Q-1. \[ \text{if } \overline{a} = 2 \hat{i} + 2 \hat{j} + 2 \hat{k} \]
then \[ |\hat{i} \times (\overline{a} \times \hat{i})|^2 + |\hat{j} \times (\overline{a} \times \hat{j})|^2 + |\hat{k} \times (\overline{a} \times \hat{k})|^2 \]

is equal to:

Ans.: 24

Sol.: \[
\hat{i} \times (\overline{a} \times \hat{i}) = (\hat{i} \cdot \hat{i}) \overline{a} - (\hat{i} \cdot \overline{a}) \hat{i}
= 1 \overline{a} - 2 \hat{i} = 2 (\hat{j} + \hat{k})
\]

Similarly:
\[
\hat{j} \times (\overline{a} \times \hat{j}) = \overline{a} - 2 \hat{j} = 2 (\hat{i} + \hat{k})
\hat{k} \times (\overline{a} \times \hat{k}) = \overline{a} - 2 \hat{k} = 2 (\hat{i} + \hat{j})
\]

then \[ |\hat{i} \times (\overline{a} \times \hat{i})|^2 + |\hat{j} \times (\overline{a} \times \hat{j})|^2 + |\hat{k} \times (\overline{a} \times \hat{k})|^2 \]
\[ = 4 [1 + 1] \times 3 = 24 \]
Q-2. Distance of a point \( (1, -2, 3) \) from plane \( x - y + z = 5 \), which measure parallel to line
\[
\frac{x}{a} = \frac{y}{b} = \frac{z}{c}
\]
is equal to.

Anno. 5.
Sol.:
\[
\frac{x}{a} = \frac{y}{b} = \frac{z}{c}
\]

Equation of line through \( P \) and \( \Pi \) to given line in
\[
\frac{x-1}{2} = \frac{y+2}{3} = \frac{z+3}{-6} = \gamma.
\]

\( Q (1+2\gamma, -2+3\gamma, -3-6\gamma) \)
lies on plane then
\[
(1+2\gamma) - (3\gamma-2) + (-3-6\gamma) = 5
\]
\[-7\gamma = 5 \quad \gamma = -\frac{5}{7}
\]

\( Q \left( -\frac{3}{7}, -\frac{29}{7}, \frac{9}{7} \right) \)

\[ |PQ| = \sqrt{100 + 225 + 900} = \frac{35}{7} = 5. \]
Q. If \( \int_0^n x^4 \, dx, \int_0^n [x] \, dx \) and \( 10(n^2-n) \) are in G.P., (where \( n \in \mathbb{N} \)). Then \( n \) is equal to:

\( \text{Ans. } 21 \)

Sol.

\[
I_2 = \int_0^n [x] \, dx = \int_0^0 0 \, dx + \int_1^1 1 \, dx + \int_2^2 2 \, dx + \cdots + \int_{n-1}^n (n-1) \, dx
\]

\[
= (x)^1 + 2(x)^2 + 3(x)^3 + \cdots + (n-1)(x)^{n-1}
\]

\[
= 1 + 2 + \cdots + (n-1)
\]

\[
= \frac{(n-1)n}{2}
\]

\[
I_2 = \int_0^n x^4 \, dx = \int_0^1 x^4 \, dx = \frac{n}{5}
\]

Let \( I_3 = 10(n^2-n) \)

\( I_1, I_2, I_3 \) are in G.P.

\[
(I_2)^2 = I_1 \cdot I_3 = \frac{10^2(n-1)^2}{2} = \frac{n^2}{2} \cdot 10(n-1)
\]

\[
(n-1)[n-1-2e] = 0
\]

\[n = 1, 21\]
Q: If \((2+w)^2 = a+bw\) where \(a, b \in \mathbb{R}\), and \(w\) is an imaginary cube root of unity, then \((a+b)\) is equal to

A: \(6\)

Solution:
\[
(2+w)^2 = a+bw
\]
\[
4+4w+w^2 = a+bw
\]
\[
u + w^2 + 4w = a + bw
\]
\[
u + (-1-w) + 4w = a + bw
\]
\[
3 + 3w = a + bw
\Rightarrow a = b = 3
Q: If \( a_1, a_2, a_3, \ldots a_n \) are in A.P. where
\[ n \in \mathbb{N} \text{ and } 15 < n \leq 50. \]
If \( a_1 = 1 \) and \( a_n = 300 \)
then \((S_{n-1}, a_{n-1})\) is

