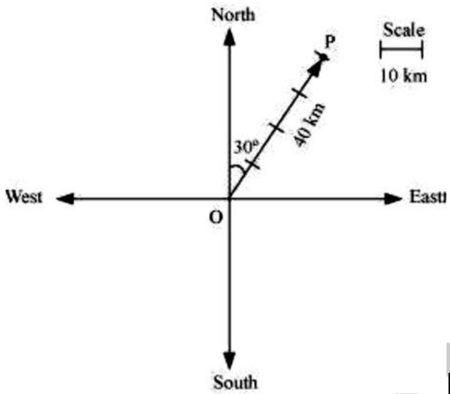
CBSE NCERT Solutions for Class 12 Maths Chapter 10

Back of Chapter Questions

Exercise: 10.1

1. Represent graphically a displacement of $40 \text{ km}, 30^{\circ}$ east of north. **[2 Marks]**

Solution:



[1 Mark]

Here, vector $\overrightarrow{OP}$ represents the displacement of $40 \text{ km}, 30^{\circ}$ East of North. **[1 Mark]**

2. Classify the following measures as scalars and vectors. **[1 Mark each]**

(i) 10 kg

(ii) 2 metres north-west

(iii) 40°

(iv) 40 watt

(v) 10^{-19} coulomb

(vi) 20 m/s^2

Solution:

(i) 10 kg is a scalar quantity because it involves only magnitude. **[1 Mark]**

(ii) 2 meters north-west is a vector quantity as it involves both magnitude and direction. **[1 Mark]**

(iii) 40° is a scalar quantity as it involves only magnitude. **[1 Mark]**

(iv) 40 watts is a scalar quantity as it involves only magnitude. **[1 Mark]**

(v) 10^{-19} coulomb is a scalar quantity as it involves only magnitude. **[1 Mark]**

(vi) $20\ \text{m/s}^2$ is a vector quantity as it involves magnitude as well as direction. **[1 Mark]**

3. Classify the following as scalar and vector quantities. **[1 Mark each]**

(i) time period

(ii) distance

(iii) force

(iv) velocity

(v) work done

Solution:

(i) Time period is a scalar quantity as it involves only magnitude. **[1 Mark]**

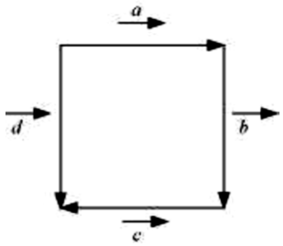
(ii) Distance is a scalar quantity as it involves only magnitude. **[1 Mark]**

(iii) Force is a vector quantity as it involves both magnitude and direction. **[1 Mark]**

(iv) Velocity is a vector quantity as it involves both magnitude as well as direction. **[1 Mark]**

(v) Work done is a scalar quantity as it involves only magnitude. **[1 Mark]**

4. In Figure, identify the following vectors.**[1 Mark each]**



(i) Coinitial

(ii) Equal

(iii) Collinear but not equal

Solution:

(i) Vectors $\vec{a}$ and $\vec{d}$are coinitial because they have the same initial point. **[1 Mark]**

(ii) Vectors $\vec{b}$ are $\vec{d}$ equal because they have the same magnitude and direction. **[1 Mark]**

(iii) Vectors $\vec{a}$ and $\vec{b}$ are collinear but not equal. This is because although they are parallel, their directions are not the same. **[1 Mark]**

5. Answer the following as true or false. **[1 Mark each]**

(i) $\vec{a}$ and $-\vec{a}$ are collinear.

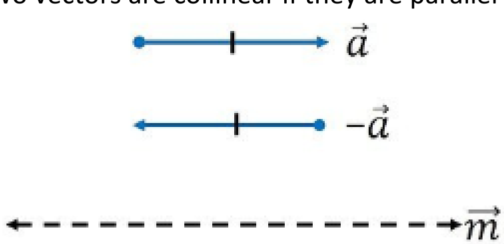
(ii) Two collinear vectors are always equal in magnitude.

(iii) Two vectors having same magnitude are collinear.

(iv) Two collinear vectors having the same magnitude are equal.

Solution:

(i) True.
Two vectors are collinear if they are parallel to line the same line.

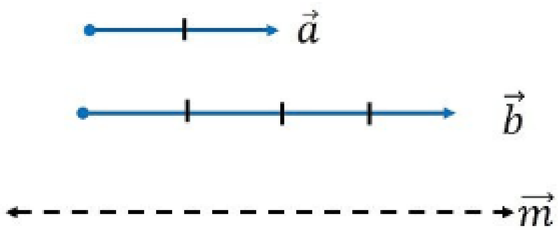


Vectors $\vec{a}$ and $-\vec{a}$ are parallel to the same line $\vec{m}$. **[1 Mark]**

So, $\vec{a}$ and $-\vec{a}$ are collinear.

(ii) False.

Collinear vectors are those vectors that are parallel to the same line.

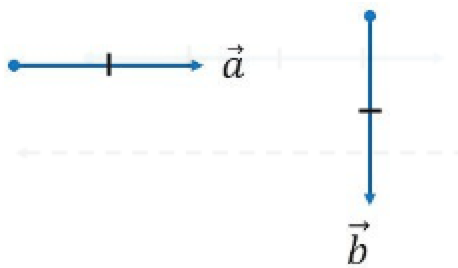


Here, $\vec{a}$ and $\vec{b}$ are parallel to $\vec{m}$

Hence, collinear.

But, $\vec{a}$ and $\vec{b}$ are not equal in magnitude. **[1 Mark]**

(iii) False.
Two or more vectors are collinear if they are parallel to the same line.



$\vec{a}$ and $\vec{b}$ are equal in magnitude but not parallel to the same line

Hence, $\vec{a}$ and $\vec{b}$ are not collinear. **[1 Mark]**

(iv) False
Two or more vectors are equal if they have the same magnitude and same direction
Collinear vectors may have the same magnitude but are not equal.

Hence false. **[1 Mark]**

Exercise: 10.2

1. Compute the magnitude of the following vectors: **[4 Marks]**

$\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$; $\vec{b} = 2\hat{\imath} - 7\hat{\jmath} - 3\hat{k}$; $\vec{c} = \frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} - \frac{1}{\sqrt{3}}\hat{k}$

Solution:

Given vectors are $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$; $\vec{b} = 2\hat{\imath} - 7\hat{\jmath} - 3\hat{k}$; $\vec{c} = \frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} - \frac{1}{\sqrt{3}}\hat{k}$ **[1 Mark]**

Magnitude of $\vec{a}$ is $|\vec{a}| = \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3}$

Hence, $|\vec{a}| = \sqrt{3}$ **[1 Mark]**

Magnitude of $\vec{b}$ is $|\vec{b}| = \sqrt{(2)^2 + (-7)^2 + (-3)^2}$

$= \sqrt{4 + 49 + 9}$

$= \sqrt{62}$

Hence, $|\vec{b}| = \sqrt{62}$ **[1 Mark]**

Magnitude of $\vec{c}$ is $|\vec{c}| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(-\frac{1}{\sqrt{3}}\right)^2}$

$= \sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = 1$

Hence, $|\vec{c}| = 1$ **[1 Mark]**

2. Write two different vectors having same magnitude. **[2 Marks]**

Solution:

Let $\vec{a} = (-\hat{\imath} - 2\hat{\jmath} + 3\hat{k})$ and $\vec{b} = (2\hat{\imath} + \hat{\jmath} - 3\hat{k})$

Now, magnitude of $\vec{a}$ is $|\vec{a}| = \sqrt{(-1)^2 + (-2)^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$ **[1 Mark]**

$|\vec{b}| = \sqrt{2^2 + 1^2 + (-3)^2} = \sqrt{4 + 1 + 9} = \sqrt{14}$ **[1 Mark]**

Hence, $\vec{a}$ and $\vec{b}$ are two different vectors having the same magnitude.

3. Write two different vectors having same direction. **[2 Marks]**

Solution:

Let $\vec{a} = (\hat{\imath} + \hat{\jmath} + \hat{k})$ and $\vec{b} = (2\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$.

The direction cosines of $\vec{a}$ are given by,

$l = \frac{1}{\sqrt{1^2+1^2+1^2}} = \frac{1}{\sqrt{3}}, m = \frac{1}{\sqrt{1^2+1^2+1^2}} = \frac{1}{\sqrt{3}}$, and $n = \frac{1}{\sqrt{1^2+1^2+1^2}} = \frac{1}{\sqrt{3}}$ **[1 Mark]**

The direction cosines of $\vec{b}$ are given by,

$l = \frac{2}{\sqrt{2^2+2^2+2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}, m = \frac{2}{\sqrt{2^2+2^2+2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}},$

And $n = \frac{2}{\sqrt{2^2+2^2+2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$ **[1 Mark]**

The direction cosines of $\vec{a}$ and $\vec{b}$ are the same. Hence, the two vectors have the same direction.

4. Find the values of x and y so that the vectors $2\hat{\imath} + 3\hat{\jmath}$ and $x\hat{\imath} + \hat{y}$ are equal **[1 Mark]**

Solution:

The two vectors $2\hat{\imath} + 3\hat{\jmath}$ and $x\hat{\imath} + \hat{y}$ will be equal if their corresponding components are equal.

Hence, the required values of x and y are 2 and 3 respectively. **[1 Mark]**

5. Find the scalar and vector components of the vector with initial point $(2,1)$ and terminal point $(-5,7)$. **[1 Mark]**

Solution:

Let the given points be $P(2, 1)$ and $Q(-5, 7)$

The vector with the initial point $P(2, 1)$ and terminal point $Q(-5, 7)$ can be given by,

$\overrightarrow{PQ} = (-5 - 2)\hat{\imath} + (7 - 1)\hat{\jmath}$ **[$\frac{1}{2}$ Mark]**

$\Rightarrow \overrightarrow{PQ} = -7\hat{\imath} + 6\hat{\jmath}$

Hence, the required scalar components are -7 and 6 while the vector components are $-7\hat{\imath}$ and $6\hat{\jmath}$. **[$\frac{1}{2}$ Mark]**

6. Find the sum of the vectors $\vec{a} = \hat{\imath} - 2\hat{\jmath} + \hat{k}, \vec{b} = -2\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$ and $\vec{c} = \hat{\imath} - 6\hat{\jmath} - 7\hat{k}$ **[1 Mark]**

Solution:

The given vectors are $\vec{a} = \hat{\imath} - 2\hat{\jmath} + \hat{k}, \vec{b} = -2\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$ and $\vec{c} = \hat{\imath} - 6\hat{\jmath} - 7\hat{k}$

$\therefore \vec{a} + \vec{b} + \vec{c} = (1 - 2 + 1)\hat{\imath} + (-2 + 4 - 6)\hat{\jmath} + (1 + 5 - 7)\hat{k}$ **[$\frac{1}{2}$ Mark]**

$= 0 \cdot \hat{\imath} - 4\hat{\jmath} - 1 \cdot \hat{k}$

$= -4\hat{\jmath} - \hat{k}$ **[$\frac{1}{2}$ Mark]**

7. Find the unit vector in the direction of the vector $\vec{a} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$. **[1 Mark]**

Solution:

The unit vector $\hat{a}$ in the direction of vector $\vec{a} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$.

