CBSE NCERT Solutions for Class 11 Mathematics Chapter 12

Back of Chapter Questions

Exercise 12.1

1. A point is on the $x$-axis. What are its $y$-coordinates and $z$-coordinates?
   Hint: If a point is on the $x$-axis, then its $y$-coordinates and $z$-coordinates are zero.

   **Solution:**

   Solutions step 1: If a point is on the $x$-axis, then its $y$-coordinates and $z$-coordinates are zero.

2. A point is in the $XZ$-plane. What can you say about its $y$-coordinate?
   Hint: If a point is in the $XZ$ plane, then its $y$-coordinate is zero.

   **Solution:**

   Solutions step1: If a point is in the $XZ$ plane, then its $y$-coordinate is zero.

3. Name the octants in which the following points lie:
   $(1,2,3), (4,-2,3), (4,-2,-5), (4,2,-5), (-4,2,-5), (-4,2,5), (-3,-1,6), (2,-4,-7)$
   Hint: Check the positive and negative of number.

   **Solution:**

   Solution step 1: The $x$-coordinate, $y$-coordinate, and $z$-coordinate of point $(1,2,3)$ are all positive. Therefore, this point lies in octant I.
   The $x$-coordinate, $y$-coordinate, and $z$-coordinate of point $(4,-2,3)$ are positive, negative, and positive respectively. Therefore, this point lies in octant IV.
   The $x$-coordinate, $y$-coordinate, and $z$-coordinate of point $(4,-2,-5)$ are positive, negative, and negative respectively. Therefore, this point lies in octant VIII.
   The $x$-coordinate, $y$-coordinate, and $z$-coordinate of point $(4,2,-5)$ are positive, positive, and negative respectively. Therefore, this point lies in octant V.
The $x$-coordinate, $y$-coordinate, and $z$-coordinate of point $(-4,2,-5)$ are negative, positive, and negative respectively. Therefore, this point lies in octant VI.

The $x$-coordinate, $y$-coordinate, and $z$-coordinate of point $(-4,2,5)$ are negative, positive, and positive respectively. Therefore, this point lies in octant II.

The $x$-coordinate, $y$-coordinate, and $z$-coordinate of point $(-3,-1,6)$ are negative, negative, and positive respectively. Therefore, this point lies in octant III.

The $x$-coordinate, $y$-coordinate, and $z$-coordinate of point $(2,-4,-7)$ are positive, negative, and negative respectively. Therefore, this point lies in octant VIII.

4. Fill in the blanks:
   (i) The $x$-axis and $y$-axis taken together determine a plane known as_______.
   Hint: $xy$-plane.

   **Solution:**
   Solution step 1: (i) The $x$-axis and $y$-axis taken together determine a plane known as $xy$-plane.

   (ii) The coordinates of points in the $XY$-plane are of the form _______.
   Hint: $XY$-plane are of the form $(x,y,0)$.

   **Solution:**
   Solution step 1: (ii) The coordinates of points in the $XY$-plane are of the form $(x,y,0)$.

   (iii) Coordinate planes divide the space into ______ octants.
   Hint: Coordinate planes divide the space into eight octants.

   **Solution:**
   Solution step 1: Coordinate planes divide the space into eight octants.

**Exercise 12.2**

1. Find the distance between the following pairs of points:
   (i) $(2,3,5)$ and $(4,3,1)$
Hint: \( PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \)

**Solution:**

Solution step 1

(i): The distance between points \( P(x_1, y_1, z_1) \) and \( P(x_2, y_2, z_2) \) is given by

\[
PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}
\]

Distance between points \( (2,3,5) \) and \( (4,3,1) \)

\[
= \sqrt{(4 - 2)^2 + (3 - 3)^2 + (1 - 5)^2}
\]

\[
= \sqrt{4 + 0 + 16}
\]

\[
= \sqrt{20}
\]

\[
= 2 \sqrt{5}
\]

1: 2

(ii): \((-3,7,2) \) and \((2,4,-1)\)

Hint: \( PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \)

**Solution:**

Solution step 1: Distance between points \((-3,7,2) \) and \((2,4,-1)\)

\[
= \sqrt{(2 + 3)^2 + (4 - 7)^2 + (-1 - 2)^2}
\]

\[
= \sqrt{25 + 9 + 9}
\]

\[
= \sqrt{46}
\]

1: 2

(iii): \((-1,3,-4) \) and \((1,-3,4)\)

