A Rod

An object weighs m_1 in liquid of density d_1 and that in liquid of density d_2 is m_2. The density d of the object is

a) $d = \frac{m_2 d_2 - m_1 d_1}{m_2 - m_1}$ b) $d = \frac{m_1 d_1 - m_2 d_2}{m_2 - m_1}$

c) $d = \frac{m_2 d_1 - m_1 d_2}{m_2 - m_1}$ d) $d = \frac{m_1 d_2 - m_2 d_1}{m_1 - m_2}$

let Volume of object V and density d

$$Vg\,\Delta d = \text{weight} = V(\rho_o - \rho_L)g \text{ or } V(d_o - d_L)g$$

$$m_1 g = \cancel{V(d_o - d_1)g} = V(d - d_1)g \quad —(1)$$

$$m_2 g = V(d - d_2)g \quad —(2)$$

divide (1) / (2)

$$\frac{m_1}{m_2} = \frac{d - d_1}{d - d_2}$$

$$m_1 d - m_1 d_2 = m_2 d - m_2 d_1$$

$$m_1 d - m_2 d = m_1 d_2 - m_2 d_1$$

$$d = \frac{m_1 d_2 - m_2 d_1}{m_1 - m_2}$$

option d is correct

Q.2 1.56×10^5 J of heat is conducted through a $2 m^2$ wall of 12 cm thick in one hour. Temperature difference b/w the two sides of the wall is $20°C$. The thermal conductivity of the material of the wall is (in $Wm^{-1}K^{-1}$)

a) 0.11 b) 0.13 c) 0.15 d) 1.2

Rate of heat transfer

given
$T = 1$ hour
$T = 3600$ sec
$\Delta\theta = 1.56 \times 10^5$ J
$A = 2 m^2$
$\Delta T = 20°C$
$t = 12 \times 10^{-2}$ m

$$\frac{\Delta\theta}{\Delta t} = \frac{KA\,\Delta T}{t}$$

$$\frac{1.56 \times 10^5}{3600} = \frac{K \times 2 \times 20°}{\cancel{12} \times 10^{-2}}$$

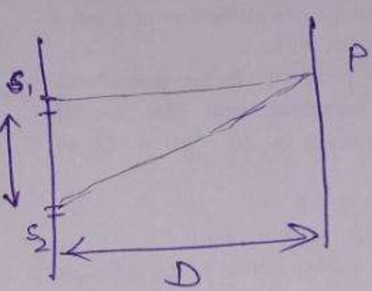
$$K = \frac{1.56 \times 10^5 \times \cancel{10^{-2}} \times \cancel{12}}{2 \times \cancel{20} \times 3\cancel{600}}$$

$$K = \frac{1.56}{12} = 0.13$$

$$K = 0.13 \ Wm^{-1}K^{-1}$$ option (B)

Q3 In Young's double slit experiment the two slits are d distance apart interference pattern is observed on a Screen at a distance D from the slits. A Dark fringe is observed on the Screen directly opposite to one of the slits. The wave length of the light is

a) $\frac{D^2}{2d}$ b) $\frac{d^2}{2D}$ c) $\frac{D^2}{d}$ d) $\frac{d^2}{D}$



$S_1P = D$

$S_2P = \sqrt{d^2 + D^2}$

$S_2P = D\sqrt{1+\frac{d^2}{D^2}}$

$= D + \frac{d^2}{2D}$

$\Delta x = S_2P - S_1P$

$\Delta x = D + \frac{d^2}{2D} - D$

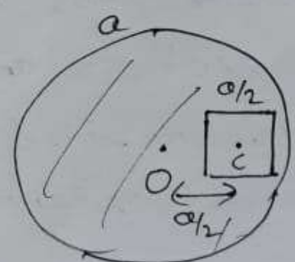
$\Delta x = \frac{d^2}{2D}$

for dark frindge

$\Delta x = \frac{\lambda}{2} = \frac{d^2}{2D}$

$$\boxed{\lambda = \frac{d^2}{D}}$$

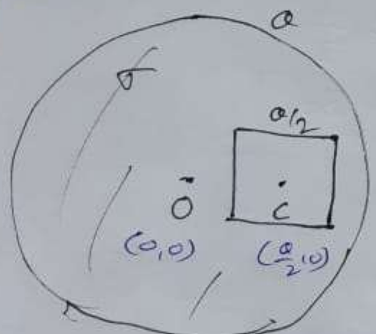
(5)



There is a square cavity of side $(a/2)$ in a uniform circular disc of radius a.
The separation ~~bew~~ between centre of disc and centre of square is $(a/2)$. If the distance from centre 'O' to com is given as (a/x).
Find 'x' upto nearest integer.

