CBSE NCERT Solutions for Class 12 Maths Chapter 08

Back of Chapter Questions

Exercise 8.1

1. Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1, x = 4$ and the x-axis in the first quadrant.

[2 Marks]

Solution:

Step 1:
Given: Equation of the curve $y^2 = x$ and the lines $x = 1, x = 4$ and the x-axis.
The given equations can be represented as follows:

\[ y^2 = x \]

Step 2:
The area of the region bounded by the curve, $y^2 = x$, the lines, $x = 1$ and $x = 4$, and the x-axis is the area $ABCD$.

Area of $ABCD = \int_1^4 y \, dx$

\[ = \int_1^4 \sqrt{x} \, dx \]  

[1 Mark]

Step 3:
\[ = \left[ \frac{3}{2} x^{\frac{3}{2}} \right]_1^4 \]

\[ = \frac{2}{3} \left( 4^{\frac{3}{2}} - 1^{\frac{3}{2}} \right) \]
2. Find the area of the region bounded by $y^2 = 9x, x = 2, x = 4$ and the $x$-axis in the first quadrant.

**Solution:**

**Step 1:**

**Given:** The equations of the curve $y^2 = 9x, x = 2, x = 4$ and the $x$-axis in the first quadrant.

The given equations can be represented as follows:

$$
\text{The area of the region bounded by the curve, } y^2 = 9x, x = 2, x = 4, \text{ and the } x\text{-axis is the area } ABCD.
$$

**Step 2:**

Area of $ABCD = \int_2^4 y \, dx$

$$
= \int_2^4 3\sqrt{x} \, dx \\
= \left[ \frac{3}{2} \right] x^{\frac{3}{2}} \bigg|_2^4 \\
= 2 \left[ \frac{3}{2} \right]^{\frac{4}{2}}
$$

Hence, the required area is $\frac{14}{3}$ square units.

$$
\begin{align*}
&= \frac{2}{3} [8 - 1] \\
&= \frac{14}{3} \text{ square units} \\
&= \left[ \frac{1}{2} \text{ Mark} \right]
\end{align*}
$$
3. Find the area of the region bounded by \( x^2 = 4y, y = 2, y = 4 \) and the \( y \)-axis in the first quadrant.

[2 Marks]

**Solution:**

**Step 1:**
Given: The equations of the curve \( x^2 = 4y, y = 2, y = 4 \) and the \( y \)-axis in the first quadrant.

The given equations can be represented as follows:

\[
\begin{align*}
\text{Area} & = \int_2^4 xy \, dy \\
& = \int_2^4 2 \sqrt{y} \, dy \\
& = 2 \left[ \sqrt{y} \right]_2^4 \\
& = 2 \left( \sqrt{4} - \sqrt{2} \right) \\
& = 2 \left( 2 - \sqrt{2} \right) \text{ square units} \quad [\frac{1}{2} \text{ Mark}]
\end{align*}
\]

**Step 2:**
The area of the region bounded by the curve, \( x^2 = 4y, y = 2, y = 4 \), and the \( y \)-axis is the area \( ABCD \).

\[
\text{Area of } ABCD = \int_2^4 xy \, dy \\
= \int_2^4 2 \sqrt{y} \, dy \\
[1 \text{ Mark}]
\]

**Step 3:**

\[
= 2 \int_2^4 \sqrt{y} \, dy \\
\]
= 2 \left[ \frac{y^2}{3} \right]_2^4
= \frac{4}{3} \left[ (4)^3 - (2)^3 \right]
= \frac{4}{3} \left[ 8 - 2\sqrt{2} \right]
= \left( \frac{32 - 8\sqrt{2}}{3} \right) \text{ square units} \quad [\frac{1}{2} \text{ Mark}]

Hence, the required area is \( \left( \frac{32 - 8\sqrt{2}}{3} \right) \) square units.

4. Find the area of the region bounded by the ellipse \( \frac{x^2}{16} + \frac{y^2}{9} = 1. \) \hspace{1cm} [2 Marks]

Solution:

Step 1:
Given: The equation of the ellipse: \( \frac{x^2}{16} + \frac{y^2}{9} = 1 \)

The given equation of the ellipse can be represented as follows:

Step 2:
It is observed that the ellipse is symmetrical about \( x \)-axis and \( y \)-axis. \( \therefore \text{Area bounded by ellipse} \)
\( = 4 \times \text{Area of } OAB \)
Area of \( OAB = \int_0^4 y \, dx \)
\( = \int_0^4 3 \sqrt{1 - \frac{x^2}{16}} \, dx \) \[1 \text{ Mark}\]

Step 3:
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$$\frac{3}{4} \int_{0}^{4} \sqrt{16 - x^2} \, dx$$

$$= \frac{3}{4} \left[ \frac{x}{2} \sqrt{16 - x^2} + \frac{16}{2} \sin^{-1} \frac{x}{4} \right]_{0}^{4}$$

$$= \frac{3}{4} \left[ 2\sqrt{16 - 16} + 18 \sin^{-1}(1) - 0 - 8 \sin^{-1}(0) \right]$$

$$= \frac{3}{4} \left[ \frac{8\pi}{2} \right]$$

$$= \frac{3}{4} [4\pi]$$

$$= 3\pi$$

Hence, area bounded by the ellipse $$= 4 \times 3\pi = 12\pi$$ square units. [1½ Mark]

5. Find the area of the region bounded by the ellipse $$\frac{x^2}{4} + \frac{y^2}{9} = 1$$. [2 Marks]

Solution:

Step 1:
Given: The equation of the ellipse $$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

The given equation of the ellipse can be represented as

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

$$\Rightarrow y = 3 \sqrt{1 - \frac{x^2}{4}} \ldots \text{(i)}$$ [1½ Mark]

Step 2:
It is observed that the ellipse is symmetrical about $x$-axis and $y$-axis.
∴ Area bounded by ellipse = $4 \times \text{Area } OAB$
∴ Area of $OAB = \int_{0}^{2} y \, dx$
\[
= \int_{0}^{2} 3 \sqrt{1 - \frac{x^2}{4}} \, dx \quad \text{[Using (i)]} \quad \text{[1 Mark]}
\]
\[\text{Step 3:}\]
\[
= \frac{3}{2} \int_{0}^{2} \sqrt{4 - x^2} \, dx
\]
\[
= \frac{3}{2} \left[ \frac{x}{2} \sqrt{4 - x^2} + \frac{4}{2} \sin^{-1}\left(\frac{-x}{2}\right) \right]_{0}^{2}
\]
\[
= \frac{3}{2} \left[ \frac{2\pi}{2} \right]
\]
\[
= \frac{3\pi}{2}
\]
Hence, area bounded by the ellipse = $4 \times \frac{3\pi}{2} = 6\pi$ square units. \[
\frac{1}{2} \text{ Mark}\]

6. Find the area of the region in the first quadrant enclosed by $x$-axis, line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$. \[
\text{[4 marks]}\]

\textbf{Solution:}\n
\textbf{Step 1:}\n
\textbf{Given:} The equations of the circle: $x^2 + y^2 = 4$ and $x$-axis, line $x = \sqrt{3}y$
The area of the region bounded by the circle, \( x^2 + y^2 = 4, x = \sqrt{3}y \) and the x-axis is the area \( OAB \).

The point of intersection of the line and the circle in the first quadrant is \( (\sqrt{3}, 1) \)

\[
\text{Area } OAB = \text{Area } \triangle OCA + \text{Area } ACB \quad \left[ \frac{1}{2} \text{Mark} \right]
\]

**Step 3:**

\[
\text{Area of } OAC = \frac{1}{2} \times OC \times AC = \frac{1}{2} \times \sqrt{3} \times 1 = \frac{\sqrt{3}}{2} \quad \left[ \frac{1}{2} \text{Mark} \right]
\]

**Step 4:**

\[
\text{Area of } ABC = \int_{\sqrt{3}}^{2} y \, dx \quad \left[ \frac{1}{2} \text{Mark} \right]
\]

**Step 5:**

\[
= \int_{\sqrt{3}}^{2} \sqrt{4 - x^2} \, dx
\]

\[
= \left[ \frac{x}{2} \sqrt{4 - x^2} + \frac{4}{2} \sin^{-1} \left( \frac{x}{2} \right) \right]_{\sqrt{3}}^{2} \quad \left[ \frac{1}{2} \text{Mark} \right]
\]

**Step 5:**

\[
= \left[ \frac{2 \times \pi}{2} - \frac{\sqrt{3}}{2} \sqrt{4 - \frac{3}{2}} - 2 \sin^{-1} \left( \frac{\sqrt{3}}{2} \right) \right]
\]

\[
= \left[ \pi - \frac{\sqrt{3}}{2} - 2 \left( \frac{\pi}{3} \right) \right]
\]

\[
= \left[ \pi - \frac{\sqrt{3}}{2} - \frac{2 \pi}{3} \right]
\]

\[
= \left[ \frac{\pi}{3} - \frac{\sqrt{3}}{2} \right] \quad \left[ \frac{1}{2} \text{Mark} \right]
\]