Hint: \( PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \)
Solution:

Solution step 1: Distance between points \((-1, 3, -4)\) and \((1, -3, 4)\)

\[= \sqrt{(1 + 1)^2 + (-3 - 3)^2 + (4 + 4)^2} \]
\[= \sqrt{2^2 + (-6)^2 + (8)^2} \]
\[= \sqrt{4 + 36 + 64} \]
\[= \sqrt{104} \]
\[= 2\sqrt{26} \]

(iv) \((2, -1, 3)\) and \((-2, 1, 3)\)

Hint: \(PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}\)

Solution:

Solution step: (iv) Distance between points \((2, -1, 3)\) and \((-2, 1, 3)\)

\[= \sqrt{(-2 - 2)^2 + (1 + 1)^2 + (3 - 3)^2} \]
\[= \sqrt{(-4)^2 + (2)^2 + (0)^2} \]
\[= \sqrt{16 + 4} \]
\[= \sqrt{20} \]
\[= 2\sqrt{5} \]

2. Show that the points \((-2, 3, 5)\), \((1, 2, 3)\) and \((7, 0, -1)\) are collinear.

Hint: \(PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}\)

Solution:

Solution step 1: Let points \((-2, 3, 5)\), \((1, 2, 3)\) and \((7, 0, -1)\) be denoted by P, Q and R respectively. Points P, Q and R are collinear if they lie on a line.

\(PQ = \sqrt{(1 + 2)^2 + (2 - 3)^2 + (3 - 5)^2} \)
\[= \sqrt{3^2 + (-1)^2 + (-2)^2} \]
\[= \sqrt{9 + 1 + 4} \]
\[ QR = \sqrt{(7 - 1)^2 + (0 - 2)^2 + (-1 - 3)^2} \]
\[ = \sqrt{6^2 + (-2)^2 + (-4)^2} \]
\[ = \sqrt{72} + 4 + 16 \]
\[ = \sqrt{56} \]
\[ = 2\sqrt{14} \]

\[ PR = \sqrt{(7 + 2)^2 + (0 - 3)^2 + (-1 - 5)^2} \]
\[ = \sqrt{(9)^2 + (-3)^2 + (-6)^2} \]
\[ = \sqrt{81 + 9 + 36} \]
\[ = \sqrt{126} \]
\[ = 3\sqrt{14} \]

Here, \( PQ + QR = \sqrt{14} + 2\sqrt{14} = 3\sqrt{14} = PR \)
Hence, points P\((-2, 3, 5)\), Q\((1, 2, 3)\) and R\((7, 0, -1)\) are collinear.

3. (i) Verify the following:
   (i) \((0, 7, -10), (1, 6, -6)\) and \((4, 9, -6)\) are the vertices of an isosceles triangle
   Hint: \( PQA = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \)

**Solution:**
Solution step 1: Let points \((0, 7, -10), (1, 6, -6)\) and \((4, 9, -6)\) be denoted by A, B and C respectively.
\[ AB = \sqrt{(1 - 0)^2 + (6 - 7)^2 + (-6 + 10)^2} \]
\[ = \sqrt{1^2 + (-1)^2 + (4)^2} \]
\[ = \sqrt{1 + 1 + 16} \]
\[ = \sqrt{18} \]
\[ = 3\sqrt{2} \]
\[ BC = \sqrt{(4 - 1)^2 + (9 - 6)^2 + (-6 + 6)^2} \]
= \sqrt{(3)^2 + (3)^2} \\
= \sqrt{9 + 9} \\
= \sqrt{18} \\
= 3\sqrt{2} \\
CA = \sqrt{(0 - 4)^2 + (7 - 9)^2 + (-10 - 6)^2} \\
= \sqrt{(-4)^2 + (-2)^2 + (-4)^2} \\
= \sqrt{16 + 4 + 16} \\
= \sqrt{36} \\
= 6 \\
Here, AB = BC \neq CA \\
Thus, the given points are the vertices of an isosceles triangle.

(ii) (0,7,10), (-1,6,6) and (-4,9,6) are the vertices of a right angled triangle.

Hint: 
PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}

Solution:
Solution step 1: Let (0,7,10), (-1,6,6) and (-4,9,6) be denoted by A, B and C respectively.

