
= \frac{(k + 1)^2(k + 4)}{4(k + 1)(k + 2)(k + 3)}
\]

\[
= \frac{(k + 1)((k + 1) + 3)}{4((k + 1) + 1)((k + 1) + 2)}
\]

Therefore $P(k + 1)$ is true whenever $P(k)$ is true.

Thus, by the principle of mathematical induction, statement $P(n)$ is true for all natural numbers i.e., $N$.

OVERALL HINT:

\[
P(n) = \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \cdots + \frac{1}{n(n + 1)(n + 2)} = \frac{n(n + 3)}{4(n + 1)(n + 2)}
\]
12. Prove the following by using the principle of mathematical induction for all \( n \in \mathbb{N} \): \( a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(r^n-1)}{r-1} \)

**Solution:**

**STEP 1**
Consider the given statement be \( P(n) \), i.e.,

\[ P(n) = a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(r^n-1)}{r-1} \]

For \( n = 1 \), we have

\[ P(1): a = \frac{a(r^1-1)}{(r-1)} = a, \text{ which is true.} \]

Consider \( P(k) \) be true for some positive integer \( k \), i.e.,

\[ a + ar + ar^2 + \ldots + ar^{k-1} = \frac{a(r^k-1)}{r-1} \ldots (i) \]

Now to prove that \( P(k + 1) \) is true.

**STEP 2**
Consider

\[ \{a + ar + ar^2 + \ldots + ar^{k-1}\} + ar^{k} = \frac{a(r^{k+1}-1)}{r-1} \]

\[ = \frac{a(r^k-1) + ar^k}{r-1} \]

\[ = \frac{a(r^k-1) + ar^k(r - 1)}{r-1} \]

\[ = \frac{a(r^k-1) + ar^{k+1} - ar^k}{r-1} \]

\[ = \frac{ar^{k+1} - a}{r-1} \]

\[ = \frac{a(r^{k+1} - 1)}{r-1} \]

Therefore, \( P(k + 1) \) is true whenever \( P(k) \) is true.

Thus, by the principle of mathematical induction, statement \( P(n) \) is true for all natural numbers i.e., \( N \).
OVERALL HINT:
USE THE GIVEN FORMULA
\[ P(n) = a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1} \]

13. Prove the following by using the principle of mathematical induction for all \( n \in N \):
\[ (1 + \frac{3}{1})(1 + \frac{5}{4})(1 + \frac{7}{9}) \ldots (1 + \frac{\frac{2n+1}{n^2}}{n^2}) = (n + 1)^2 \]

Solution:
STEP:1
Consider the given statement be \( P(n) \), i.e.,
\[ P(n): (1 + \frac{3}{1})(1 + \frac{5}{4})(1 + \frac{7}{9}) \ldots (1 + \frac{\frac{2n+1}{n^2}}{n^2}) = (n + 1)^2 \]
For \( n = 1 \), we have
\[ P(1): (1 + \frac{3}{1}) = 4 \Rightarrow (1 + 1)^2 = 2^2 = 4 \] which is true.
Consider \( P(k) \) be true for some positive integer \( k \), i.e.,
\[ (1 + \frac{3}{1})(1 + \frac{5}{4})(1 + \frac{7}{9}) \ldots (1 + \frac{\frac{2k+1}{k^2}}{k^2}) = (k + 1)^2 \ldots (i) \]
Now to prove that \( P(k + 1) \) is true.
STEP:2
Consider
\[
\left[ (1 + \frac{3}{1})(1 + \frac{5}{4})(1 + \frac{7}{9}) \ldots (1 + \frac{\frac{2(k+1)+1}{(k+1)^2}}{(k+1)^2}) \right] \left\{ 1 + \frac{\frac{2(k+1)+1}{(k+1)^2}}{(k+1)^2} \right\}
\]
\[ = (k + 1)^2 \left( 1 + \frac{\frac{2(k+1)+1}{(k+1)^2}}{(k+1)^2} \right) \text{[Using (i)]} \]
\[ = (k + 1)^2 \left[ \frac{(k + 1)^2 + 2(k + 1) + 1}{(k + 1)^2} \right] \]
\[ = (k + 1)^2 + 2(k + 1) + 1 \]
\[ = (k + 1)(k + 1)^2 \]
Therefore, \( P(k + 1) \) is true whenever \( P(k) \) is true.
Thus, by the principle of mathematical induction, statement $P(n)$ is true for all natural numbers i.e., $N$.