**Step 6:**

Hence, area enclosed by x-axis, the line \( x = \sqrt{3}y \) and the circle \( x^2 + y^2 = 4 \) in the first quadrant is \( \frac{\sqrt{3} \pi}{2} + \frac{\pi}{3} = \frac{\pi}{3} \) sq. units. \( [1 \text{ Mark}] \)

7. Find the area of the smaller part of the circle \( x^2 + y^2 = a^2 \) cut off by the line \( x = \frac{a}{\sqrt{2}} \) \( [4 \text{ Marks}] \)

**Solution:**

**Step 1:**

**Given:** The equation of the circle: \( x^2 + y^2 = a^2 \) which is cut off by a line \( x = \frac{a}{\sqrt{2}} \)
Step 2:
The area of the smaller part of the circle, \( x^2 + y^2 = a^2 \), cut off by the line, \( x = \frac{a}{\sqrt{2}} \) is the area \( ABCDA \).
It is observed that the area \( ABCD \) is symmetrical about \( x \)-axis.
\[ \therefore \text{Area } ABCD = 2 \times \text{Area } ABC \]  

Step 3:
Area of \( ABC \) = \( \int_{a}^{\frac{a}{\sqrt{2}}} y \, dx \)  
= \( \int_{\frac{a}{\sqrt{2}}}^{a} \sqrt{a^2 - x^2} \, dx \)  

Step 4:
\[ \begin{align*}
&= \left[ \frac{a^2}{2} \left( \frac{\pi}{2} \right) - \frac{a}{2\sqrt{2}} \sqrt{a^2 - a^2} - \frac{a^2}{2} \sin^{-1} \left( \frac{1}{\sqrt{2}} \right) \right] \\
&= \frac{a^2\pi}{4} - \frac{a^2}{2\sqrt{2}} - \frac{a^2}{2} \left( \frac{\pi}{4} \right) \\
&= \frac{a^2\pi}{4} - \frac{a^2}{4} \frac{\sqrt{2}}{2} \\
&= \frac{a^2}{2} \left[ \frac{\pi}{2} - 1 - \frac{\pi}{2} \right] \\
&= \frac{a^2}{4} \left[ \frac{\pi}{2} - 1 \right] \\
\end{align*} \]  

Step 5:
\[ \Rightarrow \text{Area } ABCD = 2 \left[ \frac{a^2}{4} \left( \frac{\pi}{2} - 1 \right) \right] = \frac{a^2}{2} \left( \frac{\pi}{2} - 1 \right) \]  
Hence, the area of smaller part of the circle, \( x^2 + y^2 = a^2 \), cut off by the line, \( x = \frac{a}{\sqrt{2}} \) is \( \frac{a^2}{2} \left( \frac{\pi}{2} - 1 \right) \) sq. units.  

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8. The area between \( x = y^2 \) and \( x = 4 \) is divided into two equal parts by the line \( x = a \), find the value of \( a \). \[4 \text{ marks}\]

**Solution:**

**Step 1:**

**Given:** The area between \( x = y^2 \) and \( x = 4 \) is divided into two equal parts by the line \( x = a \).

The line, \( x = a \), divides the area bounded by the parabola and \( x = 4 \) into two equal parts.

\( \therefore \) Area \( OAD = \text{Area} \ AB\!CD \)

**Step 2:**

It can be observed that the given area is symmetrical about \( x \)-axis.

\( \Rightarrow \) Area \( OED = \text{Area} \ EFCD \)

**Step 3:**

\[
\text{Area} \ OED = \int_{0}^{a} y \, dx \quad \left[ \frac{1}{2} \text{ Mark} \right]
\]

\[
= \int_{0}^{a} \sqrt{x} \, dx
\]

\[
= \left[ \frac{x^{3/2}}{3/2} \right]_{0}^{a}
\]

\[
= \frac{2}{3} (a)^{3/2} \quad \ldots \text{(i)} \quad \left[ \frac{1}{2} \text{ Mark} \right]
\]

**Step 4:**

Area of \( EFCD = \int_{a}^{4} \sqrt{x} \, dx \quad \left[ \frac{1}{2} \text{ Mark} \right]

**Step 5:**
Step 6:
From (i) and (ii), we get
\[ \frac{2}{3} (a)^3 = \frac{2}{3} \left[ 8 - (a)^{\frac{3}{2}} \right] \]
\[ \Rightarrow 2. (a)^{\frac{3}{2}} = 8 \]
\[ \Rightarrow (a)^{\frac{3}{2}} = 4 \]
\[ \Rightarrow a = (4)^{\frac{2}{3}} \]
Hence, the value of \( a \) is \( (4)^{\frac{2}{3}} \).

9. Find the area of the region bounded by the parabola \( y = x^2 \) and \( y = |x| \). [2 Marks]

Solution:

Step 1:

Given: Equation of the parabola \( y = x^2 \) and \( y = |x| \)

The area bounded by the parabola, \( x^2 = y \) and the line, \( y = |x| \), can be represented as
The given area is symmetrical about y-axis.
∴ Area $OACO = Area \ ODBO$
The point of intersection of parabola, $x^2 = y$ and line, $y = x$ is $A(1, 1)$.
Area of $OACO = Area \ ∆OAB - Area \ OBAO$ $\left[ \frac{1}{2} \text{Mark} \right]$

**Step 3:**
∴ Area of $\Delta OAB = \frac{1}{2} \times OB \times AB = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$
Area of $OBAO = \int_{0}^{1} y \, dx = \int_{0}^{1} x^2 \, dx = \left[ \frac{x^3}{3} \right]_{0}^{1} = \frac{1}{3}$ $\left[ \frac{1}{2} \text{Mark} \right]$

**Step 4:**
⇒ Area of $OACO = Area \ ∆OAB - Area \ OBAO$
\[
= \frac{1}{2} - \frac{1}{3}
= \frac{1}{6}
\]
Hence, required area $= 2 \left[ \frac{1}{6} \right] = \frac{1}{3} \text{ sq. units}$ $\left[ \frac{1}{2} \text{Mark} \right]$

10. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$. $[4 \text{ Marks}]$

**Solution:**

**Step 1:**
**Given:** Equation of the curve $x^2 = 4y$ and the line $x = 4y - 2$
The area bounded by the curve, $x^2 = 4y$ and line, $x = 4y - 2$, is represented by the shaded area $OBAO$. 
Step 2:
Let $A$ and $B$ be the points of intersection of the line and parabola. Coordinates of point $A$ are $(-1, \frac{1}{4})$. Coordinates of point $B$ are $(2, 1)$. We draw $AL$ and $BM$ perpendicular to $x$-axis.

Step 3:
It is observed that,
Area $OBAO = Area\ OBCO + Area\ OACO$ ...(i)
Then, Area $OBCO = Area\ OMBC - Area\ OMOB$
\[ \int_{0}^{2} \frac{x^2+2}{4} dx - \int_{0}^{1} \frac{x^2}{4} dx \]  
Step 4:
\[ = \left[ \frac{x^2}{2} + \frac{2x^2}{4} \right]_{0}^{2} - \left[ \frac{x^3}{4} \right]_{0}^{3} \]
\[ = \frac{1}{4} \left[ 2 + 4 \right] - \frac{1}{4} \left[ \frac{4}{3} \right] \]
\[ = \frac{1}{2} - \frac{2}{3} \]
\[ = \frac{5}{6} \]  
Step 5:
Similarly, Area $OACO = Area\ OLAC - Area\ OLAO$
\[ = \int_{-1}^{0} \frac{x^2}{4} dx - \int_{-1}^{1} \frac{x^2}{4} dx \]
Step 6:
Practice more on Application of Integrals

11. Find the area of the region bounded by the curve \( y^2 = 4x \) and the line \( x = 3 \). [2 marks]

Solution:

Step 1:

Given: Equation of the curve \( y^2 = 4x \) and the line \( x = 3 \)

The region bounded by the parabola, \( y^2 = 4x \) and the line, \( x = 3 \) is the area \( OACO \). [\( \frac{1}{2} \) Mark]

Step 2:

The area \( OACO \) is symmetrical about \( x \)-axis

\( \therefore \) Area of \( OACO = 2 \) (Area of \( OAB \))

