Q.1 Box-I contains 30 cards marked from 1 to 30 and Box-II contains 20 cards marked from 31 to 50. A box is selected and a card is drawn. If the number on card is non-prime then what is prob. that it came from box-I.

Sol:- $P(B_1) = \frac{1}{2} = P(B\text{-}2)$

$P(\text{Non-prime}) = P(B_1)\cdot P(N.P/B_1) + P(B_2)\cdot P(N.P/B_2)$

$= \frac{1}{2}\cdot\frac{20}{30} + \frac{1}{2}\cdot\frac{15}{20}$

$$P(B\text{-}1/N.P.) = \frac{\frac{1}{2}\cdot\frac{20}{30}}{\frac{1}{2}\cdot\frac{20}{30} + \frac{1}{2}\cdot\frac{15}{20}} = \frac{4}{7}$$

Q.2 ~~which~~ The contra-positive of "If I reach the station on time then I will get the train" is -

Sol:- If I don't get the Train then I don't reach station on time

(Contrapositive of $p \to q$ is $\sim q \to \sim p$)

(4) $\dfrac{1+\sin y}{1-\cos y}\dfrac{dy}{dx} = -\cos x$ s.t.

Sol: $$\left(\frac{1+\sin y}{1-\cos^2 y}\right)\frac{dy}{dx} = -\cos x$$

$$\left(\frac{1+2\sin\frac{y}{2}\cos\frac{y}{2}}{2\sin^2\frac{y}{2}}\right)dy = -\cos x\,dx$$

$$\left(\frac{1}{2}\csc^2\frac{y}{2} + \cot\frac{y}{2}\right)dy = -\cos x\,dx$$

Q.3 Evaluate $\int_0^2 \left| |x-1| + x \right| dx$

Sol: $I = \int_0^2 \left| |x-1| + x \right| dx$

$$= \int_0^1 |1-x+1| \, dx + \int_1^2 |x-1+x| \, dx$$

$$= \int_0^1 (2-x) \, dx + \int_1^2 (2x-1) \, dx$$

$$= \left[2x - \frac{x^2}{2} \right]_0^1 + \left[x^2 - x \right]_1^2$$

$$= 2(1-0) - \frac{1}{2}(1-0) + (4-2) - (1-1)$$

$$= 2 - \frac{1}{2} + 2$$

$$= \frac{7}{2}$$

(A) $\frac{9}{2}$ (B) $\frac{5}{2}$ (C) $\frac{7}{2}$ (D) $3/2$

3) Length of perpendicular and foot of perpendicular from the point $(1, 3/2, 2)$ to the plane $2x - 2y + 4z + 5 = 0$.

$$\frac{x-1}{2} = \frac{y-3/2}{-2} = \frac{z-2}{4} = \frac{-(2-3+8+5)}{20}$$

$$= -\frac{3}{5}$$

$x = -1/5 \quad y = 3/2 + 6/5 \quad z = 2 - 12/5$

$x = -1/5 \quad y = \frac{27}{10} \quad z = -2/5$

$P(-1/5, \frac{27}{10}, -2/5)$

$A(1, \frac{3}{2}, 2)$

$$AP = \sqrt{\frac{36}{25} + \left(\frac{12}{10}\right)^2 + \left(\frac{12}{5}\right)^2}$$

$$= \sqrt{\frac{36}{25} + \frac{36}{25} + \frac{144}{25}}$$

$$= \sqrt{\frac{216}{25}} = \frac{6}{5}\sqrt{6}$$

(2) Points of local maxima and local minima of the function.

$$f(x) = -\frac{3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105.$$

$$f'(x) = -3x^3 - 24x^2 - 45x.$$

$$= -3x(x^2 + 8x + 15)$$

$$= -3x(x+3)(x+5)$$

$+ \quad - \quad + \quad -$

$-5 \quad -3 \quad 0$

Local max at $x = -5, 0$

min at $x = -3,$

1. $\sin^{-1} 6x + \sin^{-1} 6\sqrt{3}x = -\pi/2$

Ans. $x = -1/12$. $x < 0$.

$$\sin^{-1} 6\sqrt{3}x + \frac{\pi}{2} = -\sin^{-1} 6x.$$

$$\cancel{6\sqrt{3}} \sin(\pi/2 + \sin^{-1} 6\sqrt{3}x) = -6x.$$

$$\sqrt{1 - 108x^2} = -6x$$

$$1 - 108x^2 = 36x^2$$

$$x = -1/12.$$

Q. $\dfrac{z-\alpha}{z+\alpha}$ is purely imaginary and $|z| = 82$

then $|\alpha|$ is :-

$$\frac{z-\alpha}{z+\alpha} + \frac{\bar{z}-\bar{\alpha}}{\bar{z}+\bar{\alpha}} = 0$$

$$2z\bar{z} \cancel{-\alpha\bar{z}} \cancel{+\bar{\alpha}z} \cancel{-z\bar{\alpha}} \cancel{+\alpha\bar{z}} - \alpha\bar{\alpha} = 0$$

$$2|z|^2 = |\alpha|^2$$

$$\therefore |\alpha| = 82\sqrt{2}$$

Q If $\lim\limits_{x\to 1} \dfrac{x+x^2+x^3+x^n-n}{x-1} = 820$ then the value of n is