OVERALL HINT:

USE THE GIVEN FORMULA

$P(n): \left(1 + \frac{3}{1}\right)\left(1 + \frac{5}{4}\right)\left(1 + \frac{7}{9}\right) \ldots \left(1 + \frac{(2n + 1)}{n^2}\right) = (n + 1)^2$

14. Prove the following by using the principle of mathematical induction for all $n \in N$: $\left(1 + \frac{1}{1}\right)\left(1 + \frac{1}{2}\right)\ldots \left(1 + \frac{1}{n}\right) = (n + 1)$

Solution:
STEP: 1
Consider the given statement be $P(n)$, i.e.,

$\left(1 + \frac{1}{1}\right)\left(1 + \frac{1}{2}\right)\ldots \left(1 + \frac{1}{n}\right) = (n + 1)$

For $n = 1$, we have

$P(1): \left(1 + \frac{1}{1}\right) = 2 = (1 + 1)$, which is true.

Consider $P(k)$ be true for some positive integer $k$, i.e.,

$P(k): \left(1 + \frac{1}{1}\right)\left(1 + \frac{1}{2}\right)\ldots \left(1 + \frac{1}{k}\right) = (k + 1) \ldots (1)$

Now to prove that $P(k + 1)$ is true.

STEP: 2
Consider

$\left(1 + \frac{1}{1}\right)\left(1 + \frac{1}{2}\right)\ldots \left(1 + \frac{1}{k}\right)\left(1 + \frac{1}{k + 1}\right)$

$= \left[\left(1 + \frac{1}{1}\right)\left(1 + \frac{1}{2}\right)\ldots \left(1 + \frac{1}{k}\right)\right] \left(1 + \frac{1}{k + 1}\right)$

$= (k + 1) \left(1 + \frac{1}{k+1}\right) [\text{Using (1)}]$

$= (k + 1) \left(\frac{k + 1}{k + 1}\right)$
= (k + 1) + 1
Therefore, \( P(k + 1) \) is true whenever \( P(k) \) is true.
Thus, by the principle of mathematical induction, statement \( P(n) \) is true for all natural numbers \( n \).

OVERALL HINT:
USE THE FORMULA: \( P(k): \left(1 + \frac{1}{1}\right)\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right) \cdots \left(1 + \frac{1}{k}\right) = (k + 1) \)

15. Prove the following by using the principle of mathematical induction for all \( n \in N \):

\[
1^2 + 3^2 + 5^2 + \cdots + (2n - 1)^2 = \frac{n(2n - 1)(2n + 1)}{3}
\]

Solution:
Consider the given statement be \( P(n) \), i.e.,

\[
P(n) = 1^2 + 3^2 + 5^2 + \cdots + (2n - 1)^2 = \frac{n(2n - 1)(2n + 1)}{3}
\]

For \( n = 1 \), we have

\[
P(1) = 1^2 = 1 \Rightarrow \frac{1(2(1) - 1)(2(1) + 1)}{3} = \frac{(1)(3)}{3} = 1, \text{which is true.}
\]

STEP:2
Consider \( P(k) \) be true for some positive integer \( k \), i.e.,

\[
P(k) = 1^2 + 3^2 + 5^2 + \cdots + (2k - 1)^2 = \frac{k(2k - 1)(2k + 1)}{3} \quad (1)
\]

Now to prove that \( P(k + 1) \) is true.

STEP:3
Consider

\[
1^2 + 3^2 + 5^2 + \cdots + (2k - 1)^2 + (2k + 1)^2
\]

\[
\Rightarrow \{1^2 + 3^2 + 5^2 + \cdots + (2k - 1)^2\} + (2k + 1)^2
\]

\[
= \frac{k(2k - 1)(2k + 1)}{3} + (2k + 2 - 1)^2 \quad \text{[Using(1)]}
\]

\[
= \frac{k(2k - 1)(2k + 1)}{3} + (2k + 1)^2
\]
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\[ k(2k - 1)(2k + 1) + 3(2k + 1)^2 \]
\[ \frac{3}{3} \]
\[ (2k + 1)[k(2k - 1) + 3(2k + 1)] \]
\[ = \frac{3}{3} \]
\[ (2k + 1)[2k^2 - k + 6k + 3] \]
\[ = \frac{3}{3} \]
\[ (2k + 1)[2k^2 + 5k + 3] \]
\[ = \frac{3}{3} \]
\[ (2k + 1)[2k^2 + 2k + 3k + 3] \]
\[ = \frac{3}{3} \]
\[ (2k + 1)[2k(k + 1) + 3(k + 1)] \]
\[ = \frac{3}{3} \]
\[ (2k + 1)(k + 1)(2k + 3) \]
\[ = \frac{3}{3} \]
\[ (k + 1)[2(k + 1) - 1][2(k + 1) + 1] \]

Therefore, \( P(k + 1) \) is true whenever \( P(k) \) is true.