Area \( OACO = 2 \left[ \int_0^3 y \, dx \right] \) [\( \frac{1}{2} \) Mark]

\[ \int_0^3 y \, dx \]

\[ = \frac{1}{4} \left[ \frac{x^2}{2} + 2x \right]_0^3 - \frac{1}{4} \left[ \frac{x^3}{3} \right]_0^3 \]

\[ = - \frac{1}{4} \left[ \frac{(-1)^2}{2} + 2(-1) \right] - \frac{1}{4} \left( \frac{(-1)^3}{3} \right) \]

\[ = - \frac{1}{4} \left[ \frac{1}{2} - 2 \right] - \frac{1}{12} \]

\[ = \frac{1}{2} - \frac{1}{8} - \frac{1}{12} \]

\[ = \frac{7}{24} \] [\( \frac{1}{2} \) Mark]
Step 3:
\[\int_{0}^{3} 2\sqrt{x} \, dx \]
\[= 2 \left[ \frac{x^{3/2}}{3/2} \right]_{0}^{3} \quad \text{[1/2 Mark]}\]

Step 4:
\[= \frac{8}{3} \left( 3^{3/2} \right) \]
\[= 8\sqrt{3} \]

Hence, the required area is \(8\sqrt{3}\) Square units. \([1/2 \text{ Mark]}\)

12. Area lying in the first quadrant and bounded by the circle \(x^2 + y^2 = 4\) and the lines \(x = 0\) and \(x = 2\) is \([2 \text{ Marks}]\)

(A) \(\pi\)
(B) \(\frac{\pi}{2}\)
(C) \(\frac{\pi}{3}\)
(D) \(\frac{\pi}{4}\)

Solution:

(A)

Step 1:
Given: The equation of the circle \(x^2 + y^2 = 4\) and the lines \(x = 0\) and \(x = 2\)

The area bounded by the given circle and the lines, \(x = 0\) and \(x = 2\) in the first quadrant is represented as
Step 2:
\[ \therefore \text{Area } OAB = \int_{0}^{2} y \, dx \quad [\frac{1}{2} \text{Mark}] \]

Step 3:
\[ = \int_{0}^{2} \sqrt{4 - x^2} \, dx \quad [\frac{1}{2} \text{Mark}] \]

Step 4:
\[ = \left[ \frac{x}{2} \sqrt{4 - x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_{0}^{2} \]
\[ = 2 \left( \frac{\pi}{2} \right) \]
\[ = \pi \text{ units} \]

Hence, (A) is the correct answer. \[\frac{1}{2} \text{Mark}] \]

13. Area of the region bounded by the curve \( y^2 = 4x \), \( y \)-axis and the line \( y = 3 \) is \[2 \text{ Marks}] \]

(A) 2
(B) \( \frac{9}{4} \)
(C) \( \frac{9}{3} \)
(D) \( \frac{9}{2} \)

Solution:

(B)

Step 1:
Given: The equation of the curve \( y^2 = 4x \), \( y \)-axis and the line \( y = 3 \)

The area bounded by the curve, \( y^2 = 4x \), \( y \)-axis and \( y = 3 \) is represented as
Step 2:
\[ \therefore \text{Area } OAB = \int_0^3 x \, dy \ [\frac{1}{2} \text{Mark}] \]

Step 3:
\[ = \int_0^3 y^3 \cdot \frac{1}{4} \, dy \ [\frac{1}{2} \text{Mark}] \]

Step 4:
\[ = \frac{1}{4} \left[ \frac{y^4}{3} \right]_0^3 \]
\[ = \frac{1}{12} (27) \]
\[ = \frac{9}{4} \text{ units} \]

Hence, \((B)\) is the correct answer. \([\frac{1}{2} \text{Mark}]\)

Exercise 8.2

1. Find the area of the circle \(4x^2 + 4y^2 = 9\) which is interior to the parabola \(x^2 = 4y\). \([4 \text{ Marks}]\)

Solution:

Step 1:
**Given:** Equation of the circle \(4x^2 + 4y^2 = 9\) and the equation of the parabola \(x^2 = 4y\). 
The required area is represented by the shaded area $OBCDO$. $\left[ \frac{1}{2} \text{ Mark} \right]$

**Step 2:**
Solving the given equation of circle, $4x^2 + 4y^2 = 9$, and parabola, $x^2 = 4y$, we obtain the point of intersection as $B \left( \sqrt{2}, \frac{1}{2} \right)$ and $D \left( -\sqrt{2}, \frac{1}{2} \right)$
It is observed that the required area is symmetrical about $y$-axis.
∴ Area $OBCDO = 2 \times$ Area $OBCO \left[ \frac{1}{2} \text{ Mark} \right]$

**Step 3:**
We draw $BM$ perpendicular to $OA$.
Hence, the coordinates of $M$ are $\left( \sqrt{2}, 0 \right)$.
Hence, Area $OBCO = $ Area $OMBCO - $ Area $OMBO$

$$
\begin{align*}
\int_0^{\sqrt{2}} \left( \frac{9 - 4x^2}{4} \right) dx - \int_0^{\sqrt{2}} \frac{x^2}{4} dx \\
= \frac{1}{2} \int_0^{\sqrt{2}} \sqrt{9 - 4x^2} dx - \frac{1}{4} \int_0^{\sqrt{2}} x^2 dx \\
= \frac{1}{4} \left[ x \sqrt{9 - 4x^2} + \frac{9}{2} \sin^{-1} \left( \frac{x}{3} \right) \right]_0^{\sqrt{2}} - \frac{1}{4} \left[ \frac{x^3}{3} \right]_0^{\sqrt{2}} \\
= \frac{\sqrt{2}}{4} \left[ 9 \frac{2\sqrt{2}}{3} - \frac{2\sqrt{2}}{3} \right] - \frac{1}{12} \sqrt{2} \\
= \frac{\sqrt{2}}{4} \left[ \frac{9}{2} \sin^{-1} \left( \frac{2\sqrt{2}}{3} \right) \right] - \frac{\sqrt{2}}{6} \\
= \frac{\sqrt{2}}{4} \left[ \frac{9}{2} \sin^{-1} \left( \frac{2\sqrt{2}}{3} \right) \right] - \frac{\sqrt{2}}{6} \\
= \frac{1}{2} \left( \frac{\sqrt{2}}{6} + \frac{9}{4} \sin^{-1} \left( \frac{2\sqrt{2}}{3} \right) \right) \quad \left[ 2 \text{ Marks} \right]
\end{align*}
$$

**Step 4:**
Hence, the required area $OBCDO$ is
$$
\left( 2 \times \frac{1}{2} \left( \frac{\sqrt{2}}{6} + \frac{9}{4} \sin^{-1} \left( \frac{2\sqrt{2}}{3} \right) \right) \right) = \left[ \frac{\sqrt{2}}{6} + \frac{9}{4} \sin^{-1} \left( \frac{2\sqrt{2}}{3} \right) \right] \text{ square units} \left[ 1 \text{ Mark} \right]
$$
2. Find the area bounded by curves \((x - 1)^2 + y^2 = 1\) and \(x^2 + y^2 = 1\). [4 Marks]

Solution:

Step 1:
Given: The equations of the curves \((x - 1)^2 + y^2 = 1\) and \(x^2 + y^2 = 1\).
The area bounded by the curves, \((x - 1)^2 + y^2 = 1\) and \(x^2 + y^2 = 1\) is represented by the shaded area as

![Shaded Area Diagram](image)

\[\text{[1 Mark]}\]

Step 2:
After solving the equations \((x - 1)^2 + y^2 = 1\) and \(x^2 + y^2 = 1\), we get the point of intersection as \(A \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)\) and \(B \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)\).
It is observed that the required area is symmetrical about \(x\)-axis.
\[\therefore \text{Area } OBCAO = 2 \times \text{Area } OCAO \quad \text{[1 Mark]}\]

Step 3:
We join \(AB\), which intersects \(OC\) at \(M\), such that \(AM\) is perpendicular to \(OC\).
The coordinates of \(M\) are \(\left(\frac{1}{2}, 0\right)\).
\[\Rightarrow \text{Area } OCAO = \text{Area } OMAO + \text{Area } MCAM \quad \text{[1 Mark]}\]