AB = \sqrt{(-1 - 0)^2 + (6 - 7)^2 + (6 - 10)^2} \\
= \sqrt{(-1)^2 + (-1)^2 + (-4)^2} \\
= \sqrt{1 + 1 + 16} \\
= \sqrt{18} \\
= 3\sqrt{2} \\
BC = \sqrt{(-4 + 1)^2 + (9 - 6)^2 + (6 - 6)^2} \\
= \sqrt{(-3)^2 + (3)^2 + (0)^2} \\
= \sqrt{9 + 9} \\
= \sqrt{18} \\
= 3\sqrt{2}
CA = \sqrt{(0 + 4)^2 + (7 - 9)^2 + (10 - 6)^2}
= \sqrt{(4)^2 + (-2)^2 + (4)^2}
= \sqrt{16 + 4 + 16}
= \sqrt{36}
= 6

Now, \ AB^2 + BC^2 = (3\sqrt{2})^2 + (3\sqrt{2})^2 = 18 + 18 = 36 = AC^2
Therefore, by Pythagoras theorem, ABC is a right triangle.
Hence, the given points are the vertices of a right-angled triangle.

(iii) \((-1, 2, 1), (1, -2, 5), (4, -7, 8)\) and \((2, -3, 4)\) are the vertices of a parallelogram.

Hint: \(PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}\)

Solution:
Solution step 1: Let \((-1, 2, 1), (1, -2, 5), (4, -7, 8)\) and \((2, -3, 4)\) be denoted by \(A, B, C\) and \(D\) respectively.
\(AB = \sqrt{(1 + 1)^2 + (-2 - 2)^2 + (5 - 1)^2}\)
= \(\sqrt{4 + 16 + 16}\)
= 6
\(BC = \sqrt{(4 - 1)^2 + (-7 + 2)^2 + (8 - 5)^2}\)
= \(\sqrt{9 + 25 + 9}\)
= \(\sqrt{43}\)
\(CD = \sqrt{(2 - 4)^2 + (-3 + 7)^2 + (4 - 8)^2}\)
= \(\sqrt{4 + 16 + 16}\)
= \(\sqrt{36}\)
= 6
\(DA = \sqrt{(-1 - 2)^2 + (2 + 3)^2 + (1 - 4)^2}\)
= \(\sqrt{9 + 25 + 9}\)
= \(\sqrt{43}\)
Here, \( AB = CD = 6, BC = AD = \sqrt{43} \)

Hence, the opposite sides of quadrilateral \( ABCD \), whose vertices are taken in order, are equal.
Therefore, \( ABCD \) is a parallelogram.
Hence, the given points are the vertices of a parallelogram.

4. Find the equation of the set of points which are equidistant from the points \((1,2,3)\) and \((3,2,-1)\).

Hint: \( PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \)

**Solution:**

Solution step 1: Let \( P(x, y, z) \) be the point that is equidistant from points \( A(1,2,3) \) and \( B(3,2,-1) \). Accordingly, \( PA = PB \)

\[
\Rightarrow PA^2 = PB^2
\]

\[
\Rightarrow x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9 = x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 + 2z + 1
\]

\[
\Rightarrow -2x - 4y - 6z + 14 = -6x - 4y + 2z + 14
\]

\[
\Rightarrow -2x - 6z + 6x - 2z = 0
\]

\[
\Rightarrow 4x - 8z = 0
\]

\[
\Rightarrow x - 2z = 0
\]

Thus, the required equation is \( x - 2z = 0 \).

5. Find the equation of the set of points \( P \), the sum of whose distances from \( A(4,0,0) \) and \( B(-4,0,0) \) is equal to \( 10 \).

Hint: \( PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \)

**Solution:**

Solution step 1: Let the coordinates of \( P \) be \((x, y, z)\).

The coordinates of points \( A \) and \( B \) are \((4,0,0)\) and \((-4,0,0)\) respectively.

It is given that \( PA + PB = 10 \).

\[
\Rightarrow \sqrt{(x - 4)^2 + y^2 + z^2} + \sqrt{(x + 4)^2 + y^2 + z^2} = 10
\]
⇒ \(\sqrt{(x - 4)^2 + y^2 + z^2} = 10 - \sqrt{(x + 4)^2 + y^2 + z^2}\)

On squaring both sides, we obtain

⇒ \((x - 4)^2 + y^2 + z^2 = 100 - 20\sqrt{(x + 4)^2 + y^2 + z^2} + (x + 4)^2 + y^2 + z^2\)

⇒ \(x^2 - 8x + 16 + y^2 + z^2 = 100 - 20\sqrt{x^2 + 8x + 16 + y^2 + z^2 + x^2 + 8x + 16 + y^2 + z^2}\)

