Q→(1) if $f(x+y) = f(x) + f(y)$ and. $f(1)=2$.

and. $g(x) = \sum_{K=1}^{n} f(K)$.

Ans(4)

Then find n. for which $g(x) = 20$.

Sol.

Let $f(x) = ax$.

$f(1) = a = 2$

then $f(x) = 2x$

$g(x) = 2\sum_{K=1}^{n} K \Rightarrow 20 = n(n+1)$

$\boxed{n=4}$

Q-(2)

Ans(1).

$\int_1^2 |2x - [3x]|\, dx$ is equal.

$\int_1^{4/3} |2x-3|\, dx + \int_{4/3}^{5/3} |2x-4|\, dx + \int_{5/3}^{2} |2x-5|\, dx$

$\int_1^{4/3} (3-2x)\, dx + \int_{4/3}^{5/3} 4-2x\, dx + \int_{5/3}^{2} 5-2x\, dx.$

$\left(\cancel{3x-x^2}\right) \quad (3x)_1^{4/3} + (4x)_{4/3}^{5/3} + (5x)_{5/3}^{2} - (x^2)_1^2$

$3\left(+\frac{1}{3}\right) + 4\left(\frac{1}{3}\right) + 5\left(\frac{1}{3}\right) - [4-1]$

$4-3 = 1.$

Q– if ratio of 3 consecutive binomial coefficients in the expansion of $(1+x)^n$ is 2:5:12 ~~2:12:20~~. then n is equal to:-

(1) 34 ② 35 ③ 118 ④ 120.

Ans. ③.

$\because$ then $T_{r+1} = {}^nC_r\, x^{n-r} a^r$

$${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 2:5:12.$$

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{5}{2} \quad \text{and.} \quad \frac{{}^nC_{r+1}}{{}^nC_r} = \frac{12}{5}.$$

$$\frac{n-r+1}{r} = \frac{5}{2} \quad \text{and.} \quad \frac{n-r}{n+1} = \frac{12}{5}$$

then $n = 118.$

Q→ if $a, b, c \in R$ such that $a^3+b^3+c^3 = 2$ and.

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$$ then abc is equal:-

(1) $\frac{2}{3}$ ② $\frac{1}{3}$ (3) $\frac{-2}{3}$ (4) 1.

Ans ①.

$$\because \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a^3+b^3+c^3-3abc) = 0.$$

then $a^3+b^3+c^3 = 3abc$

$abc = 2/3.$

Q:- value of $\lim_{x\to 0} \left[\tan(\pi/4 + x)\right]^{1/x}$ is equal:-

(1) e (2) $1/e$ (3) e^2 (4) e^4.

Ans. (3).

$$L = \lim_{x\to 0} \left[\tan\left(\frac{\pi}{4} + x\right)\right]^{1/x} = 1^{\infty}$$

$$e^{\lim_{x\to 0} \frac{1}{x}\left[\tan(\pi/4 + x) - 1\right]}$$

$$e^{\lim_{x\to 0} \frac{1}{x}\left[\frac{1+\tan x}{1-\tan x} - 1\right]}$$

$$e^{\lim_{x\to 0} \frac{1}{x}\left[\frac{2\tan x}{1-\tan x}\right]} = e^2.$$

Q ~~Ref~~ The range of α when $\sin^4\theta + \cos^4\theta + \lambda = 0$ have real solution

Ans. $\left[-1, -\frac{1}{2}\right]$

$$\lambda = -\left[(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2 x\cos^2 x\right]$$

$$\lambda = -\left[1 - \frac{\sin^2 2x}{2}\right] \Rightarrow \lambda = \frac{\sin^2 2x}{2} - 1$$

$$\because -1 \le \sin x \le 1 \Rightarrow -\frac{1}{2} \le \frac{\sin^2 x}{2} \le \frac{1}{2}$$

