1. Draw a quadrilateral in the Cartesian plane, whose vertices are \((-4, 5), (0, 7), (5, -5)\) and \((-4, -2)\). Also, find its area.

Hint: The area of a triangle whose vertices are \((x_1, y_1), (x_2, y_2)\) and \((x_3, y_3)\) is

\[
\frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|
\]

Solution:
Solution step 1: Let \(ABCD\) be the given quadrilateral with vertices \(A(-4, 5), B(0, 7), C(5, -5)\) and \(D(-4, -2)\).

Then, by plotting \(A, B, C\) and \(D\) on the Cartesian plane and joining \(AB, BC, CD\) and \(DA\) the given quadrilateral can be drawn as:

To find the area of quadrilateral \(ABCD\), we draw one diagonal, say \(AC\).

Accordingly, area \((ABCD) = area(\Delta ABC) + area(\Delta ACD)\)

We know that the area of a triangle whose vertices are \((x_1, y_1), (x_2, y_2)\) and \((x_3, y_3)\) is

\[
\frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|
\]

Therefore, area of \(\Delta ABC\)

\[
= \frac{1}{2} |-4(7 + 5) + 0(-5 - 5) + 5(5 - 7)|unit^2
\]

\[
= \frac{1}{2} |-4(12) + 5(-2)|unit^2
\]

\[
= \frac{1}{2} |-48 - 10|unit^2
\]

\[
= \frac{1}{2} |58|unit^2
\]
Thus, \[ \text{Area of } \triangle ACD = \frac{1}{2} \times 58 \text{ unit}^2 \]

\[ = 29 \text{ unit}^2 \]

Area of \( \Delta ACD \)

\[ = \frac{1}{2} | -4(-5 + 2) + 5(-2 - 5) + (-4)(5 + 5)| \text{unit}^2 \]

\[ = \frac{1}{2} | -4(-3) + 5(-7) - (10)| \text{unit}^2 \]

\[ = \frac{1}{2} | -12 - 35 - 40| \text{unit}^2 \]

\[ = \frac{1}{2} | -63| \text{unit}^2 \]

\[ = \frac{63}{2} \text{ unit}^2 \]

Thus, \[ \text{Area}(ABCD) = (29 + \frac{63}{2}) \text{ unit}^2 = \frac{58 + 63}{2} \text{ unit}^2 = \frac{121}{2} \text{ unit}^2 \]

2. The base of an equilateral triangle with side \(2a\) lies along the \(y\)-axis such that the mid-point of the base is at the origin. Find vertices of the triangle.

Hint: Pythagoras theorem to \( \triangle AOC \) is \((AC)^2 = (OA)^2 + (OC)^2 \) (\( \angle C = 90^\circ \))

Solution:

Solution step 1: Let \( \triangle ABC \) be the given equilateral triangle with side \(2a\).

Accordingly, \( AB = BC = CA = 2a \).

Assume that base \( BC \) lies along the \(y\)-axis such that the mid-point of \( BC \) is at the origin. i.e., \( BO = OC = a \), where \( O \) is the origin.

Now, it is clear that the coordinates of point \( C \) are \((0, a)\), while the coordinates of point \( B \) are \((0, -a)\).

It is known that the line joining a vertex of an equilateral triangle with the mid-point of its opposite side is perpendicular.

Hence, vertex \( A \) lies on the \(y\)-axis.

On applying Pythagoras theorem to \( \triangle AOC \), we obtain

\[ (AC)^2 = (OA)^2 + (OC)^2 \]
Thus, the vertices of the given equilateral triangle are \((0, a), (0, -a)\) and \((\sqrt{3}a, 0)\) or \((0, a), (0, -a)\) and \((-\sqrt{3}a, 0)\).

3. Find the distance between \(P(x_1, y_1)\) and \(Q(x_2, y_2)\) when:
   (i) \(PQ\) is parallel to the \(y\)-axis,

   \[P(x_1, y_1) \text{ and } Q(x_2, y_2) \text{ then } PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\]

   **Solution:**

   Solution step 1: The given points are \(P(x_1, y_1)\) and \(Q(x_2, y_2)\).

   (i) When \(PQ\) is parallel to the \(y\)-axis, \(x_1 = x_2\).

   In this case, distance between \(P\) and \(Q\) is:
   \[\sqrt{(y_2 - y_1)^2 + (y_2 - y_1)^2} = |y_2 - y_1|\]

   (ii) \(PQ\) is parallel to the \(x\)-axis.

   **Solution:**

   Solution step 1: When \(PQ\) is parallel to the \(x\)-axis, \(y_1 = y_2\).

   In this case, distance between \(P\) and \(Q\) is:
   \[\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = |x_2 - x_1|\]

4. Find a point on the \(x\)-axis, which is equidistant from the points \((7, 6)\) and \((3, 4)\).

   **Hint:** \(P(x_1, y_1)\) and \(Q(x_2, y_2)\) then \(PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\)
Solution:
Solution step 1: Let \((a, 0)\) be the point on the \(x\) axis that is equidistant from the points \((7,6)\) and \((3,4)\).

Accordingly, 
\[
\sqrt{(7-a)^2 + (6-0)^2} = \sqrt{(3-a)^2 + (4-0)^2}
\]
\[
\Rightarrow \sqrt{49 + a^2 - 14a + 36} = \sqrt{9 + a^2 - 6a + 16}
\]
\[
\Rightarrow a^2 - 14a + 85 = a^2 - 6a + 25
\]

On squaring both sides, we obtain 
\[
a^2 - 14a + 85 = a^2 - 6a + 25
\]
\[
\Rightarrow -14a + 6a = 25 - 85
\]
\[
\Rightarrow -8a = -60
\]
\[
\Rightarrow a = \frac{60}{8} = 7.5
\]

Thus, the required point on the \(x\)-axis is \((15/2, 0)\).

5. Find the slope of a line, which passes through the origin and the mid-point of the line segment joining the points \(P(0, -4)\) and \(B(8, 0)\).

Hint: Slope \(m = \frac{y_2 - y_1}{x_2 - x_1}\)

Solution:
Solution step 1: The coordinates of the mid-point of the line segment joining the points 
\(P(0, -4)\) and \(B(8, 0)\) are \(\left(\frac{0+8}{2}, \frac{-4+0}{2}\right) = (4, -2)\).

It is known that the slope \(m\) of a non-vertical line passing through the points \((x_1, y_1)\) and \((x_2, y_2)\) is given by 
\[
m = \frac{y_2 - y_1}{x_2 - x_1}, \quad x_2 \neq x_1.
\]

Therefore, the slope of the line passing through \((0, 0)\) and \((4, -2)\) is 
\[
m = \frac{-2 - 0}{4 - 0} = -\frac{1}{2}
\]

Hence, the required slope of the line is \(\frac{1}{2}\).

6. Without using the Pythagoras theorem, show that the points \((4, 4), (3, 5)\) and \((-1, -1)\) are the vertices of a right-angled triangle.

Hint: Slope \(m = \frac{y_2 - y_1}{x_2 - x_1}\)

Solution:
Solution step 1: The vertices of the given triangle are \(A(4, 4), B(3, 5)\) and \(C(-1, -1)\).
It is known that the slope \( m \) of a non-vertical line passing through the points \((x_1, y_1)\) and \((x_2, y_2)\) is given by

\[
m = \frac{y_2 - y_1}{x_2 - x_1}, \quad x_2 \neq x_1.
\]

\[\therefore \text{Slope of } AB(m_1) = \frac{5 - 4}{3 - 4} = -1
\]

\[
\text{Slope of } BC(m_2) = \frac{-1 - 5}{-1 - 3} = \frac{-6}{-4} = \frac{3}{2}
\]

\[
\text{Slope of } CA(m_3) = \frac{4 + 1}{4 + 1} = \frac{5}{5} = 1
\]

It is observed that \( m_1m_3 = -1 \)

This shows that line segments \(AB\) and \(CA\) are perpendicular to each other i.e., the given triangle is right-angled at \(A(4, 4)\).

Thus, the points \((4, 4), (3, 5)\) and \((-1, -1)\) are the vertices of a right-angled triangle.

7. Find the slope of the line, which makes an angle of 30° with the positive direction of \(Y\)-axis measure clockwise.

Hint: \(\tan 60^\circ = \sqrt{3}\)

\[\text{Solution:}\]

Solution step 1: If a line makes an angle of 30° with the positive direction of the \(Y\)-axis measured anticlockwise, then the angle made by the line with the positive direction of the \(X\)-axis measured anticlockwise is \(90^\circ + 30^\circ = 120^\circ\).

Thus, the slope of the given line is \(\tan 120^\circ = \tan (180^\circ - 60^\circ) = -\tan 60^\circ = -\sqrt{3}\)

8. Find the value of \(x\) for which the points \((x, -1), (2, 1)\) and \((4, 5)\) are collinear.

\[
\text{Hint: Slope } (m) = \frac{y_2 - y_1}{x_2 - x_1}
\]
**Solution:**

Solution step 1: If points \( A(x, -1), B(2, 1) \) and \( C(4, 5) \) are collinear, then

Slope of \( AB = \) Slope of \( BC \)

\[
\frac{1 - (-1)}{2 - x} = \frac{5 - 1}{4 - 2}
\]

\[
\frac{2}{2 - x} = \frac{4}{2} = 2
\]

\[
2 = 4 - 2x
\]

\[
2x = 2
\]

\[
x = 1
\]

Thus, the required value of \( x \) is 1.

9. Without using distance formula, show that points \((-2, -1), (4, 0), (3, 3)\) and \((-3, 2)\) are vertices of a parallelogram.

Hint: Slope

\[
(m) = \frac{y_2 - y_1}{x_2 - x_1}
\]

**Solution:**

Solution step 1: Let points \((-2, -1), (4, 0), (3, 3)\) and \((-3, 2)\) be respectively denoted by \( A, B, C \) and \( D \).

![Parallelogram Diagram]

Slope of \( AB = \frac{0 + 1}{4 + 2} = \frac{1}{6} \)

Slope of \( CD = \frac{2 - 3}{-3 - 3} = -\frac{1}{6} = \frac{1}{6} \)

\( \Rightarrow \) Slope of \( AB = \) Slope of \( CD \)

\( \Rightarrow \) \( AB \) and \( CD \) are parallel to each other.

Now, slope of \( BC = \frac{3 - 0}{3 - 4} = -3 \)

Slope of \( AD = \frac{2 + 1}{-3 + 2} = \frac{3}{-1} = -3 \)

\( \Rightarrow \) Slope of \( BC = \) Slope of \( AD \)

\( \Rightarrow \) \( BC \) and \( AD \) are parallel to each other.

Therefore, both pairs of opposite sides of quadrilateral \( ABCD \) are parallel. Hence, \( ABCD \) is a parallelogram.
Thus points \((-2, -1), (4, 0), (3, 3)\) and \((-3, 2)\) are the vertices of a parallelogram.

10. Find the angle between the \(x\)-axis and the line joining the points \((3, -1)\) and \((4, -2)\).

Hint: Slope 

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

Solution:

Solution step 1: The slope of the line joining the points \((3, -1)\) and \((4, -2)\) is 

\[
m = \frac{-2 - (-1)}{4 - 3} = -2 + 1 = -1
\]

Now, the inclination \((\theta)\) of the line joining the points \((3, -1)\) and \((4, -2)\) is given by 

\[
tan \theta = -1
\]

\[
\Rightarrow \theta = (90^\circ + 45^\circ) = 135^\circ
\]

Thus, the angle between the \(x\)-axis and the line joining the points \((3, -1)\) and \((4, -2)\) is \(135^\circ\).

11. The slope of a line is double of the slope of another line. If tangent of the angle between them is \(\frac{1}{3}\), find the slopes of the lines.

Hint: If \(\theta\) is the angle between the lines \(l_1\) and \(l_2\) with slopes \(m_1\) and \(m_2\), then 

\[
tan \theta = \left| \frac{m_2 - m_1}{1 + m_1m_2} \right|
\]

Solution:

Solution step 1: Let \(m_1\) and \(m\) be the slopes of the two given lines such that \(m_1 = 2m\).

We know that if \(\theta\) is the angle between the lines \(l_1\) and \(l_2\) with slopes \(m_1\) and \(m_2\), then 

\[
tan \theta = \left| \frac{m_2 - m_1}{1 + m_1m_2} \right|
\]

It is given that the tangent of the angle between the two lines is \(\frac{1}{3}\).

\[
\frac{1}{3} = \left| \frac{m - 2m}{1 + (2m) \cdot m} \right|
\]

\[
\Rightarrow \frac{1}{3} = \left| \frac{-m}{1 + 2m^2} \right|
\]

\[
\Rightarrow \frac{1}{3} = \frac{-m}{1 + 2m^2} \quad \text{or} \quad \frac{1}{3} = \frac{m}{1 + 2m^2}
\]

Case I

\[
\frac{1}{3} = \frac{-m}{1 + 2m^2}
\]

\[
\Rightarrow 1 + 2m^2 = -3m
\]

\[
\Rightarrow 2m^2 + 3m + 1 = 0
\]

\[
\Rightarrow 2m^2 + 2m + m + 1 = 0
\]
\[2m(m + 1) + 1(m + 1) = 0\]
\[(m + 1)(2m + 1) = 0\]
\[m = -1 \text{ or } m = -\frac{1}{2}\]

If \(m = -1\), then the slopes of the lines are \(-1\) and \(-2\).

If \(m = -\frac{1}{2}\), then the slopes of the lines are \(-\frac{1}{2}\) and \(-1\).

