

EXERCISE 5.1

2. f is continuous at $x = 3$
3. (a), (b), (c) and (d) are all continuous functions
5. f is continuous at $x = 0$ and $x = 2$; Not continuous at $x = 1$
6. Discontinuous at $x = 2$ 7. Discontinuous at $x = 3$
8. Discontinuous at $x = 0$ 9. No point of discontinuity
10. No point of discontinuity 11. No point of discontinuity
12. f is discontinuous at $x = 1$ 13. f is not continuous at $x = 1$
14. f is not continuous at $x = 1$ and $x = 3$
15. $x = 1$ is the only point of discontinuity
16. Continuous 17. $a = b + \frac{2}{3}$
18. For no value of λ , f is continuous at $x = 0$ but f is continuous at $x = 1$ for any value of λ .
20. f is continuous at $x = \pi$ 21. (a), (b) and (c) are all continuous
22. Cosine function is continuous for all $x \in \mathbf{R}$; cosecant is continuous except for $x = n\pi$, $n \in \mathbf{Z}$; secant is continuous except for $x = (2n+1)\frac{\pi}{2}$, $n \in \mathbf{Z}$ and cotangent function is continuous except for $x = n\pi$, $n \in \mathbf{Z}$
23. There is no point of discontinuity.
24. Yes, f is continuous for all $x \in \mathbf{R}$ 25. f is continuous for all $x \in \mathbf{R}$
26. $k = 6$ 27. $k = \frac{3}{4}$ 28. $k = \frac{-2}{\pi}$
29. $k = \frac{9}{5}$ 30. $a = 2, b = 1$
34. There is no point of discontinuity.

EXERCISE 5.2

1. $2x \cos(x^2 + 5)$ 2. $-\cos x \sin(\sin x)$ 3. $a \cos(ax + b)$
4. $\frac{\sec(\tan \sqrt{x}) \cdot \tan(\tan \sqrt{x}) \cdot \sec^2 \sqrt{x}}{2\sqrt{x}}$
5. $a \cos(ax + b) \sec(cx + d) + c \sin(ax + b) \tan(cx + d) \sec(cx + d)$
6. $10x^4 \sin x^5 \cos x^5 \cos x^3 - 3x^2 \sin x^3 \sin^2 x^5$

$$7. \frac{-2\sqrt{2}x}{\sin x^2 \sqrt{\sin 2x^2}} \quad 8. -\frac{\sin \sqrt{x}}{2\sqrt{x}}$$

EXERCISE 5.3

$$1. \frac{\cos x - 2}{3} \quad 2. \frac{2}{\cos y - 3} \quad 3. -\frac{a}{2by + \sin y}$$

$$4. \frac{\sec^2 x - y}{x + 2y - 1} \quad 5. -\frac{(2x + y)}{(x + 2y)} \quad 6. -\frac{(3x^2 + 2xy + y^2)}{(x^2 + 2xy + 3y^2)}$$

$$7. \frac{y \sin xy}{\sin 2y - x \sin xy} \quad 8. \frac{\sin 2x}{\sin 2y} \quad 9. \frac{2}{1 + x^2} \quad 10. \frac{3}{1 + x^2}$$

$$11. \frac{2}{1 + x^2} \quad 12. \frac{-2}{1 + x^2} \quad 13. \frac{-2}{1 + x^2} \quad 14. \frac{2}{\sqrt{1 - x^2}}$$

$$15. -\frac{2}{\sqrt{1 - x^2}}$$

EXERCISE 5.4

$$1. \frac{e^x (\sin x - \cos x)}{\sin^2 x}, x \neq n\pi, n \in \mathbf{Z} \quad 2. \frac{e^{\sin^{-1} x}}{\sqrt{1 - x^2}}, x \in (-1, 1)$$

$$3. 3x^2 e^{x^3} \quad 4. -\frac{e^{-x} \cos(\tan^{-1} e^{-x})}{1 + e^{-2x}}$$

$$5. -e^x \tan e^x, e^x \neq (2n + 1)\frac{\pi}{2}, n \in \mathbf{N} \quad 6. e^x + 2x e^{x^2} + 3x^2 e^{x^3} + 4x^3 e^{x^4} + 5x^4 e^{x^5}$$

$$7. \frac{e^{\sqrt{x}}}{4\sqrt{x} e^{\sqrt{x}}}, x > 0 \quad 8. \frac{1}{x \log x}, x > 1$$

$$9. -\frac{(x \sin x \cdot \log x + \cos x)}{x(\log x)^2}, x > 0 \quad 10. -\frac{1}{x} + e^x \sin(\log x + e^x), x > 0$$