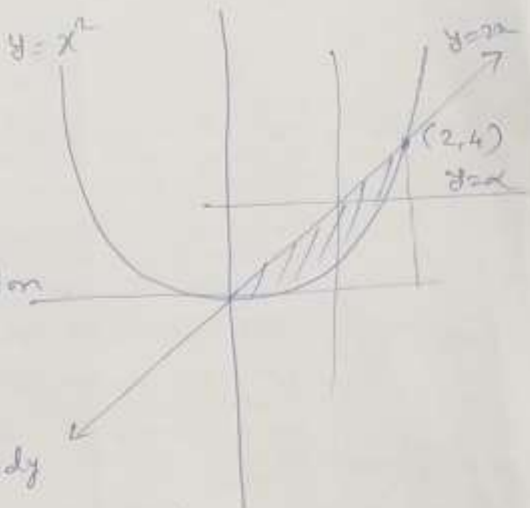
$$\therefore 0 \le \frac{\sin^2 2x}{2} \le 1/2 \quad \therefore \lambda \in \left[-1, -\frac{1}{2}\right]$$

Q) If $y=\alpha$ bisects the area bounded by region

$x^2 \le y \le 2x$ then:

(A) $4\alpha^{3/2} + 3\alpha^2 = 8$ (B) $8\alpha^{3/2} - 3\alpha^2 = 4$ (C) $8\alpha^{3/2} - 3\alpha^2 = 8$

(D) $4\alpha^{3/2} - 3\alpha^2 = 8$



Ans. (C) Here, $y - x^2 \le 0$

$\& \ 2x + y \ge 0$

Area, will be shaded region

so, for $y=\alpha$ we can write

$$\therefore \int_0^4 \left(\sqrt{y} - \frac{y}{2}\right) dy = 2\int_0^{\alpha} \left(\sqrt{y} - \frac{y}{2}\right) dy$$

$$\left[\frac{2y^{3/2}}{3} - \frac{y^2}{4}\right]_0^4 = 2\left[\frac{2y^{3/2}}{3} - \frac{y^2}{4}\right]_0^{\alpha} \Rightarrow \frac{16}{3} - 4 = 2\left(\frac{2}{3}\alpha^{3/2} - \frac{\alpha^2}{4}\right)$$

$$\therefore \quad 8\alpha^{3/2} - 3\alpha^2 = 8 \quad \text{Ans}$$

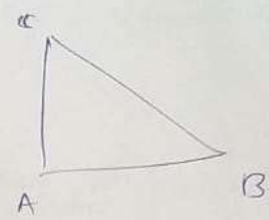
49) Ans,

Let ΔABC $\angle A = 90°$

$\therefore B + C = 90°$

Given, $B - C = \frac{2\pi}{5}$ or $B - C = 72°$

from here, $\angle B = 81°$ & $\angle C = 9°$.



Hint:- In a right angle triangle sum of two acute angles is $90°$.

Q) If point P divides the line joining $A(i+j+\hat{k})$ and $B(2i+j+3\hat{k})$ in the ratio $\lambda:1$ such that $\vec{OB}\cdot\vec{OP} - 3|\vec{OA}\times\vec{OP}|^2 = 6$ then the value of λ is;

Ans. 3/4

from here ~~position~~ $\vec{OP}$ position vector of $\vec{OP} = \frac{\vec{a}+\lambda\vec{b}}{\lambda+1}$

$$\because \vec{OB}\cdot\vec{OP} - 3|\vec{OA}\times\vec{OP}|^2 = 6$$

$$\therefore b\cdot\left(\frac{\vec{a}+\lambda\vec{b}}{\lambda+1}\right) - 3\left|\vec{a}\times\left(\frac{\vec{a}+\lambda\vec{b}}{\lambda+1}\right)\right|^2 = 6$$

$$\frac{\vec{a}\cdot\vec{b} + \lambda|\vec{b}|^2}{\lambda+1} - \frac{3\lambda^2}{(\lambda+1)^2}|\vec{a}\times\vec{b}|^2 = 6$$

$$\frac{6+\lambda 14}{\lambda+1} - \frac{3\lambda^2}{(\lambda+1)^2}\cdot 6 = 6 \Rightarrow \frac{18\lambda^2}{(\lambda+1)^2} + 6 = 6 + \frac{8\lambda}{\lambda+1}$$