⇒ \(20\sqrt{x^2 + 8x + 16 + y^2 + z^2} = 100 + 16x\)

⇒ \(5\sqrt{x^2 + 8x + 16 + y^2 + z^2} = (25 + 4x)\)

On squaring both sides again, we obtain

\(25(x^2 + 8x + 16 + y^2 + z^2) = 625 + 16x^2 + 200x\)

⇒ \(25x^2 + 200x + 400 + 25y^2 + 25z^2 = 625 + 16x^2 + 200x\)

⇒ \(9x^2 + 25y^2 + 25z^2 - 225 = 0\)

Thus, the required equation is \(9x^2 + 25y^2 + 25z^2 - 225 = 0\).

**Exercise 12.3**

1. (i) Find the coordinates of the point which divides the line segment joining the points \((-2, 3, 5)\) and \((1, -4, 6)\) in the ratio \(2 : 3\) internally,

   Hint: \(P(x_1, y_1, z_1)\) and \(Q(x_2, y_2, z_2)\) internally in the ratio \(m : n\) are

   \[
   \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)
   \]

   **Solution:**

   Solution step 1: The coordinates of point \(R\) that divides the line segment joining points \(P(x_1, y_1, z_1)\) and \(Q(x_2, y_2, z_2)\) internally in the ratio \(m : n\) are

   \[
   \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)
   \]

   Let \(R(x, y, z)\) be the point that divides the line segment joining points \((-2, 3, 5)\) and \((1, -4, 6)\) internally in the ratio \(2 : 3\)

   \[
   x = \frac{2(-2) + 3(1)}{2+3}, \quad y = \frac{2(-4) + 3(3)}{2+3} \quad \text{and} \quad z = \frac{2(6) + 3(5)}{2+3}
   \]

   i.e., \(x = \frac{-4}{5}, y = \frac{1}{5}\) and \(z = \frac{27}{5}\)

   Thus, the coordinates of the required point are \((-\frac{4}{5}, \frac{1}{5}, \frac{27}{5})\).
(ii) 2 : 3 externally

Solution:

Solution step 1: externally in the ratio \( m : n \) are \( \left( \frac{mx_2 + nx_1}{m-n}, \frac{my_2 - my_1}{m-n}, \frac{mz_2 - nz_1}{m-n} \right) \)

The coordinates of point \( R \) that divides the line segment joining points \( P(x_1, y_1, z_1) \) and \( Q(x_2, y_2, z_2) \) externally in the ratio \( m : n \) are

\[
\left( \frac{mx_2 + nx_1}{m-n}, \frac{mz_2 - nz_1}{m-n} \right)
\]

Let \( R(x, y, z) \) be the point that divides the line segment joining points \((-2,3,5)\) and \((1, -4,6)\) externally in the ratio \( 2 : 3 \)

\[
x = \frac{2(-2) - 3(-3)}{2 - 3}, \quad y = \frac{2(-4) - 3(3)}{2 - 3} \quad \text{and} \quad z = \frac{2(6) - 3(5)}{2 - 3}
\]

i.e., \( x = -8, y = 17 \) and \( z = 3 \)

Thus, the coordinates of the required point are \((-8,17,3)\).

2. Given that \( P(3,2,-4), Q(5,4,-6) \) and \( R(9,8,-10) \) are collinear. Find the ratio in which \( Q \) divides \( PR \).

Hint: \( P(x_1, y_1, z_1) \) and \( Q(x_2, y_2, z_2) \) internally in the ratio \( m : n \) are

\[
\left( \frac{mx_2 + nx_1}{m+n}, \frac{my_2 + my_1}{m+n}, \frac{mz_2 + nz_1}{m+n} \right)
\]

Solution:

Solution step: I Let point \( Q(5,4,-6) \) divide the line segment joining points \( P(3,2,-4) \) and \( R(9,8,-10) \) in the ratio \( k : 1 \).

Therefore, by section formula,

\[
(5,4,-6) = \left( \frac{k(9) + 3}{k + 1}, \frac{k(8) + 2}{k + 1}, \frac{k(-10) - 4}{k + 1} \right)
\]

\[
\Rightarrow \frac{9k + 3}{k + 1} = 5
\]

\[
\Rightarrow 9k + 3 = 5k + 5
\]

\[
\Rightarrow 4k = 2
\]

\[
\Rightarrow k = \frac{1}{2}
\]

Thus, point \( Q \) divides \( PR \) in the ratio \( 1 : 2 \).
3. Find the ratio in which the YZ-plane divides the line segment formed by joining the points 
(−2, 4, 7) and (3, −5, 8).