Step 4:
\[
= \left[ \int_{0}^{\frac{1}{2}} \sqrt{1 - (x - 1)^2} \, dx + \int_{\frac{1}{2}}^{1} \sqrt{1 - x^2} \, dx \right] \\
= \left[ \frac{x - 1}{2} \sqrt{1 - (x - 1)^2} + \frac{1}{2} \sin^{-1}(x - 1) \right]_{0}^{\frac{1}{2}} + \left[ \frac{x}{2} \sqrt{1 - x^2} + \frac{1}{2} \sin^{-1} x \right]_{\frac{1}{2}}^{1} \\
\]
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\[
= \left[ -\frac{1}{4} \sqrt{1 - \left( -\frac{1}{2} \right)^2} + \frac{1}{2} \sin^{-1} \left( \frac{1}{2} - 1 \right) - \frac{1}{2} \sin^{-1} (-1) \right] \\
+ \left[ \frac{1}{2} \sin^{-1} (1) - \frac{1}{4} \sqrt{1 - \left( \frac{1}{2} \right)^2} - \frac{1}{2} \sin^{-1} \left( \frac{1}{2} \right) \right]
\]

\[
= \left[ -\frac{\sqrt{3}}{8} + \frac{1}{2} \left( -\frac{\pi}{6} \right) - \frac{1}{2} \left( -\frac{\pi}{2} \right) \right] + \left[ \frac{1}{2} \left( \frac{\pi}{2} \right) - \frac{\sqrt{3}}{8} - \frac{1}{2} \left( \frac{\pi}{2} \right) \right]
\]

\[
= \left[ -\frac{\sqrt{3}}{4} + \frac{\pi}{12} + \frac{\pi}{4} + \frac{\pi}{4} - \frac{\pi}{12} \right]
\]

\[
= \left[ -\frac{\sqrt{3}}{4} + \frac{\pi}{6} + \frac{\pi}{2} \right]
\]

\[
= \left[ \frac{2\pi}{6} - \frac{\sqrt{3}}{4} \right] \quad [1 \frac{1}{2} \text{ Mark}]
\]

**Step 5:**

Hence, required area \( OBCAO = 2 \times \left( \frac{2\pi}{6} - \frac{\sqrt{3}}{4} \right) \) sq. units \[1 \text{ Mark}\]

3. Find the area of the region bounded by the curves \( y = x^2 + 2, y = x, x = 0 \) and \( x = 3 \) \[2 \text{ Marks}\]

**Solution:**

**Step 1:**

Given: The equations \( y = x^2 + 2, y = x, x = 0 \) and \( x = 3 \)

The area bounded by the curves \( y = x^2 + 2, y = x, x = 0 \) and \( x = 3 \) is represented by the shaded area \( OCBACO \) as

**Step 2:**

Then, Area \( OCBACO = \text{Area } ODBACO - \text{Area } ODCO \)

\[
= \int_0^3 (x^2 + 2) \, dx - \int_0^3 x \, dx \quad [1 \text{ Mark}]
\]
Step 4:
\[
\frac{x^3}{3} + 2x \bigg|_0^3 - \frac{x^2}{2} \bigg|_0^3
= [9 + 6] - \left(\frac{9}{2}\right)
= 15 - \frac{9}{2}
= \frac{21}{2} \text{ square units} \quad \left[\frac{1}{2} \text{ Mark}\right]
\]
Hence, the required area is \(\frac{21}{2}\) square units.

4. Using integration find the area of the region bounded by the triangle whose vertices are \((-1, 0), (1, 3)\) and \((3, 2)\). \([6 \text{ Marks}]\)

Solution:

Step 1:
Given: The vertices of a triangle \((-1, 0), (1, 3)\) and \((3, 2)\).

Step 2:
\(BL\) and \(CM\) are drawn perpendicular to \(x\)-axis.

It is observed in the figure that
\[
\text{Area}(\Delta ACM) = \text{Area}(ALBA) + \text{Area}(BLMCB) - \text{Area}(AMCA) \quad \cdots(i) \quad \left[1 \text{ Mark}\right]
\]

Step 3:
Equation of line segment \(AB\) is
\[
y - 0 = \frac{3 - 0}{1 + 1}(x + 1)
\]
\[
y = \frac{3}{2}(x + 1) \quad \left[\frac{1}{2} \text{ Mark}\right]
\]

Step 4:
\[
\therefore \text{Area}(ALBA) = \int_{-\frac{1}{2}}^{1} \frac{3}{2} (x + 1)dx = \frac{3}{2} \left[\frac{x^2}{2} + x\right]_{-\frac{1}{2}}^{1} = \frac{3}{2} \left[\frac{1}{2} + 1 - \frac{1}{2} + 1\right] = 3 \text{ sq. units.} \quad \left[1 \text{ Mark}\right]
\]
Step 5:
Equation of line segment BC is
\[ y - 3 = \frac{2 - 3}{3 - 1}(x - 1) \]
\[ y = \frac{1}{2}(-x + 7) \quad \left\{ \frac{1}{2} \text{ Mark} \right\} \]

Step 6:
\[ \therefore \text{Area}(BLMCB) = \int_{1}^{3} \frac{1}{2}(-x + 7)dx = \frac{1}{2} \left[ -\frac{x^2}{2} + 7x \right]_{1}^{3} = \frac{1}{2} \left[ -\frac{9}{2} + 21 + \frac{1}{2} - 7 \right] = 5 \text{ square units.} \]
\[ [1 \text{ Mark}] \]

Step 7:
Equation of line segment AC is
\[ y - 0 = \frac{2 - 0}{3 + 1}(x + 1) \]
\[ y = \frac{1}{2}(x + 1) \quad \left\{ \frac{1}{2} \text{ Mark} \right\} \]
\[ \therefore \text{Area}(AMCA) = \frac{1}{2} \int_{1}^{3}(x + 1)dx = \frac{1}{2} \left[ \frac{x^2}{2} + x \right]_{1}^{3} = \frac{1}{2} \left[ \frac{9}{2} + 3 - \frac{1}{2} + 1 \right] = 4 \text{ square units.} \quad \left\{ 1 \text{ Mark} \right\} \]

Step 8:
Hence, from equation (i), we obtain \( \text{Area}(\Delta ABC) = (3 + 5 - 4) = 4 \text{ square units.} \quad \left\{ \frac{1}{2} \text{ Mark} \right\} \]

---

5. Using integration find the area of the triangular region whose sides have the equations \( y = 2x + 1, y = 3x + 1 \) and \( x = 4 \). \[ 4 \text{ Marks} \]

Solution:

Step 1:
Given: The equations of the sides of a triangle: \( y = 2x + 1, y = 3x + 1 \) and \( x = 4 \).

Step 2:
On solving these equations, we obtain the vertices of triangle as \( A(0, 1), B(4, 13) \) and \( C(4, 9) \).
Step 3:
It is observed that,
\[ \text{Area}(\Delta ACB) = \text{Area}(OLBAO) - \text{Area}(OLCAO) \] \[ \left[ \frac{1}{2} \text{Mark} \right] \]

Step 4:
\[ = \int_{0}^{4} (3x + 1)dx - \int_{0}^{4} (2x + 1)dx \] \[ \left[ 1 \text{ Mark} \right] \]

Step 5:
\[ = \left[ \frac{3x^2}{2} + x \right]_{0}^{4} - \left[ \frac{2x^2}{2} + x \right]_{0}^{4} \]
\[ = (24 + 4) - (16 + 4) \]
\[ = 28 - 20 \]
\[ = 8 \text{ square units} \] \[ \left[ 1 \text{ Mark} \right] \]
Hence, the area of the triangular region is 8 square units.

6. Smaller area enclosed by the circle \( x^2 + y^2 = 4 \) and the line \( x + y = 2 \) is \( \left( A \right) 2(\pi - 2) \) \( \left( B \right) \pi - 2 \) \( \left( C \right) 2\pi - 1 \) \( \left( D \right) 2(\pi + 2) \) \[ 2 \text{ Marks} \]

Solution:
(B)

Step 1:
Given: The equation of the circle \( x^2 + y^2 = 4 \) and the line \( x + y = 2 \).
Step 2:
The smaller area enclosed by the circle, \( x^2 + y^2 = 4 \) and the line \( x + y = 2 \) is represented by the shaded area \( ACBA \). \([\frac{1}{2} \text{ Mark}]\)

**Step 3:**
It is observed that,
\[
\text{Area } ACBA = \text{Area } OACBO - \text{Area } (\Delta OAB) \quad [\frac{1}{2} \text{ Mark}]
\]

**Step 4:**
\[
\begin{align*}
&= \int_{0}^{2} \sqrt{4 - x^2} \, dx - \int_{0}^{2} (2 - x) \, dx \\
&= \left[ \frac{x}{2} \sqrt{4 - x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_{0}^{2} - \left[ 2x - \frac{x^2}{2} \right]_{0}^{2} \\
&= \left[ 2 \cdot \frac{\pi}{2} \right] - [4 - 2] \quad [\frac{1}{2} \text{ Mark}]
\end{align*}
\]

**Step 5:**
\[
= (\pi - 2) \text{ square units}
\]

Hence, (B) is the correct answer. \([\frac{1}{2} \text{ Mark}]\)

7. Area lying between the curve \( y^2 = 4x \) and \( y = 2x \) is \([2 \text{ Marks}]\)

(A) \( \frac{2}{3} \)

(B) \( \frac{1}{3} \)

(C) \( \frac{1}{4} \)

(D) \( \frac{3}{4} \)

**Solution:**

(B)

**Step 1:**
Given: The equations of the curve $y^2 = 4x$ and $y = 2x$

Step 2:
The area lying between the curve $y^2 = 4x$ and $y = 2x$ is represented by the shaded area $OBAO$ as

The points of intersection of these curves are $O(0, 0)$ and $A(1, 2).$ [\frac{1}{2} \text{ Mark}]

Step 3:
We draw $AC$ perpendicular to $x$-axis such that the coordinates of $C$ are $(1, 0)$.