**Case II**

\[
\frac{1}{3} = \frac{m}{1 + 2m^2}
\]
\[2m^2 + 1 = 3m\]
\[2m^2 - 3m + 1 = 0\]
\[2m^2 - 2m - m + 1 = 0\]
\[2m(m - 1) - 1(m - 1) = 0\]
\[(m - 1)(2m - 1) = 0\]
\[m = 1 \text{ or } m = \frac{1}{2}\]

If \(m = 1\), then the slopes of the lines are \(1\) and \(2\).

If \(m = \frac{1}{2}\), then the slopes of the lines are \(\frac{1}{2}\) and \(1\).

Hence, the slopes of the lines are \(-1\) and \(-2\) or \(-\frac{1}{2}\) and \(-1\) or \(1\) and \(2\) or \(\frac{1}{2}\) and \(1\).

**12.** A line passes through \((x_1, y_1)\) and \((h, k)\). If slope of the line is \(m\), show that \(k - y_1 = m(h - x_1)\).

**Hint:** Slope \(m = \frac{y_2 - y_1}{x_2 - x_1}\)

**Solution:**

Solution step 1: The slope of the line passing through \((x_1, y_1)\) and \((h, k)\) is \(\frac{k - y_1}{h - x_1}\).

It is given that the slope of the line is \(m\).

\[\therefore \frac{k - y_1}{h - x_1} = m\]
\[\Rightarrow k - y_1 = m(h - x_1)\]

Hence, \(k - y_1 = m(h - x_1)\)

**13.** If three point \((h, 0), (a, b)\) and \((0, k)\) lie on a line, show that \(\frac{a}{h} + \frac{b}{k} = 1 \cdot m(h - x_1)\).

**Hint:** Slope \(m = \frac{y_2 - y_1}{x_2 - x_1}\)
**Solution:**

Solution step 1: If the points \( A(h, 0), B(a, b) \) and \( C(0, k) \) lie on a line, then slope of \( AB = \) slope of \( BC \)

\[
\frac{b - 0}{a - h} = \frac{k - b}{0 - a}
\]

\[
\Rightarrow \frac{b}{a - h} = \frac{k - b}{-a}
\]

\[
\Rightarrow -ab = (k - b)(a - h)
\]

\[
\Rightarrow -ab = ka - kh - ab + bh
\]

\[
\Rightarrow ka + bh = kh
\]

On dividing both sides by \( kh \), we obtain

\[
\frac{ka}{kh} + \frac{bh}{kh} = \frac{kh}{kh}
\]

\[
\Rightarrow \frac{a}{h} + \frac{b}{k} = 1
\]

Hence, \( \frac{a}{h} + \frac{b}{k} = 1 \)

---

14. Consider the given population and year graph. Find the slope of the line \( AB \) and using it, find what will be the population in the year 2010?

Hint: Slope \( m = \frac{y_2 - y_1}{x_2 - x_1} \)

**Solution:**

Solution step 1: Since line \( AB \) passes through points \( A(1985, 92) \) and \( B(1995, 97) \), its slope is

\[
\text{Slope of } AB = \frac{97 - 92}{1995 - 1985} = \frac{5}{10} = \frac{1}{2}
\]

Let \( Y \) be the population in the year 2010. Then, according to the given graph, line \( AB \) must pass through point \( C(2010, y) \).

\[
\Rightarrow \text{Slope of } AB = \text{Slope of } BC
\]

\[
\Rightarrow \frac{1}{2} = \frac{y - 97}{2010 - 1995}
\]

\[
\Rightarrow \frac{1}{2} = \frac{y - 97}{15}
\]
Thus, the slope of line $AB$ is $\frac{1}{2}$ while in the year 2010, the population will be 104.5 crores.

**Exercise 10.2**

1. Write the equations for the $x$ and $y$-axes.
   Hint: Every point of $y$ is 0 on $x$ axis and every point of $x$ is 0 on $y$ axis.

   **Solution:**
   Solution step 1: The $y$-coordinate of every point on the $x$-axis is 0.
   Therefore, the equation of the $x$-axis is $y = 0$.
   The $x$-coordinate of every point on the $y$-axis is 0.
   Therefore, the equation of the $y$-axis is $x = 0$.

2. Find the equation of the line which passes through the point $(-4, 3)$ with slope $\frac{1}{2}$.
   Hint: Equation of line: $(y - y_0) = m(x - x_0)$

   **Solution:**
   Solution step 1: We know that the equation of the line passing through point $(x_0, y_0)$, whose slope is $m$, is $(y - y_0) = m(x - x_0)$
   Thus, the equation of the line passing through point $(-4, 3)$, whose slope is $\frac{1}{2}$ is
   
   $(y - 3) = \frac{1}{2}(x + 4)$
   $2(y - 3) = x + 4$
   $2y - 6 = x + 4$
   i.e., $x - 2y + 10 = 0$

3. Find the equation of the line which passes through $(0, 0)$ with slope $m$.
   Hint: Equation of line: $(y - y_0) = m(x - x_0)$

   **Solution:**
   Solution step 1: We know that the equation of the line passing through point $(x_0, y_0)$, whose slope is $m$, is
   $(y - y_0) = m(x - x_0)$. 
Thus, the equation of the line passing through point \((0, 0)\), whose slope is \(m\), is
\[(y - 0) = m(x - 0)\]
i.e., \(y = mx\)

4. Find the equation of the line which passes through \((2, 2\sqrt{3})\) and is inclined with the \(x\)-axis at an angle of \(75^\circ\).

Hint: Equation of line: \((y - y_0) = m(x - x_0)\)

**Solution:**

Solution step 1: The slope of the line that inclines with the \(x\)-axis at an angle of \(75^\circ\) is \(m = \tan 75^\circ\)

\[
\Rightarrow m = \tan (45^\circ + 30^\circ) = \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \cdot \tan 30^\circ} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \cdot \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}
\]

We know that the equation of the line passing through point \((x_0, y_0)\) whose slope is \(m\), is \((y - y_0) = m(x - x_0)\).

Thus, if a line passes through \((2, 2\sqrt{3})\) and inclines with the \(x\)-axis at an angle of \(75^\circ\), then the equation of the line is given as

\[
(y - 2\sqrt{3}) = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}(x - 2)
\]

\[
(y - 2\sqrt{3})(\sqrt{3} - 1) = (\sqrt{3} + 1)(x - 2)
\]

\[
y(\sqrt{3} - 1) - 2\sqrt{3}(\sqrt{3} - 1) = x(\sqrt{3} + 1) - 2(\sqrt{3} + 1)
\]

\[
(\sqrt{3} + 1)x - (\sqrt{3} - 1)y = 2\sqrt{3} + 2 - 6 + 2\sqrt{3}
\]

\[
(\sqrt{3} + 1)x - (\sqrt{3} - 1)y = 4\sqrt{3} - 4
\]

i.e., \((\sqrt{3} + 1)x - (\sqrt{3} - 1)y = 4(\sqrt{3} - 1)\)

5. Find the equation of the line which intersects the \(x\)-axis at a distance of 3 units to the left of origin with slope \(-2\).

Hint: A line with slope \(m\) makes \(x\)-intercept \(d\), then the equation of the line is given as \(y = m(x - d)\)

**Solution:**

Solution step 1: It is known that if a line with slope \(m\) makes \(x\)-intercept \(d\), then the equation of the line is given as \(y = m(x - d)\)

For the line intersecting the \(x\)-axis at a distance of 3 units to the left of the origin, \(d = -3\).

The slope of the line is given as \(m = -2\)

Thus, the required equation of the given line is \(y = -2[x - (-3)]\) \(y = -2x - 6\)

i.e., \(2x + y + 6 = 0\)
6. Find the equation of the line which intersects the \(y\)-axis at a distance of 2 units above the origin and makes an angle of \(30^\circ\) with the positive direction of the \(x\)-axis.
Hint: A line with slope \(m\) makes \(y\)-intercept \(c\), then the equation of the line is given as \(y = mx + c\).

Solution:
Solution step 1: It is known that if a line with slope \(m\) makes \(y\)-intercept \(c\), then the equation of the line is given as \(y = mx + c\).

Here, \(c = 2\) and \(m = \tan 30^\circ = \frac{1}{\sqrt{3}}\).

Thus, the required equation of the given line is

\[ y = \frac{1}{\sqrt{3}}x + 2 \]

\[ y = \frac{\sqrt{3}x + 6}{\sqrt{3}} \]

\[ \sqrt{3}y = x + 2\sqrt{3} \]

i.e., \(x - \sqrt{3}y + 2\sqrt{3} = 0\)

7. Find the equation of the line which passes through the points \((-1, 1)\) and \((2, -4)\).

Hint: The equation of the line passing through points \((x_1, y_1)\) and \((x_2, y_2)\) is \(y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)\).

Solution:
Solution step 1: It is known that the equation of the line passing through points \((x_1, y_1)\) and \((x_2, y_2)\) is

\[ y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \]

Therefore, the equation of the line passing through the points \((-1, 1)\) and \((2, -4)\) is

\[ (y - 1) = \frac{-4 - 1}{2 + 1} (x + 1) \]

\[ (y - 1) = \frac{-5}{3} (x + 1) \]

\[ 3(y - 1) = -5(x + 1) \]

\[ 3y - 3 = -5x - 5 \]

i.e., \(5x + 3y + 2 = 0\)

8. Find the equation of the line which is at a perpendicular distance of 5 units from the origin and the angle made by the perpendicular with the positive \(x\)-axis is \(30^\circ\).

Hint: If \(P\) is the length of the normal from the origin to a line and \(\omega\) is the angle made by the normal with the positive direction of the \(x\)-axis, then the equation of the line is given by \(\cos \omega x + \sin \omega y = P\).

Practice more on Straight Lines
Solution:
Solution step 1: If \( P \) is the length of the normal from the origin to a line and \( \omega \) is the angle made by the normal with the positive direction of the \( x \)-axis, then the equation of the line is given by \( x \cos \omega + y \sin \omega = p. \)

Here, \( p = 5 \) units and \( \omega = 30^\circ \)
Thus, the required equation of the given line is \( x \)
\[
\cos 30^\circ + y \sin 30^\circ = 5
\]
\[
\frac{x \sqrt{3}}{2} + \frac{y}{2} = 5
\]
i.e., \( \sqrt{3}x + y = 10 \)

9. The vertices of \( \Delta PQR \) are \( P(2,1), Q(-2,3) \) and \( R(4,5) \). Find equation of the median through the vertex \( R \).

Hint: The equation of the line passing through points \( (x_1, y_1) \) and \( (x_2, y_2) \) is \( y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \)

Solution:
Solution step 1: It is given that the vertices of \( \Delta PQR \) are \( P(2,1), Q(-2,3) \) and \( R(4,5) \).
Let \( RL \) be the median through vertex \( R \).
Accordingly, \( L \) is the mid-point of \( PQ \).

By mid-point formula, the coordinates of point \( L \) are given by \( \left( \frac{2 + (-2)}{2}, \frac{1 + 3}{2} \right) = (0, 2) \)

It is known that the equation of the line passing through points \( (x_1, y_1) \) and \( (x_2, y_2) \) is \( y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \)

Therefore, the equation of \( RL \) can be determined by substituting \( (x_1, y_1) = (4, 5) \) and \( (x_2, y_2) = (0, 2) \).

Hence, \( y - 5 = \frac{2 - 5}{0 - 4}(x - 4) \)
\[\Rightarrow y - 5 = \frac{-3}{-4}(x - 4)\]
\[\Rightarrow 4(y - 5) = 3(x - 4)\]
\[\Rightarrow 4y - 20 = 3x - 12\]
\[\Rightarrow 3x - 4y + 8 = 0\]
Thus, the required equation of the median through vertex \( R \) is \( 3x - 4y + 8 = 0 \).
10. Find the equation of the line passing through \((-3, 5)\) and perpendicular to the line through the points \((2, 5)\) and \((-3, 6)\).

Hint: Equation of line: \((y - y_0) = m(x - x_0)\)

Solution:

Solution step 1: The slope of the line joining the points \((2, 5)\) and \((-3, 6)\) is \(m = \frac{6 - 5}{-3 - 2} = \frac{1}{-5}\)

We know that two non-vertical lines are perpendicular to each other if and only if their slopes are negative reciprocals of each other.

Therefore, slope of the line perpendicular to the line through the points\((-3, 6)\) and \((2, 5)\)

\((-3, 6)\) \(\Rightarrow\) \(-m = -\frac{1}{5}\) \(\Rightarrow\) \(m = \frac{1}{5}\)

Now, the equation of the line passing through point \((-3, 5)\), whose slope is \(\frac{1}{5}\), is

\((y - 5) = 5(x + 3)\)
\(y - 5 = 5x + 15\)
i.e., \(5x - y + 20 = 0\)

11. A line perpendicular to the line segment joining the points \((1, 0)\) and \((2, 3)\) divides it in the ratio \(1:n\). Find the equation of the line.

Hint: section formula

Solution:

Solution step 1: According to the section formula, the coordinates of the point that divides the line segment joining the points \((1, 0)\) and \((2, 3)\) in the ratio \(1:n\) is given by

\(\left(\frac{m(x_1) + 1(x_2)}{1+n}, \frac{n(y_1) + 1(y_2)}{1+n}\right)\)

The slope of the line joining the points \((1, 0)\) and \((2, 3)\) is

\(m = \frac{3 - 0}{2 - 1} = 3\)

We know that two non-vertical lines are perpendicular to each other if and only if their slopes are negative reciprocals of each other.