$$18\left(\frac{\lambda}{\lambda+1}\right)^2 - \frac{8\lambda}{\lambda+1} = 0 \Rightarrow \left(\frac{\lambda}{\lambda+1}\right)\left[\frac{18\lambda}{\lambda+1} - 8\right) = 0$$

from here, $18\lambda - 8(\lambda+1) = 0$

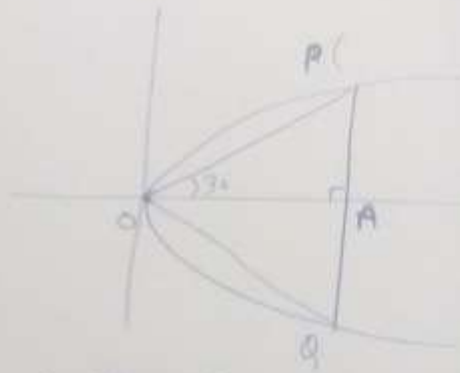
$10\lambda = 8$ or $\boxed{\lambda = 4/5}$ Ans

Q5 An equilateral triangle is inscribled inside parabala $y^2 = 8x$ whose one vertex coinside with vertex of parabola then the area of triangle is :-

Ans $(192\sqrt{3})$

$y = 8x$

$\therefore P(2t^2, 4t)$



Now, In ΔOAP

$\tan 30° = \dfrac{4t}{2t^2}$

$\Rightarrow t = 2\sqrt{3}$

$\therefore P = (24, 8\sqrt{3})$ $\qquad \therefore OP = 16\sqrt{3}$

$\therefore Ar \Delta OPQ = \dfrac{\sqrt{3}}{4} \cdot OP^2 = \dfrac{\sqrt{3}}{4} \cdot 256 \times 3 \Rightarrow Area = 192\sqrt{3}$ sq-unit

Q) Let $a_1, a_2, a_3 \ldots a_{11}$ be an increasing AP. If varience of these numbers is 90. then the value of common difference of A.P is.

Ans. ($d=3$)

Given $a_1, a_2 - - a_{11} \rightarrow A.P$

and variance $= 90$

$$\therefore \frac{\sum_{i=1}^{11} a_i^2}{11} - \left(\frac{\sum_{i=1}^{11} a_i}{11}\right)^2 = 90 \Rightarrow \cancel{\frac{d^2}{10}\left(\frac{10\times11\times21}{6}\right)}$$

$$\frac{d^2}{10} \cdot \frac{10\times11\times21}{6} - d^2\left(\frac{55}{11}\times\frac{55}{11}\right) = 90$$

$$d^2(35-25) = 90 \Rightarrow d^2 = 9 \Rightarrow d = \pm 3$$

for increasing AP $\Rightarrow d = 3$

Q:- if $f(x) = \dfrac{\ln(1+x)}{x}$, $x \in (-1,\infty)$ and. $f(0) = 1$ then $f(x)$ is:-

(1) decreasing in $(-1,0)$ and. increasing in. $(0,\infty)$

(2) Increasing in $(-1,0)$ and decreasing in $(0,\infty)$

(3) Always increasing

(4) Always decreasing.

Ans. (4).

$$f(x) = \frac{\ln(1+x)}{x}$$

$$f'(x) = \frac{x - (1+x)\ln(1+x)}{x^2 \cdot (1+x)} \quad \text{—} (1)$$

Let $g(x) = x - (1+x)\ln(1+x)$

then $g'(x) = \cancel{1} - \cancel{1} - \ln(1+x) \begin{cases} < & x \in (0,\infty) \\ > & x \in (-1,0) \end{cases}$

then $g(x) < 0 \ \forall\, x \in (-1,\infty)$

then $f'(x) < 0$ for $x \in (-1,\infty)$

Hence $f(x)$ is always decreasing in domain

Q:- If there are n station in a circle. Two consecutive stations are connected by blue line and. two non-consecutive stations are connected by red lines. If total number of red lines is equal to 99. times the number of blue lines. Then n is equal.

(1) 199 (2) 200 (3) 201 (4) 202.