$\therefore$ Area $OBAO = \text{Area (}\triangle OCA) - \text{Area (OCABO)}$ [\frac{1}{2} \text{ Mark}]

Step 4:
$$= \int_0^1 2x \, dx - \int_0^1 2\sqrt{x} \, dx$$ [\frac{1}{2} \text{ Mark}]

Step 5:
$$= 2 \left[ \frac{x^2}{2} \right]_0^1 - 2 \left[ \frac{x^{3/2}}{3/2} \right]_0^1$$
$$= \left| 1 - \frac{4}{3} \right|$$
$$= \left| -\frac{1}{3} \right|$$
$$= \frac{1}{3} \text{ square units}$$

hence, $(B)$ is the correct answer. [\frac{1}{2} \text{ Mark}]

Miscellaneous

1. Find the area under the given curves and given lines:
   (i) $y = x^2, x = 1, x = 2$ and $x$-axis [\text{2 Marks}]
   (ii) $y = x^4, x = 1, x = 5$ and $x$-axis [\text{2 Marks}]

Practice more on Application of Integrals
Solution:

(i) Step 1:
Given: The equations of the curves: \( y = x^2, x = 1, x = 2 \) and \( x \)-axis

Step 2:
The required area is represented by the shaded area \( ADCBA \) as

\[
\int_{1}^{2} y \, dx
\]

Step 3:
Area \( ADCBA = \int_{1}^{2} y \, dx \)

Step 4:
\[
= \int_{1}^{2} x^2 \, dx
\]
\[
= \left[ \frac{x^3}{3} \right]_{1}^{2}
\]
\[
= \frac{8}{3} - \frac{1}{3}
\]
\[
= \frac{7}{3}	ext{ square units}
\]

Hence, the required area is \( \frac{7}{3} \) square units.

(ii) Step 1:
Given: The equations of the curves are: \( y = x^4, x = 1, x = 5 \) and \( x \)-axis.

Step 2:
The required area is represented by the shaded area \( ADCBA \)
2. Find the area between the curves $y = x$ and $y = x^2$. [4 Marks]

Solution:

Step 1:
Given: The equation of the curve are $y = x$ and $y = x^2$

Step 2:
The required area is represented by the shaded area $OBAO$ as
2. Find the area of the region lying in the first quadrant and bounded by \( y = x^2 \), \( x = 0 \), \( y = 1 \) and \( y = 4 \).

Solution:

Step 1:
Given: The equations of the curves are: \( y = x^2 \), \( x = 0 \), \( y = 1 \) and \( y = 4 \).

Step 2:
The area in the first quadrant bounded by \( y = x^2 \), \( x = 0 \), \( y = 1 \) and \( y = 4 \) is represented by the shaded area \( ABCDA \) as...
4. Sketch the graph of \( y = |x + 3| \) and evaluate \( \int_{-6}^{0} |x + 3| \, dx \). [4 Marks]

Solution:

Step 1:
Given: The equation is \( y = |x + 3| \)

Step 2:
Some corresponding values of \( x \) and \( y \) are given in the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-6</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
</tr>
</thead>
</table>

Hence, the required area is \( \frac{7}{3} \) square units.
Step 3:
After plotting these points, we obtain the graph of $y = |x + 3|$ as follows.

\[
\int_{-6}^{0} |x + 3| \, dx = -\int_{-3}^{0} (x + 3) \, dx + \int_{-3}^{0} (x + 3) \, dx
\]

Step 4:
As we know that $(x + 3) \leq 0$ for $-6 \leq x \leq -3$ and $(x + 3) \geq 0$ for $-3 \leq x \leq 0$
\[\int_{-6}^{0} |x + 3| \, dx = -\int_{-3}^{0} (x + 3) \, dx + \int_{-3}^{0} (x + 3) \, dx \quad [1 \text{ mark}]\]

Step 5:
\[= - \left[ \frac{x^2}{2} + 3x \right]_{-6}^{-3} + \left[ \frac{x^2}{2} + 3x \right]_{-3}^{0}
\]
\[= - \left[ \left( \frac{(-3)^2}{2} + 3(-3) \right) - \left( \frac{(-6)^2}{2} + 3(-6) \right) \right] + \left[ 0 - \left( \frac{(-3)^2}{2} + 3(-3) \right) \right]
\]
\[= - \left[ -\frac{9}{2} \right] - \left[ -\frac{9}{2} \right]
\]
\[= 9 \quad [1 \text{ Mark}]
\]
Hence, the required area is 9 square units.

5. Find the area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$.

Solution:

Step 1:
Given: The equation of the curve: $y = \sin x$ bounded between $x = 0$ and $x = 2\pi$

Step 2:
The graph of $y = \sin x$ is drawn as
**: Required area = Area $OABO + Area BCDB$ \[\frac{1}{2} \text{Mark}\]

**Step 3:**
\[
= \int_0^\pi \sin x \, dx + \left| \int_\pi^{2\pi} \sin x \, dx \right| \quad \left[\frac{1}{2} \text{Mark}\right]
\]

**Step 4:**
\[
= [- \cos x]_0^\pi + |[- \cos x]_\pi^{2\pi}|
\]
\[
= [- \cos \pi + \cos 0] + |- \cos 2\pi + \cos \pi| \quad \left[\frac{1}{2} \text{Mark}\right]
\]

**Step 5:**
\[
= 1 + 1 + |(-1 - 1)|
\]
\[
= 2 + |2| = 4 \text{ square units} \quad \left[\frac{1}{2} \text{Mark}\right]
\]
Hence, the required area is 4 square units.

6. Find the area enclosed between the parabola $y^2 = 4ax$ and the line $y = mx$. \[6 \text{ marks}\]

**Solution:**

**Step 1:**
**Given:** The equation of parabola: $y^2 = 4ax$ and the line $y = mx$

**Step 2:**
The area enclosed between the parabola, $y^2 = 4ax$ and the line $y = mx$ is represented by the shaded area $OABO$ as
Step 3:
The points of intersection of both the curves are \((0, 0)\) and \(\left(\frac{4a}{m^2}, \frac{4a}{m}\right)\). [\(\frac{1}{2}\) Mark]

Step 4:
We draw \(AC\) perpendicular to \(x\)-axis.
\[\therefore \text{Area } OABO = \text{Area } OCABO - \text{Area } \triangle OCA\] [1 Mark]

Step 5:
\[= \int_{0}^{\frac{4a}{m^2}} 2\sqrt{ax} \, dx - \int_{0}^{\frac{4a}{m^2}} mx \, dx\] [1 Mark]

Step 6:
\[= 2\sqrt{a} \left[ \frac{x^2}{3} \right]_{0}^{\frac{4a}{m^2}} - m \left[ \frac{x^2}{2} \right]_{0}^{\frac{4a}{m^2}}\]
\[= \frac{4}{3} \sqrt{a} \left( \frac{4a}{m^2} \right)^{\frac{3}{2}} - m \left[ \frac{(4a)^2}{2m^2} \right]\]
\[= \frac{32a^2}{3m^3} - \frac{m}{2} \left( \frac{16a^2}{m^4} \right)\]
\[= \frac{32a^2}{3m^3} - \frac{8a^2}{m^3}\] [2 Marks]

Step 7:
\[= \frac{8a^2}{3m^3} \text{ square units}\] [1 Mark]

Hence, the required area is \(\frac{8a^2}{3m^3}\) square units

7. Find the area enclosed by the parabola \(4y = 3x^2\) and the line \(2y = 3x + 12\). [4 Marks]

Solution:
Step 1:
**Given:** Equation of the parabola: \(4y = 3x^2\). Equation of the line: \(2y = 3x + 12\)

Step 2:
The area enclosed between the parabola, \(4y = 3x^2\) and the line \(2y = 3x + 12\) is represented by the shaded area \(OBAO\) as

Step 3:
The points of intersection of the given curves are \(A(-2, 3)\) and \(B(4, 12)\). \([\frac{1}{2} \text{Mark}]\)

Step 4:
We draw \(AC\) and \(BD\) perpendicular to \(x\)-axis.
\[ \therefore \text{Area } OBAO = \text{Area } CDBA - (\text{Area } ODBO + \text{Area } OACO) \] \([\frac{1}{2} \text{Mark}]\)

Step 5:
\[
\begin{align*}
&= \int_{-2}^{1} \frac{1}{2}(3x + 12)dx - \int_{-2}^{3} \frac{3x^2}{4}dx \\
&= \frac{1}{2} \left[3x^2 + 12x\right]_{-2}^{1} - \frac{3}{4} \left[x^3\right]_{-2}^{3} \\
&= \frac{1}{2} [24 + 48 - 6 + 24] - \frac{3}{4} [64 + 8] \\
&= \frac{1}{2} [90] - \frac{3}{4} [72] \\
&= 45 - 18 \\
&= 27 \text{ square units} \quad [2 \text{Marks}]
\end{align*}
\]
Hence, the required area is 27 square units.