Therefore, slope of the line that is perpendicular to the line joining the points \((1, 0)\) and \((2, 3)\) is \(-\frac{1}{3}\)

Now, the equation of the line passing through \((\frac{n+2}{n+1}, 3)\) and whose slope is \(-\frac{1}{3}\) is given by

\(-\frac{1}{3} = \frac{3 - \frac{n+2}{n+1}}{n+1}\)
\(\Rightarrow 3[(n+1)y - 3] = -(n+1)x + n + 2\)
\(\Rightarrow (1+n)x + 3(1+n)y = n + 11\)
12. Find the equation of a line that cuts off equal intercepts on the coordinate axes and passes through the point \((2, 3)\).

The equation of a line in the intercept form is \( \frac{x}{a} + \frac{y}{b} = 1 \).

Solution:

Solution step 1: The equation of a line in the intercept form is \( \frac{x}{a} + \frac{y}{b} = 1 \) \(\text{...(i)}\)

Here, \(a\) and \(b\) are the intercepts on \(x\) and \(y\) axes respectively. It is given that the line cuts off equal intercepts on both the axes. This means that \(a = b\). Accordingly, equation \(\text{(i)}\) reduces to

\[ \frac{x}{a} + \frac{y}{a} = 1 \]

\[ \Rightarrow x + y = a \quad \text{...(ii)} \]

Since the given line passes through point \((2, 3)\), equation \(\text{(ii)}\) reduces to \(2 + 3 = a \Rightarrow a = 5\)

On substituting the value of \(a\) in equation \(\text{(ii)}\), we obtain \(x + y = 5\), which is the required equation of the line.

13. Find equation of the line passing through the point \((2, 2)\) and cutting off intercepts on the axes whose sum is 9.

Hint: The equation of a line in the intercept form is \( \frac{x}{a} + \frac{y}{b} = 1 \).

Solution:

Solution step 1: The equation of a line in the intercept form is \( \frac{x}{a} + \frac{y}{b} = 1 \) \(\text{...(i)}\)

Here, \(a\) and \(b\) are the intercepts on \(x\) and \(y\) axes respectively.

It is given that \(a + b = 9 \Rightarrow b = 9 - a\) \(\text{...(ii)}\) from equations \(\text{(i)}\) and \(\text{(ii)}\), we obtain

\[ \frac{x}{a} + \frac{y}{9 - a} = 1 \quad \text{...(iii)} \]

It is given that the line passes through point \((2, 2)\). Therefore, equation \(\text{(iii)}\) reduces to \(\frac{2}{a} + \frac{2}{9 - a} = 1 \)

\[ \Rightarrow \frac{1}{a} + \frac{1}{9 - a} = 1 \]

\[ \Rightarrow \frac{18}{a(9 - a)} = 1 \]

\[ \Rightarrow 18 = a(9 - a) \]

\[ \Rightarrow 18 = 9a - a^2 \]

\[ \Rightarrow a^2 - 9a + 18 = 0 \]

\[ \Rightarrow a^2 - 6a - 3a + 18 = 0 \]

\[ \Rightarrow a(\alpha - 6) - 3(\alpha - 6) = 0 \]

\[ \Rightarrow (\alpha - 6)(\alpha - 3) = 0 \]

\[ \Rightarrow \alpha = 6 \text{ or } \alpha = 3 \]

If \(a = 6\) and \(b = 9 - 6 = 3\), then the equation of the line is
14. Find equation of the line through the point \((0, 2)\) making an angle \(\frac{2\pi}{3}\) with the positive \(x\)-axis. Also, find the equation of line parallel to it and crossing the \(y\)-axis at a distance of 2 units below the origin.

Hint: Equation of line: \((y - y_0) = m(x - x_0)\)

**Solution:**

Solution step 1: The slope of the line making an angle \(\frac{2\pi}{3}\) with the positive \(x\)-axis is 
\[
m = \tan \left(\frac{2\pi}{3}\right) = -\sqrt{3}
\]

Now, the equation of the line passing through point \((0, 2)\) and having a slope \(-\sqrt{3}\) is 
\[
(y - 2) = -\sqrt{3}(x - 0)
\]
\[
y - 2 = -\sqrt{3}x
\]
i.e., \(\sqrt{3}x + y - 2 = 0\).

The slope of line parallel \(\sqrt{3}x + y - 2 = 0\) is \(-\sqrt{3}\) to line.

It is given that the line parallel to line \(\sqrt{3}x + y - 2 = 0\) crosses the \(y\)-axis 2 units below the origin i.e., it passes through point \((0, -2)\).

Hence, the equation of the line passing through point \((0, -2)\) and having a slope \(-\sqrt{3}\) is 
\[
y - (-2) = -\sqrt{3}(x - 0)
\]
\[
y + 2 = -\sqrt{3}x
\]
\[
\sqrt{3}x + y + 2 = 0
\]

15. The perpendicular from the origin to a line meets it at the point \((-2, 9)\), find the equation of the line.

Hint: Equation of line: \((y - y_0) = m(x - x_0)\)

**Solution:**

Solution step 1: The slope of the line joining the origin \((0, 0)\) and point \((-2, 9)\) is 
\[
m_1 = \frac{9 - 0}{-2 - 0} = -\frac{9}{2}
\]

Accordingly, the slope of the line perpendicular to the line joining the origin and point \((-2, 9)\) is 
\[
m_2 = -\frac{1}{m_1} = -\frac{1}{-\frac{9}{2}} = \frac{2}{9}
\]

Now, the equation of the line passing through point \((-2, 9)\) and having a slope \(m_2\) is 
\[
(y - 9) = \frac{2}{9}(x + 2)
\]
\[
9y - 81 = 2x + 4
\]
i.e., \(2x - 9y + 85 = 0\)
16. The length \( L \) (in centimetre) of a copper rod is a linear function of its Celsius temperature \( C \). In an experiment, if \( L = 124.942 \) when \( C = 20 \) and \( L = 125.134 \) when \( C = 110 \), express \( L \) in terms of \( C \).

Hint: Equation of line: \((y - y_0) = m(x - x_0)\)

Solution:

Solution step 1: It is given that when \( C = 20 \), the value of \( L \) is 124.942, whereas when \( C = 110 \), the value of \( L \) is 125.134.

Accordingly, points (20,124.942) and (110, 125.134) satisfy the linear relation between \( L \) and \( C \).

Now, assuming \( C \) along the \( x \)-axis and \( L \) along the \( y \)-axis, we have two points i.e., (20, 124.942) and (110, 125.134) in the \( XY \) plane.

Therefore, the linear relation between \( L \) and \( C \) is the equation of the line passing through points (20, 124.942) and (110, 125.134).

\[
(L - 124.942) = \frac{125.134 - 124.942}{110 - 20}(C - 20)
\]

\[
L - 124.942 = \frac{0.192}{90}(C - 20)
\]

i.e. \( L = \frac{0.192}{90}(C - 20) + 124.942 \), which is the required linear relation.

17. The owner of a milk store finds that, he can sell 980 litres of milk each week at ₹14/litre and 1220 litres of milk each week at ₹16/litre. Assuming a linear relationship between selling price and demand, how many litres could he sell weekly at ₹17/litre?

Hint: Equation of line: \((y - y_0) = m(x - x_0)\)

Solution:

Solution step 1: The relationship between selling price and demand is linear.

Assuming selling price per litre along the \( x \)-axis and demand along the \( y \)-axis, we have two points i.e., (14, 980) and (16, 1220) in the \( XY \) plane that satisfy the linear relationship between selling price and demand.

Therefore, the linear relationship between selling price per litre and demand is the equation of the line passing through points (14, 980) and (16, 1220).

\[
y - 980 = \frac{1220 - 980}{16 - 14}(x - 14)
\]

\[
y - 980 = \frac{240}{2}(x - 14)
\]

\[
y - 980 = 120(x - 14)
\]

i.e., \( y = 120(x - 14) + 980 \)

When \( x = ₹17/litre \),

\[
y = 120(17 - 14) + 980
\]

\[
y = 120 \times 3 + 980 = 360 + 980 = 1340
\]

Thus, the owner of the milk store could sell 1340 litres of milk weekly at ₹17/litre.
18. \( P(a, b) \) is the mid-point of a line segment between axes. Show that equation of the line is \( \frac{x}{a} + \frac{y}{b} = 2 \)

Hint: Equation of line: \( (y - y_0) = m(x - x_0) \)

Solution:
Solution step 1: Let \( AB \) be the line segment between the axes and let \( P(a, b) \) be its mid-point.

Let the coordinates of \( A \) and \( B \) be \((0, y)\) and \((x, 0)\) respectively.
Since \( P(a, b) \) is the mid-point of \( AB \),
\[
\left( \frac{0 + x}{2}, \frac{y + 0}{2} \right) = (a, b)
\]
\[
\Rightarrow \left( \frac{x}{2}, \frac{y}{2} \right) = (a, b)
\]
\[
\Rightarrow \frac{x}{2} = a \text{ and } \frac{y}{2} = b
\]
\[
\therefore x = 2a \text{ and } y = 2b
\]
Thus, the respective coordinates of \( A \) and \( B \) are \((0, 2b)\) and \((2a, 0)\).
The equation of the line passing through points \((0, 2b)\) and \((2a, 0)\) is
\[
(y - 2b) = \frac{(0 - 2b)}{(2a - 0)}(x - 0)
\]
\[
y - 2b = \frac{-2b}{2a}(x)
\]
\[
a(y - 2b) = -bx
\]
\[
ay - 2ab = -bx
\]
i.e., \( bx + ay = 2ab \)
On dividing both sides by \( ab \), we obtain
\[
\frac{bx}{ab} + \frac{ay}{ab} = \frac{2ab}{ab}
\]
\[
\frac{x}{a} + \frac{y}{b} = 2
\]
Thus, the equation of the line is \( \frac{x}{a} + \frac{y}{b} = 2 \).

19. Point \( R(h, k) \) divides a line segment between the axes in the ratio \( 1:2 \). Find equation of the line.

Hint: Equation of line: \( (y - y_0) = m(x - x_0) \)
Solution:
Solution step 1: Let $AB$ be the line segment between the axes such that point $R(h, k)$ divides $AB$ in the ratio 1:2.

Let the respective coordinates of $A$ and $B$ be $(x, 0)$ and $(0, y)$.

Since point $R(h, k)$ divides $AB$ in the ratio 1:2, according to the section formula,

$$h = \frac{1 \times 0 + 2 \times x}{1 + 2}, \quad k = \frac{1 \times y + 2 \times 0}{1 + 2}$$

$$\Rightarrow h = \frac{2x}{3}, \quad k = \frac{y}{3}$$

$$\Rightarrow x = \frac{3h}{2}, \quad y = 3k$$

Therefore, the respective coordinates of $A$ and $B$ are $\left(\frac{3h}{2}, 0\right)$ and $(0, 3k)$.

Now, the equation of line $AB$ passing through points $\left(\frac{3h}{2}, 0\right)$ and $(0, 3k)$ is

$$y - 0 = \frac{3k - 0}{0 - \frac{3h}{2}} \left(x - \frac{3h}{2}\right)$$

$$\Rightarrow y = \frac{2k}{h} \left(x - \frac{3h}{2}\right)$$

$$\Rightarrow hy = 2kx + 3hk$$

i.e., $2kx + hy = 3hk$

Thus, the required equation of the line is $2kx + hy = 3hk$

20. By using the concept of equation of a line, prove that the three points $(3, 0), (-2, -2)$ and $(8, 2)$ are collinear.

Hint: Equation of line: $(y - y_0) = m(x - x_0)$

Solution:
Solution step 1: In order to show that points $(3, 0), (-2, -2)$ and $(8, 2)$ are collinear, it suffices to show that the line passing through points $(3, 0)$ and $(-2, -2)$ also passes through point $(8, 2)$.

The equation of the line passing through points $(3, 0)$ and $(-2, -2)$ is

$$(y - 0) = \frac{(-2 - 0)}{(-2 - 3)}(x - 3)$$
\[ y = \frac{-2}{-5} (x - 3) \]
\[ 5y = 2x - 6 \]
i.e., \[ 2x - 5y = 6 \]
It is observed that at \[ x = 8 \text{ and } y = 2, \]
L.H.S. \[ = 2 \times 8 - 5 \times 2 = 16 - 10 = 6 = \text{R.H.S.} \]
Therefore, the line passing through points \((3, 0)\) and \((-2, -2)\) also passes through point \((8, 2)\).
Hence, points \((3, 0), (-2, -2)\) and \((8, 2)\) are collinear.

**Exercise 10.3**

1. Reduce the following equations into slope-intercept form and find their slopes and the \(Y\) intercepts.
   (i) \[ x + 7y = 0 \]
   Hint: Slope intercept form \[ y = mx + c \]
   **Solution:**
   Solution step 1: The given equation is \( x + 7y = 0 \). It can be written as
   \[ y = -\frac{1}{7}x + 0 \] ...(i)
   This equation is of the form \( y = mx + c \), where \( m = -\frac{1}{7} \) and \( c = 0 \).
   Therefore, equation (i) is in the slope-intercept form, where the slope and the \(Y\)-intercept are \( -\frac{1}{7} \) and \( 0 \) respectively.
   (ii) \[ 6x + 3y - 5 = 0 \]
   Hint: Slope intercept form \[ y = mx + c \]
   **Solution:**
   Solution step 1: The given equation is \( 6x + 3y - 5 = 0 \).
   It can be written as
   \[ y = \frac{1}{3}(-6x + 5) \]
   \[ y = -2x + \frac{5}{3} \] ...(ii)
   This equation is of the form \( y = mx + c \), where \( m = -2 \) and \( c = \frac{5}{3} \).
   Therefore, equation (ii) is in the slope-intercept form, where the slope and the \(Y\)-intercept are \(-2\) and \(\frac{5}{3}\) respectively.
   (iii) \[ y = 0 \]
Hint: Slope intercept form $y = mx + c$

**Solution:**
Solution step 1: The given equation is $y = 0$.
It can be written as

$$y = 0 \cdot x + 0 \quad \ldots(\text{iii})$$

This equation is of the form $y = mx + c$, where $m = 0 \text{ and } c = 0$.
Therefore, equation (iii) is in the slope-intercept form, where the slope and the $Y$-intercept are $0$ and $0$ respectively.