$|\vec{a}| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{1 + 1 + 4} = \sqrt{6}$ **[$\frac{1}{2}$ Mark]**

$\therefore \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{\imath}+\hat{\jmath}+2\hat{k}}{\sqrt{6}} = \frac{1}{\sqrt{6}}\hat{\imath} + \frac{1}{\sqrt{6}}\hat{\jmath} + \frac{2}{\sqrt{6}}\hat{k}$ **[$\frac{1}{2}$ Mark]**

8. Find the unit vector in the direction of vector, $\overrightarrow{PQ}$, where P and Q are the points $(1, 2, 3)$ and $(4, 5, 6)$, respectively. **[2 Marks]**

Solution:

The given points are $P(1, 2, 3)$ and $Q(4, 5, 6)$.

$\therefore \overrightarrow{PQ} = (4 - 1)\hat{\imath} + (5 - 2)\hat{\jmath} + (6 - 3)\hat{k} = 3\hat{\imath} + 3\hat{\jmath} + 3\hat{k}$

$|\overrightarrow{PQ}| = \sqrt{3^2 + 3^2 + 3^2} = \sqrt{9 + 9 + 9} = \sqrt{27} = 3\sqrt{3}$ **[1 Mark]**

Hence, the unit vector in the direction of $\overrightarrow{PQ}$ is

$\frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|} = \frac{3\hat{\imath}+3\hat{\jmath}+3\hat{k}}{3\sqrt{3}} = \frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} + \frac{1}{\sqrt{3}}\hat{k}$ **[1 Mark]**

9. For given vectors, $\vec{a} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $\vec{b} = -\hat{\imath} + \hat{\jmath} - \hat{k}$find the unit vector in the direction of the vector $\vec{a} + \vec{b}$ **[2 Marks]**

Solution:

The given vectors are $\vec{a} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $\vec{b} = -\hat{\imath} + \hat{\jmath} - \hat{k}$.

$\vec{a} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$

$\vec{b} = -\hat{\imath} + \hat{\jmath} - \hat{k}$

$\therefore \vec{a} + \vec{b} = (2-1)\hat{\imath} + (-1+1)\hat{\jmath} + (2-1)\hat{k} = 1\hat{\imath} + 0\hat{\jmath} + 1\hat{k} = \hat{\imath} + \hat{k}$ **[1 Mark]**

$|\vec{a} + \vec{b}| = \sqrt{1^2 + 1^2} = \sqrt{2}$

Hence, the unit vector in the direction of $(\vec{a} + \vec{b})$ is

$\frac{(\vec{a}+\vec{b})}{|\vec{a}+\vec{b}|} = \frac{\hat{\imath}+\hat{k}}{\sqrt{2}} = \frac{1}{2}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{k}$ **[1 Mark]**

10. Find a vector in the direction of vector $5\hat{\jmath} - \hat{\jmath} + 2\hat{k}$ which has magnitude 8 units.**[2 Marks]**

Solution:

Let $\vec{a} = 5\hat{\jmath} - \hat{\jmath} + 2\hat{k}$

$\therefore |\vec{a}| = \sqrt{5^2 + (-1)^2 + 2^2} = \sqrt{25 + 1 + 4} = \sqrt{30}$

$\therefore \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{5\hat{\imath} - \hat{\jmath} + 2\hat{k}}{\sqrt{30}}$ **[1 Mark]**

Hence, the unit vector in the direction of vector $5\hat{\jmath} - \hat{\jmath} + 2\hat{k}$ which has magnitude 8 units is given by,

$$8\hat{a} = 8\left(\frac{5\hat{\imath} - \hat{\jmath} + 2\hat{k}}{\sqrt{30}}\right) = \frac{40}{\sqrt{30}}\hat{\imath} - \frac{8}{\sqrt{30}}\hat{\jmath} + \frac{16}{\sqrt{30}}\hat{k}$$

$$= 8\left(\frac{5\vec{\imath} - \vec{\jmath} + 2\vec{k}}{\sqrt{30}}\right)$$

$= \frac{40}{\sqrt{30}}\vec{\imath} - \frac{8}{\sqrt{30}}\vec{\jmath} + \frac{16}{\sqrt{30}}\vec{k}$ **[1 Mark]**

11. Show that the vectors $2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $-4\hat{\imath} + 6\hat{\jmath} - 8\hat{k}$ are collinear. **[2 Marks]**

Solution:

Let $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $\vec{b} = -4\hat{\imath} + 6\hat{\jmath} - 8\hat{k}$.

It is observed that $\vec{b} = -4\hat{\imath} + 6\hat{\jmath} - 8\hat{k} = -2(2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}) = -2\vec{a}$ **[1 Mark]**

$\therefore \vec{b} = \lambda\vec{a}$

Where,

$\lambda = -2$

Hence, the given vectors are collinear. **[1 Mark]**

12. Find the direction cosines of the vector $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ **[1 Mark]**

Solution:

Let $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\therefore |\vec{a}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$ **[$\frac{1}{2}$ Mark]**

Hence, the direction cosines of $\vec{a}$ are $\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$. **[$\frac{1}{2}$ Mark]**

13. Find the direction cosines of the vector joining the points $A(1,2,-3)$ and $B(-1,-2,1)$ directed from A to B. **[1 Mark]**

Solution:

The given points are $A(1,2,-3)$ and $B(-1,-2,1)$.

$\therefore \overrightarrow{AB} = (-1-1)\hat{\imath} + (-2-2)\hat{\jmath} + \{1-(-3)\}\hat{k}$

$\Rightarrow \overrightarrow{AB} = -2\hat{\imath} - 4\hat{\jmath} + 4\hat{k}$ **[$\frac{1}{2}$ Mark]**

$\therefore |\overrightarrow{AB}| = \sqrt{(-2)^2 + (-4)^2 + 4^2} = \sqrt{4 + 16 + 16} = \sqrt{36} = 6$

Hence, the direction cosines of $\overrightarrow{AB}$ are $\left(-\frac{2}{6}, -\frac{4}{6}, \frac{4}{6}\right) = \left(-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right)$. **[$\frac{1}{2}$ Mark]**

14. Show that the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$ is equally inclined to the axes OX, OY, and OZ.**[1 Mark]**

Solution:

Let $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$

Then,

$|\vec{a}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

Therefore, the direction cosines of $\vec{a}$ are $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$. **[$\frac{1}{2}$ Mark]**

Now, let α, β, and γ be the angles formed by $\vec{a}$ with the positive directions of x, y, and z axes.

Then, we have

$$\cos\alpha = \frac{1}{\sqrt{3}}, \cos\beta = \frac{1}{\sqrt{3}}, \cos\gamma = \frac{1}{\sqrt{3}}$$

Hence, the given vector is equally inclined to axes OX, OY, and OZ. **[$\frac{1}{2}$ Mark]**

15. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $-\hat{\imath} + \hat{\jmath} + \hat{k}$ respectively, in the ration $2:1$

(i) internally

(ii) externally

Solution:

The position vector of point R dividing the line segment joining two points P and Q in the ratio $\mathrm{m}:\mathrm{n}$ is given by:

(i) Internally: $\frac{m\vec{b}+n\vec{a}}{m+n}$ **[1 Mark]**

(ii) Externally: $\frac{m\vec{b}-n\vec{a}}{m-n}$ **[1 Mark]**

Position vectors of P and Q are given as:

$\overrightarrow{OP} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $\overrightarrow{OQ} = -\hat{\imath} + \hat{\jmath} + \hat{k}$

(i) The position vector of point R which divides the line joining two points P and Q internally in the ratio $2:1$ is given by,

$$\overrightarrow{OR} = \frac{2(-\hat{\imath} + \hat{\jmath} + \hat{k}) + 1(\hat{\imath} + 2\hat{\jmath} - \hat{k})}{2+1} = \frac{(-2\hat{\imath} + 2\hat{\jmath} + 2\hat{k}) + (\hat{\imath} + 2\hat{\jmath} - \hat{k})}{3}$$

$= \frac{-\hat{\imath}+4\hat{\jmath}+\hat{k}}{3} = -\frac{1}{3}\hat{\imath} + \frac{4}{3}\hat{\jmath} + \frac{1}{3}\hat{k}$ **[1 Mark]**

(ii) The position vector of point R which divides the line joining two points P and Q externally in the ratio $2:1$ is given by,

$$\overrightarrow{OR} = \frac{2(-\hat{\imath} + \hat{\jmath} + \hat{k}) - 1(\hat{\imath} + 2\hat{\jmath} - \hat{k})}{2-1} = (-2\hat{\imath} + 2\hat{\jmath} + 2\hat{k}) - (\hat{\imath} + 2\hat{\jmath} - \hat{k})$$

$= -3\hat{\imath} + 3\hat{k}$ **[1 Mark]**

16. Find the position vector of the mid-point of the vector joining the points $P(2, 3, 4)$ and $Q(4, 1, -2)$. **[1 Mark]**

Solution:

The position vector of mid-point R of the vector joining points $P(2, 3, 4)$ and $Q(4, 1, -2)$ is given by,

$\overrightarrow{OR} = \frac{(2\hat{\imath}+3\hat{\jmath}+4\hat{k})+(4\hat{\imath}+\hat{\jmath}-2\hat{k})}{2} = \frac{(2+4)\hat{\imath}+(3+1)\hat{\jmath}+(4-2)\hat{k}}{2}$ **[$\frac{1}{2}$ Mark]**

$= \frac{6\hat{\imath}+4\hat{\jmath}+2\hat{k}}{2} = 3\hat{\imath} + 2\hat{\jmath} + \hat{k}$ **[$\frac{1}{2}$ Mark]**

17. Show that the points A, B and C with position vectors,

$\vec{a} = 3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}, \vec{b} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{c} = \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ respectively form the vertices of a right-angled triangle. **[2 Marks]**

Solution:

Position vectors of points A, B and C are respectively given as:

$\vec{a} = 3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}, \vec{b} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{c} = \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$

$\therefore \overrightarrow{\mathrm{AB}} = \vec{b} - \vec{a} = (2-3)\hat{\imath} + (-1+4)\hat{\jmath} + (1+4)\hat{k} = -\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$

$\overrightarrow{\mathrm{BC}} = \vec{c} - \vec{b} = (1-2)\hat{\imath} + (-3+1)\hat{\jmath} + (-5-1)\hat{k} = -\hat{\imath} - 2\hat{\jmath} - 6\hat{k}$

$\overrightarrow{\mathrm{CA}} = \vec{a} - \vec{c} = (3-1)\hat{\imath} + (-4+3)\hat{\jmath} + (-4+5)\hat{k} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

$\therefore \left|\overrightarrow{AB}\right|^2 = (-1)^2 + 3^2 + 5^2 = 1 + 9 + 25 = 35$ **[1 Mark]**

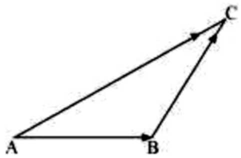
$\left|\overrightarrow{BC}\right|^2 = (-1)^2 + (-2)^2 + (-6)^2 = 1 + 4 + 36 = 41$

$\left|\overrightarrow{CA}\right|^2 = 2^2 + (-1)^2 + 1^2 = 4 + 1 + 1 = 6$

$\therefore \left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{CA}\right|^2 = 36 + 6 = 41 = \left|\overrightarrow{BC}\right|^2$