Thus, by the principle of mathematical induction, statement \( P(n) \) is true for all natural numbers i.e., \( N \).

OVERALL HINT: USE THE FORMULA
\[ P(n) = 1^2 + 3^2 + 5^2 + \cdots + (2n - 1)^2 \]
\[ = \frac{n(2n - 1)(2n + 1)}{3} \]

AND \( P(k) = 1^2 + 3^2 + 5^2 + \cdots + (2k - 1)^2 = \frac{k(2k-1)(2k+1)}{3} \)

16. Prove the following by using the principle of mathematical induction for all \( n \in N \):
\[ \frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \cdots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{(3n+1)} \]

Solution:

STEP:1

Consider the given statement be \( P(n) \), i.e.,
\[ P(n): \frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \cdots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{(3n+1)} \]

For \( n = 1 \) we get,
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\[ P(1) = \frac{1}{1.4} = \frac{1}{4} \Rightarrow \frac{1}{3(1)+1} = \frac{1}{4} \text{, which is true} \]

Consider \( P(k) \) be true for some positive integer \( k \), i.e.,

\[ P(k) = \frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \ldots + \frac{1}{(3k-2)(3k+1)} = \frac{k}{3k+1} \ldots (1) \]

Now to prove that \( P(k+1) \) is true.

STEP 2

Consider

\[ \left( \frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \ldots + \frac{1}{(3k-2)(3k+1)} \right) + \frac{1}{(3k+2)(3k+1)} \]

\[ = \frac{k}{3k+1} + \frac{1}{(3k+2)(3k+4)} \left[ \text{Using (1)} \right] \]

\[ = \frac{1}{3k+1} \left\{ k + \frac{1}{3k+4} \right\} \]

\[ = \frac{1}{3k+1} \left\{ k(3k+4) + 1 \right\} \]

\[ = \frac{1}{3k+1} \left\{ 3k^2 + 4k + 1 \right\} \]

\[ = \frac{1}{3k+1} \left\{ 3k^2 + 3k + k + 1 \right\} \]

\[ = \frac{(3k+1)(k+1)}{(3k+1)(3k+4)} \]

\[ = \frac{(k+1)}{3k+1} + 1 \]

Therefore, \( P(k+1) \) is true whenever \( P(k) \) is true.

Thus, by the principle of mathematical induction, statement \( P(n) \) is true for all natural numbers i.e., \( N \).

OVERALL HINT:

USE THE FORMULA:

\[ P(n): \frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \ldots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{(3n+1)} \AND \frac{k}{3k+1} + \frac{1}{(3k+1)(3k+4)} \]
17. Prove the following by using the principle of mathematical induction for all \( n \in \mathbb{N} \):
\[
\frac{1}{7.9} + \cdots + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}
\]

**Solution:**

**STEP:1**

Consider the given statement be \( P(n) \), i.e.,
\[
P(n): \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \cdots + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}
\]

For \( n = 1 \), we get
\[
P(1): \frac{1}{3.5} = \frac{1}{3(2\times1+3)} = \frac{1}{3\times5} \text{, which is true.}
\]

**STEP:2**

Consider \( P(k) \) be true for some positive integer \( k \), i.e.,
\[
P(k): \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \cdots + \frac{1}{(2k+1)(2k+3)} = \frac{k}{3(2k+3)} \tag{1}
\]

Now to prove that \( P(k+1) \) is true.