Ans = $\underline{\underline{23}}$

Solution



Let surface mass density be σ.
Coordinates of com be $(x, 0)$

Now,

$x_{com} = \cancel{M_1 M_1 - m}$

$$x_{com} = \frac{Mx - m_1 x_1}{M - m_1}$$

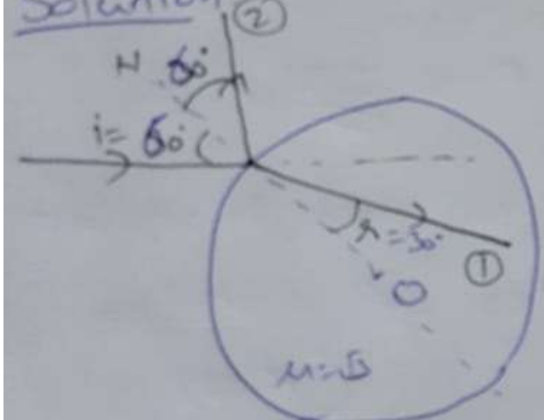
$$= \frac{\sigma \times \pi a^2 \times 0 - \sigma \times \frac{a^2}{4} \times \frac{a}{2}}{\sigma \pi a^2 - \frac{\sigma a^2}{4}}$$

$$= \frac{-a/8}{\pi - 1/4} = \frac{-a}{8\pi - 2} = \frac{-a}{23.12}$$

(4) A ray of light enters spherical glass ($\mu=\sqrt{3}$) at 60° and partially reflects and refracts from the opposite surface. Find the angle between reflected and refracted rays.

A) 30° B) 60° C) 90° D) 120°

Solution:



Angle of ~~reflec~~ refraction (r)

$$1\times\sin 60^\circ=\sqrt{3}\times\sin r$$

$$\sin r=\frac{1}{2}$$

$$\boxed{r=30^\circ}$$

Angle between ① & ②

$$=180-30-60=\underline{\underline{90^\circ}}$$

(7) If the change in value of 'g' at height 'h' above earth is same as at 'h' depth inside the earth surface. Find 'h' in terms of Earth's Radius 'R'.

A) $(\sqrt{2}+1)R$

B) $(\sqrt{2}-1)R$

C) $(2^{1/3}+1)R$

D) $(2^{1/3}-1)R$

Solution

Value of 'g' inside the Earth,

$$g_1 = \frac{GMr}{R^3} \Rightarrow \frac{\Delta g_1}{\Delta r} = \frac{GM}{R^3} \quad \text{——— (1)}$$

Value of 'g' outside the Earth,

$$g_2 = \frac{GM}{r^2}, \quad \frac{\Delta g_2}{\Delta r} = \frac{-2GM}{r^3} \quad \text{——— (2)}$$

In (1) & (2), $\Delta r = h$ and

$$\Delta g_1 = \Delta g_2$$

$$\Rightarrow \frac{GM}{R^3} = \frac{2GM}{r^3}$$

$$\Rightarrow \frac{1}{R^3} = \frac{2}{(R+h)^3}$$

$$\Rightarrow R+h = 2^{1/3}R$$

$$\Rightarrow \boxed{h = (2^{1/3}-1)R}$$

(6) If 'f' ~~be~~ is the frequency associated with transition state from $(n-1)^{th}$ transition to n^{th} transition state, where $(n>>1)$. Find relation between 'f' and 'n'.