Ans. \((2490, 248)\)

Sol.: \[ a_n = a_1 + (n-1)d \]
\[ 300 = 1 + (n-1)d \]
\[ n = \frac{299}{d} \]
\[ n = 15.2 \approx \frac{18.2}{d} \]
\[ d = 13 \]
\[ n = 24. \]
\[ a_{n-1} = a_{24} = 1 + 19 \times 13 = 248 \]
\[ S_{n-1} = S_{24} = 2490. \]

Q: Minimum value of \( (2 \sin x + 2 \cos x) \) is equal to:

Ans. \( 1 + \sqrt{2} \)

Solution: AM \( \geq \) GM,
\[ \frac{2 \sin x + 2 \cos x}{2} \geq \left(2 \sin x + 2 \cos x\right)^{1/2} \]
\[ -\sqrt{2} \leq (\sin x + \cos x) \leq \sqrt{2} \]
then \[ 2 \sin x + 2 \cos x \geq 2 \times \sqrt{1 - \frac{1}{2}} = 2 - \frac{\sqrt{2}}{2} \]
\[
\frac{18}{3} = \frac{8}{9}
\]

\[
\int_0^\infty e^{-t^2} dt = \frac{\sqrt{\pi}}{2}
\]

Put tan2x sin3x = t

3 \int_0^\infty e^{-t^2} dt = \frac{\sqrt{\pi}}{2}

\int_{-\infty}^\infty e^{-t^2} dt = \sqrt{\pi}

\int_{-\infty}^\infty e^{-t^2} dt = \frac{\sqrt{\pi}}{2}

\text{Sol.:} \quad I = \int_{-\infty}^\infty e^{-t^2} dt

\text{Ans.:} \quad \frac{1}{\sqrt{\pi}}

equal to:

- \frac{\sqrt{\pi}}{2}

- \int_{-\infty}^\infty e^{-t^2} dt = \sqrt{\pi}

value of Cauchy trick.
Q: Find the distance below the cloud, and the image of the cloud formed in the lake.

If angle of elevation of the cloud from a point 200 m. above the lake is 30° and the angle of depression of the image is 60°.

Ans. 800m.

Sol.

\[ \tan 30° = \frac{h}{x} \]

\[ x = h \sqrt{3} \]  \hspace{1cm} (1)

Also \[ \tan 60° = \frac{200 + h}{x} \]

\[ \sqrt{3} x = 200 + h \]

\[ 8h = 400 + h \Rightarrow h = 200 \]

distance below cloud and its image in 800m.
Q:- If the ratio of three consecutive terms in the expansion of \((1+x)^{n+5}\) is 5:10:14. Then greatest coefficient is.

Ans: (1) 252 (2) 462 (3) 792 (4) 320.

Ans: (2).

Solution:- Let \(T_r : T_{r+1} : T_{r+2} = 5 : 10 : 14\),

Then \(\frac{T_{r+1}}{T_r} = \frac{10}{5}\) and \(\frac{T_{r+2}}{T_{r+1}} = \frac{14}{10}\)

\(\frac{n+5}{n+5} \cdot \frac{c_r}{c_{r-1}} = 2\) and \(\frac{n+5}{n+5} \cdot \frac{c_{r+1}}{c_r} = \frac{7}{5}\)

Then \(\frac{n+5-\text{r+1}}{\text{r}} = 2\) and \(\frac{n+5-\text{r-1}+1}{\text{r+1}} = \frac{7}{5}\)

\(\Rightarrow\ n-3\text{r}+6 = 0 \quad \text{(1)}\)

and \(5n - 12\text{r} + 18 = 0 \quad \text{(2)}\)

On solving we get \(n=6\) and \(r=4\). We know that coefficient of middle term is greatest = \(n+5c_5 = 11c_5 = 462\).
Q: - if \[
\lim_{t \to x} \frac{x^2 f^2(t) - t^2 f^2(x)}{t - x} = 0 \quad \text{and} \quad f(0) = e
\]

then solution of eqn. \( f(x) = 1 \) is

\( 1. \) \( \frac{1}{e} \) \( 2. \) \( \frac{1}{2e} \) \( 3. \) e \( 4. \) 2e.