8. Find the area of the smaller region bounded by the ellipse \(\frac{x^2}{9} + \frac{y^2}{4} = 1\) and the line \(\frac{x}{3} + \frac{y}{2} = 1\).
[6 Marks]

**Solution:**

**Step 1:**

*Given:* Equation of the ellipse: \( \frac{x^2}{9} + \frac{y^2}{4} = 1 \).
Equation of the line: \( \frac{x}{3} + \frac{y}{2} = 1 \)

**Step 2:**

The area of the smaller region bounded by the ellipse, \( \frac{x^2}{9} + \frac{y^2}{4} = 1 \) and the line, \( \frac{x}{3} + \frac{y}{2} = 1 \) is represented by the shaded region \( BCAB \)

**Step 3:**

The points of intersection obtained after solving the two equations are \( A(0,2) \) and \( B(3,0) \). \([ \frac{1}{2} \text{ Mark} \]

**Step 4:**

\( \therefore \) Area \( BCAB = \text{Area} \ (OBCAO) - \text{Area} \ (OBAO) \) \([ \frac{1}{2} \text{ Mark} \]

**Step 5:**

\( \frac{x^2}{9} + \frac{y^2}{4} = 1 \)
Hence, \( y = 2\sqrt{1 - \frac{x^2}{9}} = y_1 \)

\( \frac{x}{3} + \frac{y}{2} = 1 \)
Hence, \( y = 2 \left(1 - \frac{x}{3}\right) = y_2 \) \([1 \text{ Mark} \]

**Step 6:**

\[ = \int_{0}^{3} y_1 \, dx - \int_{0}^{3} y_2 \, dx \] \([1 \text{ Mark} \]

**Step 7:**

\[ = \int_{0}^{3} 2\sqrt{1 - \frac{x^2}{9}} \, dx - \int_{0}^{3} 2 \left(1 - \frac{x}{3}\right) \, dx \]
\[ = \frac{2}{3} \left[ \int_{0}^{3} (9 - x^2) \, dx \right] - \frac{2}{3} \left[ \int_{0}^{3} (3 - x) \, dx \right] \]
\[ \frac{2}{3} \left[ \frac{x}{2} \sqrt{9-x^2} + \frac{9}{2} \sin^{-1} \left( \frac{x}{3} \right) \right]_0^3 - \frac{2}{3} \left[ 3x - \frac{x^2}{2} \right]_0^3 = \frac{2}{3} \left[ \frac{9}{2} \left( \frac{\pi}{2} \right) \right] - \frac{2}{3} \left[ 9 - \frac{9}{2} \right] \\
= \frac{2}{3} \left[ \frac{9}{2} \pi - \frac{9}{2} \right] \\
= \frac{2}{3} \times \frac{9}{4} (\pi - 2) \quad [1 \frac{1}{2} \text{Marks}]
\]

**Step 8:**

\[ \frac{3}{2} (\pi - 2) \text{ square units.} \quad [\frac{1}{2} \text{Mark}] \]

Hence, the required area is \( \frac{3}{2} (\pi - 2) \) square units.

---

9. Find the area of the smaller region bounded by the ellipse \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) and the line \( \frac{x}{a} + \frac{y}{b} = 1 \). [6 Marks]

**Solution:**

**Step 1:**

Given: The equation of the ellipse: \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) and the line \( \frac{x}{a} + \frac{y}{b} = 1 \)

**Step 2:**

The area of the smaller region bounded by the ellipse, \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) and the line, \( \frac{x}{a} + \frac{y}{b} = 1 \) is represented by the shaded region \( BCA'B \)

\[
\begin{align*}
(0,b) & \\
C & \\
(0,0) & \\
A & \\
B(a,0) \\
(\frac{x}{a} + \frac{y}{b} = 1) & \\
(x^2 + y^2 = 1) & \\
Y & \\
X & \\
\end{align*}
\]

**[1 Mark]**

**Step 3:**

The points of intersection after solving the two given equations: \( A(0, b) \) and \( B(a, 0) \) \([\frac{1}{2} \text{Mark}]\)

**Step 4:**
\[ \therefore \text{Area } BCAB = \text{Area } (OBCAO) - \text{Area } (OBAO) \quad \left[ \frac{1}{2} \text{ Mark} \right] \]

**Step 5:**
\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
\]
Hence, \( y = b \sqrt{1 - \frac{x^2}{a^2}} = y_1 \)

\[
\frac{x}{a} + \frac{y}{b} = 1
\]
Hence, \( y = b \left( 1 - \frac{x}{a} \right) = y_2 \) \quad \left[ 1 \text{ Mark} \right]

**Step 6:**
\[
= \int_0^a y_1 \, dx - \int_0^a y_2 \, dx \quad \left[ 1 \text{ Mark} \right]
\]

**Step 7:**
\[
= \int_0^a b \sqrt{1 - \frac{x^2}{a^2}} \, dx - \int_0^a b \left( 1 - \frac{x}{a} \right) \, dx
\]
\[
= \frac{b}{a} \int_0^a \sqrt{a^2 - x^2} \, dx - \frac{b}{a} \int_0^a (a - x) \, dx
\]
\[
= \frac{b}{a} \left[ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]_0^a - \left[ \frac{ax}{2} - \frac{x^2}{2} \right]_0^a
\]
\[
= \frac{b}{a} \left[ \frac{a^2}{2} \left( \frac{\pi}{2} \right) - \left( a^2 - \frac{a^2}{2} \right) \right]
\]
\[
= \frac{b}{a} \left[ \frac{a^2 \pi}{4} - \frac{a^2}{2} \right]
\]
\[
= \frac{b a^2}{2a} \left[ \frac{\pi}{2} - 1 \right]
\]
\[
= \frac{ab}{2} \left[ \frac{\pi}{2} - 1 \right] \quad \left[ 1 \frac{1}{2} \text{ Mark} \right]
\]

**Step 8:**
\[
= \frac{ab}{4} (\pi - 2) \text{ square units} \quad \left[ \frac{1}{2} \text{ Mark} \right]
\]
Hence, the required area is \( \frac{ab}{4} (\pi - 2) \) square units.

10. Find the area of the region enclosed by the parabola \( x^2 = y \) the line \( y = x + 2 \) and the \( x \)-axis. \quad [6 \text{ Marks}]

**Solution:**
Step 1:
**Given:** Equation of the parabola \( x^2 = y \)
Equation of the line: \( y = x + 2 \) and \( x \)-axis.

Step 2:
The area of the region enclosed by the parabola \( x^2 = y \) the line, \( y = x + 2 \) and \( x \)-axis is represented by the shaded region \( OABCO \) as

\[
\text{Area } OABCO = \text{Area } (BCA) + \text{Area } COAC \quad [\frac{1}{2} \text{ Mark}]
\]

Step 3:
The point of intersection of the parabola \( x^2 = y \) and the line \( y = x + 2 \) is \( A(-1, 1) \). [\frac{1}{2} \text{ Mark}]

Step 4:
\[\therefore \text{ Area } OABCO = \text{Area } (BCA) + \text{Area } COAC \quad [\frac{1}{2} \text{ Mark}]
\]

Step 5:
Let \( y_1 = x^2 \) and \( y_2 = x + 2 \)
Required area = \[\int_{-2}^{-1} y_2 \, dx + \int_{-1}^{0} y_1 \, dx\] [1 Mark]

Step 6:
\[
\begin{align*}
\int_{-2}^{-1} (x + 2) \, dx + \int_{-1}^{0} x^2 \, dx \\
= \left[ \frac{x^2}{2} + 2x \right]_{-2}^{-1} + \left[ \frac{x^3}{3} \right]_{-1}^{0} \\
\end{align*}
\] [1 Mark]

Step 7:
\[
\begin{align*}
\left[ \frac{(-1)^2}{2} + 2(-1) - \frac{(-2)^2}{2} - 2(-2) \right] + \left[ \frac{(-1)^3}{3} \right] \\
= \left[ \frac{1}{2} - 2 + 2 + 4 + \frac{1}{3} \right] \\
\end{align*}
\] [1 Mark]

Step 8:
\[\frac{5}{6} \text{ square units} \quad [1 \text{ Mark}]
\]

Hence, the required area is \( \frac{5}{6} \) square units.
11. Using the method of integration find the area bounded by the curve $|x| + |y| = 1$. [Hint: The required region is bounded by lines $x + y = 1$, $x - y = 1$, $-x + y = 1$ and $-x - y = 1$.] [4 Marks]

**Solution:**

**Step 1:**
Given: Equation of the curve: $|x| + |y| = 1$

**Step 2:**
The given curve can be visualized as four different lines.
- $x + y = 1$ for $x > 0, y > 0$
- $-x + y = 1$ for $x < 0, y > 0$
- $x - y = 1$ for $x > 0, y < 0$
- $-x - y = 1$ for $x < 0, y < 0$ [1 Mark]

**Step 3:**
The area bounded by the curve, $|x| + |y| = 1$ is represented by the shaded region $ADCB$ as

![Diagram showing the shaded region ABCD]

**Step 4:**
The curve intersects the axes at points $A(0, 1), B(1, 0), C(0, -1)$ and $D(-1, 0)$. It can be observed that the given curve is symmetrical about $x$-axis and $y$-axis.