**STEP:3**

Consider
\[
\left[ \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \cdots + \frac{1}{(2k+1)(2k+3)} \right] + \frac{1}{(2k+3)(2k+5)}
\]

\[
= \frac{k}{3(2k+3)} + \frac{1}{(2k+3)(2k+5)} \quad \text{[Using (1)]}
\]

\[
= \frac{1}{(2k+3)} \left[ \frac{k}{3} + \frac{1}{(2k+5)} \right]
\]

\[
= \frac{1}{(2k+3)} \left[ \frac{k(2k+5) + 3}{3(2k+5)} \right]
\]

\[
= \frac{1}{(2k+3)} \left[ \frac{2k^2 + 5k + 3}{3(2k+5)} \right]
\]

\[
= \frac{1}{(2k+3)} \left[ \frac{2k(k+1) + 3(k+1)}{3(2k+5)} \right]
\]

\[
= \frac{1}{(2k+3)} \left[ \frac{2k(k+1) + 3(k+1)}{3(2k+5)} \right]
\]

\[
= \frac{1}{(2k+3)} \left[ \frac{2k(k+1) + 3(k+1)}{3(2k+5)} \right]
\]

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Therefore, \( P(k + 1) \) is true whenever \( P(k) \) is true.

Thus, by the principle of mathematical induction, statement \( P(n) \) is true for all natural numbers i.e., \( N \).

OVERALL HINT:

Use the formula:

\[
P(n): \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \cdots + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)} + \frac{k}{3(2k+3)} + \frac{1}{(2k+3)(2k+5)}\]

18. Prove the following by using the principle of mathematical induction for all \( n \in N: 1 + 2 + 3 + \cdots + n < \frac{1}{8}(2n + 1)^2 \)

Solution:

Step: 1

Consider the given statement be \( P(n) \), i.e.,

\( P(n): 1 + 2 + 3 + \cdots + n < \frac{1}{8}(2n + 1)^2 \)

this note d that \( P(n) \)is true for \( n = 1 \) since

\[ 1 < \frac{1}{8}(2 \times 1 + 1)^2 = \frac{9}{8} \]

Consider \( P(k) \) be true for some positive integer \( k \), i.e.,

\[ 1 + 2 + \cdots + k < \frac{1}{8}(2k + 1)^2 \ldots (1) \]

We shall now prove that \( P(k + 1) \) is true whenever \( P(k) \) is true.

STEP: 2

Consider

\[
(1 + 2 + \cdots + k) + (k + 1) < \frac{1}{8}(2k + 1)^2 + (k + 1) [Adding (k + 1) both the sides]
\]
(1 + 2 + \cdots + k) + (k + 1) < \frac{1}{8}(2k + 1)^2 + 8(k + 1)\\
(1 + 2 + \cdots + k) + (k + 1) < \frac{1}{8}(4k^2 + 4k + 1 + 8k + 8)\\
(1 + 2 + \cdots + k) + (k + 1) < \frac{1}{8}(4k^2 + 12k + 9)\\
(1 + 2 + \cdots + k) + (k + 1) < \frac{1}{8}(2k + 3)^2\\
(1 + 2 + \cdots + k) + (k + 1) < \frac{1}{8}(2(k + 1) + 1)^2

Therefore, (1 + 2 + 3 + \cdots + k) + (k + 1) < \frac{1}{8}(2k + 1)^2 + (k + 1)

Therefore, \(P(k + 1)\) is true when \(P(k)\) is true.

Thus, by the principle of mathematical induction, statement \(P(n)\) is true for all natural numbers i.e., \(N\).

OVERALL HINT: use the formula:

\(P(n): 1 + 2 + 3 + \cdots + n < \frac{1}{8}(2n + 1)^2\)

19. Prove the following by using the principle of mathematical induction for all \(n \in N: n(n + 1)(n + 5)\) is a multiple of 3.

Solution:

Step: 1

Consider the given statement be \(P(n)\), i.e.,

\(P(n): n(n + 1)(n + 5)\), which is a multiple of 3.

It is noted that \(P(n)\) is true for \(n = 1\) since,

\(1(1 + 1)(1 + 5) = 12\), which is a multiple of 3.

Consider, \(P(k)\) be true for some positive integer \(k\), i.e.,

\(k(k + 1)(k + 5)\) is a multiple of 3.

\(\therefore k(k + 1)(k + 5) = 3m\), where \(m \in N \ldots (1)\)

Now to prove that \(P(k + 1)\) is true when \(P(k)\) is true.
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STEP:2
Consider

\[(k + 1)((k + 1) + 1)(k + 1) + 1\]

\[= (k + 1)(k + 2)((k + 5) + 1)\]

\[= (k + 1)(k + 2)(k + 1) + (k + 1)(k + 2)\]

\[= k(k + 1)(k + 5) + 2(k + 1)(k + 5) + (k + 1)(k + 2)\]

\[= 3m + (k + 1){2(k + 5) + (k + 2)}\]

\[= 3m + (k + 1)(2k + 10 + k + 2)\]

\[= 3m + (k + 1)(3k + 12)\]

\[= 3m + 3(k + 1)(k + 4)\]

\[= 3\{m + (k + 1)(k + 4)\} = 3 \times q, \text{where } q = \{m + (k + 1)(k + 4)\} \text{is some natural number}\]

Thus, \((k + 1)((k + 1) + 1)(k + 1) + 1\) is a multiple of 3.