2. Reduce the following equations into intercept form and find their intercepts on the axes.
   (i) $3x + 2y - 12 = 0$
   Hint: Intercepts on the axes form $\frac{x}{a} + \frac{y}{b} = 1$.

**Solution:**
Solution step 1: The given equation is $3x + 2y - 12 = 0$
It can be written as

$$3x + 2y = 12$$
$$\frac{3x}{12} + \frac{2y}{12} = 1$$
$$\frac{x}{4} + \frac{y}{6} = 1 \quad \ldots(\text{i})$$

This equation is of the form $\frac{x}{a} + \frac{y}{b} = 1$, where $a = 4 \text{ and } b = 6$.
Therefore, equation (i) is in the intercept form, where the intercepts on the $x$ and $Y$ axes are $4$ and $6$ respectively.

(ii) $4x - 3y = 6$
Hint: Intercepts on the axes form $\frac{x}{a} + \frac{y}{b} = 1$.

**Solution:**
Solution step 1: The given equation is $4x - 3y = 6$
It can be written as

$$4x - 3y = 6$$
$$\frac{4x}{6} - \frac{3y}{6} = 1$$
$$\frac{2x}{3} - \frac{y}{2} = 1$$
$$\frac{x}{(\frac{3}{2})} + \frac{y}{(-\frac{2}{3})} = 1 \quad \ldots(\text{ii})$$

This equation is of the form $\frac{x}{a} + \frac{y}{b} = 1$, where $a = \frac{3}{2} \text{ and } b = -2$. 

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Therefore, equation (ii) is in the intercept form, where the intercepts on the $x$ and $y$ axes are $\frac{3}{2}$ and $-2$ respectively.

(iii) $3y + 2 = 0$

Hint: Intercepts on the axes form $\frac{x}{a} + \frac{y}{b} = 1$.

Solution:
Solution step 1: The given equation is $3y + 2 = 0$.
It can be written as

$$3y = -2$$

$$\frac{y}{-2} = 1$$

i.e., $\left(-\frac{3}{2}\right)$ …(iii)

The equation is of the form $\frac{x}{a} + \frac{y}{b} = 1$, where $a = 0$ and $b = -\frac{2}{3}$.

Therefore, equation (iii) is in the intercept form, where the intercept on the $y$-axis $-\frac{2}{3}$ is and it has no intercept on the $x$-axis.

3. Reduce the following equations into normal form. Find their perpendicular distances from the origin and angle between perpendicular and the positive $x$-axis.
(i) $x - \sqrt{3}y + 8 = 0$

Hint: Normal form equation $y - y_1 = m(x - x_1)$

Solution:
Solution step 1: The given equation is $x - \sqrt{3}y + 8 = 0$.
It can be reduced as:

$x - \sqrt{3}y = -8$

$\Rightarrow x + \sqrt{3}y = 8$

On dividing both sides by $\sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{4} = 2$, we obtain

$$\frac{x}{2} + \frac{\sqrt{3}}{2}y = \frac{8}{2}$$

$\Rightarrow \left(-\frac{1}{2}\right)x + \left(\frac{\sqrt{3}}{2}\right)y = 4$

$\Rightarrow x\cos 120^\circ + y\sin 120^\circ = 4$ …(i)

Equation (i) is in the normal form.

On comparing equation (i) with the normal form of equation of line $xcos \omega + ysin \omega = p$, we obtain

$\omega = 120^\circ$ and $p = 4$.

Thus, the perpendicular distance of the line from the origin is 4, while the angle between the perpendicular and the positive $x$-axis is $120^\circ$. 
(ii) $y - 2 = 0$
Hint: Normal form equation $= y - y_1 = m(x - x_1)$

**Solution:**
Solution step 1: The given equation is $y - 2 = 0$.
It can be reduced as $0x + 1y = 2$.
On dividing both sides by $\sqrt{0^2 + 1^2} = 1$, we obtain $0x + 1y = 2$
$\Rightarrow x\cos 90^\circ + y\sin 90^\circ = 2 \quad ...(i)$
Equation (i) is in the normal form.
On comparing equation (i) with the normal form of equation of line $x\cos \omega + y\sin \omega = p$, we obtain
$\omega = 90^\circ$ and $p = 2$.
Thus, the perpendicular distance of the line from the origin is $2$, while the angle between the perpendicular and the positive $x$-axis is $90^\circ$.

(iii) $x - y = 4$
Hint: Normal form equation $= y - y_1 = m(x - x_1)$

**Solution:**
Solution step 1: The given equation is $x - y = 4$.
It can be reduced as $1x + (-1)y = 4$.
On dividing both sides by $\sqrt{1^2 + (-1)^2} = \sqrt{2}$, we obtain
$\frac{1}{\sqrt{2}}x + \left( -\frac{1}{\sqrt{2}} \right)y = \frac{4}{\sqrt{2}}$
$\Rightarrow x\cos \left( 2\pi - \frac{\pi}{4} \right) + y\sin \left( 2\pi - \frac{\pi}{4} \right) = 2\sqrt{2}$
$\Rightarrow x\cos 315^\circ + y\sin 315^\circ = 2\sqrt{2} \quad ...(i)$
Equation (i) is in the normal form.
On comparing equation (i) with the normal form of equation of line $x\cos \omega + y\sin \omega = p$, we obtain $\omega = 315^\circ$ and $p = 2\sqrt{2}$.
Thus, the perpendicular distance of the line from the origin is $2\sqrt{2}$, while the angle between the perpendicular and the positive $x$-axis is $315^\circ$.

4. Find the distance of the point $(-1, 1)$ from the line $12(x + 6) = 5(y - 2)$.
Hint: The perpendicular distance $(d)$ of a line $Ax + By + C = 0$ from a point $(x_1, y_1)$ is given by
$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$
Solution:

Solution step 1: The given equation of the line is $12(x + 6) = 5(y - 2)$.

\[\Rightarrow 12x + 72 = 5y - 10\]
\[\Rightarrow 12x - 5y + 82 = 0 \quad \cdots \text{(i)}\]

On comparing equation (i) with general equation of line $Ax + By + C = 0$, we obtain $A = 12$, $B = -5$, and $C = 82$.

It is known that the perpendicular distance $(d)$ of a line $Ax + By + C = 0$ from a point $(x_1, y_1)$ is given by

\[d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.

The given point is $(x_1, y_1) = (-1, 1)$.

Therefore, the distance of point $(-1, 1)$ from the given line

\[= \frac{|12(-1) + (-5)(1) + 82|}{\sqrt{(12)^2 + (-5)^2}} = \frac{|-12 - 5 + 82|}{\sqrt{169}} = \frac{65}{13} \text{ units} = 5 \text{ units}.

5. Find the points on the $x$-axis, whose distances from the line $\frac{x}{3} + \frac{y}{4} = 1$ are 4 units.

Hint: the perpendicular distance $(d)$ of a line $Ax + By + C = 0$ from a point $(x_1, y_1)$ is given by

\[d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.

Solution:

Solution step 1: The given equation of line is $\frac{x}{3} + \frac{y}{4} = 1$

or $4x + 3y - 12 = 0 \quad \cdots \text{(i)}$

On comparing equation (i) with general equation of line $Ax + By + C = 0$, we obtain $A = 4$, $B = 3$, and $C = -12$.

Let $(a, 0)$ be the point on the $x$-axis whose distance from the given line is 4 units.

It is known that the perpendicular distance $(d)$ of a line $Ax + By + C = 0$ from a point $(x_1, y_1)$ is given by

\[d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.

Therefore,

\[4 = \frac{|4a + 3 \times 0 - 12|}{\sqrt{4^2 + 3^2}}\]
\[\Rightarrow 4 = \frac{|4a - 12|}{5}\]
\[\Rightarrow |4a - 12| = 20\]
\[\Rightarrow \pm (4a - 12) = 20\]
\[\Rightarrow (4a - 12) = 20 \text{ or } - (4a - 12) = 20\]
\[\Rightarrow 4a = 20 + 12 \text{ or } 4a = -20 + 12\]
\[\Rightarrow a = 8 \text{ or } -2\]

Thus, the required points on the $x$-axis are $(−2, 0)$ and $(8, 0)$.  

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6. Find the distance between parallel lines
   (i) $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$
   Hint: the perpendicular distance ($d$) of a line $Ax + By + C = 0$ from a point $(x_1, y_1)$ is given by
   $$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$ 

   Solution:
   Solution step 1: It is known that the distance ($d$) between parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is given by
   $$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}.$$ 

   The given parallel lines are $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$. Here, $A = 15$, $B = 8$, $C_1 = -34$ and $C_2 = 31$. Therefore, the distance between the parallel lines is
   $$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}} = \frac{|-34 - 31|}{\sqrt{(15)^2 + (8)^2}} = \frac{65}{\sqrt{337}} = \frac{65}{17} \text{ units}.$$ 

   (ii) $l(x + y) + p = 0$ and $l(x + y) - r = 0$
   Hint: the perpendicular distance ($d$) of a line $Ax + By + C = 0$ from a point $(x_1, y_1)$ is given by
   $$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$ 

   Solution:
   Solution step 1 : (ii) The given parallel lines are $l(x + y) + p = 0$ and $l(x + y) - r = 0$. Here, $A = l$, $B = l$, $C_1 = p$ and $C_2 = -r$. Therefore, the distance between the parallel lines is
   $$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}} = \frac{|p + r|}{\sqrt{l^2 + l^2}} = \frac{|p + r|}{\sqrt{2l^2}} = \frac{|p + r|}{l\sqrt{2}} \text{ units}.$$ 

7. Find equation of the line parallel to the line $3x - 4y + 2 = 0$ and passing through the point $(-2, 3)$.
   Hint: Normal form equation $\frac{y - y_1}{l} = m(x - x_1)$

   Solution:
   Solution step 1: The equation of the given line is $3x - 4y + 2 = 0$
which is of the form $y = mx + c$

∴ Slope of the given line $= \frac{3}{4}$

It is known that parallel lines have the same slope.

∴ Slope of the other line $= m = \frac{3}{4}$

Now, the equation of the line that has a slope of $\frac{3}{4}$ and passes through the point $(-2, 3)$ is

$$(y - 3) = \frac{3}{4}(x - (-2))$$

$4y - 12 = 3x + 6$

i.e., $3x - 4y + 18 = 0$

8. Find equation of the line perpendicular to the line $x - 7y + 5 = 0$ and having $x$-intercept $3$.

Hint: Normal form equation $= y - y_1 = m(x - x_1)$

Solution:

Solution step 1: The given equation of line is $x - 7y + 5 = 0$ or $y = \frac{1}{7}x + \frac{5}{7}$

which is of the form $y = mx + c$

∴ Slope of the given line $= \frac{1}{7}$

The slope of the line perpendicular to the line having a slope of $\frac{1}{7}$ is $-7$.

The equation of the line with slope $-7$ and $x$-intercept $3$ is given by $y = m(x - d)$

$\Rightarrow y = -7(x - 3)$

$\Rightarrow y = -7x + 21$

$\Rightarrow 7x + y = 21$

9. Find angles between the lines $\sqrt{3}x + y = 1$ and $x + \sqrt{3}y = 1$

Hint: Normal form equation $= y - y_1 = m(x - x_1)$

Solution:

Solution step 1: The given lines are $\sqrt{3}x + y = 1$ and $x + \sqrt{3}y = 1$.

$y = -\sqrt{3}x + 1 \quad \ldots(i)$ and

$y = -\frac{1}{\sqrt{3}}x + \frac{1}{\sqrt{3}} \quad \ldots(ii)$
The slope of line (i) is \( m_1 = -\sqrt{3} \), while the slope of line (ii) is \( m_2 = -\frac{1}{\sqrt{3}} \).

The acute angle i.e., \( \theta \) between the two lines is given by

\[
\tan \theta = \frac{m_1 - m_2}{1 + m_1m_2}
\]

\[
\tan \theta = \frac{-\sqrt{3} + \frac{1}{\sqrt{3}}}{1 + (-\sqrt{3})\left(-\frac{1}{\sqrt{3}}\right)}
\]

\[
\tan \theta = \frac{-3 + 1}{1 + 1} + \left| \frac{-2}{2 \times \sqrt{3}} \right|
\]

\[
\tan \theta = \frac{1}{\sqrt{3}}
\]

\[\theta = 30^\circ\]

Thus, the angle between the given lines is either 30° or 180° - 30° = 150°.

10. The line through the points \((h, 3)\) and \((4, 1)\) intersects the line \(7x - 9y - 19 = 0\) at right angle. Find the value of \(h\).

Hint: Normal form equation = \(y - y_1 = m(x - x_1)\)

**Solution:**

Solution step 1: The slope of the line passing through points \((h, 3)\) and \((4, 1)\) is

\[
m_1 = \frac{1 - 3}{4 - h} = \frac{-2}{4 - h}
\]

The slope of line \(7x - 9y - 19 = 0\) or \(y = \frac{7}{9}x - \frac{19}{9}\) is \(m_2 = \frac{7}{9}\).