Ans.(40)

Using L-hospital limit becomes

$$\lim_{x\to 1} \frac{1+2x+3x^2+\cdots+nx^{n-1}}{1} = 820$$

$$\therefore \quad 1+2+3+\cdots n = 820$$

$$\frac{n(n+1)}{2} = 820$$

$$n^2+n-1640 = 0$$

$$(n+41)(n-40) = 0$$

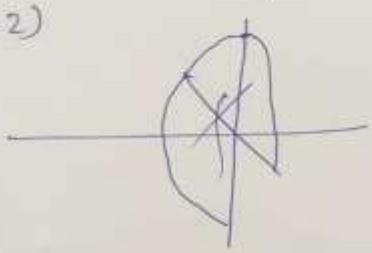
$$n = 40, \cancel{-41}$$

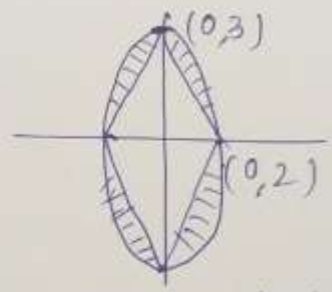
$\frac{}{7}$

(10) Find the area of region given by

$$\frac{x^2}{4} + \frac{y^2}{9} \leq 1 \quad \text{and} \quad 3|x| + 2|y| \geq 6$$

Ans. $6(\pi - 2)$





Required area $= \pi \cdot 2 \cdot 3 - 4 \cdot \left(\frac{1}{2} \cdot 2 \cdot 3\right)$

$= (6\pi - 12)$ sq. unit

Q) Find the rank of word ~~mother~~ MOTHER if all words are written in alphabetical order.

Ans. 309) E, H, M, O, R, T

Words start with E are $\lfloor 5$

H are $\lfloor 5$

ME are $\lfloor 4$

MH are $\lfloor 4$

MOE are $\lfloor 3$

MOH are $\lfloor 3$

MOR are $\lfloor 3$

MOTE are $\lfloor 2$, 1

MOTHER

Rank is :- 309

Solve differential equation:-

Q→ If $\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx} = -\cos x$

Ans. $y = \frac{c}{\sin x + 2} - 1$.

$$\frac{dy}{1+y} = -\left(\frac{\cos x}{2+\sin x}\right)dx$$

Integrate both sides

$$\ln(1+y) = -\ln(2+\sin x) + \ln c.$$

$$(1+y) = \frac{c}{2+\sin x}.$$

Q:- if eqn. $6x^2 + 5x - 2 = 0$. has roots α, β and. $S_n = \alpha^n + \beta^n$. then select the Correct option:-

Ans (1) ~~(A)~~

(1) $6 S_6 + 5 S_5 = 2 S_4$

(2) $6 S_4 + 2 S_5 = 5 S_6$

(3) $6 S_2 + 5 S_6 = 6 S_4$

(4) $6 S_2 + 5 S_4 = 2 S_6$

$\because 6\alpha^2 + 5\alpha = 2$

$$6 S_6 + 5 S_5 = \alpha^4 (6\alpha^2 + 5\alpha) + \beta^4 (6\beta^2 + 5\beta)$$

$$= 2(\alpha^4 + \beta^4)$$

$$= 2 S_4$$

Q:- Let $a^3+b^2=2$. Then in the expansion of $\left(ax^{1/9}+bx^{-1/6}\right)^{10}$. if the term independent of x is 10K. Find the value of K.

Ans (21) in $\left(ax^{1/9}+bx^{-1/6}\right)^{10}$

$$T_{r+1} = {}^{10}C_r \left(ax^{1/9}\right)^{10-r}\left(bx^{-1/6}\right)^{r}$$

$$= {}^{10}C_r \, a^{10-r}\, b^r \, x^{\left(\frac{10-r}{9}-\frac{r}{6}\right)}$$

$$\frac{10-r}{9}-\frac{r}{6}=0 \Rightarrow r=4$$

then $T_5 = {}^{10}C_4 \, a^6 b^4 = 10K$

$$\frac{\lfloor 10}{\lfloor 4 \lfloor 6}\, a^6 b^4 = 10K$$

$$K = 21\, a^6 b^4.$$

Now, $a^3+b^2=2$.