Hence, ABC is a right-angled triangle. **[1 Mark]**

18. In triangle ABC which of the following is not true: **[4 Marks]**



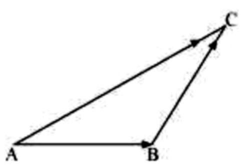
A. $\overrightarrow{\mathrm{AB}} + \overrightarrow{\mathrm{BC}} + \overrightarrow{\mathrm{CA}} = \vec{0}$

B. $\overrightarrow{\mathrm{AB}} + \overrightarrow{\mathrm{BC}} - \overrightarrow{\mathrm{AC}} = \vec{0}$

C. $\overrightarrow{\mathrm{AB}} + \overrightarrow{\mathrm{BC}} - \overrightarrow{\mathrm{CA}} = \vec{0}$

D. $\overrightarrow{\mathrm{AB}} - \overrightarrow{\mathrm{CB}} + \overrightarrow{\mathrm{CA}} = \vec{0}$

Solution:



On applying the triangle law of addition in the given triangle, we have:

$\overrightarrow{\mathrm{AB}} + \overrightarrow{\mathrm{BC}} = \overrightarrow{\mathrm{AC}} \quad \ldots(1)$

$\Rightarrow \overrightarrow{\mathrm{AB}} + \overrightarrow{\mathrm{BC}} = -\overrightarrow{\mathrm{CA}}$

$\Rightarrow \overrightarrow{\mathrm{AB}} + \overrightarrow{\mathrm{BC}} + \overrightarrow{\mathrm{CA}} = \vec{0} \quad \ldots(2)$

$\therefore$ The equation given in alternative A is true. **[1 Mark]**

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$

$\therefore$ The equation given in alternative B is true. **[1 Mark]**

From equation (2), we have:

$\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \vec{0}$

$\therefore$ The equation given in alternative D is true. **[1 Mark]**

Now, consider the equation given in alternative C:

$\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \vec{0}$

$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{CA} \quad \ldots(3)$

From equations (1) and (3), we have:

$\overrightarrow{AC} = \overrightarrow{CA}$

$\Rightarrow \overrightarrow{AC} = -\overrightarrow{AC}$

$\Rightarrow \overrightarrow{AC} + \overrightarrow{AC} = \vec{0}$

$\Rightarrow 2\overrightarrow{AC} = \vec{0}$

$\Rightarrow \overrightarrow{AC} = \vec{0}$, which is not true.

Hence, the equation given in alternative C is incorrect. **[1 Mark]**

The correct Answer is C.

19. If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of the following are incorrect: **[4 Marks]**

A. $\vec{b} = \lambda\vec{a}$, for some scalar λ

B. $\vec{a} = \pm\vec{b}$

C. the respective components of $\vec{a}$ and $\vec{b}$ are proportional

D. both the vectors $\vec{a}$ and $\vec{b}$ have same direction, but different magnitudes

Solution:

If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then they are parallel.

Therefore, we have:

$\vec{b} = \lambda\vec{a}$ (For some scalar λ) **[1 Mark]**

If $\lambda = \pm 1$, then $\vec{a} = \pm\vec{b}$

If $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$, then

$\vec{b} = \lambda\vec{a}$

$\Rightarrow b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k} = \lambda\left(a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}\right)$

$\Rightarrow b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k} = (\lambda a_1)\hat{\imath} + (\lambda a_2)\hat{\jmath} + (\lambda a_3)\hat{k}$

$\Rightarrow b_1 = \lambda a_1, b_2 = \lambda a_2, b_3 = \lambda a_3$

$\Rightarrow \frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_3}{a_3} = \lambda$ **[1 Mark]**

Thus, the respective components of $\vec{a}$ and $\vec{b}$ are proportional.

However, vectors $\vec{a}$ and $\vec{b}$ can have different directions.

Hence, the statement given in D is incorrect.

The correct Answer is D. **[2 Marks]**

Exercise: 10.3

1. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ and with magnitudes $\sqrt{3}$ and 2, respectively having $\vec{a} \cdot \vec{b} = \sqrt{6}$. **[2 Marks]**

Solution:

It is given that,

$|\vec{a}| = \sqrt{3}, |\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{6}$

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Now, we know that

$\therefore \sqrt{6} = \sqrt{3} \times 2 \times \cos\theta$ **[1 Mark]**

$$\Rightarrow \cos\theta = \frac{\sqrt{6}}{\sqrt{3} \times 2}$$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$$

$\Rightarrow \theta = \frac{\pi}{4}$

Hence, the angle between the given vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$ **[1 Mark]**

2. Find the angle between the vectors $\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ **[2 Marks]**

Solution:

The given vectors are $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$

$|\vec{a}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$

$|\vec{b}| = \sqrt{3^2 + (-2)^2 + 1^2} = \sqrt{9 + 4 + 1} = \sqrt{14}$

Now, $\vec{a} \cdot \vec{b} = (\hat{\imath} - 2\hat{\jmath} + 3\hat{k})(3\hat{\imath} - 2\hat{\jmath} + \hat{k})$

$= 1.3 + (-2)(-2) + 3.1$

$= 10$ **[1 Mark]**

Also, we know that $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\therefore 10 = \sqrt{14}\sqrt{14}\cos\theta$

$\Rightarrow \cos\theta = \frac{10}{14}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{5}{7}\right)$ **[1 Mark]**

3. Find the projection of the vector $\hat{\imath} - \hat{\jmath}$ on the vector $\hat{\imath} + \hat{\jmath}$. **[1 Mark]**

Solution:

Let $\vec{a} = \hat{\imath} - \hat{\jmath}$ and $\vec{b} = \hat{\imath} + \hat{\jmath}$. **[$\frac{1}{2}$ Mark]**

Now, projection of vector $\vec{a}$ on $\vec{b}$ is given by,

$\frac{1}{|\vec{b}|}(\vec{a} \cdot \vec{b}) = \frac{1}{\sqrt{1+1}}\{1.1 + (-1)(1)\} = \frac{1}{\sqrt{2}}(1 - 1) = 0$ **[$\frac{1}{2}$ Mark]**

Hence, the projection of vector $\vec{a}$ on $\vec{b}$ is 0.

4. Find the projection of the vector $\hat{\imath} + 3\hat{\jmath} + 7\hat{k}$ on the vector $7\hat{\imath} - \hat{\jmath} + 8\hat{k}$. **[1 Mark]**

Solution:

Let $\vec{a} = \hat{\imath} + 3\hat{\jmath} + 7\hat{k}$ and $\hat{b} = 7\hat{\imath} - \hat{\jmath} + 8\hat{k}$.

Now, projection of vector $\vec{a}$ on $\vec{b}$ is given by,

$$\frac{1}{|\vec{b}|}(\vec{a} \cdot \vec{b}) = \frac{1}{\sqrt{7^2+(-1)^2+8^2}}\{1(7) + 3(-1) + 7(8)\} = \frac{7-3+56}{\sqrt{49+1+64}} = \frac{60}{\sqrt{114}}$$ **[1 Mark]**

5. Show that each of the given three vectors is a unit vector:

$$\frac{1}{7}(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}), \frac{1}{7}(3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}), \frac{1}{7}(6\hat{\imath} + 2\hat{\jmath} - 3\hat{k})$$

Also, show that they are mutually perpendicular to each other. **[2 Marks]**

Solution:

Let $\vec{a} = \frac{1}{7}(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}) = \frac{2}{7}\hat{\imath} + \frac{3}{7}\hat{\jmath} + \frac{6}{7}\hat{k}$,

$$\vec{b} = \frac{1}{7}(3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}) = \frac{3}{7}\hat{\imath} - \frac{6}{7}\hat{\jmath} + \frac{2}{7}\hat{k}$$

$$\vec{c} = \frac{1}{7}(6\hat{\imath} + 2\hat{\jmath} - 3\hat{k}) = \frac{6}{7}\hat{\imath} + \frac{2}{7}\hat{\jmath} - \frac{3}{7}\hat{k}$$

$$|\vec{a}| = \sqrt{\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{6}{7}\right)^2} = \sqrt{\frac{4}{49} + \frac{9}{49} + \frac{36}{49}} = 1$$

$$|\vec{b}| = \sqrt{\left(\frac{3}{7}\right)^2 + \left(-\frac{6}{7}\right)^2 + \left(\frac{2}{7}\right)^2} = \sqrt{\frac{9}{49} + \frac{36}{49} + \frac{4}{49}} = 1$$

$$|\vec{c}| = \sqrt{\left(\frac{6}{7}\right)^2 + \left(\frac{2}{7}\right)^2 + \left(-\frac{3}{7}\right)^2} = \sqrt{\frac{36}{49} + \frac{4}{49} + \frac{9}{49}} = 1$$ **[1 Mark]**

Thus, each of the given three vectors is a unit vector.

$$\vec{a}\cdot\vec{b}=\frac{2}{7}\times\frac{3}{7}+\frac{3}{7}\times\left(\frac{-6}{7}\right)+\frac{6}{7}\times\frac{2}{7}=\frac{6}{49}-\frac{18}{49}+\frac{12}{49}=0$$

$$\vec{b}\cdot\vec{c}=\frac{3}{7}\times\frac{6}{7}+\left(\frac{-6}{7}\right)\times\frac{2}{7}+\frac{2}{7}\times\left(\frac{-3}{7}\right)=\frac{18}{49}-\frac{12}{49}-\frac{6}{49}=0$$

$$\vec{c}\cdot\vec{a}=\frac{6}{7}\times\frac{2}{7}+\frac{2}{7}\times\frac{3}{7}+\left(\frac{-3}{7}\right)\times\frac{6}{7}=\frac{12}{49}+\frac{6}{49}-\frac{18}{49}=0$$

Hence, the given three vectors are mutually perpendicular to each other. **[1 Mark]**

6. Find

$|\vec{a}|$ and $|\vec{b}|$, if $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})=8$ and $|\vec{a}|=8|\vec{b}|$. **[2 Marks]**

Solution:

$(\vec{a}\cdot\vec{b})\cdot(\vec{a}-\vec{b})=8$

$\Rightarrow \vec{a}\cdot\vec{a}-\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}-\vec{b}\cdot\vec{b}=8$

$\Rightarrow |\vec{a}|^2-|\vec{b}|^2=8$

$\Rightarrow (8|\vec{b}|)^2-|\vec{b}|^2=8$ $\quad [|\vec{a}|=8|\vec{b}|]$

$\Rightarrow 64|\vec{b}|^2-|\vec{b}|^2=8$

$\Rightarrow 63|\vec{b}|^2=8$

$\Rightarrow |\vec{b}|^2=\frac{8}{63}$

$\Rightarrow |\vec{b}|=\sqrt{\frac{8}{63}}$ [Magnitude of a vector is non-negative]

$\Rightarrow |\vec{b}|=\frac{2\sqrt{2}}{3\sqrt{7}}$ **[1 Mark]**

$|\vec{a}|=8|\vec{b}|=\frac{8\times2\sqrt{2}}{3\sqrt{7}}=\frac{16\sqrt{2}}{3\sqrt{7}}$ **[1 Mark]**

7. Evaluate the product $(3\vec{a}-5\vec{b})\cdot(2\vec{a}+7\vec{b})$. **[1 Mark]**

Solution:

$(3\vec{a}-5\vec{b})\cdot(2\vec{a}+7\vec{b})$

$=3\vec{a}\cdot 2\vec{a}+3\vec{a}\cdot 7\vec{b}-5\vec{b}\cdot 2\vec{a}-5\vec{b}\cdot 7\vec{b}$ **[$\frac{1}{2}$ Mark]**

$=6\vec{a}\cdot\vec{a}+21\vec{a}\cdot\vec{b}-10\vec{a}\cdot\vec{b}-35\vec{b}\cdot\vec{b}$

$=6|\vec{a}|^2+11\vec{a}\cdot\vec{b}-35|\vec{b}|^2$ **[$\frac{1}{2}$ Mark]**

8. Find the magnitude of two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude and such that the angle between them is 60^o and their scalar product is $\frac{1}{2}$. **[1 Mark]**

Solution:

Let θ be the angle between the vectors $\vec{a}$ and $\vec{b}$.