∴ Area $ADCB = 4 \times$ Area $OBAO$ [1 Mark]

**Step 5:**
\[
= 4 \int_{0}^{1} (1 - x)dx \\
= 4 \left( x - \frac{x^2}{2} \right)_{0}^{1} \\
= 4 \left( 1 - \frac{1}{2} \right) \\
= 2 \text{ square units} \quad \text{[1 Mark]}
\]
Hence, the required area is 2 square units.

12. Find the area bounded by curves \( \{(x, y): y \geq x^2 \text{ and } y = |x|\} \). \[6 \text{ Marks}\]

**Solution:**

**Step 1:**

Given: The equations of the curves: \( \{(x, y): y \geq x^2 \text{ and } y = |x|\} \)

**Step 2:**

\[ y = |x| \]
\[ = x, x \geq 0 \]
\[ = -x, x \leq 0 \]

Hence, the area bounded by the curves, \( \{(x, y): y \geq x^2 \text{ and } y = |x|\} \) is represented by the shaded region:

\[1\frac{1}{2} \text{ Mark}\]

**Step 3:** The points of intersection obtained by solving the given equations are: \( A(1,1) \) and \( B(-1,1) \).

It is observed that the required area is symmetrical about \( y \)-axis.

Required area = \( 2[\text{Area (OCAO) - Area(OCADO)}] \) \[1 \text{ Mark}\]

**Step 4:**

\[ y_1 = y = x \]
\[ y_2 = y = x^2 \]

**Step 5:**

\[ = 2 \left[ \int_0^1 y_1 \, dx - \int_0^1 y_2 \, dx \right] \] \[1 \text{ mark}\]

**Step 6:**

\[ = 2 \left[ \int_0^1 x \, dx - \int_0^1 x^2 \, dx \right] \]
13. Using the method of integration find the area of the triangle $ABC$, coordinates of whose vertices are $A(2, 0), B(4, 5)$ and $C(6, 3)$. [6 Marks]

Solution:

Step 1:
Given: The vertices of $\Delta ABC$ are $A(2, 0), B(4, 5)$ and $C(6, 3)$.

Step 2:

Equation of line segment $AB$ is

$$\frac{y - 0}{5 - 0} = \frac{x - 2}{4 - 2}$$

$$2y = 5x - 10$$

$$y = \frac{5}{2}(x - 2) \quad ...(i) \quad [\text{1 Mark}]$$

Step 3:
Equation of line segment $BC$ is

$$y - 5 = \frac{3 - 5}{6 - 4}(x - 4)$$
Step 4:
Equation of line segment $CA$ is

$y - 3 = \frac{0 - 3}{2 - 6} (x - 6)$

$-4y + 12 = -3x + 18$

$4y = 3x - 6$

$y = \frac{3}{4} (x - 2)$ …(iii)  \[\frac{1}{2} \text{ Mark}\]

Step 5:
Area $(\Delta ABC) = \text{Area (ABLA)} + \text{Area (BLMBC)} - \text{Area (ACMA)}$  \[1 \text{ Mark}\]

Step 6:

$= \int_2^4 \frac{5}{2} (x - 2) dx + \int_4^6 (-x + 9) dx - \int_2^6 \frac{3}{4} (x - 2) dx$  \[1 \text{ Mark}\]

Step 7:

$= \frac{5}{2} \left[ \frac{x^2}{2} - 2x \right]_2^4 + \left[ -\frac{x^2}{2} + 9x \right]_4^6 - \frac{3}{4} \left[ \frac{x^2}{2} - 2x \right]_2^6$  \[1 \text{ Mark}\]

$= \frac{5}{2} [8 - 8 - 2 + 4] + [-18 + 54 + 8 - 36] - \frac{3}{4} [18 - 12 - 2 + 4]$

$= 5 + 8 - \frac{3}{4} (8)$

$= 13 - 6$

Step 8:

$= 7$ square units  \[\frac{1}{2} \text{ Mark}\]

Hence, the required area is 7 square units.

14. Using the method of integration find the area of the region bounded by lines: $2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$  \[6 \text{ Marks}\]

Solution:

Step 1:
Given:
The given equations of lines are

$2x + y = 4$ …(i)

$3x - 2y = 6$ …(ii)

And $x - 3y + 5 = 0$ …(iii)

Step 2:
Step 3: The points of intersection of the three lines obtained by solving them simultaneously are: $A(1,2), B(2,0)$ and $C(4,3)$

The area of the region bounded by the lines is the area of $\Delta ABC$.

$AL$ and $CM$ are the perpendiculars on $x$-axis. [1 Mark]

Step 4:

$\text{Area}(\Delta ABC) = \text{Area}(ALMCA) - \text{Area}(ALB) - \text{Area}(CMB)$ [1 Mark]

Step 5:

$$\int_2^4 \left(\frac{x+5}{3}\right) dx - \int_1^2 (4 - 2x) dx - \int_2^4 \left(\frac{3x-6}{2}\right) dx$$ [1 Mark]

Step 6:

$$\frac{1}{3}\left[\frac{x^2}{2} + 5x\right]_1^4 - \left[4x - \frac{x^2}{2}\right]_1^2 - \frac{1}{2}\left[\frac{3x^2}{2} - 6x\right]_2^4$$ [1 Mark]

$$= \frac{1}{3}\left[8 + 20 - \frac{1}{2} - 5\right] - \left[8 - 4 - 4 + 1\right] - \frac{1}{2}\left[24 - 24 - 6 + 12\right]$$

Step 7:

$$= \left(\frac{1}{3} \times \frac{45}{2}\right) - (1) - \frac{1}{2}(6)$$

$$= \frac{15}{2} - 1 - 3$$

$$= \frac{15}{2} - 4 = \frac{15 - 8}{2} = \frac{7}{2} \text{ square units}$$ [1 Mark]

hence, the required area is $\frac{7}{2}$ square units

15. Find the area of the region $\{(x, y): y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$ [6 Marks]

Solution:
Step 1:
Given: Equation of the curve: \((x, y): y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\)

Step 2:
The equation \(y^2 \leq 4x\) represents the region interior to a parabola, symmetric about \(x\)-axis.
The equation \(4x^2 + 4y^2 \leq 9\) represents the region interior to a circle with center at the origin and radius \(\frac{3}{2}\).
The area bounded by the curves \((x, y): y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\) is represented as

![Diagram of curves and points of intersection]

The points of intersection of both the curves obtained by solving the given equations simultaneously are \(\left(\frac{1}{2}, \sqrt{2}\right)\) and \(\left(\frac{1}{2}, -\sqrt{2}\right)\). [1 Mark]

Step 3:
The required area is given by \(OABC\).
It is observed that area \(OABC\) is symmetrical about \(x\)-axis.
\[\therefore \text{Area } OABC = 2 \times \text{Area } OBC\] [1 Mark]

Step 4:
Area \(OBCO = \text{Area } OMC + \text{Area } MBC\) [1 mark]