It is given that the two lines are perpendicular.

\[
\therefore m_1 \times m_2 = -1
\]

\[
\Rightarrow \left(\frac{-2}{4 - h}\right) \times \left(\frac{7}{9}\right) = -1
\]

\[
\Rightarrow \frac{-14}{36 - 9h} = -1
\]

\[
\Rightarrow 14 = 36 - 9h
\]

\[
\Rightarrow 9h = 36 - 14
\]

\[
\Rightarrow h = \frac{22}{9}
\]

Thus, the value of \(h\) is \(\frac{22}{9}\).
11. Prove that the line through the point \((x_1, y_1)\) and parallel to the line \(Ax + By + C = 0\) is 
\[A(x - x_1) + B(y - y_1) = 0.\]

Hint: Normal form equation= \(y - y_1 = m(x - x_1)\)

Solution:

Solution step 1: The slope of line \(Ax + By + C = 0\) or \(y = \left(-\frac{A}{B}\right)x + \left(-\frac{C}{B}\right)\) is \(m = -\frac{A}{B}\)

It is known that parallel lines have the same slope.

\[\therefore\text{Slope of the other line} = m = -\frac{A}{B}\]

The equation of the line passing through point \((x_1, y_1)\) and having a slope \(m = -\frac{A}{B}\) is

\[y - y_1 = -\frac{A}{B}(x - x_1)\]

\[B(y - y_1) = -A(x - x_1)\]

\[A(x - x_1) + B(y - y_1) = 0\]

Hence, the line through point \((x_1, y_1)\) and parallel to line \(Ax + By + C = 0\) is 
\[A(x - x_1) + B(y - y_1) = 0\]

12. Two lines passing through the point \((2, 3)\) intersects each other at an angle of 60°. If slope of one line is 2, find equation of the other line.

Hint: The angle between the two lines is

\[\theta^0 \Rightarrow \tan \theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|\]

Solution:

Solution step 1: It is given that the slope of the first line, \(m_1 = 2\).

Let the slope of the other line be \(m_2\).

The angle between the two lines is 60°.

\[\therefore \tan 60^0 = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|\]

\[\Rightarrow \sqrt{3} = \left|\frac{2 - m_2}{1 + 2m_2}\right|\]

\[\Rightarrow \sqrt{3} = \pm \left|\frac{2 - m_2}{1 + 2m_2}\right|\]

\[\Rightarrow \sqrt{3} = \frac{2 - m_2}{1 + 2m_2} \text{ or } \sqrt{3} = -\frac{2 - m_2}{1 + 2m_2}\]

\[\Rightarrow \sqrt{3}(1 + 2m_2) = 2 - m_2 \text{ or } \sqrt{3}(1 + 2m_2) = -(2 - m_2)\]

\[\Rightarrow \sqrt{3} + 2\sqrt{3}m_2 + m_2 = 2 \text{ or } \sqrt{3} + 2\sqrt{3}m_2 - m_2 = -2\]
The equation of the line passing through point \((2, 3)\) and having a slope of \(\frac{2 - \sqrt{3}}{2(\sqrt{3} + 1)}\) is given by

\[(y - 3) = \frac{2 - \sqrt{3}}{2\sqrt{3} + 1}(x - 2)\]

\[(2\sqrt{3} + 1)y - 3(2\sqrt{3} + 1) = (2 - \sqrt{3})x - 2(2 - \sqrt{3})\]

\[(\sqrt{3} - 2)x + (2\sqrt{3} + 1)y = -4 + 2\sqrt{3} + 6\sqrt{3} + 3\]

\[(\sqrt{3} - 2)x + (2\sqrt{3} + 1)y = -1 + 8\sqrt{3}\]

In this case, the equation of the other line is \((\sqrt{3} - 2)x + (2\sqrt{3} + 1)y = -1 + 8\sqrt{3}\).

Case II: \(m_2 = \frac{-2 + \sqrt{3}}{2(\sqrt{3} - 1)}\)

The equation of the line passing through point \((2, 3)\) and having a slope of \(\frac{-2 + \sqrt{3}}{2(\sqrt{3} - 1)}\) is

\[(y - 3) = \frac{-2 + \sqrt{3}}{2\sqrt{3} - 1}(x - 2)\]

\[(2\sqrt{3} - 1)y - 3(2\sqrt{3} - 1) = -(2 + \sqrt{3})x + 2(2 + \sqrt{3})\]

\[(2\sqrt{3} - 1)y + (2 + \sqrt{3})x = 4 + 2\sqrt{3} + 6\sqrt{3} - 3\]

\[(2 + \sqrt{3})x + (2\sqrt{3} - 1)y = 1 + 8\sqrt{3}\]

In this case, the equation of the other line is \((2 + \sqrt{3})x + (2\sqrt{3} - 1)y = 1 + 8\sqrt{3}\)

Thus, the required equation of the other line is \((\sqrt{3} - 2)x + (2\sqrt{3} + 1)y = -1 + 8\sqrt{3}\) or \((2 + \sqrt{3})x + (2\sqrt{3} - 1)y = 1 + 8\sqrt{3}\).

13. Find the equation of the right bisector of the line segment joining the points \((3, 4)\) and \((-1, 2)\).

Hint: Normal form equation = \(y - y_1 = m(x - x_1)\)

Solution:

Solution step 1: The right bisector of a line segment bisects the line segment at \(90^\circ\).

The end-points of the line segment are given as \(A(3, 4)\) and \(B(-1, 2)\).

Accordingly, mid-point of \(AB = \left(\frac{3 - 1 + 4}{2}, \frac{2}{2}\right) = (1, 3)\)

Slope of \(AB = \frac{4 - 2}{3 - (-1)} = \frac{2}{4} = \frac{1}{2}\)

\[\therefore \text{Slope of the line perpendicular to} \]

The equation of the line passing through \((1, 3)\) and having a slope of \(-2\) is

\[(y - 3) = -2(x - 1)y\]

\[-3 = -2x + 2\]
2x + y = 5
Thus, the required equation of the line is 2x + y = 5.

14. Find the coordinates of the foot of perpendicular from the point \((-1, 3)\) to the line 3x - 4y - 16 = 0.

Hint: If two lines are perpendicular, then \(m_1m_2 = -1\)

Solution:
Solution step 1: Let \((a, b)\) be the coordinates of the foot of the perpendicular from the point \((-1, 3)\) to the line 3x - 4y - 16 = 0.

Slope of the line joining \((-1, 3)\) and \((a, b)\),

\[m_1 = \frac{b - 3}{a + 1}\]

Slope of the line 3x - 4y - 16 = 0 or \(y = \frac{3}{4}x - 4\),

\[m_2 = \frac{3}{4}\]

Since these two lines are perpendicular,

\[
\frac{b - 3}{a + 1} \times \frac{3}{4} = -1
\]

\[
3b - 9 = -4a + 4
\]

\[
3b - 9 = -4a + 4
\]

\[
4a + 3b = 5 \hspace{1cm} (i)
\]

Point \((a, b)\) lies on line 3x - 4y = 16.

\[
3a - 4b = 16 \hspace{1cm} (ii)
\]

On solving equations (i) and (ii), we obtain

\[a = \frac{68}{25} \quad \text{and} \quad b = -\frac{49}{25}\]

Thus, the required coordinates of the foot of the perpendicular are \(\left(\frac{68}{25}, -\frac{49}{25}\right)\).

15. The perpendicular from the origin to the line \(y = mx + c\) meets it at the point \((-1, 2)\). Find the values of \(m\) and \(c\).

Hint: If two lines are perpendicular, then \(m_1m_2 = -1\)
Solution:
Solution step 1: The given equation of line is \( y = mx + c \).
It is given that the perpendicular from the origin meets the given line at \((-1, 2)\).
Therefore, the line joining the points \((0, 0)\) and \((-1, 2)\) is perpendicular to the given line.
\[ \therefore \text{Slope of the line joining } (0, 0) \text{ and } (-1, 2) = \frac{2}{-1} = -2 \]
The slope of the given line is \( m \).
\[ \therefore m \times -2 = -1 \quad [\text{The two lines are perpendicular}] \]
\[ \Rightarrow m = \frac{1}{2} \]
Since point \((-1, 2)\) lies on the given line, it satisfies the equation \( y = mx + c \).
\[ \therefore 2 = m(-1) + c \]
\[ \Rightarrow 2 = \frac{1}{2}(-1) + c \]
\[ \Rightarrow c = 2 + \frac{1}{2} = \frac{5}{2} \]
Thus, the respective values of \( m \) and \( c \) are \( \frac{1}{2} \) and \( \frac{5}{2} \).

16. If \( P \) and \( Q \) are the lengths of perpendiculars from the origin to the lines \( x \cos \theta - y \sin \theta = k \cos 2\theta \) and \( x \sec \theta + y \cosec \theta = k \), respectively, prove that \( p^2 + 4q^2 = k^2 \).

Hint: The perpendicular distance \((d)\) of a line \( Ax + By + C = 0 \) from a point \((x_1, y_1)\) is
\[ d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}. \]

Solution:
Solution step 1: The equations of given lines are \( x \cos \theta - y \sin \theta = k \cos 2\theta \) \ ...(i)
\( x \sec \theta + y \cosec \theta = k \) \ ...(ii)

The perpendicular distance \((d)\) of a line \( Ax + By + C = 0 \) from a point \((x_1, y_1)\) is given by
\[ d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}. \]

On comparing equation (i) to the general equation of line i.e., \( Ax + By + C = 0 \), we obtain
\[ A = \cos \theta, B = -\sin \theta \text{ and } C = -k \cos 2\theta. \]
It is given that \( P \) is the length of the perpendicular from \((0, 0)\) to line (i).
\[ \therefore P = \frac{|A(0) + B(0) + C|}{\sqrt{A^2 + B^2}} = \frac{|C|}{\sqrt{A^2 + B^2}} = \frac{|-k \cos 2\theta|}{\sqrt{\cos^2 \theta + \sin^2 \theta}} = |-k \cos 2\theta| \quad \text{...(iii)} \]

On comparing equation (ii) to the general equation of line i.e., \( Ax + By + C = 0 \), we obtain
\[ A = \sec \theta, B = \cosec \theta \text{ and } C = -k. \]
It is given that \( Q \) is the length of the perpendicular from \((0, 0)\) to line (ii).
\[ \therefore Q = \frac{|A(0) + B(0) + C|}{\sqrt{A^2 + B^2}} = \frac{|C|}{\sqrt{A^2 + B^2}} = \frac{|-k|}{\sqrt{\sec^2 \theta + \cosec^2 \theta}} \quad \text{...(iv)} \]
From (iii) and (iv), we have
\[ p^2 + 4q^2 = (|k - k\cos 2\theta|)^2 + 4\left(\frac{|-k|}{\sec^2 \theta + \cosec^2 \theta}\right)^2 \]
\[ = k^2\cos^2 2\theta + \frac{4k^2}{\sec^2 \theta + \cosec^2 \theta} \]
\[ = k^2\cos^2 2\theta + \frac{1}{\left(\frac{\cos^2 \theta + 1}{\sin^2 \theta}\right)} \]
\[ = k^2\cos^2 2\theta + \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta \cos^2 \theta} \frac{4k^2}{\sin^2 \theta \cos^2 \theta} \]
\[ = k^2\cos^2 2\theta + \frac{1}{\sin^2 \theta \cos^2 \theta} \]
\[ = k^2\cos^2 2\theta + 4k^2\sin^2 \theta \cos^2 \theta \]
\[ = k^2\cos^2 2\theta + k^2(2\sin \theta \cos \theta)^2 \]
\[ = k^2\cos^2 2\theta + k^2 \sin^2 2\theta \]
\[ = k^2(\cos^2 2\theta + \sin^2 2\theta) \]
\[ = k^2 \]

Hence, we proved that \( p^2 + 4q^2 = k^2 \).

17. In the triangle \( ABC \) with vertices \( A(2, 3), B(4, -1) \) and \( C(1, 2) \), find the equation and length of altitude from the vertex \( A \).

Hint: Normal form equation \( y - y_1 = m(x - x_1) \)

Solution:
Solution step 1: Let \( AD \) be the altitude of triangle \( ABC \) from vertex \( A \).
Accordingly, \( AD \perp BC \).

The equation of the line passing through point \( (2, 3) \) and having a slope of \( 1 \) is
\[ (y - 3) = 1(x - 2) \]
\[ \Rightarrow x - y + 1 = 0 \]
Therefore, equation of the altitude from vertex $A = y - x = 1$.

Length of $AD = \text{Length of the perpendicular from } A(2, 3) \text{ to } BC$. The equation of $BC$ is 

$$(y + 1) = \frac{2 + 1}{1 - 4}(x - 4)$$

$$\Rightarrow (y + 1) = -\frac{3}{3}(x - 4)$$

$$\Rightarrow y + 1 = -x + 4$$

$$\Rightarrow x + y - 3 = 0 \quad (i)$$

The perpendicular distance $(d)$ of a line $Ax + By + C = 0$ from a point $(x_1, y_1)$ is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$   

On comparing equation (i) to the general equation of line $Ax + By + C = 0$, we obtain $A = 1$, $B = 1$ and $C = -3$.

$$\therefore \text{Length of}$$

$$AD = \frac{|1 \times 2 + 1 \times 3 - 3|}{\sqrt{1^2 + 1^2}} \text{units} = \frac{2}{\sqrt{2}} \text{units} = \sqrt{2} \text{units}$$

Thus, the equation and the length of the altitude from vertex $A$ are $y - x = 1$ and $\sqrt{2}$ units respectively.