It is given that $|\vec{a}|=|\vec{b}|, \vec{a}\cdot\vec{b}=\frac{1}{2}$, and $\theta=60^{\circ}$ $\quad\ldots(1)$

We know that $\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta$

$\therefore \frac{1}{2}=|\vec{a}||\vec{a}|\cos 60^{\circ}$ [Using (1)] **[$\frac{1}{2}$ Mark]**

$\Rightarrow \frac{1}{2}=|\vec{a}|^2\times\frac{1}{2}$

$\Rightarrow |\vec{a}|^2=1$

$\Rightarrow |\vec{a}|=|\vec{b}|=1$ **[$\frac{1}{2}$ Mark]**

9. Find

$|\vec{x}|$ if for a unit vector $\vec{a}$, $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a})=12$. **[1 Mark]**

Solution:

$(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a})=12$

$\Rightarrow \vec{x}\cdot\vec{x}+\vec{x}\cdot\vec{a}-\vec{a}\cdot\vec{x}-\bar{a}\cdot\vec{a}=12$ **[$\frac{1}{2}$ Mark]**

$\Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 12$

$\Rightarrow |\vec{x}|^2 - 1 = 12$ $[|\vec{a}| = 1$ as $\vec{a}$ is a unit vector$]$

$\Rightarrow |\vec{x}|^2 = 13$

$\therefore |\vec{x}| = \sqrt{13}$ **[$\frac{1}{2}$ Mark]**

10. If $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}, \vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{c} = 3\hat{\imath} + \hat{\jmath}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ. **[2 Marks]**

Solution:

The given vectors are $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}, \vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{c} = 3\hat{\imath} + \hat{\jmath}$.

Now,

$\vec{a} + \lambda\vec{b} = (2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(-\hat{\imath} + 2\hat{\jmath} + \hat{k}) = (2 - \lambda)\hat{\imath} + (2 + 2\lambda)\hat{\jmath} + (3 + \lambda)\hat{k}$

If $(\vec{a} + \lambda\vec{b})$ is perpendicular to $\vec{c}$, then

$(\vec{a} + \lambda\vec{b}) \cdot \vec{c} = 0$ **[1 Mark]**

$\Rightarrow [(2 - \lambda)\hat{\imath} + (2 + 2\lambda)\hat{\jmath} + (3 + \lambda)\hat{k}] \cdot (3\hat{\imath} + \hat{\jmath}) = 0$

$\Rightarrow (2 - \lambda)3 + (2 + 2\lambda)1 + (3 + \lambda)0 = 0$

$\Rightarrow 6 - 3\lambda + 2 + 2\lambda = 0$

$\Rightarrow -\lambda + 8 = 0$

$\Rightarrow \lambda = 8$

Hence, the required value of λ is 8. **[1 Mark]**

11. Show that:

$|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$,

for any two nonzero vectors $\vec{a}$ and $\vec{b}$ **[1 Mark]**

Solution:

$(|\vec{a}|\vec{b} + |\vec{b}|\vec{a}) \cdot (|\vec{a}|\vec{b} - |\vec{b}|\vec{a})$

$= |\vec{a}|^2\vec{b} \cdot \vec{b} - |\vec{a}||\vec{b}|\vec{b} \cdot \vec{a} + |\vec{b}||\vec{a}|\vec{a} \cdot \vec{b} - |\vec{b}|^2\vec{a} \cdot \vec{a}$ **[$\frac{1}{2}$ Mark]**

$= |\vec{a}|^2|\vec{b}|^2 - |\vec{b}|^2|\vec{a}|^2$

$= 0$

Hence, $|\vec{a}|\vec{b} + |\vec{b}|\bar{a}$ and $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$ are perpendicular to each other. **[$\frac{1}{2}$ Mark]**

12. If, $\vec{a} \cdot \vec{a} = 0$ and $\vec{a} \cdot \vec{b} = 0$, then what can be concluded about the vector $\vec{b}$? **[1 Mark]**

Solution:

It is given that $\vec{a} \cdot \vec{a} = 0$ and $\vec{a} \cdot \vec{b} = 0$.

Now,

$\vec{a} \cdot \vec{a} = 0 \Rightarrow |\vec{a}|^2 = 0 \Rightarrow |\vec{a}| = 0$ **[$\frac{1}{2}$ Mark]**

$\therefore \vec{a}$ is a zero vector.

Hence, vector $\vec{b}$ satisfying $\vec{a} \cdot \vec{b} = 0$ can be any vector. **[$\frac{1}{2}$ Mark]**

13. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$,

find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ **[1 Mark]**

Solution:

$|\vec{a} + \vec{b} + \vec{c}|^2 = (\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$

$\Rightarrow 0 = 1 + 1 + 1 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$ **[$\frac{1}{2}$ Mark]**

$\Rightarrow (\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = \frac{-3}{2}$ **[$\frac{1}{2}$ Mark]**

14. If either vector $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$, then $\vec{a} \cdot \vec{b} = 0$. But the converse need not be true. Justify your Answer with an example. **[2 Marks]**

Solution:

Consider $\vec{a} = 2\hat{\imath} + 4\hat{\jmath} + 3\hat{k}$ and $\vec{b} = 3\hat{\imath} + 3\hat{\jmath} - 6\hat{k}$.

Then,

$\vec{a} \cdot \vec{b} = 2.3 + 4.3 + 3(-6) = 6 + 12 - 18 = 0$.**[1 Mark]**

We now observe that:

$|\vec{a}| = \sqrt{2^2 + 4^2 + 3^2} = \sqrt{29}$

$\therefore \vec{a} \neq \vec{0}$

$|\vec{b}| = \sqrt{3^2 + 3^2 + (-6)^2} = \sqrt{54}$

$\therefore \vec{b} \neq \vec{0}$

Hence, the converse of the given statement need not be true. **[1 Mark]**

15. If the vertices A, B, C of a triangle ABC are $(1,2,3), (-1,0,0), (0,1,2)$, respectively, then find $\angle ABC$. [$\angle ABC$ is the angle between the vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$] **[2 Marks]**

Solution:

The vertices of ΔABC are given as $A(1,2,3), B(-1,0,0)$, and $C(0,1,2)$.

Also, it is given that $\angle ABC$ is the angle between the vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$.

$\overrightarrow{BA} = \{1 - (-1)\}\hat{\imath} + (2 - 0)\hat{\jmath} + (3 - 0)\hat{k} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\overrightarrow{BC} = \{0 - (-1)\}\hat{\imath} + (1 - 0)\hat{\jmath} + (2 - 0)\hat{k} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$

$\therefore \overrightarrow{BA} \cdot \overrightarrow{BC} = (2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) \cdot (\hat{\imath} + \hat{\jmath} + 2\hat{k})$

$= 2 \times 1 + 2 \times 1 + 3 \times 2 = 2 + 2 + 6 = 10$ **[1 Mark]**

$|\overrightarrow{BA}| = \sqrt{2^2 + 2^2 + 3^2} = \sqrt{4 + 4 + 9} = \sqrt{17}$

$|\overrightarrow{BC}| = \sqrt{1 + 1 + 2^2} = \sqrt{6}$

Now, it is known that:

$\overrightarrow{BA}\cdot\overrightarrow{BC}=|\overrightarrow{BA}||\overrightarrow{BC}|\cos(\angle ABC)$

$\therefore 10=\sqrt{17}\times\sqrt{6}\cos(\angle ABC)$

$\Rightarrow \cos(\angle ABC)=\dfrac{10}{\sqrt{17}\times\sqrt{6}}$

$\Rightarrow \angle ABC=\cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$ **[1 Mark]**

16. Show that the points $A(1,2,7), B(2,6,3)$ and $C(3,10,-1)$ are collinear. **[2 Marks]**

Solution:

The given points are $A(1,2,7), B(2,6,3)$ and $C(3,10,-1)$.

$\therefore \overrightarrow{AB}=(2-1)\hat{\imath}+(6-2)\hat{\jmath}+(3-7)\hat{k}=\hat{\imath}+4\hat{\jmath}-4\hat{k}$

$\overrightarrow{BC}=(3-2)\hat{\imath}+(10-6)\hat{\jmath}+(-1-3)\hat{k}=\hat{\imath}+4\hat{\jmath}-4\hat{k}$

$\overrightarrow{AC}=(3-1)\hat{\imath}+(10-2)\hat{\jmath}+(-1-7)\hat{k}=2\hat{\imath}+8\hat{\jmath}-8\hat{k}$ **[1 Mark]**

$|\overrightarrow{AB}|=\sqrt{1^2+4^2+(-4)^2}=\sqrt{1+16+16}=\sqrt{33}$

$|\overrightarrow{BC}|=\sqrt{1^2+4^2+(-4)^2}=\sqrt{1+16+16}=\sqrt{33}$

$|\overrightarrow{AC}|=\sqrt{2^2+8^2+8^2}=\sqrt{4+64+64}=\sqrt{132}=2\sqrt{33}$

$\therefore |\overrightarrow{AC}|=|\overrightarrow{AB}|+|\overrightarrow{BC}|$

Hence, the given points A, B, and C are collinear. **[1 Mark]**

17. Show that the vectors $2\hat{\imath}-\hat{\jmath}+\hat{k}, \hat{\imath}-3\hat{\jmath}-5\hat{k}$ and $3\hat{\imath}-4\hat{\jmath}-4\hat{k}$ form the vertices of a right angled triangle. **[2 Marks]**

Solution:

Let vectors $2\hat{\imath}-\hat{\jmath}+\hat{k}, \hat{\imath}-3\hat{\jmath}-5\hat{k}$ and $3\hat{\imath}-4\hat{\jmath}-4\hat{k}$ be position vectors of points A, B, and C respectively.

i.e., $\overrightarrow{OA}=2\hat{\imath}-\hat{\jmath}+\hat{k}, \overrightarrow{OB}=\hat{\imath}-3\hat{\jmath}-5\hat{k}$ and $\overrightarrow{OC}=3\hat{\imath}-4\hat{\jmath}-4\hat{k}$

Now, vectors $\overrightarrow{AB}, \overrightarrow{BC}$ and $\overrightarrow{AC}$ represent the sides of ΔABC.

i.e., $\overrightarrow{OA} = 2\hat{\imath} - \hat{\jmath} + \hat{k}, \overrightarrow{OB} = \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ and $\overrightarrow{OC} = 3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$.