Step 5:
\[
\begin{align*}
&= \int_{0}^{1/2} 2\sqrt{x} \, dx + \int_{1/2}^{3/2} \sqrt{\frac{9}{4} - x^2} \, dx \\
&= 2 \int_{0}^{1/2} \sqrt{x} \, dx + \int_{1/2}^{3/2} \sqrt{\left(\frac{3}{2}\right)^2 - x^2} \, dx
\end{align*}
\]
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\[= 2 \left[ \frac{x^3}{2} \right]_0^\frac{1}{2} + \left[ \frac{x \sqrt{\left(\frac{3}{2}\right)^2} - x^2 + \left(\frac{3}{2}\right)^2 \sin^{-1} \frac{x}{3} \right]_0^{\frac{1}{2}} \]

\[= \frac{4}{3} \left[ \frac{x^3}{2} \right]_0^\frac{1}{2} + \left[ \frac{x \sqrt{\left(\frac{3}{2}\right)^2} - x^2 + \left(\frac{3}{2}\right)^2 \sin^{-1} \frac{x}{3} \right] - \left[ \frac{x \sqrt{\left(\frac{3}{2}\right)^2} - x^2 + \left(\frac{3}{2}\right)^2 \sin^{-1} \frac{x}{3} \right]_0^{\frac{1}{2}} \]

\[= \frac{4}{3} \left[ \frac{1}{2}^\frac{3}{2} - 0 \right] + \left[ \frac{3}{4} \sqrt{3} + \frac{9}{8} \sin^{-1} \frac{1}{3} \right] - \left[ \frac{1}{4} \sqrt{2} + \frac{9}{8} \sin^{-1} \frac{1}{3} \right] \]

\[= \sqrt{\frac{2}{3}} + \frac{9\pi}{16} - \sqrt{\frac{2}{4}} - \frac{9}{8} \sin^{-1} \frac{1}{3} \quad [2 \text{ Marks}] \]

**Step 6:**

Hence, the required area = \(2 \times \left[ \frac{\sqrt{2}}{3} + \frac{9\pi}{16} - \frac{\sqrt{2}}{4} - \frac{9}{8} \sin^{-1} \frac{1}{3} \right] \)

\[= \frac{1}{3\sqrt{2}} + \frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \frac{1}{3} \quad [1 \text{ Mark}] \]

16. Area bounded by the curve \(y = x^3\), the \(x\)-axis and the ordinates \(x = -2\) and \(x = 1\) is \(\quad [4 \text{ Marks}]\)

(A) \(-9\)

(B) \(-\frac{15}{4}\)

(C) \(\frac{15}{4}\)

(D) \(\frac{17}{4}\)

**Solution:**

(D)

**Step 1:**

\textbf{Given:} The equations of the curve is \(y = x^3\) and the ordinates are \(x = -2\) and \(x = 1\).

**Step 2:** The given equations can be represented as follows:
Step 3:
Required area \(= \int_{-2}^{1} |y| \, dx \)  
\(= \int_{-2}^{1} |x^3| \, dx \) \hspace{1cm} [1 Mark]

Step 4:
\(= \int_{-2}^{0} |x^3| \, dx + \int_{0}^{1} |x^3| \, dx \)  
\(= -\int_{-2}^{0} x^3 \, dx + \int_{0}^{1} x^3 \, dx \)  
\(= \left[-\frac{x^4}{4}\right]_{-2}^{0} + \left[\frac{x^4}{4}\right]_{0}^{1} \)  
\(= 4 + \frac{1}{4} = \frac{17}{4} \) square units
Hence, \((D)\) is the correct answer. \(\quad \text{[2 Marks]}\)

17. The area bounded by the curve \(y = x|x|\), \(x\)-axis and the ordinates \(x = -1\) and \(x = 1\) is given by \([\text{Hint: } y = x^2 \text{ if } x > 0 \text{ and } y = -x^2 \text{ if } x < 0]\). \(\quad \text{[4 Marks]}\)

(A) 0  
(B) \(\frac{1}{3}\)  
(C) \(\frac{2}{3}\)  
(D) \(\frac{4}{3}\)

Solution:

(C)
Step 1:
Given: The equation of the curve: \( y = x|x| \), \( x \)-axis and the ordinates \( x = -1 \) and \( x = 1 \)
Step 2: The given equation can be represented as follows:

Step 3:
Required area = \( \int_{-1}^{1} |y| \, dx \) [1 Mark]

Step 4:
\[
= \int_{-1}^{1} x |x| \, dx
= \int_{-1}^{0} x^2 \, dx + \int_{0}^{1} x^2 \, dx \quad [1 \text{ Mark}]
\]

Step 5:
\[
= \left[ \frac{x^3}{3} \right]_{-1}^{0} + \left[ \frac{x^3}{3} \right]_{0}^{1}
= -\left( \frac{-1}{3} \right) + \frac{1}{3}
= \frac{2}{3} \text{ square units}
\]
Hence, (C) is the correct answer. [1 Mark]

18. The area of the circle \( x^2 + y^2 = 16 \) exterior to the parabola \( y^2 = 6x \) is

(A) \( \frac{4}{3} \left( 4\pi - \sqrt{3} \right) \)

(B) \( \frac{4}{3} \left( 4\pi + \sqrt{3} \right) \)

(C) \( \frac{4}{3} \left( 8\pi - \sqrt{3} \right) \) [6 Marks]
Solution:

Step 1:
Given: Equation of the circle: \(x^2 + y^2 = 16\)
Equation of the parabola: \(y^2 = 6x\)

Step 2:
The given equations is represented as follows:

![Diagram of circle and parabola]

The points of intersection are marked as shown.

Area bounded exterior to the parabola and the circle is as shaded above. 

Step 3: The area
\[
= 2 \left[ \text{Area}(OMCO) + \text{Area}(CMBC) \right]
= 2 \left[ \int_0^2 \sqrt{6} x \, dx + \int_2^4 \sqrt{16 - x^2} \, dx \right] \quad [1 \text{ Mark}]
\]

Step 4:
\[
= 2 \left[ \sqrt{6} \left( \frac{3}{2} \right) \right] + 2 \left[ \frac{x}{2} \sqrt{16 - x^2} + \frac{16}{2} \sin^{-1} x \right]_0^2
= 2\sqrt{6} \times \frac{3}{2} + 2 \left[ \frac{8\pi}{2} - \sqrt{16 - 4} - 8 \sin^{-1} \left( \frac{1}{2} \right) \right]
= \frac{4\sqrt{6}}{3} \left( 2\sqrt{2} \right) + 2 \left[ 4\pi - \sqrt{12} - 8 \frac{\pi}{6} \right]
= \frac{16\sqrt{3}}{3} + 8\pi - 4\sqrt{3} - 8 \frac{\pi}{3}
= \frac{4}{3} \left[ 4\sqrt{3} + 6\pi - 3\sqrt{3} - 2\pi \right]
= \frac{4}{3} \left[ \sqrt{3} + 4\pi \right]
= \frac{4}{3} [4\pi + \sqrt{3}] \text{ square units} \quad [2 \text{ marks}]
\]

Step 5:
Area of circle = \(\pi r^2\)
= \(\pi (4)^2\)
= 16\(\pi\) square units \([\frac{1}{2}\text{ Mark}]\)

\[
\therefore \text{Required area} = 16\pi - \frac{4}{3}[4\pi + \sqrt{3}]
\]
\[
= \frac{4}{3}(4 \times 3\pi - 4\pi - \sqrt{3})
\]
\[
= \frac{4}{3}(8\pi - \sqrt{3}) \text{ square units}
\]
Hence, (C) is the correct answer. \([1 \frac{1}{2}\text{ Mark}]\)

19. The area bounded by the \(y\)-axis, \(y = \cos x\) and \(y = \sin x\) when \(0 \leq x \leq \frac{\pi}{2}\) \([4\text{ Marks}]\)

(A) \(2(\sqrt{2} - 1)\)
(B) \(\sqrt{2} - 1\)
(C) \(\sqrt{2} + 1\)
(D) \(\sqrt{2}\)

Solution:

(B)

Step 1:
The given equations are \(y = \cos x\) \(...(i)\)
And \(y = \sin x\) \...(ii)

Step 2: The given equations is represented as follows. The point of intersection for given curve is as:
\[
\sin x = \cos x \\
\Rightarrow \tan x = 1 \\
\Rightarrow x = \frac{\pi}{4} \\
\Rightarrow y = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}
\]
Step 3:
Required area = Area (ABLA) + area (OBLO)
\[ = \int_{\frac{1}{\sqrt{2}}}^{1} x\,dy + \int_{0}^{\frac{1}{\sqrt{2}}} x\,dy \]  
[1 mark]

Step 4:
\[ = \int_{\frac{1}{\sqrt{2}}}^{1} \cos^{-1} y\,dy + \int_{0}^{\frac{1}{\sqrt{2}}} \sin^{-1} y\,dy \]

Integrating by parts, we obtain
\[ = \left[ y\cos^{-1}y - \sqrt{1 - y^2}\right]_{\frac{1}{\sqrt{2}}}^{1} + \left[ y\sin^{-1}y + \sqrt{1 - y^2}\right]_{0}^{\frac{1}{\sqrt{2}}} \]
\[ = \left[ \cos^{-1}(1) - \frac{1}{\sqrt{2}}\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sqrt{1 - \frac{1}{2}}\right] + \left[ \frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sqrt{1 - \frac{1}{2}} - 1 \right] \]
\[ = \frac{-\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 \]
\[ = \frac{2}{\sqrt{2}} - 1 \]
\[ = \sqrt{2} - 1 \text{ square units} \]

Hence, (B) is the correct answer.  
[2 marks]