A) $\frac{1}{n}$ B) $\frac{1}{n^2}$ C) $\frac{1}{n^3}$ D) $\frac{1}{n^4}$

Solution,

Rydberg's formula;

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \quad \& \quad v = f\lambda$$

$$\frac{f}{v} = RZ^2\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right)$$

$$f = RvZ^2\left[\frac{n^2 - n^2 - 1 + 2n}{[n(n-1)]^2}\right]$$

$$= RvZ^2\left[\frac{(2n-1)}{n^2(n-1)^2}\right]$$

If $n >> 1$,

$$f \approx \frac{2RvZ^2}{n^2} \propto \frac{1}{n^2}$$

Q. length of a rod is changed by 0.02 %. when heated from 0 to 10°C. what is the % change in mass density?

① 0.02 ② 0.03 ③ 0.06 ④ 0.10

$$\alpha = \frac{\Delta L}{L \Delta T} = \frac{0.02}{100 \times 10}$$

given : $\Delta L = 0.02\%$

$$\frac{\Delta L}{L} = \frac{0.02}{100}$$

$\Delta T = 10°C$

$$\alpha = 2 \times 10^{-5}$$

$$\eta = 3\alpha = 3 \times 2 \times 10^{-5}$$

$$= 6 \times 10^{-5}$$

$$\frac{\Delta V}{V} = \eta \times \Delta T$$

% change in mass density $= \frac{\Delta V}{V} \times 100 = \eta \times \Delta T \times 100$

$$= 6 \times 10^{-5} \times 10 \times 100$$

$$= 6 \times 10^{-2} \%$$

$$= 0.06 \%$$

if Area (A) Time (T) and Momentum (P) Then find the dimensional formula of energy:

(1) $A^{1/2} T^{-1} P^{1}$ (2) $A^{1/2} T^{-1} P^{2}$

(3) $A T^{-1/2} P^{2}$ (4) $A T^{-2} P^{1}$

Dimension formula. of Energy

$$E = M^1 L^2 T^{-2} \quad \text{--- (1)}$$

let Dimensonal formula of Energy in form of (A) (T) & (P)

$$E = A^a T^b P^c \quad \text{--- (2)}$$

from eqn (1) & (2)

$$M^1 L^2 T^{-2} = A^a T^b P^c$$

$$= [L^2]^a [T]^b [M^1 L^1 T^{-1}]^c$$

$$M^1 L^2 T^{-2} = L^{2a} T^b M^c L^c T^{-c}$$

$$M^1 L^2 T^{-2} = M^c L^{2a+c} T^{b-c}$$

$c = 1$

$2a + c = 2 \Rightarrow a = 1/2$

$b - c = -2 \Rightarrow b = -1$

$E = A^{1/2} T^{-1} P^{1}$

option (1) is Correct

Q. A capillary of radius 0.15mm is dipped in liquid of density $\rho = 667\ kg/m^3$. find the height up to which liquid rises in capillary when Surface Tension of liquid is $\frac{1}{20}\ Nm^{-1}$ and angle of Contact is $60°$.

(1) 0.01 m (2) 0.03 m (3) 0.07 m (4) 0.05 m

we know

$$h = \frac{2T\cos\theta}{\rho g r}$$

given

$T = \frac{1}{20}\ Nm^{-1}$

$\rho = 667\ kg/m^3$

$r = 0.15 \times 10^{-3}\ m$

$g = 10\ m/s^2$

$$h = \frac{2 \times \frac{1}{20} \times \frac{1}{2}}{667 \times 10 \times 0.15 \times 10^{-3}}$$

$$h = \frac{10^3}{667 \times 200 \times 0.15}$$

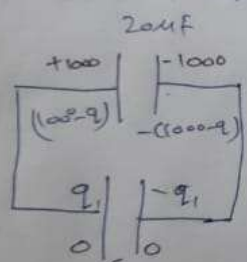
$$h = \frac{100\cancel{0}}{667 \times 3\cancel{0}} = \frac{1\cancel{00}}{2\cancel{000}} = 0.05\ m$$

(11) 20 μF capacitor is charged by a battery of 50V and then battery is removed. The charged capacitor is connected to another capacitor 'C' and on equilibrium potential difference of 20V is acheived.
Find 'C'?