$\therefore \overrightarrow{AB} = (1-2)\hat{\imath} + (-3+1)\hat{\jmath} + (-5-1)\hat{k} = -\hat{\imath} - 2\hat{\jmath} - 6\hat{k}$

$\overrightarrow{BC} = (3-1)\hat{\imath} + (-4+3)\hat{\jmath} + (-4+5)\hat{k} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

$\overrightarrow{AC} = (2-3)\hat{\imath} + (-1+4)\hat{\jmath} + (1+4)\hat{k} = -\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$ **[1 Mark]**

$|\overrightarrow{AB}| = \sqrt{(-1)^2 + (-2)^2 + (-6)^2} = \sqrt{1+4+36} = \sqrt{41}$

$|\overrightarrow{BC}| = \sqrt{2^2 + (-1)^2 + I^2} = \sqrt{4+1+1} = \sqrt{6}$

$|\overrightarrow{AC}| = \sqrt{(-1)^2 + 3^2 + 5^2} = \sqrt{1+9+25} = \sqrt{35}$

$\therefore |\overrightarrow{BC}|^2 + |\overrightarrow{AC}|^2 = 6 + 35 = 41 = |\overrightarrow{AB}|^2$

Hence, ΔABC is a right-angled triangle. **[1 Mark]**

18. If $\vec{a}$ is a nonzero vector of magnitude 'a' and λ a nonzero scalar, then $\lambda\vec{a}$ is unit vector

If **[1 Mark]**

(A) $\lambda = 1$

(B) $\lambda =- 1$

(C) $a = |\lambda|$

(D) $a = \frac{1}{|\lambda|}$

Solution:

Vector $\lambda\vec{a}$ is a unit vector if $|\lambda\vec{a}| = 1$.

Now,

$|\lambda\vec{a}| = 1$

$\Rightarrow |\lambda||\vec{a}| = 1$ **[$\frac{1}{2}$ Mark]**

$\Rightarrow |\vec{a}| = \frac{1}{|\lambda|} [\lambda \neq 0]$

$\Rightarrow a = \frac{1}{|\lambda|} [|\vec{a}| = a]$

Hence, vector $\lambda\vec{a}$ is a unit vector if $a = \frac{1}{|\lambda|}$

The correct Answer is D. [$\frac{1}{2}$ Mark]

Exercise: 10.4

1. Find

$|\vec{a} \times \vec{b}|$, if $\vec{a} = \hat{\imath} - 7\hat{\jmath} + 7\hat{k}$ and $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ **[1 Mark]**

Solution:

We have, $\vec{a} = \hat{\imath} - 7\hat{\jmath} + 7\hat{k}$ and $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix}$$

$= \hat{\imath}(-14 + 14) - \hat{\jmath}(2 - 21) + \hat{k}(-2 + 21) = 19\hat{\jmath} + 19\hat{k}$ [$\frac{1}{2}$ Mark]

$\therefore |\vec{a} \times \vec{b}| = \sqrt{(19)^2 + (19)^2} = \sqrt{2 \times (19)^2} = 19\sqrt{2}$ [$\frac{1}{2}$ Mark]

2. Find a unit vector perpendicular to each of the vector $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, where

$\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$ **[2 Marks]**

Solution:

We have,

$\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$

$\therefore \vec{a} + \vec{b} = 4\hat{\imath} + 4\hat{\jmath}, \vec{a} - \vec{b} = 2\hat{\imath} + 4\hat{k}$

$$(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix} = \hat{\imath}(16) - \hat{\jmath}(16) + \hat{k}(-8) = 16\hat{\imath} - 16\hat{\jmath} - 8\hat{k}$$

$\therefore |(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})| = \sqrt{16^2 + (-16)^2 + (-8)^2}$

$= \sqrt{2^2 \times 8^2 + 2^2 \times 8^2 + 8^2}$

$= 8\sqrt{2^2 + 2^2 + 1} = 8\sqrt{9} = 8 \times 3 = 24$ **[1 Mark]**

Hence, the unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, is given by the relation,

$$=\pm\frac{(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})}{|(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})|}=\pm\frac{16\hat{\imath}-16\hat{\jmath}-8\hat{k}}{24}$$

$=\pm\frac{2\hat{\imath}-2\hat{\jmath}-\hat{k}}{3}=\pm\frac{2}{3}\hat{\imath}\mp\frac{2}{3}\hat{\jmath}\mp\frac{1}{3}\hat{k}$ **[1 Mark]**

3. If a unit vector $\vec{a}$ makes an $\frac{\pi}{3}$ with i, $\frac{\pi}{4}$ angle with $\hat{\jmath}$ and an acute angle θ with $\hat{k}$, then find θ and hence, the compounds of $\vec{a}$. **[4 Marks]**

Solution:

Let unit vector $\vec{a}$ have (a_1, a_2, a_3) components.

$\vec{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$

Since $\vec{a}$ is a unit vector, $|\vec{a}|=1$.

Also, it is given that $\vec{a}$ makes angles $\frac{\pi}{3}$ with i, $\frac{\pi}{4}$ with $\hat{\jmath}$ and an acute angle θ with $\hat{k}$. Then, we have:

$$\cos\frac{\pi}{3}=\frac{a_1}{|\vec{a}|}$$

$\Rightarrow\frac{1}{2}=a_1$ $[|\vec{a}|=1]$ **[$\frac{1}{2}$ Mark]**

$$\cos\frac{\pi}{4}=\frac{a_2}{|\vec{a}|}$$

$\Rightarrow\frac{1}{\sqrt{2}}=a_2$ $[|\vec{a}|=1]$ **[$\frac{1}{2}$ Mark]**

Also, $\cos\theta=\frac{a_3}{|\vec{a}|}$.

$\Rightarrow a_3=\cos\theta$ **[$\frac{1}{2}$ Mark]**

Now,

$|a|=1$

$\Rightarrow\sqrt{a_1^2+a_2^2+a_3^2}=1$ **[$\frac{1}{2}$ Mark]**

$$\Rightarrow\left(\frac{1}{2}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2+\cos^2\theta=1$$

$\Rightarrow \frac{1}{4}+\frac{1}{2}+\cos^2\theta = 1$

$\Rightarrow \frac{3}{4}+\cos^2\theta = 1$

$\Rightarrow \cos^2\theta = 1-\frac{3}{4} = \frac{1}{4}$

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

$\therefore a_3 = \cos\frac{\pi}{3} = \frac{1}{2}$

Hence, $\theta = \frac{\pi}{3}$ and the components of $\vec{a}$ are $\left(\frac{1}{2},\frac{1}{\sqrt{2}},\frac{1}{2}\right)$. **[2 Marks]**

4. Show that

$(\vec{a}-\vec{b})\times(\vec{a}+\vec{b}) = 2(\vec{a}\times\vec{b})$ **[1 Mark]**

Solution:

$(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})$

$= (\vec{a}-\vec{b})\times\vec{a} + (\vec{a}-\vec{b})\times\vec{b}$ [By distributive of vector product over addition] **[$\frac{1}{2}$ Mark]**

$= \vec{a}\times\vec{a} - \vec{b}\times\vec{a} + \vec{a}\times\vec{b} - \vec{b}\times\vec{b}$ [Again, by distributivity of vector product over addition]

$= \vec{0} + \vec{a}\times\vec{b} + \vec{a}\times\vec{b} - \vec{0}$

$= 2\vec{a}\times\vec{b}$ **[$\frac{1}{2}$ Mark]**

5. Find λ and μ if $(2\hat{\imath}+6\hat{\jmath}+27\hat{k})\times(\hat{\imath}+\lambda\hat{\jmath}+\mu\hat{k}) = \vec{0}$. **[2 Marks]**

Solution:

$(2\hat{\imath}+6\hat{\jmath}+27\hat{k})\times(\hat{\imath}+\lambda\hat{\jmath}+\mu\hat{k}) = \vec{0}$

$\Rightarrow \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{vmatrix} = 0\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$

$\Rightarrow \hat{\imath}(6\mu - 27\lambda) - \hat{\jmath}(2\mu - 27) + \hat{k}(2\lambda - 6) = 0\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$ **[1 Mark]**

On comparing the corresponding components, we have:

$6\mu - 27\lambda = 0$

$2\mu - 27 = 0$

$2\lambda - 6 = 0$

Now,

$2\lambda - 6 = 0 \Rightarrow \lambda = 3$

$2\mu - 27 = 0 \Rightarrow \mu = \frac{27}{2}$

Hence $\lambda = 3$ and $\mu = \frac{27}{2}$. **[1 Mark]**

6. Given that $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} \times \vec{b} = \vec{0}$.

What can you conclude about the vectors $\vec{a}$ and $\vec{b}$? **[1 Mark]**

Solution:

$\vec{a} \cdot \vec{b} = 0$

Then,

(i) Either $|\vec{a}| = 0$ or $|\vec{b}| = 0$, or $\vec{a} \perp \vec{b}$ (in case $\vec{a}$ and $\vec{b}$ are non-zero)

(ii) Either $|\vec{a}| = 0$ or $|\vec{b}| = 0$, or $\vec{a} \parallel \vec{b}$ (in case $\vec{a}$ and $\vec{b}$ are non-zero) **[$\frac{1}{2}$ Mark]**

But, $\vec{a}$ and $\vec{b}$ cannot be perpendicular and parallel simultaneously.

Hence, $|\vec{a}| = 0$ or $|\vec{b}| = 0$ **[$\frac{1}{2}$ Mark]**

7. Let the vectors $\vec{a}, \vec{b}, \vec{c}$ given as $a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}, b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}, c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then show that $= \vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$ **[4 Marks]**

Solution:

We have,

$\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}, \vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}, \vec{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

$(\vec{b} + \vec{c}) = (b_1 + c_1)\hat{\imath} + (b_2 + c_2)\hat{\jmath} + (b_3 + c_3)\hat{k}$

Now, $\vec{a} \times (\vec{b} + \vec{c}) \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 + c_1 & b_2 + c_2 & b_3 + c_3 \end{vmatrix}$

$= \hat{\imath}[a_2(b_3 + c_3) - a_3(b_2 + c_2)] - \hat{\jmath}[a_1(b_3 + c_3) - a_3(b_1 + c_1)] + \hat{k}[a_1(b_2 + c_2) - a_2(b_1 + c_1)]$

$= \hat{\imath}[a_2b_3 + a_2c_3 - a_3b_2 - a_3c_2] + \hat{\jmath}[-a_1b_3 - a_3c_3 + a_3b_1 + a_3c_1] + \hat{k}[a_1b_2 + a_1c_2 - a_2b_1 - a_2c_1] \quad \ldots(1)$ **[1 Mark]**

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

$= \hat{\imath}[a_2b_3 - a_3b_2] + \hat{\jmath}[b_1a_3 - a_1b_3] + \hat{k}[a_1b_2 - a_2b_1] \quad \ldots(2)$ **[1 Mark]**

$\vec{a} \times \vec{c} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

$= \hat{\imath}[a_2c_3 - a_3c_2] + \hat{\jmath}[a_3c_1 - a_1c_3] + \hat{k}[a_1c_2 - a_2c_1] \quad \ldots(3)$ **[1 Mark]**

On adding (2) and (3), we get:

$(\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c})$
$= \hat{\imath}[a_2b_3 + a_2c_3 - a_3b_2 - a_3c_2] + \hat{\jmath}[b_1a_3 + a_3c_1 - a_1b_3 - a_1c_3] + \hat{k}[a_1b_2 + a_1c_2 - a_2b_1 - a_2c_1] \quad \ldots(4)$

Now, from (1) and (4), we have:

$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

Hence, the given result is proved. **[1 Mark]**

8. If either $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$, then $\vec{a} \times \vec{b} = \vec{0}$.

Is the converse true? Justify your Answer with an example. **[2 Marks]**

Solution:

Take any parallel non-zero vectors so that $\vec{a} \times \vec{b} = \vec{0}$.