Ans: (1)

Solution:

\[
\lim_{t \to x} \frac{x^2 f^2(t) - t^2 f^2(x)}{t - x} = 0 \quad \text{form}
\]

Then by L-Hopital rule:

\[
\lim_{t \to x} \frac{2x^2 f(x) f'(x) - 2t f^2(x)}{1} = 0
\]

\[
2x^2 f(x) f'(x) - 2xf^2(x) = 0
\]

\[
\int \frac{2x f(x) f'(x)}{f^2(x)} \, dx = \int \frac{p}{x} \, du.
\]

\[
\ln |f^2(x)| = \ln x + \ln c. \quad \therefore f(x) = e
\]

\( c \ln e = \ln c \Rightarrow c = e. \)

Then, \( f(x) = e^x. \)

Then, \( f(x) = 1 \Rightarrow x = \frac{1}{e}. \)
Q:- The Contrapositive of statement:
   "If \( f(x) \) is continuous at \( x=a \) then
   \( f(x) \) is differentiable at \( x=a \)" is

(1) If \( f(x) \) is continuous at \( x=a \) then \( f'(x) \) is not differentiable at \( x=a \).

(2) If \( f(x) \) is not differentiable at \( x=a \),
    then \( f(x) \) is not continuous at \( x=a \).

(3) If \( f(x) \) is differentiable at \( x=a \) then
    it is continuous at \( x=a \).

(4) If \( f(x) \) is differentiable at \( x=a \) then
    it is not continuous at \( x=a \).

Ans. (2)

Sol. "Contrapositive of \( p \Rightarrow q \) is \( \neg q \Rightarrow \neg p \)."
Q: If circle $s=0$ passes through the intersection points of circles $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 - 4y = 0$.

If centre of circle $s=0$ lies on the line $2x - 3y + 12 = 0$, then circle $s=0$ passes through:

(1) $(-3, 1)$, (2) $(-4, -2)$, (3) $(1, 2)$, (4) $(-3, 6)$

Ans. (4)

Sol. $s=0 = (x^2 + y^2 - 6x) + d(x^2 + y^2 - 4y) = 0$

$(1 + d)(x^2 + y^2) - 6x - 4y d = 0$

Centre $\left( \frac{3}{1+d}, \frac{2d}{1+d} \right)$ lies on the line $2x - 3y + 12 = 0$

Then $6 - 6d + 12(1 + d) = 0$

$d = -3$

Hence eq. of circle $s=0$ is $x^2 + y^2 + 3x - 6y = 0$

which passes through $(-3, 6)$. 
Q: If \( \alpha, \beta \) are the roots of equation \( \alpha^2 - \alpha + 2d = 0 \) and \( \alpha, \gamma \) are the roots of eq. \( 3\alpha^2 - 10\alpha + 2d = 0 \)

Then value of \( \frac{\beta \gamma}{d} \) is equal to:

(1) 9  (2) 15  (3) 18  (4) 27.

\( \text{Ans. (3)} \)

Sol: Equations \( \alpha^2 - \alpha + 2d = 0 \) \( \text{---(1)} \)
and \( \alpha^2 - \frac{10}{3}\alpha + 9d = 0 \) \( \text{---(2)} \)

have one root common then \( \text{---(1)} \) \( \text{---(2)} \)

\[ \frac{7\alpha}{3} = 7\alpha \]
\[ \alpha = \alpha = 3d \]

From eq. \( \text{---(1)} \)

\( 9d^2 - 3d + 2d = 0 \)

\( d = 0, \frac{1}{9}, \text{ and } d \neq 0 \).

Then eq. are \( \alpha^2 - \alpha + \frac{2}{9} = 0 \) and
\[ 3\alpha^2 - 10\alpha + 3 = 0, \text{ then} \]
\[ \alpha = \frac{1}{3}, \beta = \frac{2}{3}, \gamma = 3. \]

\[ \frac{\beta \gamma}{d} = 18. \]
If $x = a$ is the directrix of an ellipse \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \)
whose eccentricity is $\frac{1}{2}$.
Then the normal to the ellipse at point $(1, \beta)$ where $\beta > 0$, passed through the point $(1, \frac{3}{2})$.