Hint: P(𝑥_1, 𝑦_1, 𝑧_1) and Q(𝑥_2, 𝑦_2, 𝑧_2) internally in the ratio 𝑚 : 𝑛 are
\[
\left(\frac{𝑚𝑥_2+𝑛𝑥_1}{𝑚+𝑛}, \frac{𝑚𝑦_2+𝑛𝑦_1}{𝑚+𝑛}, \frac{𝑚𝑧_2+𝑛𝑧_1}{𝑚+𝑛}\right)
\]

Solution:

Solution step: 1 Let the YZ plane divide the line segment joining points (−2, 4, 7) and (3, −5, 8) in 
the ratio 𝑘 : 1.

Hence, by section formula, the coordinates of point of intersection are given by
\[
\left(\frac{𝑘(3)−2}{𝑘+1}, \frac{𝑘(−5)+4}{𝑘+1}, \frac{𝑘(8)+7}{𝑘+1}\right)
\]

On the YZ plane, the 𝑥-coordinate of any point is zero.

\[
\frac{3𝑘−2}{𝑘+1} = 0
\]

⇒ 3𝑘 − 2 = 0

⇒ 𝑘 = \frac{2}{3}

Thus, the YZ plane divides the line segment formed by joining the given points in the ratio 2 : 3.

4. Using section formula, show that the points A(2, −3, 4), B(−1, 2, 1) and C\left(\frac{0}{3}, 2\right) are collinear.

Hint: P(𝑥_1, 𝑦_1, 𝑧_1) and Q(𝑥_2, 𝑦_2, 𝑧_2) internally in the ratio 𝑚 : 𝑛 are
\[
\left(\frac{𝑚𝑥_2+𝑛𝑥_1}{𝑚+𝑛}, \frac{𝑚𝑦_2+𝑛𝑦_1}{𝑚+𝑛}, \frac{𝑚𝑧_2+𝑛𝑧_1}{𝑚+𝑛}\right)
\]

Solution:

Solution step: 1 The given points are A(2, −3, 4), B(−1, 2, 1) and C\left(\frac{0}{3}, 2\right).

Let P be a point that divides AB in the ratio 𝑘 : 1.

Hence, by section formula, the coordinates of P are given by
\[
\left(\frac{𝑘(−1)+2}{𝑘+1}, \frac{𝑘(2)−3}{𝑘+1}, \frac{𝑘(1)+4}{𝑘+1}\right)
\]

Now, we find the value of 𝑘 at which point P coincides with point C.

By taking \(\frac{−𝑘+2}{𝑘+2}\), we obtain 𝑘 = 2.
For $k = 2$, the coordinates of point $P$ are $\left(0, \frac{1}{3}, 2\right)$.

i.e., $C \left(0, \frac{1}{3}, 2\right)$ is a point that divides $AB$ externally in the ratio $2 : 1$ and is the same as point $P$.

Hence, points $A, B$ and $C$ are collinear.

5. Find the coordinates of the points which trisect the line segment joining the points $P(4,2, -6)$ and $Q(10, -16, 6)$.

Hint: $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ internally in the ratio $m : n$ are $(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n})$

Solution:
Solution step 1: Let $A$ and $B$ be the points that trisect the line segment joining points $P(4,2, -6)$ and $Q(10, -16, 6)$

Point $A$ divides $PQ$ in the ratio $1 : 2$. Therefore, by section formula, the coordinates of point $A$ are given by

$$\left(\frac{1(10) + 2(4)}{1 + 2}, \frac{1(-16) + 2(2)}{1 + 2}, \frac{1(6) + 2(-6)}{1 + 2}\right) = (6, -4, -2)$$

Point $B$ divides $PQ$ in the ratio $2 : 1$. Therefore, by section formula, the coordinates of point $B$ are given by

$$\left(\frac{2(10) + 1(4)}{2 + 1}, \frac{2(-16) + 1(2)}{2 + 1}, \frac{2(6) - 1(6)}{2 + 1}\right) = (8, -10, 2)$$

Thus, $(6, -4, -2)$ and $(8, -10, 2)$ are the points that trisect the line segment joining points $P(4,2, -6)$ and $Q(10, -16, 6)$. 

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