Let $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}, \vec{b} = 4\hat{\imath} + 6\hat{\jmath} + 8\hat{k}$

Then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 4 \\ 4 & 6 & 8 \end{vmatrix} = \hat{\imath}(24 - 24) - \hat{\jmath}(16 - 16) + \hat{k}(12 - 12) = 0\hat{\imath} + 0\hat{\jmath} + 0\hat{k} = \vec{0}$$

It can now be observed that:

$|\vec{a}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$ **[1 Mark]**

$\therefore \vec{a} \neq \vec{0}$

$|\vec{b}| = \sqrt{4^2 + 6^2 + 8^2} = \sqrt{116}$

$\therefore \vec{b} \neq \vec{0}$

Hence, the converse of the given statement need not be true. **[1 Mark]**

9. Find the area of the triangle with vertices $A(1,1,2), B(2,3,5)$ and $C(1,5,5)$. **[2 Marks]**

Solution:

The vertices of triangle ABC are given as $A(1,1,2), B(2,3,5)$ and $C(1,5,5)$.

The adjacent sides $\overrightarrow{AB}$ and $\overrightarrow{BC}$ of ΔABC are given as:

$\overrightarrow{\mathrm{AB}} = (2 - 1)\hat{\imath} + (3 - 1)\hat{\jmath} + (5 - 2)\hat{k} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\overrightarrow{\mathrm{BC}} = (1 - 2)\hat{\imath} + (5 - 3)\hat{\jmath} + (5 - 5)\hat{k} = -\hat{\imath} + 2\hat{\jmath}$ **[1 Mark]**

Area of $\Delta\mathrm{ABC} = \frac{1}{2}\left|\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{BC}}\right|$

$$\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{BC}} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 2 & 0 \end{vmatrix} = \hat{\imath}(-6) - \hat{\jmath}(3) + \hat{k}(2 + 2) = -6\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$$

$\therefore \left|\overrightarrow{AB} \times \overrightarrow{BC}\right| = \sqrt{(-6)^2 + (-3)^2 + 4^2} = \sqrt{36 + 9 + 16} = \sqrt{61}$ **[1 Mark]**

Hence, the area of ΔABC is $\frac{\sqrt{61}}{2}$ square units.

10. Find the area of the parallelogram whose adjacent sides are determined by the vector $\vec{a} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{b} = 2\hat{\imath} - 7\hat{\jmath} + \hat{k}$. **[2 Marks]**

Solution:

The area of the parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$ is $|\vec{a} \times \vec{b}|$.

Adjacent sides are given as:

$\vec{a} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{b} = 2\hat{\imath} - 7\hat{\jmath} + \hat{k}$

$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix} = \hat{\imath}(-1+21) - \hat{\jmath}(1-6) + \hat{k}(-7+2) = 20\hat{\imath} + 5\hat{\jmath} - 5\hat{k}$ **[1 Mark]**

$|\vec{a} \times \vec{b}| = \sqrt{20^2 + 5^2 + 5^2} = \sqrt{400 + 25 + 25} = 15\sqrt{2}$

Hence, the area of the given parallelogram is $15\sqrt{2}$ square units. **[1 Mark]**

11. Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$, then $\vec{a} \times \vec{b}$ is a unit vector, if the angle between $\vec{a}$ and $\vec{b}$ is **[2 Marks]**

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

Solution:

It is given that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$

We know that $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\hat{n}$, where is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ and θ is the angle between $\vec{a}$ and $\vec{b}$.

Now, $\vec{a} \times \vec{b}$ is a unit vector if $|\vec{a} \times \vec{b}| = 1$

$|\vec{a} \times \vec{b}| = 1$

$\Rightarrow \left||\vec{a}||\vec{b}|sin\theta\hat{n}\right| = 1$

$\Rightarrow |\left||\vec{a}|\left|\vec{b}\right|sin\theta\right| = 1$

$\Rightarrow 3 \times \frac{\sqrt{2}}{3} \times \sin\theta = 1$ **[1 Mark]**

$\Rightarrow \sin\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{\pi}{4}$

Hence, $\vec{a} \times \vec{b}$ is a unit vector if the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$.

The correct Answer is B. **[1 Mark]**

12. Area of a rectangle having vertices A, B, C, and D with position vectors $-\hat{\imath} + \frac{1}{2}\hat{\jmath} + 4\hat{k}, \hat{\imath} + \frac{1}{2}\hat{\jmath} + 4\hat{k}, \hat{\imath} - \frac{1}{2}\hat{\jmath} + 4\hat{k}$ and $-\hat{\imath} - \frac{1}{2}\hat{\jmath} + 4\hat{k}$ respectively is **[2 Marks]**

(A) $\frac{1}{2}$

(B) 1

(C) 2

(D) 4

Solution:

The position vectors of vertices A, B, C, and D of rectangle $ABCD$ are given as:

$$\overrightarrow{OA} = -\hat{\imath} + \frac{1}{2}\hat{\jmath} + 4\hat{k}, \overrightarrow{OB} = \hat{\imath} + \frac{1}{2}\hat{\jmath} + 4\hat{k}, \overrightarrow{OC} = \hat{\imath} - \frac{1}{2}\hat{\jmath} + 4\hat{k}, \overrightarrow{OD} = -\hat{\imath} - \frac{1}{2}\hat{\jmath} + 4\hat{k}$$

The adjacent sides $\overrightarrow{AB}$ and $\overrightarrow{BC}$ of the given rectangle are given as:

$$\overrightarrow{AB} = (1+1)\hat{\imath} + \left(\frac{1}{2} - \frac{1}{2}\right)\hat{\jmath} + (4-4)\hat{k} = 2\hat{\imath}$$

$$\overrightarrow{BC} = (1-1)\hat{\imath} + \left(-\frac{1}{2} - \frac{1}{2}\right)\hat{\jmath} + (4-4)\hat{k} = -\hat{\jmath}$$ **[1 Mark]**

$$\therefore \overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} = \hat{k}(-2) = -2\hat{k}$$

$$\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \sqrt{(-2)^2} = 2$$

Now, it is known that the area of a parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$ is $\left|\vec{a} \times \vec{b}\right|$.

Hence, the area of the given rectangle is $|\overrightarrow{AB} \times \overrightarrow{BC}| = 2$ square units.

The correct Answer is C. **[1 Mark]**

Miscellaneous exercise

1. Write down a unit vector in XY-plane, making an angle of 30^o with the positive direction of x-axis. **[1 Mark]**

Solution:

If $\vec{r}$ is a unit vector in the XY-plane, then $\vec{r} = \cos\theta\hat{\jmath} + \sin\theta\hat{\jmath}$.

Here, θ is the angle made by the unit vector with the positive direction of the x-axis.

Therefore, for $\theta = 30^o$: **[$\frac{1}{2}$ Mark]**

$$\vec{r} = \cos 30^\circ\hat{\imath} + \sin 30^\circ\hat{\jmath} = \frac{\sqrt{3}}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath}$$

Hence, the required unit vector is $= \frac{\sqrt{3}}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath}$ **[$\frac{1}{2}$ Mark]**

2. Find the scalar components and magnitude of the vector joining the points

$P(x_1, y_1, z_1)$.and $Q(x_2, y_2, z_2)$. **[2 Marks]**

Solution:

The vector joining the points $P(x_1, y_1, z_1)$.and $Q(x_2, y_2, z_2)$

$\overrightarrow{PQ}$ =Position vector of Q −Position vector of P.

$= (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath} + (z_2 - z_1)\hat{k}$ **[1 Mark]**

$|\overrightarrow{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Hence, the scalar components and the magnitude of the vector joining the given points are respectively $\{(x_2 - x_1), (y_2 - y_1), (z_2 - z_1)\}$ and $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

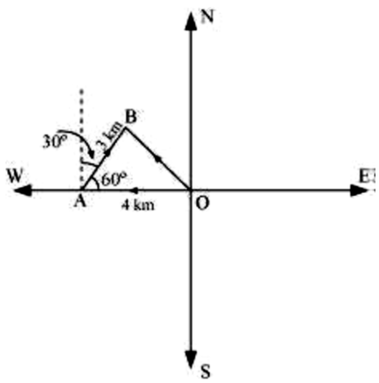
[1 Mark]

3. A girl walks 4 km towards west, then she walks 3 km in a direction 30^{o} east of north and stops. Determine the girl's displacement from her initial point of departure. **[2 Marks]**

Solution:

Let O and B be the initial and final positions of the girl respectively.

Then, the girl's position can be shown as:



Now, we have:

$\overrightarrow{OA} = -4\hat{\imath}$

$\overrightarrow{AB} = \hat{\imath}|\overrightarrow{AB}|\cos 60^{\circ} + \hat{\jmath}|\overrightarrow{AB}|\sin 60^{\circ}$

$= \hat{\imath}3 \times \frac{1}{2} + \hat{\jmath}3 \times \frac{\sqrt{3}}{2}$

$= \frac{3}{2}\hat{\imath} + \frac{3\sqrt{3}}{2}\hat{\jmath}$ **[1 Mark]**

By the triangle law of vector addition, we have:

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$= (-4\hat{\imath}) + \left(\frac{3}{2}\hat{\imath} + \frac{3\sqrt{3}}{2}\hat{\jmath}\right)$

$= \left(-4 + \frac{3}{2}\right)\hat{\imath} + \frac{3\sqrt{3}}{2}\hat{\jmath}$

$= \left(\frac{-8+3}{2}\right)\hat{\imath} + \frac{3\sqrt{3}}{2}\hat{\jmath}$

$= \frac{-5}{2}\hat{\imath} + \frac{3\sqrt{3}}{2}\hat{\jmath}$

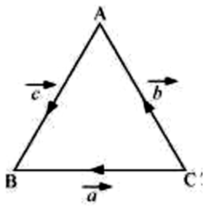
Hence, the girl's displacement from her initial point of departure is

$= \frac{-5}{2}\hat{\imath} + \frac{3\sqrt{3}}{2}\hat{\jmath}.$ **[1 Mark]**

4. If $\vec{a} = \vec{b} + \vec{c}$, then is it true that $|\vec{a}| = |\vec{b}| + |\vec{c}|$? Justify your Answer. **[2 Marks]**

Solution:

In ΔABC, let $\overrightarrow{CB} = \vec{a}, \overrightarrow{CA} = \vec{b}$ and $\overrightarrow{AB} = \vec{c}$ (as shown in the following figure)



Now, by the triangle law of vector addition, we have $\vec{a} = \vec{b} + \vec{c}$. **[1 Mark]**

It is clearly known that $|\vec{a}|, |\vec{b}|$ and $|\vec{c}|$ represent the sides of ΔABC.

Also, it is known that the sum of the lengths of any two sides of a triangle is greater than the third side.

$\therefore |\vec{a}| < |\vec{b}| + |\vec{c}|$

Hence, it is not true that $|\vec{a}| = |\vec{b}| + |\vec{c}|$. **[1 Mark]**

5. Find the value of x for which $x(\hat{\imath} + \hat{\jmath} + \hat{k})$ is a unit vector. **[1 Mark]**

Solution:

$x(\hat{\imath} + \hat{\jmath} + \hat{k})$ is a unit vector if $|x(\hat{\imath} + \hat{\jmath} + \hat{k})| = 1$.