Ans: (3).

Sol:—
\[ \text{Eqn. of directrix } x = \frac{a}{e} = a \]
\[ a = 2. \]
\[ \text{Then } b^2 = u \left(1 - \frac{1}{4}\right) = 3/4 \cdot 3 \]
\[ \text{Eqn. of ellipse } \frac{x^2}{u} + \frac{y^2}{3} = 1 \text{ and } (1, \beta) \]
and $(1, \beta)$ lies on ellipse then
\[ \frac{1}{u} + \frac{\beta^2}{3} = 1 \quad \Rightarrow \beta = \frac{3}{2} \]
then eqn. of normal at $(1, \frac{3}{2})$ is
\[ \frac{u \cdot x - 3y}{\frac{3}{2}} = 1 \quad \Rightarrow \text{ux} - 2y = 1 \]
$(1, \frac{3}{2})$ lies on their normal.
Q: If \( \frac{dy}{dx} = \frac{(y-3x)}{\ln(y-3x)} = 3 \) then.

\[
(1) \quad \ln(y-3x)^2 = x^2 + c \\
(2) \quad \frac{\ln(y-3x)}{2} = u + c \\
(3) \quad \ln^2(y-3x)^2 = u + c \\
(4) \quad \frac{\ln^2(y-3x)}{2} = x^2 + c
\]

ANSA (3)

Solution:

Given, \( \frac{dy}{dx} = \frac{(y-3x)}{\ln(y-3x)} = 3 \).

Then \( \frac{dy}{dx} - 3 = \frac{(y-3x)}{\ln(y-3x)} \)

Put \( y-3x = t \)

\( \frac{dt}{dx} = \frac{t}{\ln t} \)

\[ \int \frac{\ln t}{t} dt = \int dx \]

\[ \frac{1}{x} \ln t = x + c \]

\[ \frac{1}{2} \ln^2(y-3x) = x + c \]
Q: if \( f(x) = \begin{cases} \frac{1}{2} (|x| - 1) & |x| > 1 \\ \tan^{-1} x & |x| \leq 1 \end{cases} \) then \( f(x) \) is:

1. Continuous for \( x \in \mathbb{R} - \{0\} \)
2. Continuous for \( x \in \mathbb{R} - \{0, 1, -1\} \)
3. Not continuous for \( x \in \{-1, 0, 1\} \)
4. Continuous for \( x \in \mathbb{R} - \{-1, 1\} \)

Ans(4)

\[
\begin{align*}
  f(x) &= \begin{cases} 
    \frac{|x| - 1}{2} & x \in (-\infty, -1) \cup (1, \infty) \\
    \tan^{-1} x & -1 < x \leq 1 \\
    \frac{x - 1}{2} & x > 1
  \end{cases}
\end{align*}
\]

\[
\begin{align*}
  f(-1) &= 0 \quad \text{and} \quad f(-1) = -\frac{\pi}{4} \\
  f(1) &= \frac{\pi}{4} \quad \text{and} \quad f(1^+)=0
\end{align*}
\]

Hence \( f(x) \) is discontinuous at

\( x = \{-1, 1\} \)
Q: Let $A$ be a $3 \times 3$ matrix such that
\[ A \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = B_1, \quad A \cdot \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = B_2 \quad \text{and} \quad A \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = B_3 \]

where
\[ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \]

and $B_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $B_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$, $B_3 = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$

Then value of $|A|$ is equal to:

Ans: $|A| = 3$

Sol.

Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$.

Then for $AX_1 = B_1$
\[ a_1 + b_2 + a_3 = 1, \quad b_1 + b_2 + b_3 = c_1 + c_2 + c_3 = 0 \]

By $AX_2 = B_2$,
\[ 2a_2 + a_3 = 0, \quad 2b_2 + b_3 = 2, \quad 2c_2 + c_3 = 0 \]

and by $AX_3 = B_3$
\[ a_3 = b_3 = 0 \quad \text{and} \quad c_3 = 2 \]

Then, $a_2 = 0, \quad a_1 = 1, \quad b_2 = 1, \quad b_1 = -1 \quad \text{and} \quad c_2 = -1, \quad c_1 = -1$

Then $|A| = 2$. 
Q: For grouped frequency distribution:

<table>
<thead>
<tr>
<th>Class</th>
<th>0-10</th>
<th>10-20</th>
<th>20-30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>2</td>
<td>x</td>
<td>2</td>
</tr>
</tbody>
</table>

If variance of variable is 50, then x is equal to:

(1) 3  (2) 4  (3) 5  (4) 6.