18. If $P$ is the length of perpendicular from the origin to the line whose intercepts on the axes are $a$ and $b$, then show that: $P^2 = \frac{1}{a^2} + \frac{1}{b^2}$

Hint: The perpendicular distance $(d)$ of a line $Ax + By + C = 0$ from a point $(x_1, y_1)$ is 

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$   

Solution:

Solution step 1: It is known that the equation of a line whose intercepts on the axes are $a$ and $b$ is

$$\frac{x}{a} + \frac{y}{b} = 1$$

or $bx + ay = ab$

or $bx + ay - ab = 0 \quad (i)$

The perpendicular distance $(d)$ of a line $Ax + By + C = 0$ from a point $(x_1, y_1)$ is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$   

On comparing equation (i) to the general equation of line $Ax + By + C = 0$, we obtain $A = b$, $B = a$ and $C = -ab$.

Therefore, if $P$ is the length of the perpendicular from point $(x_1, y_1) = (0, 0)$ to line (i), we obtain.

$$P = \frac{|A(0) + B(0) - ab|}{\sqrt{b^2 + a^2}}$$

$$\Rightarrow P = \frac{|-ab|}{\sqrt{a^2 + b^2}}$$

On squaring both sides, we obtain
Hence, we showed that \[ \frac{1}{b^2} \leq \frac{1}{a^2 + b^2} \]

### Miscellaneous

1. Find the values of \( k \) for which the line \( (k - 3)x - (4 - k^2)y + k^2 - 7k + 6 = 0 \) is
   (a) Parallel to the \( x \)-axis,
   Hint: Slope of the \( x \)-axis = 0.

**Solution:**

Solution step 1: The given equation of line is \( (k - 3)x - (4 - k^2)y + k^2 - 7k + 6 = 0 \) \(...(i)\)

If the given line is parallel to the \( x \)-axis, then
Slope of the given line = Slope of the \( x \)-axis
The given line can be written as
\[
\frac{(4 - k^2)y}{(k - 3)x + k^2 - 7k + 6} = \frac{(k - 3)}{(4 - k^2)}, \text{ which is of the form } y = mx + c.
\]

Slope of the given line
Slope of the \( x \)-axis = 0
\[
\frac{(k - 3)}{(4 - k^2)} = 0 \Rightarrow k - 3 = 0 \Rightarrow k = 3
\]
Thus, if the given line is parallel to the \( x \)-axis, then the value of \( k \) is 3.

(b) Parallel to the \( y \)-axis,
Hint: Slope of the \( y \)-axis = \( \frac{1}{0} \)

**Solution:**

Solution step 1: (b) If the given line is parallel to the \( y \)-axis, it is vertical. Hence, its slope will be undefined.
The slope of the given line is \(\frac{(k - 3)}{(4 - k^2)}\).

Now, \((4 - k^2)\) is undefined at \(k^2 = 4\)

\[k^2 = 4\]

\[\Rightarrow k = \pm 2\]

Thus, if the given line is parallel to the \(y\)-axis, then the value of \(k\) is \(\pm 2\).

(c) Passing through the origin.

Hint: If the line passing through the origin the equation of line should satisfy the point \((0, 0)\).

Solution:

Solution step 1: If the given line is passing through the origin, then point \((0, 0)\) satisfies the given equation of line.

\[(k - 3)(0) - (4 - k^2)(0) + k^2 - 7k + 6 = 0\]

\[k^2 - 7k + 6 = 0\]

\[k^2 - 6k - k + 6 = 0\]

\[(k - 6)(k - 1) = 0\]

\[k = 1\] or \[6\]

Thus, if the given line is passing through the origin, then the value of \(k\) is either \(1\) or \(6\).

2. Find the values of \(\theta\) and \(p\), if the equation \(x\cos \theta + y\sin \theta = p\) is the normal form of the line \(\sqrt{3}x + y + 2 = 0\).

Hint: Check the given in the question carefully.

Solution:

Solution step 1: The equation of the given line is \(\sqrt{3}x + y + 2 = 0\).

This equation can be reduced as

\[\sqrt{3}x + y + 2 = 0\]

\[\Rightarrow -\sqrt{3}x - y = 2\]

On dividing both sides by \(\sqrt{\left(\sqrt{3}\right)^2 + (-1)^2} = 2\), we obtain

\[-\frac{\sqrt{3}}{2}x - \frac{1}{2}y = \frac{2}{2}\]

\[\Rightarrow \left(-\frac{\sqrt{3}}{2}\right)x + \left(-\frac{1}{2}\right)y = 1\] …(i)

Comparing equation (i) to \(x\cos \theta + y\sin \theta = p\) we obtain

\[\cos \theta = -\frac{\sqrt{3}}{2}, \sin \theta = \frac{1}{2}\] and \(p = 1\)

Since the values of \(\sin \theta\) and \(\cos \theta\) are negative,
Thus, the respective values of $\theta$ and $P$ are $\frac{7\pi}{6}$ and 1.

3. Find the equations of the lines, which cut-off intercepts on the axes whose sum and product are 1 and $-6$, respectively.

Hint: Check the given in the question carefully.

Solution:
Solution step 1: Let the intercepts cut by the given lines on the axes be $a$ and $b$.

It is given that

$a + b = 1$  ...(i)

$ab = -6$  ...(ii)

On solving equations (i) and (ii), we obtain:

$a = 3$ and $b = -2$ or $a = -2$ and $b = 3$

It is known that the equation of the line whose intercepts on the axes are $a$ and $b$ is

$$\frac{x}{a} + \frac{y}{b} = 1$$

or $bx + ay - ab = 0$

Case I: $a = 3$ and $b = -2$

In this case, the equation of the line is $-2x + 3y + 6 = 0$, i.e., $2x - 3y = 6$.

Case II: $a = -2$ and $b = 3$

In this case, the equation of the line is $3x - 2y + 6 = 0$, i.e., $-3x + 2y = 6$.

Thus, the required equation of the lines are $2x - 3y = 6$ and $-3x + 2y = 6$.

4. What are the points on the $x$-axis whose distance from the line $\frac{x}{3} + \frac{y}{4} = 1$ is 4 units.

Hint: Check the given in the question carefully.

Solution:

Let $(0, b)$ be the point on the $y$-axis whose distance from line $\frac{x}{3} + \frac{y}{4} = 1$ is 4 units.

Solution:

Let $(0, b)$ be the point on the $y$-axis whose distance from line $\frac{x}{3} + \frac{y}{4} = 1$ is 4 units.

The given line can be written as

$4x + 3y - 12 = 0$  ...(i)

On comparing equation (i) to the general equation of line $Ax + By + C = 0$, we obtain $A = 4$, $B = 3$ and $C = -12$. 
It is known that the perpendicular distance \((d)\) of a line \(Ax + By + C = 0\) from a point \((x_1, y_1)\) is given by
\[
d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.
\]
Therefore, if \((0, b)\) is the point on the \(y\)-axis whose distance from line \(\frac{x}{2} + \frac{y}{4} = 1\) is 4 units, then:
\[
4 = \frac{|4(0) + 3(b) - 12|}{\sqrt{4^2 + 3^2}}
\]
\[
\Rightarrow 4 = \frac{|3b - 12|}{5}
\]
\[
\Rightarrow 20 = |3b - 12|
\]
\[
\Rightarrow 20 = \pm (3b - 12)
\]
\[
\Rightarrow 20 = (3b - 12) \text{ or } 20 = -(3b - 12)
\]
\[
\Rightarrow 3b = 20 + 12 \text{ or } 3b = -20 + 12
\]
\[
\Rightarrow b = \frac{32}{3} \text{ or } b = -\frac{8}{3}
\]
Thus, the required points are \(\left(0, \frac{32}{3}\right)\) and \(\left(0, -\frac{8}{3}\right)\).

5. Find the perpendicular distance from the origin to the line joining the points \((\cos \theta, \sin \theta)\) and \((\cos \phi, \sin \phi)\).

Hint: The perpendicular distance \((d)\) of a line \(Ax + By + C = 0\) from a point \((x_1, y_1)\) is given by
\[
d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.
\]

Solution:
Solution step 1: The equation of the line joining the points \((\cos \theta, \sin \theta)\) and \((\cos \phi, \sin \phi)\) is given by
\[
y - \sin \theta = \frac{\sin \phi - \sin \theta}{\cos \phi - \cos \theta}(x - \cos \theta)
\]
\[
y(\cos \phi - \cos \theta) - \sin \theta(\cos \phi - \cos \theta) = x(\sin \phi - \sin \theta) - \sin \theta(\sin \phi - \sin \theta)
\]
\[
x(\sin \theta - \sin \phi) + y(\cos \phi - \cos \theta) + \cos \theta \sin \phi - \cos \theta \sin \theta - \sin \theta \cos \phi + \sin \theta \cos \theta = 0
\]
\[
x(\sin \theta - \sin \phi) + y(\cos \phi - \cos \theta) + \sin (\phi - \theta) = 0
\]
\[
Ax + By + C = 0, \text{ where } A = \sin \theta - \sin \phi, B = \cos \phi - \cos \theta \text{ and } C = \sin (\phi - \theta)
\]
It is known that the perpendicular distance \((d)\) of a line \(Ax + By + C = 0\) from a point \((x_1, y_1)\) is given by
\[
d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.
\]
Therefore, the perpendicular distance \((d)\) of the given line from point \((x_1, y_1) = (0, 0)\) is
\[
d = \frac{|(\sin \theta - \sin \phi)(0) + (\cos \phi - \cos \theta)(0) + \sin (\phi - \theta)|}{\sqrt{(\sin \theta - \sin \phi)^2 + (\cos \phi - \cos \theta)^2}}
\]
\[
= \frac{\sqrt{\sin^2 \theta + \sin^2 \phi - 2 \sin \theta \sin \phi + \cos^2 \phi + \cos^2 \theta - 2 \cos \phi \cos \theta}}{|\sin (\phi - \theta)|}
\]
\[
= \frac{\sqrt{(\sin^2 \theta + \cos^2 \phi) + (\sin^2 \phi + \cos^2 \theta) - 2(\sin \theta + \sin \phi + \cos \theta \cos \phi)}}{|\sin (\phi - \theta)|}
\]
6. Find the equation of the line parallel to $Y$-axis and drawn through the point of intersection of the lines $x - 7y + 5 = 0$ and $3x + y = 0$.

Hint: The equation of any line parallel to the $Y$-axis is $x = a$.

**Solution:**

Solution step 1: The equation of any line parallel to the $Y$-axis is of the form $x = a$ ...(i)

The two given lines are

$x - 7y + 5 = 0$ ...(ii)

$3x + y = 0$ ...(iii)

On solving equations (ii) and (iii), we obtain $x = \frac{-5}{22}$ and $y = \frac{15}{22}$.

Therefore, $\left(\frac{-5}{22}, \frac{15}{22}\right)$ is the point of intersection of lines (ii) and (iii).

Since line $x = a$ passes through point $\left(\frac{-5}{22}, \frac{15}{22}\right)$, $a = \frac{-5}{22}$.

Thus, the required equation of the line is $x = \frac{-5}{22}$.

7. Find the equation of a line drawn perpendicular to the line $\frac{x}{4} + \frac{y}{6} = 1$ through the point, where it meets the $Y$-axis.

Hint: Slope of line perpendicular to the given line $m = \frac{-1}{m}$.

**Solution:**

Solution step 1: The equation of the given line is $\frac{x}{4} + \frac{y}{6} = 1$.

This equation can also be written as $3x + 2y - 12 = 0$.
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\[ y = \frac{-3}{2}x + 6, \text{ which is of the form } y = mx + c \]

\[ \therefore \text{Slope of the } \frac{-3}{2} \text{ given line} \]

\[ \therefore \text{Slope of line perpendicular to the given line} \]

\[ \text{Let the given line intersect the } y\text{-axis at } (0, y). \]

On substituting \( x \) with 0 in the equation of the given line, we obtain \( \frac{y}{6} = 1 \Rightarrow y = 6 \)

\[ \therefore \text{The given line intersects the } y\text{-axis at } (0, 6). \]

The equation of the line that has a slope of \( \frac{2}{3} \) and passes through point (0, 6) is \( (y - 6) = \frac{2}{3}(x - 0) \)

\[ 3y - 18 = 2x \]

\[ 2x - 3y + 18 = 0 \]

Thus, the required equation of the line is \( 2x - 3y + 18 = 0 \).

8. Find the area of the triangle formed by the lines \( y - x = 0, x + y = 0 \) and \( x - k = 0 \).

Hint: the area of a triangle whose vertices are \((x_1, y_1), (x_2, y_2)\) and \((x_3, y_3)\)

\[ \text{is } \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|. \]

Solution:

Solution step 1: The equations of the given lines are

\[ y - x = 0 \quad \text{...(i)} \]
\[ x + y = 0 \quad \text{...(ii)} \]
\[ x - k = 0 \quad \text{...(iii)} \]

The point of intersection of lines (i) and (ii) is given by \( x = 0 \) and \( y = 0 \).

The point of intersection of lines (ii) and (iii) is given by \( x = k \) and \( y = -k \).

The point of intersection of lines (iii) and (i) is given by \( x = k \) and \( y = k \).

Thus, the vertices of the triangle formed by the three given lines are \((0, 0), (k, -k)\) and \((k, k)\).