Now,

$|x(\hat{\imath} + \hat{\jmath} + \hat{k})| = 1$

$\Rightarrow \sqrt{x^2 + x^2 + x^2} = 1$

$\Rightarrow \sqrt{3x^2} = 1$

$\Rightarrow \sqrt{3}x = 1$ **[$\frac{1}{2}$ Mark]**

$\Rightarrow x = \pm\frac{1}{\sqrt{3}}$

Hence, the required value of x is $\pm\frac{1}{\sqrt{3}}$. **[$\frac{1}{2}$ Mark]**

6. Find a vector of magnitude 5 units, and parallel to the resultant of the vectors $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $\vec{b} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$. **[2 Marks]**

Solution:

We have,

$\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $\vec{b} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$

Let $\vec{c}$ be the resultant of $\vec{a}$ and $\vec{b}$.

Then,

$\vec{c} = \vec{a} + \vec{b} = (2+1)\hat{\imath} + (3-2)\hat{\jmath} + (-1+1)\hat{k} = 3\hat{\imath} + \hat{\jmath}$ **[1 Mark]**

$\therefore |\vec{c}| = \sqrt{3^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$

$\therefore \hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{(3\hat{\imath} + \hat{\jmath})}{\sqrt{10}}$

Hence, the vector of magnitude 5 units and parallel to the resultant of vectors $\vec{a}$ and $\vec{b}$ is

$\pm 5 \cdot \hat{c} = \pm 5 \cdot \frac{1}{\sqrt{10}}(3\hat{\imath} + \hat{\jmath}) = \pm \frac{3\sqrt{10}i}{2} \pm \frac{\sqrt{10}}{2}\hat{\jmath}$ **[1 Mark]**

7. If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}, \vec{b} = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{c} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$, find a unit vector parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$. **[2 Marks]**

Solution:

We have,

$\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}, \vec{b} = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{c} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$

$2\vec{a} - \vec{b} + 3\vec{c} = 2(\hat{\imath} + \hat{\jmath} + \hat{k}) - (2\hat{\imath} - \hat{\jmath} + 3\hat{k}) + 3(\hat{\imath} - 2\hat{\jmath} + \hat{k})$

$= 2\hat{\imath} + 2\hat{\jmath} + 2\hat{k} - 2\hat{\imath} + \hat{\jmath} - 3\hat{k} + 3\hat{\imath} - 6\hat{\jmath} + 3\hat{k}$

$= 3\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ **[1 Mark]**

$|2\vec{a} - \vec{b} + 3\vec{c}| = \sqrt{3^2 + (-3)^2 + 2^2} = \sqrt{9+9+4} = \sqrt{22}$

Hence, the unit vector along $2\vec{a} - \vec{b} + 3\vec{c}$ is

$\frac{2\vec{a}-\vec{b}+3\vec{c}}{|2\vec{a}-\vec{b}+3\vec{c}|} = \frac{3\hat{\imath}-3\hat{\jmath}+2\hat{k}}{\sqrt{22}} = \frac{3}{\sqrt{22}}\hat{\imath} - \frac{3}{\sqrt{22}}\hat{\jmath} + \frac{2}{\sqrt{22}}\hat{k}$ **[1 Mark]**

8. Show that the points $A(1,-2,-8), B(5,0,-2)$ and $C(11,3,7)$ are collinear, and find the ratio in which B divides AC. **[4 Marks]**

Solution:

The given points are $A(1,-2,-8), B(5,0,-2)$ and $C(11,3,7)$.

$\therefore \overrightarrow{AB} = (5-1)\hat{\imath} + (0+2)\hat{\jmath} + (-2+8)\hat{k} = 4\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$

$\overrightarrow{BC} = (11-5)\hat{\imath} + (3-0)\hat{\jmath} + (7+2)\hat{k} = 6\hat{\imath} + 3\hat{\jmath} + 9\hat{k}$

$\overrightarrow{AC} = (11-1)\hat{\imath} + (3+2)\hat{\jmath} + (7+8)\hat{k} = 10\hat{\imath} + 5\hat{\jmath} + 15\hat{k}$ **[1 Mark]**

$|\overrightarrow{AB}| = \sqrt{4^2+2^2+6^2} = \sqrt{16+4+36} = \sqrt{56} = 2\sqrt{14}$

$|\overrightarrow{BC}| = \sqrt{6^2+3^2+9^2} = \sqrt{36+9+81} = \sqrt{126} = 3\sqrt{14}$

$|\overrightarrow{AC}| = \sqrt{10^2+5^2+15^2} = \sqrt{100+25+225} = \sqrt{350} = 5\sqrt{14}$

$\therefore |\overrightarrow{AC}| = |\overrightarrow{AB}| + |\overrightarrow{BC}|$ **[1 Mark]**

Thus, the given points A, B and C are collinear.

Now, let point B divide AC in the ratio $\lambda: 1$. Then, we have:

$$\overrightarrow{OB} = \frac{\lambda\overrightarrow{OC} + \overrightarrow{OA}}{(\lambda+1)}$$

$$\Rightarrow 5\hat{\imath} - 2\hat{k} = \frac{\lambda(11\hat{\imath} + 3\hat{\jmath} + 7\hat{k}) + (\hat{\imath} - 2\hat{\jmath} - 8\hat{k})}{\lambda+1}$$

$\Rightarrow (\lambda+1)(5\hat{\imath} - 2\hat{k}) = 11\lambda\hat{\imath} + 3\lambda\hat{\jmath} + 7\lambda\hat{k} + \hat{\imath} - 2\hat{\jmath} - 8\hat{k}$

$\Rightarrow 5(\lambda+1)\hat{\imath} - 2(\lambda+1)\hat{k} = (11\lambda+1)\hat{\imath} + (3\lambda-2)\hat{\jmath} + (7\lambda-8)\hat{k}$ **[1 Mark]**

On equating the corresponding components, we get:

$5(\lambda+1) = 11\lambda + 1$

$\Rightarrow 5\lambda + 5 = 11\lambda + 1$

$\Rightarrow 6\lambda = 4$

$\Rightarrow \lambda = \frac{4}{6} = \frac{2}{3}$

Hence, point B divides AC in the ratio $2:3$. **[1 Mark]**

9. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $(2\vec{a}+\vec{b})$ and $(\vec{a}-3\vec{b})$ externally in the ratio $1:2$. Also, show that P is the mid point of the line segment RQ. **[2 Marks]**

Solution:

It is given that $\overrightarrow{\mathrm{OP}}=2\vec{a}+\vec{b}, \overrightarrow{\mathrm{OQ}}=\vec{a}-3\vec{b}$.

It is given that point R divides a line segment joining two points P and Q externally in the ratio $1:2$. Then, on using the section formula, we get:

$\overrightarrow{\mathrm{OR}}=\frac{2(2\vec{a}+\vec{b})-(\vec{a}-3\vec{b})}{2-1}=\frac{4\vec{a}+2\vec{b}-\vec{a}+3\vec{b}}{1}=3\vec{a}+5\vec{b}$ **[1 Mark]**

Therefore, the position vector of point R is $3\vec{a}+5\vec{b}$

Position vector of the mid-point of RQ $=\frac{\overrightarrow{\mathrm{OQ}}+\overrightarrow{\mathrm{OR}}}{2}$

$$=\frac{(\vec{a}-3\vec{b})+(3\vec{a}+5\vec{b})}{2}$$

$=2\vec{a}+\vec{b}$

$=\overrightarrow{\mathrm{OP}}$

Hence, P is the mid-point of the line segment RQ. **[1 Mark]**

10. The two adjacent sides of a parallelogram are $2\hat{\imath}-4\hat{\jmath}+5\hat{k}$ and $\hat{\imath}-2\hat{\jmath}-3\hat{k}$.

Find the unit vector parallel to its diagonal. Also, find its area. **[2 Marks]**

Solution:

Adjacent sides of a parallelogram are given as: $2\hat{\imath}-4\hat{\jmath}+5\hat{k}$ and $\hat{\imath}-2\hat{\jmath}-3\hat{k}$

Then, the diagonal of a parallelogram is given by $\vec{a}+\vec{b}$

$\vec{a}+\vec{b}=(2+1)\hat{\imath}+(-4-2)\hat{\jmath}+(5-3)\hat{k}=3\hat{\imath}-6\hat{\jmath}+2\hat{k}$

Thus, the unit vector parallel to the diagonal is

$\frac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|} = \frac{3\hat{\imath}-6\hat{\jmath}+2\hat{k}}{\sqrt{3^2+(-6)^2+2^2}} = \frac{3\hat{\imath}-6\hat{\jmath}+2\hat{k}}{\sqrt{9+36+4}} = \frac{3\hat{\imath}-6\hat{\jmath}+2\hat{k}}{7} = \frac{3}{7}\hat{\imath} - \frac{6}{7}\hat{\jmath} + \frac{2}{7}\hat{k}$ **[1 Mark]**

∴ Area of parallelogram ABCD $= |\vec{a} \times \vec{b}|$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -4 & 5 \\ 1 & -2 & -3 \end{vmatrix}$

$= \hat{\imath}(12+10) - \hat{\jmath}(-6-5) + \hat{k}(-4+4)$

$= 22\hat{\imath} + 11\hat{\jmath}$

$= 11(2\hat{\imath} + \hat{\jmath})$

$\therefore |\vec{a} \times \vec{b}| = 11\sqrt{2^2+1^2} = 11\sqrt{5}$

Hence, the area of the parallelogram is $11\sqrt{5}$ square units. **[1 Mark]**

11. Show that the direction cosines of a vector equally inclined to the axes OX, OY and OZ are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$. **[1 Mark]**

Solution:

Let a vector be equally inclined to axes OX, OY and OZ at angle α.

Then, the direction cosines of the vector are $cos\ \alpha, cos\ \alpha$ and $cos\ \alpha$.

Now,

$\cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$

$\Rightarrow 3\cos^2\alpha = 1$

$\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$ **[$\frac{1}{2}$ Mark]**

Hence, the direction cosines of the vector

which are equally inclined to the axes are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$. **[$\frac{1}{2}$ Mark]**

12. Let $\vec{a} = \hat{\imath} + 4\hat{\jmath} + 2\hat{k}, \vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$ and $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$. Find a vector $\vec{a}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}.\vec{d} = 15$. **[2 Marks]**

Solution:

Let $\vec{d} = d_1\hat{\imath} + d_2\hat{\jmath} + d_3\hat{k}$.