then by AM $\geq$ GM

$$\frac{a^3}{2}+\frac{a^3}{2}+\frac{b^2}{2}+\frac{b^2}{2} \geq 4\left(\frac{a^6b^4}{16}\right)^{1/4}$$

$$2^4 \geq \frac{4^4 a^6 b^4}{16} \Rightarrow a^6 b^4 \leq 1$$

$\cancel{a^6 b^4 \leq 256 \quad a^6 b^4}$

then K max = 21

Q:- if $\hat{a}, \hat{b}, \hat{c}$ are three unit vector such that $|\bar{a}-\bar{b}|^2 + |\bar{a}-\bar{c}|^2 + \cancel{|\bar{c}-\bar{a}|}^2 = 8$.

then $|\hat{a}+2\hat{b}|^2 + |\hat{a}+2\hat{c}|^2$ is equals:

Ans = (2)

given. $|\hat{a}-\hat{b}|^2 + |\hat{a}-\hat{c}|^2 \cancel{+|\hat{c}-\hat{a}|}^2 = 8$

$$4 - 2(\bar{a}\cdot\bar{b} + \bar{a}\cdot\bar{c} + \cancel{\bar{c}\cdot\bar{a}}) = 8.$$

$$\bar{a}\cdot\bar{b} + \bar{b}\cdot\bar{c} + \bar{c}\cdot\bar{a} = -2$$

then. $|\bar{a}+2\bar{b}|^2 + |\bar{a}+2c|^2$

$$= 2|\bar{a}|^2 + 4|\bar{b}|^2 + 4|\bar{c}|^2 + 4(\bar{a}\cdot\bar{b} + \bar{a}\cdot\bar{c})$$

$$= 10 - 8$$

$$= 2.$$

Q) The sum of three consecutive positive terms of a G.P. is S and their product is 27. Then the minimum ~~maximum~~ value of S is.

Ans. (9)

let terms are $\frac{a}{r}, a, ar$.

then. $a^3 = 27 \Rightarrow a = 3$.

Now $\frac{3}{r} + 3 + 3r = S$.

$3\left(\frac{1}{r} + r\right) + 3 = S$

$\because r + \frac{1}{r} \geq 2. \Rightarrow S \geq 9$.

Q:- If a, b, c are the A.M. b/w two numbers such that $a+b+c = 15$ and p, q, r be the H.M b/w the same numbers such that $\frac{1}{p} + \frac{1}{q} + \frac{1}{r} = \frac{5}{3}$. Then numbers are

(A) $(3, 3)$ (B) $(-1, -9)$, (C) $(3, -3)$

(D) $(9, 1)$

Sol. Let Numbers are x and y.

~~x~~ then

x, a, b, c, y are in A P

then. $2b = a+c = x+y$.

$b = 5$ and $a+c = 10 = x+y$.

$\frac{1}{x}, \frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{y}$ are in A P

$$\frac{2}{q} = \frac{1}{p} + \frac{1}{r} = \frac{1}{x} + \frac{1}{y}.$$

$$\frac{3}{q} = \frac{5}{3} \Rightarrow q = 9/5$$

$$\frac{1}{x} + \frac{1}{y} = \frac{10}{9}.$$

~~x+~~ $xy = 9$.

then. $x = 1$ $y = 9$

or $x = 9$, $y = 1$

Q) Let their be a plane passing through. A(1, 2, 1) and (2, 1, 2) and parallel to line $2x = 3y$, $z = 1$ then plane passes through which of the following points

(I) (6, 0, −1) (II) (−6, −1, 0) (III) (0, −6, 2)

(IV) (0, 6, −2)

Ans. (II)

eqⁿ. of plane

$a(x-1) + b(y-2) + c(z-1) = 0$

then $a - b + c = 0$ — (1)

line. $\frac{x}{3} = \frac{y}{2} = \frac{z-1}{0}$

then $3a + 2b + 0 \cdot c = 0$ — (2)

$$\frac{a}{-2} = \frac{b}{3} = \frac{c}{5}$$

$-2(x-1) + 3(y-2) + 5(z-1) = 0$

$-2x + 3y + 5z - 9 = 0$

then pt $(-6, -1, 0)$ lies on plane.

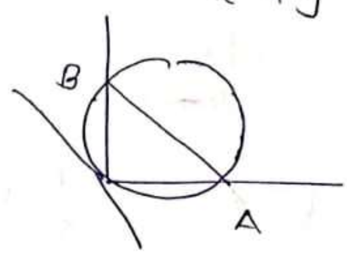
Q:- A straight line $x+2y=1$ cuts the x and y axes at A and B respectively. A circle passes through the pts. origin and A and B. Then sum of lengths of perpendiculars from A and B on the tangent of the circle at origin is

(I) $\sqrt{5}$ (II) $\frac{\sqrt{5}}{2}$ (III) $\frac{\sqrt{5}}{4}$ (IV) $\frac{\sqrt{5}}{3}$

Sol. (II)

$A(1,0) \quad B(0, 1/2)$.