We know that the area of a triangle whose vertices are \((x_1, y_1), (x_2, y_2)\) and \((x_3, y_3)\) is

\[ \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|. \]

Therefore, area of the triangle formed by the three given lines

\[ = \frac{1}{2} |0(-k - k) + k(k - 0) + k(0 + k)| \quad \text{square units} \]

\[ = \frac{1}{2} |k^2 + k^2| \quad \text{square units} \]

\[ = \frac{1}{2} |2k^2| \quad \text{square units} \]

\[ = k^2 \quad \text{square units} \]
9. Find the value of \( P \) so that the three lines \( 3x + y - 2 = 0 \), \( px + 2y - 3 = 0 \) and \( 2x - y - 3 = 0 \) may intersect at one point.

Hint: Check the given in question carefully.

**Solution:**
Solution step 1: The equations of the given lines are
\[
3x + y - 2 = 0 \quad \text{(i)}
\]
\[
px + 2y - 3 = 0 \quad \text{(ii)}
\]
\[
2x - y - 3 = 0 \quad \text{(iii)}
\]
On solving equations (i) and (iii), we obtain
\[
x = 1 \text{ and } y = -1
\]
Since these three lines may intersect at one point, the point of intersection of lines (i) and (iii) will also satisfy line (ii).
\[
p(1) + 2(-1) - 3 = 0
\]
\[
p - 2 - 3 = 0 \quad p = 5
\]
Thus, the required value of \( P \) is 5.

10. If three lines whose equations are \( y = m_1 x + c_1 \), \( y = m_2 x + c_2 \) and \( y = m_3 x + c_3 \) are concurrent, then show that \( m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0 \).

Hint: Check the given in question carefully.

**Solution:**
Solution step 1: The equations of the given lines are
\[
y = m_1 x + c_1 \quad \text{(i)}
\]
\[
y = m_2 x + c_2 \quad \text{(ii)}
\]
\[
y = m_3 x + c_3 \quad \text{(iii)}
\]
On subtracting equation (i) from (ii), we obtain
\[
0 = (m_2 - m_1)x + (c_2 - c_1)
\]
\[
(m_2 - m_1)x = c_2 - c_1
\]
\[
x = \frac{c_2 - c_1}{m_2 - m_1}
\]
On substituting this value of \( x \) in (i), we obtain
\[
y = m_1 \left( \frac{c_2 - c_1}{m_1 - m_2} \right) + c_1
\]
\[
y = \frac{m_1 c_2 - m_1 c_1}{m_1 - m_2} + c_1
\]
It is given that line (i), (ii) and (iii) are concurrent. Hence, the point of intersection of lines (i) and (ii) will also satisfy equation (iii).

\[
\begin{align*}
\frac{m_1 c_2 - m_2 c_1}{m_1 - m_2} &= m_3 \left(\frac{c_2 - c_1}{m_1 - m_2}\right) + c_3 \\
\frac{m_1 c_2 - m_2 c_1}{m_1 - m_2} &= m_3 c_2 - m_3 c_1 + c_3 m_1 - c_3 m_2 \\
\frac{m_1 c_2 - m_2 c_1}{m_1 - m_2} &= \frac{m_1 - m_2}{m_1 - m_2} \\
\frac{m_1 c_2 - m_2 c_1}{m_1 - m_2} &= m_1 - m_2 \\
\frac{m_1 c_2 - m_2 c_1 - m_3 c_2 + m_3 c_1 + c_3 m_1 + c_3 m_2}{m_1 - m_2} &= 0 \\
m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) &= 0
\end{align*}
\]

11. Find the equation of the lines through the point \((3, 2)\) which make an angle of \(45^\circ\) with the line \(x - 2y = 3\).

Hint: If \(\theta\) is the acute angle between lines \(l_1\) and \(l_2\) with slopes \(m_1\) and \(m_2\), then

\[
\tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|
\]

Solution:

Solution step 1: Let the slope of the required line be \(m_1\).

The given line can be written as \(y = \frac{1}{2}x - \frac{3}{2}\) which is of the form \(y = mx + c\)

\[
\therefore \text{Slope of the given line} = m_2 = \frac{1}{2}
\]

It is given that the angle between the required line and line \(x - 2y = 3\) is \(45^\circ\).

We know that if \(\theta\) is the acute angle between lines \(l_1\) and \(l_2\) with slopes \(m_1\) and \(m_2\), then

\[
\tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|
\]

\[
\therefore \tan 45^\circ = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|
\]

\[
\Rightarrow 1 = \left| \frac{1}{2} - m_1 \right| \left| \frac{1}{1 + m_1} \right|
\]

\[
\Rightarrow 1 = \left| \frac{1}{2} - m_1 \right|
\]
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Straight Lines

Case

The equation of the line passing through \( (3, 2) \) and having a slope of \( 3 \) is:

\[
y - 2 = 3(x - 3)
\]

\[
y - 2 = 3x - 9
\]

\[
3x - y = 7
\]

Case \( II: m_1 = -\frac{1}{3} \)

The equation of the line passing through \( (3, 2) \) and having a slope of \( -\frac{1}{3} \) is:

\[
y - 2 = \frac{1}{3}(x - 3)
\]

\[
3y - 6 = x + 3
\]

\[
x + 3y = 9
\]

Thus, the equations of the lines are \( 3x - y = 7 \) and \( x + 3y = 9 \).

12. Find the equation of the line passing through the point of intersection of the lines \( 4x + 7y - 3 = 0 \) and \( 2x - 3y + 1 = 0 \) that has equal intercepts on the axes.

Hint: Check the given in question carefully.

Solution:

Solution step 1: Let the equation of the line having equal intercepts on the axes be \( \frac{x}{a} + \frac{y}{a} = 1 \) or \( x + y = a \) \( \ldots \) (i)

On solving equations \( 4x + 7y - 3 = 0 \) and \( 2x - 3y + 1 = 0 \), we obtain \( x = \frac{1}{13} \) and \( y = \frac{5}{13} \).

\( \therefore \) \( (\frac{1}{13}, \frac{5}{13}) \) is the point of intersection of the two given lines.

Since equation (i) passes through \( (\frac{1}{13}, \frac{5}{13}) \) point, \( \frac{1}{13} + \frac{5}{13} = a \)
\[ a = \frac{6}{13} \]

\[ \therefore \text{Equation (i) becomes } x + y = \frac{6}{13}, \text{i.e., } 13x + 13y = 6 \]

Thus, the required equation of the line is \[ 13x + 13y = 6. \]

13. Show that the equation of the line passing through the origin and making an angle \( \theta \) with the line \[ y = mx + c \]

\[ \text{Hint: Check the given in the question carefully.} \]

**Solution:**

Solution step 1: Let the equation of the line passing through the origin be \( y = mx \).

If this line makes an angle of \( \theta \) with line \( y = mx + c \), then angle \( \theta \) is given by

\[ \tan \theta = \left| \frac{m_1 - m}{1 + m_1m} \right| \]

\[ \Rightarrow \tan \theta = \frac{\frac{y}{x} - m}{1 + \frac{y}{x}m} \]

\[ \Rightarrow \tan \theta = \pm \left( \frac{\frac{y}{x} - m}{1 + \frac{y}{x}m} \right) \]

\[ \Rightarrow \tan \theta = \frac{\frac{y}{x} - m}{1 + \frac{y}{x}m} \text{ or } \tan \theta = -\frac{\frac{y}{x} - m}{1 + \frac{y}{x}m} \]

**Case I:**

\[ \frac{y}{x} - m = \frac{\frac{y}{x} - m}{1 + \frac{y}{x}m} \]

\[ \Rightarrow \frac{y}{x} + \frac{y}{x}m \tan \theta = \frac{y}{x} - m \]

\[ \Rightarrow m + \tan \theta = \frac{y}{x}(1 - m \tan \theta) \]

\[ \Rightarrow \frac{y}{x} = \frac{m + \tan \theta}{1 - m \tan \theta} \]

**Case II:**
Therefore, the required line is given by
\[
\frac{y}{x} = \frac{m \pm \tan \theta}{1 + m \tan \theta}
\]

14. In what ratio, the line joining \((-1, 1)\) and \((5, 7)\) is divided by the line \(x + y = 4\)?

**Hint:** section formula
\[
\left(\frac{k(x_1) + 1(x_2)}{1 + k}, \frac{k(y_1) + 1(y_2)}{1 + k}\right)
\]

**Solution:**

Solution step 1: The equation of the line joining the points \((-1, 1)\) and \((5, 7)\) is given by
\[
y - 1 = \frac{7 - 1}{5 + 1}(x + 1)
\]
\[
y - 1 = \frac{6}{6}(x + 1)
\]
\[
x - y + 2 = 0 \quad \ldots(\text{i})
\]
The equation of the given line is
\[
x + y - 4 = 0 \quad \ldots(\text{ii})
\]
The point of intersection of lines (i) and (ii) is given by \(x = 1\) and \(y = 3\)

Let point \((1, 3)\) divide the line segment joining \((-1, 1)\) and \((5, 7)\) in the ratio \(1:k\). Accordingly, by section formula,
\[
(1, 3) = \left(\frac{k(-1) + 1(5)}{1 + k}, \frac{k(1) + 1(7)}{1 + k}\right)
\]
\[
\Rightarrow (1, 3) = \left(\frac{-k + 5}{1 + k}, \frac{k + 7}{1 + k}\right)
\]
\[
\Rightarrow \frac{-k + 5}{1 + k} = 1 \quad \text{and} \quad \frac{k + 7}{1 + k} = 3
\]
\[
\Rightarrow -k + 5 = 1 + k
\]
\[
\Rightarrow 2k = 4
\]
\[
\Rightarrow k = 2
\]
Thus, the line joining the points \((-1, 1)\) and \((5, 7)\) is divided by line \(x + y = 4\) in the ratio \(1:2\).
15. Find the distance of the line \(4x + 7y + 5 = 0\) from the point \((1, 2)\) along the line \(2x - y = 0\).

Hint: Distance formula \(pq = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\)

**Solution:**

Solution step 1: The given lines are
\(2x - y = 0 \ldots (i)\)
\(4x + 7y + 5 = 0 \ldots (ii)\)
\(A(1, 2)\) is a point on line (i).
Let \(B\) be the point of intersection of lines (i) and (ii).

On solving equations (i) and (ii), we obtain \(x = \frac{-5}{18}\) and \(y = \frac{-5}{9}\)

\(\therefore\) Coordinates of point \(B\) are \(\left(\frac{-5}{18}, \frac{-5}{9}\right)\).

By using distance formula, the distance between points \(A\) and \(B\) can be obtained as
\[AB = \sqrt{(1 + \frac{5}{18})^2 + (2 + \frac{5}{9})^2}\ units\]
\[= \sqrt{(\frac{23}{18})^2 + (\frac{23}{9})^2}\ units\]
\[= \sqrt{(\frac{23}{18})^2 + (\frac{23}{9})^2}\ units\]
\[= \sqrt{(\frac{23}{9})^2 (\frac{1}{4} + 1)}\ units\]
\[= \frac{23\sqrt{5}}{9}\ units\]
\[= \frac{23\sqrt{5}}{18}\ units\]

Thus, the required distance is \(\frac{23\sqrt{5}}{18}\) units

16. Find the direction in which a straight line must be drawn through the point \((-1, 2)\) so that its point of intersection with the line \(x + y = 4\) may be at a distance of \(3\) units from this point.
Hint: Check the given in question carefully.

**Solution:**

Solution step 1: Let \( y = mx + c \) be the line through point \((-1, 2)\).

Accordingly, \( 2 = m(-1) + c \).

\[ \Rightarrow 2 = -m + c \]

\[ \Rightarrow c = m + 2 \]

\[ \therefore y = mx + m + 2 \quad \text{...(i)} \]

The given line is

\[ x + y = 4 \quad \text{...(ii)} \]

On solving equations (i) and (ii), we obtain

\[ x = \frac{2 - m}{m + 1} \quad \text{and} \quad y = \frac{5m + 2}{m + 1} \]

\[ \therefore \left( \frac{2 - m}{m + 1}, \frac{5m + 2}{m + 1} \right) \] is the point of intersection of lines (i) and (ii).

Since this point is at a distance of 3 units from point \((-1, 2)\), according to distance formula,

\[ \sqrt{\left( \frac{2 - m}{m + 1} \right)^2 + \left( \frac{5m + 2}{m + 1} - 2 \right)^2} = 3 \]

\[ \Rightarrow \left( \frac{2 - m + m + 1}{m + 1} \right)^2 + \left( \frac{5m + 2 - 2m - 2}{m + 1} \right)^2 = 3^2 \]

\[ \Rightarrow \frac{9}{(m + 1)^2} + \frac{9m^2}{(m + 1)^2} = 9 \]

\[ \Rightarrow 1 + m^2 = 1 \]

\[ \Rightarrow 1 + m^2 = 1 + 2m \]

\[ \Rightarrow 2m = 0 \]

\[ \Rightarrow m = 0 \]

Thus, the slope of the required line must be zero i.e., the line must be parallel to the \(x\)-axis.

17. Find the image of the point \((3, 8)\) with respect to the line \(x + 3y = 7\) assuming the line to be a plane mirror.

Hint: Slope of perpendicular line \( m = -\frac{1}{m} \)

**Solution:**

Solution step 1: The equation of the given line is \(x + 3y = 7 \quad \text{...(i)} \)

Let point \( B(a, b) \) be the image of point \( A(3, 8) \).

Accordingly, line (i) is the perpendicular bisector of \( AB \).
Slope of $AB = \frac{b - 8}{a - 3}$ while the slope of line (i) $= -\frac{1}{3}$.