Ans:- 30 μF

Solution:

Initial Charge, $q = CV = 20 \times 50 = 1000\ \mu C$



~~KVL at steady state,~~

$\cancel{\frac{q}{c} = 0}$

$\cancel{\frac{q}{4} = CV =}$

$(1000 - q_1) = 20 \times 20 = 400$

$\boxed{q_1 = 600\ \mu C}$

Now, on capacitance 'C':

$q_1 = CV$

$600 = C \times 20$

$\boxed{C = 30\ \mu F}$

(12) A string of length 1m, density 900 kg/m³ and Young's modulus 9×10^{9} N/m² depeloped a strain of 4.9×10^{-4}.
Calculate the minimum Resonance frequency that can be produced in the string (Hz).

Ans = 35 Hz

Solution

$L = 1m, \quad \rho = 900\ kg/m^3, \quad Y = 9\times10^{9}\ N/m^2$

$\epsilon = 4.9\times10^{-4}$

Young's Modulus,

$$Y = \frac{\sigma}{\epsilon} = \frac{F/A}{\epsilon} \Rightarrow \frac{F}{A} = Y\epsilon$$

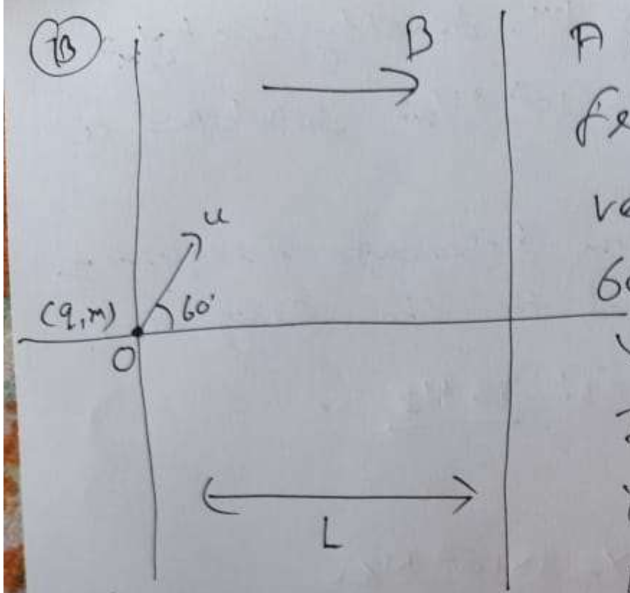
Fundamental frequency,

$$f_0 = \frac{V}{2L} = \frac{1}{2L}\sqrt{\frac{F}{\mu}} = \frac{1}{2L}\sqrt{\frac{F}{\rho A}}$$

$$= \frac{1}{2L}\sqrt{\frac{Y\epsilon}{\rho}} = \frac{1}{2\times1}\sqrt{\frac{9\times10^{9}\times4.9\times10^{-4}}{900}}$$

$$= \frac{70}{2} = 35\ Hz$$

(13)



A particle is projected from point 'O' with velocity 'u' at an angle 60° with magnetic field 'B'.

If it revolves 10 rev. in the given region before coming out. Find L?

A) $\frac{40\pi mv}{qB}$ B) $\frac{2\pi mv}{qB}$ C) $\frac{20\pi mv}{qB}$ D) $\frac{10\pi mv}{qB}$ ✓

Solution

$$R = \frac{mV_\perp}{qB} = \frac{mu\sin 60}{qB}$$

$$\text{pitch} = (V\cos 60^\circ)T = (V\cos 60)\frac{2\pi R}{V_\perp}$$

$$= (V\cos 60)\, 2\pi \times \frac{m}{qB}$$

$$L = 10 \times \text{pitch} = \frac{10\pi mV}{qB}$$

(14) A cyclic process have four steps with heat Q_1, Q_2, Q_3 & Q_4. The efficiency of the cyclic process is 50%, Find Q_4.

Given.
$Q_1 = 1915 J$, $Q_2 = -40 J$, $Q_3 = 125 J$.

A) −1080 ~~B)~~ −980J C) −1280J D) +1180J

Solution

Efficiency, $\eta = \frac{W}{Q_{supplied}}$

$$\eta = \frac{Q_1 + Q_2 + Q_3 + Q_4}{Q_1 + Q_3}$$

$$\Rightarrow 0.5 = \frac{1915 - 40 + 125 + Q_4}{1915 + 125}$$

$$\Rightarrow 1020 = 2000 + Q_4$$

$$\Rightarrow \boxed{Q_4 = -980 J}$$