Circle $x^2+y^2-x-\frac{y}{2}=0$



tangent. $-\frac{(x)}{2}-\frac{y}{4}=0$,

$2x+y=0$

$\text{Sum}=\frac{2}{\sqrt{5}}+\frac{1}{2\sqrt{5}} = \frac{5}{2\sqrt{5}}$

Q:- The value of $\left(\dfrac{1+\sin 2\frac{\pi}{9}+i\cos 2\frac{\pi}{9}}{1+\sin 2\frac{\pi}{9}-i\cos 2\frac{\pi}{9}}\right)^3$ is equal to:-

Sol. $\left[\dfrac{1+\cos\left(\frac{\pi}{2}-\frac{2\pi}{9}\right)+i\sin\left(\frac{\pi}{2}-\frac{2\pi}{9}\right)}{1+\cos\left(\frac{\pi}{2}-\frac{2\pi}{9}\right)-i\sin\left(\frac{\pi}{2}-\frac{2\pi}{9}\right)}\right]^3$.

$$\left[\frac{\cancel{\cos\frac{5\pi}{36}}\left(\cos\frac{5\pi}{36}+i\sin\frac{5\pi}{36}\right)}{\cancel{\cos\frac{5\pi}{36}}\left(\cos\frac{5\pi}{36}-i\sin\frac{5\pi}{36}\right)}\right]^3$$

$$\left(\cos\frac{5\pi}{36}+i\sin\frac{5\pi}{36}\right)^6.$$

$$\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}$$

$$\frac{-\sqrt{3}+i}{2}.$$

Q-2 ~~[illegible]~~ The contra-positive of "If I reach the station on time then I will get the train" is -

Ans:- If I don't get the train then I don't reach station on time

(Contrapositive of $p \to q$ is $\sim q \to \sim p$)

(Q) if the line $3x+4y=k$ is tangent to the circle $x^2+y^2-2x-4y+4=0$ then possible value of k is/~~are~~ :-

(I) -6 (II) -16 (III) 16 (IV) 5

Ans (III)

$C(1,2)$ centre of circle

Radius $= 1$

then. $\left|\dfrac{3+8-k}{5}\right| = 1 \Rightarrow k = 16, 6.$

Q→ Sum of series.

$$(x+y)+(x^2+xy+y^2)+(x^3+x^2y+xy^2+y^3) + \text{———} \infty \text{ is.}$$

(I) $\dfrac{x+y-xy}{(1-x)(1-y)}$ (II) $\dfrac{x+y+xy}{(1-x)(1-y)}$ (III) $\dfrac{x-y+xy}{(1+x)(1+y)}$

(IV) $\dfrac{x-y-xy}{(1+x)(1+y)}$

(Ans. (I))

Sol. $S = \dfrac{1}{x-y}\left[(x^2-y^2)+(x^3-y^3)+(x^4-y^4)+ - \infty\right]$

$$= \frac{1}{x-y}\left[\frac{x^2}{1-x} - \frac{y^2}{1-y}\right] = \frac{\cancel{(x-y)}(x+y-xy)}{\cancel{(x-y)}(1-x)(1-y)}$$

Q:- Let $P(x)$ is a polynomial of degree 3. which have maximum value 8 at $x=1$, minimum 6 at $x=2$. Then find $P(0)$.

Ans (−2).

Sol. $P'(x) = A(x-1)(x-2)$.

then $P(x) = A\left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right] + C$.

then. $8 = A\left(\frac{1}{3} - \frac{3}{2} + 2\right) + C \Rightarrow \frac{5A}{6} + C = 8$.

and. $6 = A\left(\frac{8}{3} - 6 + 4\right) + C = 6 \Rightarrow \frac{2}{3}A + C = 6$

then $C = -2$.

(Q.) if $A = \{x \in N : 1 \le x \le 17\}$

$B = \{(ax+b) : x \in A\}$ $a > 0$

if variance of set B is 216 and mean is 17 then $a+b$ is:.

(I) -7 (II) 7 (III) 6 (IV) -6

Ans(A)

Sol.

$$B(\bar{x}) = a(\bar{x}) + b = \frac{a\sum_{r=1}^{17} r}{17} + b = 17.$$

$$\frac{a \cdot \cancel{17} \cdot 18}{\cancel{17} \cdot 2} + b = 17$$

$$9a + b = 17 \quad (1)$$

$$\sigma^2 = a^2\left[\frac{1}{17}\sum_{r=1}^{17} r^2 - \left(\frac{\sum r}{17}\right)^2\right] = 216.$$

$$a^2\left[\frac{18 \cdot 35}{6} - 81\right] = 216.$$

$$24a^2 = 216 \qquad a = 3 \Rightarrow b = -10$$

$$a+b = -7$$