Since line (i) is perpendicular to $AB$,

$$\frac{b - 8}{a - 3} \times \left(-\frac{1}{3}\right) = -1$$

$$\Rightarrow 3a - 9 = 1$$

$$\Rightarrow b - 8 = 3a - 9$$

$$\Rightarrow 3a - b = 1 \quad \ldots (ii)$$

Mid-point of $AB = \left(\frac{a + 3}{2}, \frac{b + 8}{2}\right)$

The mid-point of line segment $AB$ will also satisfy line (i).

Hence, from equation (i), we have

$$\frac{a + 3}{2} + 3 \left(\frac{b + 8}{2}\right) = 7$$

$$\Rightarrow a + 3 + 3b + 24 = 14$$

$$\Rightarrow a + 3b = -13 \quad \ldots (iii)$$

On solving equations (ii) and (iii), we obtain $a = -1$ and $b = -4$.

Thus, the image of the given point with respect to the given line is $(-1, -4)$.

18. If the lines $y = 3x + 1$ and $2y = x + 3$ are equally inclined to the line $y = mx + 4$, find the value of $m$.

Hint: Check the given in question carefully.

**Solution:**

Solution step 1: The equations of the given lines are

$y = 3x + 1 \quad \ldots (i)$

$2y = x + 3 \quad \ldots (ii)$

$y = mx + 4 \quad \ldots (iii)$

Slope of line (i), $m_1 = 3$

Slope of line (ii), $m_2 = \frac{1}{2}$

Slope of line (iii), $m_3 = m$
It is given that lines (i) and (ii) are equally inclined to line (iii). This means that the angle between lines (i) and (iii) equals the angle between lines (ii) and (iii).

\[
\frac{|m_1 - m_3|}{1 + m_1 m_3} = \frac{|m_2 - m_3|}{1 + m_2 m_3}
\]

\[
\Rightarrow \frac{3 - m}{1 + 3m} = \frac{1 - m}{1 + m}
\]

\[
\Rightarrow \frac{3 - m}{1 + 3m} = \frac{1 - 2m}{m + 2}
\]

\[
\Rightarrow \frac{3 - m}{1 + 3m} = \pm \frac{1 - 2m}{m + 2}
\]

\[
\Rightarrow \frac{3 - m}{1 + 3m} = \frac{1 - 2m}{m + 2}
\]

\[
\Rightarrow \frac{3 - m}{1 + 3m} = \frac{1 - 2m}{m + 2}
\]

If \(\frac{3 - m}{1 + 3m} = \frac{1 - 2m}{m + 2}\) then

\[
(3 - m)(m + 2) = (1 - 2m)(1 + 3m)
\]

\[
\Rightarrow -m^2 + m + 6 = 1 + m - 6m^2
\]

\[
5m^2 + 5 = 0
\]

\[
\Rightarrow (m^2 + 1) = 0
\]

\[
\Rightarrow m = \sqrt{-1}, \text{ which is not real}
\]

Hence, this case is not possible.

If \(\frac{3 - m}{1 + 3m} = -\frac{1 - 2m}{m + 2}\) then

\[
(3 - m)(m + 2) = -(1 - 2m)(1 + 3m)
\]

\[
\Rightarrow -m^2 + m + 6 = -(1 + m - 6m^2)
\]

\[
7m^2 - 2m - 7 = 0
\]

\[
\Rightarrow m = \frac{2 \pm \sqrt{4 - 4(7)(-7)}}{14}
\]

\[
\Rightarrow m = \frac{1 \pm 5\sqrt{2}}{7}
\]

Thus, the required value of \(m\) is \(\frac{1 \pm 5\sqrt{2}}{7}\)

19. If sum of the perpendicular distances of a variable point \(P(x,y)\) from the lines \(x + y - 5 = 0\) and \(3x - 2y + 7 = 0\) is always 10. Show that \(P\) must move on a line.

Hint: Check the given in question carefully.

**Solution:**

Solution step 1: The equations of the given lines are \(x + y - 5 = 0\) \(\ldots (i)\)
The perpendicular distances of \( P(x, y) \) from lines (i) and (ii) are respectively given by

\[
d_1 = \frac{|x + y - 5|}{\sqrt{(1)^2 + (1)^2}} \text{ and } d_2 = \frac{|3x - 2y + 7|}{\sqrt{(3)^2 + (-2)^2}}
\]

i.e.,

\[
d_1 = \frac{|x + y - 5|}{\sqrt{2}} \text{ and } d_2 = \frac{|3x - 2y + 7|}{\sqrt{13}}
\]

It is given that \( d_1 + d_2 = 10 \)

\[
\frac{|x + y - 5|}{\sqrt{2}} + \frac{|3x - 2y + 7|}{\sqrt{13}} = 10
\]

\[
\Rightarrow \sqrt{13}|x + y - 5| + \sqrt{2}|3x - 2y + 7| - 10\sqrt{26} = 0
\]

\[
\Rightarrow 13(x + y - 5) + 2(3x - 2y + 7) - 10\sqrt{26} = 0
\]

[Assuming \((x + y - 5)\) and \((3x - 2y + 7)\) are positive]

\[
\Rightarrow \sqrt{13}(x + \sqrt{13}y - 5\sqrt{13} + 3\sqrt{2}x - 2\sqrt{2}y + 7\sqrt{2} - 10\sqrt{26} = 0
\]

\[
\Rightarrow x(\sqrt{13} + 3\sqrt{2}) + y(\sqrt{13} - 2\sqrt{2} + 7\sqrt{2} - 5\sqrt{13} - 10\sqrt{26} = 0
\]

which is the equation of a line. Similarly, we can obtain the equation of line for any sign of \((x + y - 5)\) and \((3x - 2y + 7)\). Thus, point \( P \) must move on a line.

20. Find equation of the line which is equidistant from parallel lines \( 9x + 6y - 7 = 0 \) and \( 3x + 2y + 6 = 0 \).

Hint: Check the given in question carefully.

**Solution:**

Solution step 1: The equations of the given lines are

\[
9x + 6y - 7 = 0 \quad \ldots \text{(i)}
\]

\[
3x + 2y + 6 = 0 \quad \ldots \text{(ii)}
\]

Let \( P(h, k) \) be the arbitrary point that is equidistant from lines (i) and (ii). The perpendicular distance of \( P(h, k) \) from line (i) is given by

\[
d_1 = \frac{|9h + 6k - 7|}{\sqrt{(9)^2 + (6)^2}} = \frac{|9h + 6k - 7|}{\sqrt{117}} = \frac{|9h + 6k - 7|}{3\sqrt{13}}
\]

The perpendicular distance of \( P(h, k) \) from line (ii) is given by

\[
d_2 = \frac{|3h + 2k + 6|}{\sqrt{(3)^2 + (2)^2}} = \frac{|3h + 2k + 6|}{\sqrt{13}}
\]

Since \( P(h, k) \) is equidistant from lines (i) and (ii), \( d_1 = d_2 \)

\[
\Rightarrow \frac{|9h + 6k - 7|}{3\sqrt{13}} = \frac{|3h + 2k + 6|}{\sqrt{13}}
\]

\[
\Rightarrow |9h + 6k - 7| = 3|3h + 2k + 6|
\]

\[
\Rightarrow |9h + 6k - 7| = 3(3h + 2k + 6)
\]

\[
\Rightarrow 9h + 6k - 7 = 3(3h + 2k + 6) \text{ or } 9h + 6k - 7 = -3(3h + 2k + 6)
\]

The case \( 9h + 6k - 7 = 3(3h + 2k + 6) \) is not possible as \( 9h + 6k - 7 = 3(3h + 2k + 6) \Rightarrow 7 = 18 \) (which is absurd)

\[
\Rightarrow 9h + 6k - 7 = -3(3h + 2k + 6)
\]
Thus, the required equation of the line is

21. A ray of light passing through the point \((1, 2)\) reflects on the \(x\)-axis at point \(A\) and the reflected ray passes through the point \((5, 3)\). Find the coordinates of \(A\).

Hint: Check the given in question carefully.

Solution:
Solution step 1:

Let the coordinates of point \(A\) be \((a, 0)\).

Draw a line (AL) perpendicular to the \(x\)-axis.

We know that angle of incidence is equal to angle of reflection. Hence, let \(\angle BAL = \angle CAL = \phi\)

Let \(\angle CAX = \theta\)

\(\therefore \angle OAB = 180^\circ - (\theta + 2\phi) = 180^\circ - [\theta + 2(90^\circ - \theta)]\)

\(= 180^\circ - \theta - 180^\circ + 2\theta = \theta\)

\(\therefore \angle BAX = 180^\circ - \theta\)

Now, slope of line \(AC = \frac{3 - 0}{5 - a}\)

\(\Rightarrow \tan \theta = \frac{3}{5 - a} \ldots (i)\)

Slope of line \(AB = \frac{2 - 0}{1 - a}\)

\(\Rightarrow \tan (180^\circ - \theta) = \frac{2}{1 - a}\)

\(\Rightarrow - \tan \theta = \frac{2}{1 - a}\)

\(\Rightarrow \tan \theta = \frac{2}{a - 1} \ldots (ii)\)

From equations (i) and (ii), we obtain
Thus, the coordinates of point $A$ are \( \left( \frac{13}{5}, 0 \right) \).

22. Prove that the product of the lengths of the perpendiculars drawn from the points \( \left( \sqrt{a^2 - b^2}, 0 \right) \) and \( \left( -\sqrt{a^2 - b^2}, 0 \right) \) to the line \( \frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1 \) is $b^2$.

Hint: Check the given in question carefully.

Solution:
Solution step 1: The equation of the given line is
\[ \frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1 \]
or
\[ bx \cos \theta + ay \sin \theta - ab = 0 \]...(i)

Length of the perpendicular from point \( \left( \sqrt{a^2 - b^2}, 0 \right) \) to line (i) is
\[ p_1 = \frac{|bcos \theta(\sqrt{a^2 - b^2}) + a\sin \theta(0) - ab|}{\sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta}} \]...(ii)

Length of the perpendicular from point \( \left(-\sqrt{a^2 - b^2}, 0 \right) \) to line (ii) is
\[ p_2 = \frac{|bcos \theta(-\sqrt{a^2 - b^2}) + a\sin \theta(0) + ab|}{\sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta}} \]...(iii)

On multiplying equations (ii) and (iii), we obtain
\[ p_1 p_2 = \frac{|bcos \theta(\sqrt{a^2 - b^2}) + a\sin \theta(0) - ab|}{\sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta}} \]
\[ \cdot \frac{|bcos \theta(-\sqrt{a^2 - b^2}) + a\sin \theta(0) + ab|}{\sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta}} \]

\[ = \frac{(b^2 \cos^2 \theta + a^2 \sin^2 \theta)}{(b^2 \cos^2 \theta + a^2 \sin^2 \theta)} \]

\[ = \frac{|b^2 \cos^2 \theta + a^2 \sin^2 \theta|^2}{(ab)^2} \]

\[ = \frac{|b^2 \cos^2 \theta + a^2 \sin^2 \theta|^2}{|b^2 |a^2 \cos^2 \theta - b^4 \cos^2 \theta - a^2 b^2|} \]

\[ = \frac{b^2 |a^2 \cos^2 \theta - b^4 \cos^2 \theta - a^2 b^2|}{b^2 |a^2 \cos^2 \theta - b^2 \cos^2 \theta - a^2|} \]

\[ = \frac{b^2 |a^2 \cos^2 \theta - b^2 \cos^2 \theta - a^2|}{b^2 |a^2 \cos^2 \theta - b^2 \cos^2 \theta - a^2|} \]

\[ = \frac{b^2 |a^2 \cos^2 \theta - b^2 \cos^2 \theta - a^2|}{b^2 |a^2 \cos^2 \theta - b^2 \cos^2 \theta - a^2|} \]

\[ = \frac{|b^2 \cos^2 \theta + a^2 \sin^2 \theta|^2}{|b^2 \cos^2 \theta + a^2 \sin^2 \theta|^2} \]

\[ = 1 \]

\[ \Rightarrow p_1 p_2 = b^2 \]
Hence proved.

23. A person standing at the junction (crossing) of two straight paths represented by the equations $2x - 3y + 4 = 0$ and $3x + 4y - 5 = 0$ wants to reach the path whose equation is $6x - 7y + 8 = 0$ in the least time. Find equation of the path that he should follow.

Hint: Check the given in question carefully.

Solution:
Solution step 1: The equations of the given lines are
$2x - 3y + 4 = 0$  ...(i)
$3x + 4y - 5 = 0$  ...(ii)
$6x - 7y + 8 = 0$  ...(iii)
The person is standing at the junction of the paths represented by lines (i) and (ii).

On solving equations (i) and (ii), we obtain $x = \frac{1}{17}$ and $y = \frac{22}{17}$.

Thus, the person is standing at point $\left(-\frac{1}{17}, \frac{22}{17}\right)$.

The person can reach path (iii) in the least time if he walks along the perpendicular line to (iii) from point $\left(-\frac{1}{17}, \frac{22}{17}\right)$.

Slope of the line (iii) $= \frac{6}{7}$

∴ Slope of the line perpendicular to line (iii) $= \frac{-1}{\frac{6}{7}} = \frac{-7}{6}$

The equation of the line passing through $\left(-\frac{1}{17}, \frac{22}{17}\right)$ and having a slope of $\frac{-7}{6}$ is given by

\[
(y - \frac{22}{17}) = \frac{-7}{6}(x + \frac{1}{17})
\]

$6(17y - 22) = -7(17x + 1)$

$102y - 132 = -119x - 7$

$119x + 102y = 125$

Hence, the path that the person should follow is $119x + 102y = 125$. 