Therefore, \(P(k + 1)\) is true whenever \(P(k)\) is true.

Thus, by the principle of mathematical induction, statement \(P(n)\) is true for all natural numbers i.e., \(N\).

OVERALL HINT:
Use the given formula: \(P(n): n(n + 1)(n + 5)\) and \(k(k + 1)(k + 5) = 3m, \text{where } m \in N\)

20. Prove the following by using the principle of mathematical induction for all \(n \in N: 10^{2n-1} + 1\) is divisible by 11.

Solution:

STEP:1
Consider the given statement be \(P(n)\), i.e.,

\(P(n): 10^{2n-1} + 1\) is divisible by 11.

It is observed that \(P(n)\) is true for \(n = 1\)

since \(P(1) = 10^{2\times1-1} + 1 = 11\), which is divisible by 11.

STEP:2
Consider \(P(k)\) be true for some positive integer \(k\),

\[\text{over 23}\]
Step: 1
Consider \(x^{2n} - y^{2n}\) as \(P(n)\).
It is observed that \(P(n)\) is true for \(n = 1\).
It’s because \(x^{2 \times 1} - y^{2 \times 1} = x^2 - y^2 = (x + y)(x - y)\) is divisible by \((x + y)\)
Consider \( p(k) \) be true for some positive integer \( k \), i.e.,
\[ x^{2k} - y^{2k} \] is divisible by \( x + y \)
Therefore, let \( x^{2k} - y^{2k} = m(x + y) \), where \( m \in \mathbb{N} \) ...(1)
Now to prove that \( P(k + 1) \) is true whenever \( P(k) \) is true.

STEP:2
Consider
\[ x^{2(k+1)} - y^{2(k+1)} \]
\[ = x^{2k} \cdot x^2 - y^{2k} \cdot y^2 \]
\[ = x^2(x^{2k} - y^{2k} + y^{2k}) - y^{2k} \cdot y^2 \quad [\because \text{adding and subtracting with } y^{2k}] \]
\[ = x^2(m(x + y) + y^{2k}) - y^{2k} \cdot y^2 \quad \text{[Using(1)]} \]
\[ m(x + y)x^2 + y^{2k} \cdot x^2 - y^{2k} \cdot y^2 \]
\[ m(x + y)x^2 + y^{2k}(x^2 - y^2) \]
\[ m(x + y)x^2 + y^{2k}(x + y)(x - y) \]
\[ (x + y)(mx^2 + y^{2k}(x - y)), \text{which is a factor of } (x + y). \]
Therefore, \( P(k + 1) \) is true whenever \( P(k) \) is true.

Thus, by the principle of mathematical induction, statement \( P(n) \) is true for all natural numbers i.e., \( \mathbb{N} \).

OVERALL HINT:
USING THE FORMULA: \( x^{2n} - y^{2n} \) as \( P(n) \).

22. Prove the following by using the principle of mathematical induction for all \( n \in \mathbb{N} \): \( 3^{2n+2} - 8n - 9 \) is divisible by 8.

Solution:
Step:1
Consider the given statement be \( P(n) \), i.e.,
\[ P(n): 3^{2n+2} - 8n - 9 \text{ is divisible by 8.} \]
It is observed that \( P(n) \) is true for \( n = 1 \).
Since \(3^{2x+2} - 8 \times 1 - 9 = 64\), which is divisible by 8

Consider \(P(k)\) be true for some positive integer \(k\), i.e., \(3^{2k+2} - 8k - 9\) is divisible by 8

\[\therefore 3^{2k+2} - 8k - 9 = 8m; \text{ where } m \in N \ldots(1)\]

Now to prove that \(P(k + 1)\) is true whenever \(P(k)\) is true.