EXERCISE 5.5

1. $-\cos x \cos 2x \cos 3x [\tan x + 2 \tan 2x + 3 \tan 3x]$
2. $\frac{1}{2} \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}} \left[\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5} \right]$
3. $(\log x)^{\cos x} \left[\frac{\cos x}{x \log x} - \sin x \log(\log x) \right]$
4. $x^x (1 + \log x) - 2^{\sin x} \cos x \log 2$
5. $(x+3)(x+4)^2(x+5)^3(9x^2+70x+133)$
6. $\left(x + \frac{1}{x}\right)^x \left[\frac{x^2-1}{x^2+1} + \log\left(x + \frac{1}{x}\right) \right] + x^{1+\frac{1}{x}} \left(\frac{x+1-\log x}{x^2} \right)$
7. $(\log x)^{x-1} [1 + \log x \cdot \log(\log x)] + 2x^{\log x - 1} \cdot \log x$
8. $(\sin x)^x (x \cot x + \log \sin x) + \frac{1}{2} \frac{1}{\sqrt{x-x^2}}$
9. $x^{\sin x} \left[\frac{\sin x}{x} + \cos x \log x \right] + (\sin x)^{\cos x} [\cos x \cot x - \sin x \log \sin x]$
10. $x^{x \cos x} [\cos x \cdot (1 + \log x) - x \sin x \log x] - \frac{4x}{(x^2-1)^2}$
11. $(x \cos x)^x [1 - x \tan x + \log(x \cos x)] + (x \sin x)^{\frac{1}{x}} \left[\frac{x \cot x + 1 - \log(x \sin x)}{x^2} \right]$
12. $-\frac{yx^{y-1} + y^x \log y}{x^y \log x + xy^{x-1}}$
13. $\frac{y}{x} \left(\frac{y-x \log y}{x-y \log x} \right)$
14. $\frac{y \tan x + \log \cos y}{x \tan y + \log \cos x}$
15. $\frac{y(x-1)}{x(y+1)}$
16. $(1+x)(1+x^2)(1+x^4)(1+x^8) \left[\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8} \right]; f'(1) = 120$
17. $5x^4 - 20x^3 + 45x^2 - 52x + 11$

EXERCISE 5.6

1. t^2
2. $\frac{b}{a}$
3. $-4 \sin t$
4. $-\frac{1}{t^2}$

5. $\frac{\cos \theta - 2 \cos 2\theta}{2 \sin 2\theta - \sin \theta}$ 6. $-\cot \frac{\theta}{2}$ 7. $-\cot 3t$ 8. $\tan t$
 9. $\frac{b}{a} \operatorname{cosec} \theta$ 10. $\tan \theta$

EXERCISE 5.7

1. 2 2. $380 x^{18}$ 3. $-x \cos x - 2 \sin x$
 4. $-\frac{1}{x^2}$ 5. $x(5 + 6 \log x)$ 6. $2e^x(5 \cos 5x - 12 \sin 5x)$
 7. $9 e^{6x}(3 \cos 3x - 4 \sin 3x)$ 8. $-\frac{2x}{(1+x^2)^2}$
 9. $-\frac{(1 + \log x)}{(x \log x)^2}$ 10. $-\frac{\sin(\log x) + \cos(\log x)}{x^2}$
 12. $-\cot y \operatorname{cosec}^2 y$

Miscellaneous Exercise on Chapter 5

1. $27(3x^2 - 9x + 5)^8(2x - 3)$ 2. $3 \sin x \cos x (\sin x - 2 \cos^4 x)$
 3. $(5x)^{3 \cos 2x} \left[\frac{3 \cos 2x}{x} - 6 \sin 2x \log 5x \right]$
 4. $\frac{3}{2} \sqrt{\frac{x}{1-x^3}}$ 5. $-\left[\frac{1}{\sqrt{4-x^2} \sqrt{2x+7}} + \frac{\cos^{-1} \frac{x}{2}}{(2x+7)^{\frac{3}{2}}} \right]$
 6. $\frac{1}{2}$ 7. $(\log x)^{\log x} \left[\frac{1}{x} + \frac{\log(\log x)}{x} \right], x > 1$
 8. $(a \sin x - b \cos x) \sin(a \cos x + b \sin x)$
 9. $(\sin x - \cos x)^{\sin x - \cos x} (\cos x + \sin x) (1 + \log(\sin x - \cos x)), \sin x > \cos x$
 10. $x^x(1 + \log x) + ax^{a-1} + a^x \log a$
 11. $x^{x^2-3} \left[\frac{x^2-3}{x} + 2x \log x \right] + (x-3)^{x^2} \left[\frac{x^2}{x-3} + 2x \log(x-3) \right]$

12. $\frac{6}{5} \cot \frac{t}{2}$

13. 0

17. $\frac{\sec^3 t}{at}, 0 < t < \frac{\pi}{2}$

EXERCISE 6.1

1. (a) $6\pi \text{ cm}^2/\text{cm}$

(b) $8\pi \text{ cm}^2/\text{cm}$

2. $\frac{8}{3} \text{ cm}^2/\text{s}$

3. $60\pi \text{ cm}^2/\text{s}$

4. $900 \text{ cm}^3/\text{s}$

5. $80\pi \text{ cm}^2/\text{s}$

6. $1.4\pi \text{ cm}/\text{s}$

7. (a) $-2 \text{ cm}/\text{min}$

(b) $2 \text{ cm}^2/\text{min}$

8. $\frac{1}{\pi} \text{ cm}/\text{s}$

9. $400\pi \text{ cm}^3/\text{cm}$

10. $\frac{8}{3} \text{ cm}/\text{s}$

11. $(4, 11)$ and $\left(-4, \frac{-31}{3}\right)$

12. $2\pi \text{ cm}^3/\text{s}$

13. $\frac{27}{8} \pi (2x+1)^2$

14. $\frac{1}{48\pi} \text{ cm}/\text{s}$

15. ₹ 20.967

16. ₹ 208

17. B

18. D

EXERCISE 6.2

4. (a) $\left(\frac{3}{4}, \infty\right)$

(b) $\left(-\infty, \frac{3}{4}\right)$

5. (a) $(-\infty, -2)$ and $(3, \infty)$

(b) $(-2, 3)$

6. (a) decreasing for $x < -1$ and increasing for $x > -1$

(b) decreasing for $x > -\frac{3}{2}$ and increasing for $x < -\frac{3}{2}$

(c) increasing for $-2 < x < -1$ and decreasing for $x < -2$ and $x > -1$

(d) increasing for $x < -\frac{9}{2}$ and decreasing for $x > -\frac{9}{2}$