Since $\vec{d}$ is perpendicular to both $\vec{a}$ and $\vec{b}$

$\vec{d} \cdot \vec{a} = 0$

$\Rightarrow d_1 + 4d_2 + 2d_3 = 0 \quad \dots(i)$

And,

$\vec{d} \cdot \vec{b} = 0$

$\Rightarrow 3d_1 - 2d_2 + 7d_3 = 0 \quad \dots(ii)$ **[1 Mark]**

Also, it is given that:

$\vec{c} \cdot \vec{d} = 15$

$\Rightarrow 2d_1 - d_2 + 4d_3 = 15 \quad (iii)$

On solving $(i), (ii)$ and (iii) we get:

$d_1 = \frac{160}{3}, d_2 = -\frac{5}{3}$ and $d_3 = -\frac{70}{3}$

$$\therefore \vec{d} = \frac{160}{3}\hat{\imath} - \frac{5}{3}\hat{\jmath} - \frac{70}{3}\hat{k} = \frac{1}{3}\left(160\hat{\imath} - 5\hat{\jmath} - 7\hat{0}\hat{k}\right)$$

Hence, the required vector is $\frac{1}{3}\left(160\hat{\imath} - 5\hat{\jmath} - 70\hat{k}\right)$. **[1 Mark]**

13. The scalar product of the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ and $\lambda\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ is equal to one. Find the value of λ. **[2 Marks]**

Solution:

$\left(2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}\right) + \left(\lambda\hat{\imath} + 2\hat{\jmath} + 3\hat{k}\right)$

$= (2 + \lambda)\hat{\imath} + 6\hat{\jmath} - 2\hat{k}$

Therefore, unit vector along $\left(2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}\right) + \left(\lambda\hat{\imath} + 2\hat{\jmath} + 3\hat{k}\right)$ is given as:

$\frac{(2+\lambda)\hat{\imath}+6\hat{\jmath}-2\hat{k}}{\sqrt{(2+\lambda)^2+6^2+(-2)^2}} = \frac{(2+\lambda)\hat{\imath}+6\hat{\jmath}-2\hat{k}}{\sqrt{4+4\lambda+\lambda^2+36+4}} = \frac{(2+\lambda)\hat{\imath}+6\hat{\jmath}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}$ **[1 Mark]**

Scalar product of $(\hat{\imath}+\hat{\jmath}+\hat{k})$ with this unit vector is 1.

$\Rightarrow (\hat{\imath}+\hat{\jmath}+\hat{k})\cdot\dfrac{(2+\lambda)\hat{\imath}+6\hat{\jmath}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}=1$

$\Rightarrow \dfrac{(2+\lambda)+6-2}{\sqrt{\lambda^2+4\lambda+44}}=1$

$\Rightarrow \sqrt{\lambda^2+4\lambda+44}=\lambda+6$

$\Rightarrow \lambda^2+4\lambda+44=(\lambda+6)^2$

$\Rightarrow \lambda^2+4\lambda+44=\lambda^2+12\lambda+36$

$\Rightarrow 8\lambda=8$

$\Rightarrow \lambda=1$

Hence, the value of λ is 1. **[1 Mark]**

14. If $\vec{a},\vec{b},\vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a},\vec{b}$ and $\vec{c}$. **[4 Marks]**

Solution:

Since $\vec{a},\vec{b}$ and $\vec{c}$ are mutually perpendicular vectors, we have $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$. It is given that:$|\vec{a}|=|\vec{b}|=|\vec{c}|$.

Let vector $\vec{a}+\vec{b}+\vec{c}$ be inclined to $\vec{a},\vec{b}$ and $\vec{c}$ at angles θ_1,θ_2 and $\theta_3$respectively. **[$\frac{1}{2}$ Mark]**

Then, we have:

$\cos\theta_1=\dfrac{(\vec{a}+\vec{b}+\vec{c})\cdot\vec{a}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}=\dfrac{\vec{a}\cdot\vec{a}+\vec{b}\cdot\vec{a}+\vec{c}\cdot\vec{a}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}$

$=\dfrac{|\vec{a}|^2}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}\quad[\vec{b}\cdot\vec{a}=\vec{c}\cdot\vec{a}=0]$

$=\frac{|\vec{a}|^2}{|\vec{a}+\vec{b}+\vec{c}|}$ **[1 Mark]**

$\cos\theta_2=\dfrac{(\vec{a}+\vec{b}+\vec{c})\cdot\vec{b}}{|\vec{a}+\vec{b}+\vec{c}||\vec{b}|}=\dfrac{\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{b}+\vec{c}\cdot\vec{b}}{|\vec{a}+\vec{b}+\vec{c}|\cdot|\vec{b}|}$

$=\dfrac{|\vec{b}|^2}{|\vec{a}+\vec{b}+\vec{c}|\cdot|\vec{b}|}\quad[\vec{a}\cdot\vec{b}=\vec{c}\cdot\vec{b}=0]$

$= \frac{|\vec{b}|^2}{|\vec{a}+\vec{b}+\vec{c}|}$ **[1 Mark]**

$\cos\theta_3 = \frac{(\vec{a}+\vec{b}+\vec{c})\cdot\vec{c}}{|\vec{a}+\vec{b}+\vec{c}||\vec{c}|} = \frac{\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{c}}{|\vec{a}+\vec{b}+\vec{c}||\vec{c}|}$

$= \frac{|\vec{c}|^2}{|\vec{a}+\vec{b}+\vec{c}||\vec{c}|}$ $[\vec{a}\cdot\vec{c} = \vec{b}\cdot\vec{c} = 0]$

$= \frac{|\vec{c}|^2}{|\vec{a}+\vec{b}+\vec{c}|}$ **[1 Mark]**

Now, as $|\vec{a}| = |\vec{b}| = |\vec{c}|$, $\cos\theta_1 = \cos\theta_2 = \cos\theta_3$

$\therefore \theta_1 = \theta_2 = \theta_3$

Hence, the vector $(\vec{a}+\vec{b}+\vec{c})$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$. **[$\frac{1}{2}$ Mark]**

15. Prove that $(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = |\vec{a}|^2 + |\vec{b}|^2$,if and only if $\vec{a}, \vec{b}$ are perpendicular, given $\vec{a} \neq \vec{0}, \vec{b} \neq \vec{0}$. **[2 Marks]**

Solution:

$(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = |\vec{a}|^2 + |\vec{b}|^2$

$\Leftrightarrow \vec{a}\cdot\vec{a} + \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{a} + \vec{b}\cdot\vec{b} = |\vec{a}|^2 + |\vec{b}|^2$ [Distributivity of scalar products over addition]

$\Leftrightarrow |\vec{a}|^2 + 2\vec{a}\cdot\vec{b} + |\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$ $[\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$ (Scalar product is commutative)]**[1 Mark]**

$\Leftrightarrow 2\vec{a}\cdot\vec{b} = 0$

$\Leftrightarrow \vec{a}\cdot\vec{b} = 0$

$\therefore$ $\vec{a}$ and $\vec{b}$ are perpendicular. $[\vec{a} \neq \vec{0}, \vec{b} \neq \vec{0}$ (Given)$]$ **[1 Mark]**

16. If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then $\vec{a}.\vec{b} \geq 0$ only when **[1 Mark]**

(A) $0 < \theta < \frac{\pi}{2}$

(B) $0 \leq \theta \leq \frac{\pi}{2}$

(C) $0 < \theta < \pi$

(D) $0 \leq \theta \leq \pi$

Solution:

Let θ be the angle between two vectors $\vec{a}$ and $\vec{b}$.

Then, without loss of generality, $\vec{a}$ and $\vec{b}$ are non-zero vectors so that. $|\vec{a}|$ and $|\vec{b}|$ are positive.

It is known that $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\therefore \vec{a} \cdot \vec{b} \geq 0$

$\Rightarrow |\vec{a}||\vec{b}|\cos\theta \geq 0$

$\Rightarrow \cos\theta \geq 0\ [|\vec{a}| \text{and} |\vec{b}| \text{arepositive}]$ **[$\frac{1}{2}$ Mark]**

$\Rightarrow 0 \leq \theta \leq \frac{\pi}{2}$

Hence, $\vec{a} \cdot \vec{b} \geq 0$ when $0 \leq \theta \leq \frac{\pi}{2}$.

The correct Answer is B. **[$\frac{1}{2}$ Mark]**

17. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. Then $\vec{a} + \vec{b}$ is a unit vector if **[2 Marks]**

(A) $\theta = \frac{\pi}{4}$

(B) $\theta = \frac{\pi}{3}$

(C) $\theta = \frac{\pi}{2}$

(D) $\theta = \frac{2\pi}{3}$

Solution:

Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ be the angle between them.

Then, $|\vec{a}| = |\vec{b}| = 1$.

Now, $\vec{a} + \vec{b}$ is a unit vector if $|\vec{a} + \vec{b}| = 1$.

$|\vec{a} + \vec{b}| = 1$

$\Rightarrow (\vec{a}+\vec{b})^2 = 1$

$\Rightarrow (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = 1$

$\Rightarrow \vec{a}\cdot\vec{a} + \vec{a}\vec{b} + \vec{b}\cdot\vec{a} + \vec{b}\vec{b} = 1$ **[$\frac{1}{2}$ Mark]**

$\Rightarrow |\vec{a}|^2 + 2\vec{a}\cdot\vec{b} + |\vec{b}|^2 = 1$

$\Rightarrow 1^2 + 2|\vec{a}||\vec{b}|\cos\theta + 1^2 = 1$

$\Rightarrow 1 + 2.1.1\cos\theta + 1 = 1$ **[$\frac{1}{2}$ Mark]**

$\Rightarrow \cos\theta = -\frac{1}{2}$

$\Rightarrow \theta = \frac{2\pi}{3}$ **[$\frac{1}{2}$ Mark]**

Hence, $\vec{a}+\vec{b}$ is a unit vector if $\theta = \frac{2\pi}{3}$

The correct Answer is D. **[$\frac{1}{2}$ Mark]**

18. The value of $\hat{\imath}.(\hat{\jmath}\times\hat{k}) + \hat{\jmath}.(\hat{\imath}\times\hat{k}) + \hat{k}.(\hat{\imath}\times\hat{\jmath})$ is **[2 Marks]**

(A) 0

(B) – 1

(C) 1

(D) 3

Solution:

$\hat{\imath}.(\hat{\jmath}\times\hat{k}) + \hat{\jmath}.(\hat{\imath}\times\hat{k}) + \hat{k}.(\hat{\imath}\times\hat{\jmath})$

$= \hat{\imath}\cdot\hat{\imath} + \hat{\jmath}\cdot(-\hat{\jmath}) + \hat{k}\cdot\hat{k}$

$= 1 - \hat{\jmath}\cdot\hat{\jmath} + 1$ **[$\frac{1}{2}$ Mark]**

$= 1 - 1 + 1$

$= 1$ **[$\frac{1}{2}$ Mark]**

The correct Answer is C.

19. If θ is the angle between any two vectors $\vec{a}$ and $\vec{b}$, then $|\vec{a}\cdot\vec{b}| = |\vec{a}\times\vec{b}|$ when θ is equal to **[1 Mark]**

(A) 0

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$

(D) π

Solution:

Let θ be the angle between two vectors $\vec{a}$ and $\vec{b}$.

Then, without loss of generality, $\vec{a}$ and $\vec{b}$ are non-zero vectors, so that $|\vec{a}|$ and $|\vec{b}|$ are positive.

$|\vec{a}\cdot\vec{b}| = |\vec{a}\times\vec{b}|$

$\Rightarrow |\vec{a}||\vec{b}|\cos\theta = |\vec{a}||\vec{b}|\sin\theta$

$\Rightarrow \cos\theta = \sin\theta \ [|\vec{a}| \text{and} |\vec{b}| \text{arepositive}]$

$\Rightarrow \tan\theta = 1$ **[$\frac{1}{2}$ Mark]**

$\Rightarrow \theta = \frac{\pi}{4}$

Hence, $|\vec{a}\cdot\vec{b}| = |\vec{a}\times\vec{b}|$ when θ is equal to $\frac{\pi}{4}$

The correct Answer is B. **[$\frac{1}{2}$ Mark]**