STEP: 2

Consider

\[3^{2(k+1)+2} - 8(k + 1) - 9\]

\[= 3^{2k+2} \cdot 3^2 - 8k - 8 - 9\]

\[= 3^2(3^{2k+2} - 8k - 9 + 8k + 9) - 8k - 17 \quad [\text{adding and subtracting} 8k + 9]\]

\[3^2(3^{2k+2} - 8k - 9) + 3^2(8k + 9) - 8k - 17\]

\[9 \times 8m + 9(8k + 9) - 8k - 17\]

\[9 \times 8m + 72k + 81 - 8k - 17\]

\[9 \times 8m + 64k + 64\]

\[8(9m + 8k + 8)\]

\[= 8r, \text{ Where } r = (9m + 8k + 8) \text{ is a natural number}\]

Thus, \(3^{2(k+1)+2} - 8(k + 1) - 9\) is divisible by 8.

Therefore, \(P(k + 1)\) is true when \(P(k)\) is true.

Thus, by the principle of mathematical induction, statement \(P(n)\) is true for all natural numbers i.e., \(N\).

OVERALL HINT:

USE THE FORMULA: \(P(n) : 3^{2n+2} - 8n - 9\) AND \(r = (9m + 8k + 8)\)

23. Prove the following by using the principle of mathematical induction for all \(n \in N: 41^n - 14^n\) is a multiple of 27.

Solution:

STEP: 1

Consider the given statement be \(P(n)\), i.e.,

\(P(n): 41^n - 14^n\) is a multiple of 27.
It is observed that $P(n)$ is true for $n = 1$
Since $41^n - 14^n = 27$, which is a multiple of 27.
Consider $P(k)$ be true for some positive integer $k$, i.e.,
$41^k - 14^k$ is a multiple of 27.
\[ \therefore 41^k - 14^k = 27m, \text{ where } m \in N \quad \cdots (1) \]
Now to prove that $P(k + 1)$ is true whenever $P(k)$ is true.

**STEP: 2**
Consider
\[ 41^{k+1} - 14^{k+1} \]
\[ = 41^k \cdot 41 - 14^k \cdot 14 \]
\[ = 41(41^k - 14^k + 14^k) - 14^k \times 14 \]
\[ = 41(41^k - 14^k) + 14 \times 14^k - 14^k \times 14 \]
\[ = 41 \times 27m + 14^k (41 - 14) \]
\[ = 41 \times 27m + 27 \times 14^k \]
\[ = 27(41m - 14^k) \]
\[ = 27 \times r, \text{ where } r = (41m - 14^k) \text{ is a natural number} \]
Thus, $41^{k+1} - 14^{k+1}$ is a multiple of 27
Therefore, $P(k + 1)$ is true whenever $P(k)$ is true.

Thus, by the principle of mathematical induction, statement $P(n)$ is true for all natural numbers i.e., $N$.

**OVERALL HINT:**
Use the given formula: $P(n): 41^n - 14^n$

24. Prove the following by using the principle of mathematical induction for all $n \in N: (2n + 7) < (n + 3)^2$

**Solution:**

**Step: 1**
Consider the given statement be $P(n)$, i.e.,
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\[ P(n): (2n + 7) < (n + 3)^2 \]

It is observed that \( P(n) \) is true for \( n = 1 \)

Since \( 2.1 + 7 = 9 < (1 + 3)^2 = 16 \), which is true

Consider \( P(k) \) be true for some positive integer \( k \), i.e.,

\[ (2k + 7) < (k + 3)^2 \ldots \text{(1)} \]

Now to prove that \( P(k + 1) \) is true whenever \( P(k) \) is true.

STEP: 2

Consider

\[ \{2(k + 1) + 7\} = (2k + 7) + 2 \]

\[ \therefore \{2(k + 1) + 7\} = (2k + 7) + 2 < (k + 3)^2 + 2 \quad \text{[using (1)]} \]

\[ 2(k + 1) + 7 < k^2 + 6k + 9 + 2 \]

\[ 2(k + 1) + 7 < k^2 + 6k + 11 \]

And, \( k^2 + 6k + 1 < k^2 + 8k + 16 \)

Therefore, \( 2(k + 1) + 7 < (k + 4)^2 \)

\[ 2(k + 1) + 7 < ((k + 1) + 3)^2 \]

Therefore, \( P(k + 1) \) is true whenever \( P(k) \) is true.

Thus, by the principle of mathematical induction, statement \( P(n) \) is true for all natural numbers i.e., \( N \).

OVERALL HINT: use the given formula:

\[ P(n): (2n + 7) < (n + 3)^2 \]

\[ \{2(k + 1) + 7\} = (2k + 7) + 2 \]

\[ \therefore \{2(k + 1) + 7\} = (2k + 7) + 2 < (k + 3)^2 + 2 \]