Ans. (3)

No. of ways for selecting. two consecutive stations = n.

then no. of ways for selecting two non-consecutive stations = ${}^nC_2 - n$.

then.

$$ {}^nC_2 - n = 99 \cdot n. $$

$$ \frac{n(n-1) - 2n}{2} = 99n $$

$$ n^2 - 3n = 198n \Rightarrow n(n-201) = 0 $$

$$ n = 201 $$

Q:- If sum of series.

$(x+ka)+(x^2+(k-2)a)+(x^3+(k-4)a)+ \cdots + \cdots$ upto 9 terms is equal to $\frac{x^{10}-x-45a(x-1)}{x-1}$. Then value of k is.

Ans. (03)

Sol. given.

$S = (x+x^2+x^3+ \quad$ upto 9 terms$) + a[k+(k-2)+(k-4)+ \cdots$ upto 9 terms$]$

$S = \frac{x(x^9-1)}{x-1} + \frac{9}{2}a[2k+8(-2)]$

$\frac{x^{10}-x}{x-1} + 9a(k-8) = \frac{x^{10}-x}{x-1} - 45a$

than ~~$k-8=5$~~ $k-8=-5$

Q. if Equation of hyperbola is $x^2 - y^2\sec^2\theta = 10$ and equation of ellipse is $x^2\sec^2\theta + y^2 = 5$, are such that eccentricity of hyperbola is $\sqrt{5}$ (eccentricity of ellipse). then length of Latus-rectum of ellipse is :-

(1) $\frac{4\sqrt{5}}{3}$ (2) $\frac{4}{3\sqrt{5}}$ (3) $\frac{20\sqrt{5}}{3}$ (4) $\sqrt{30}$

Ans (1)

Hyperbola $\frac{x^2}{10} - \frac{y^2}{10\cos^2\theta} = 1$.

then $e_{(Hy)} = \sqrt{1+\cos^2\theta}$

and ellipse, $\frac{x^2}{(\sqrt{5}\cos\theta)^2} + \frac{y^2}{5} = 1$

then $e_{(el.)} = \sqrt{1-\cos^2\theta}$

then $e_{(Hy)} = \sqrt{5}\,(e_{el})$

$1+\cos^2\theta = 5(1-\cos^2\theta)$

$\cos^2\theta = 4/6 = \frac{2}{3}$

Length of L.R. of ellipse $= \frac{10\cos^2\theta}{\sqrt{5}} = \frac{10}{\sqrt{5}}\cdot\frac{2}{3}$

Q:- Which of the following is a tautology:-

(1) $\sim p \wedge (p \vee q) \to q$ (2) $\sim p \vee (p \wedge q) \to q$

(3) $\sim p \vee (p \vee q) \to q$. (4) None of these.

Ans (1)

$$\sim p \wedge (p \vee q) \to q \Rightarrow (\sim p \wedge p) \vee (\sim p \wedge q) \to q$$
$$= c \vee (\sim p \wedge q) \to q$$
$$= (\sim p \wedge q) \to q$$
$$= \sim(\sim p \wedge q) \vee q$$
$$= (p \vee \sim q) \vee q$$
$$= p \vee t = t.$$

Q:- if $y = (1+x)^{2y} + \cos^2(\sin^{-1}x)$ be a curve, then equation of normal ~~at~~ at $x=0$ is.

(1) $x+4y=8$ (2) $2x+y=2$ (3) $2x-y=2$ (4) $x+4y=2$.

Ans (1)

Sol. put $x=0$ in $y = (1+x)^{2y} + \cos^2(\sin^{-1}x)$

we get $y=2$.

Now.

$$y = e^{2y\ln(1+x)} + 1 - x^2.$$

$$\frac{dy}{dx} = (1+x)^{2y}\left[\frac{2y}{1+x} + 2\ln(1+x)\frac{dy}{dx}\right] - 2x.$$

$$\left(\frac{dy}{dx}\right)_{(x=0)} = 4$$

then eq^n. of normal is

$$(y-2) = \frac{-1}{4}(x-0)$$