Ans (2)

\[ \mu = \frac{\sum fx}{\sum f} = \frac{2.5 + x\cdot15 + 2.25}{x+4} \]

Where \( x_i \) is midvalue of each class.

\[ \mu = \frac{15(x+4)}{x+4} = 15 \]

Now

\[ \sigma^2 = \frac{1}{n} \sum fi x_i^2 - \mu^2 \]

\[ 50 = \frac{1}{x+4} \left[ 2.5^2 + x \cdot 15^2 + 2.25^2 \right] - (15)^2. \]

\[ 2.75(x+4) = 25 \left[ 8 + 9x + 50 \right] \]

\[ 15x = 200 \Rightarrow x = 4. \]
Q: Two persons A and B play a game of throwing a pair of dice until one of them wins the game. A will win if sum of numbers on dice appear to be 6, and B will win if sum is 7. Then, probability of A winning the game, if A starts is:

\[
\begin{align*}
(1) & \quad \frac{30}{61} \\
(2) & \quad \frac{29}{61} \\
(3) & \quad \frac{31}{61} \\
(4) & \quad \frac{28}{61}
\end{align*}
\]

Ans. (1).

Solution:

Let

\[ P(A) = \frac{5}{36} \]

\[ P(B) = \frac{6}{36} \]

The probability of A winning the game is:

\[ P(A) = \frac{x}{1 - x^2 y^2} \]

\[ P(B) = \frac{y}{1 - x^2 y^2} \]

\[ P(A) = \frac{x}{1 - x^2 y^2} = \frac{5/36}{1 - \frac{31}{36} \frac{30}{36}} = \frac{30}{61} \]
Q. Let \( x_1, x_2, \ldots, x_{50} \) are 50 sets each having 10 elements and \( y_1, y_2, \ldots, y_n \) are \( n \) sets each having 5 elements. Let \( \bigcup_{i=1}^{50} x_i = \bigcup_{i=1}^{n} y_i = z \), and each element of \( z \) belongs to exactly 25 of \( x_i \) and exactly 6 of \( y_i \). Then value of \( n \) is

Ans. \( n = 24 \).

Sol. Given \( \bigcup_{i=1}^{50} x_i = \bigcup_{i=1}^{n} y_i = z \)

Then, \( \frac{10 \times 50}{25} = \frac{5 \times n}{6} \) \( \Rightarrow n = 24 \).

Q. If \( P Q \) is a diameter of circle \( x^2 + y^2 = a^2 \). If perpendicular distance of \( P \) and \( Q \) from the line \( x + y = 2 \) are \( \alpha \) and \( \beta \) respectively, then maximum value of \( \sqrt{\alpha \beta} \) is

Ans. \( 2 \)

Let \( P (2 \cos \alpha, 2 \sin \alpha) \Rightarrow Q (-2 \cos \alpha, -2 \sin \alpha) \)

Then, \( \alpha = \frac{2 \cos \alpha + 2 \sin \alpha - 2}{\sqrt{2}} \) and \( \beta = \frac{2 \cos \alpha + 2 \sin \alpha + 2}{\sqrt{2}} \)

\( \alpha \beta = \left( \sqrt{2} \right)^2 \left( \cos \alpha + \sin \alpha - 1 \right) \left( \cos \alpha + \sin \alpha + 1 \right) \)

\( \alpha \beta = 2 \cdot \sin 2\alpha \) \( \max = 1 \)

\( \left( \sqrt{\alpha \beta} \right) \max = 2 \)
Q:— If points A and B lie on x axis, and points C and D lie on the curve 

\[ y = x^2 - 1 \], below the x axis, then 

maximum area of rectangle ABCD in :-

(1) \( \frac{\sqrt{3}}{3} \)  
(2) \( \frac{\sqrt{3}}{9} \)  
(3) \( \frac{\sqrt{3}}{27} \)  
(4) \( \frac{8}{3} \sqrt{3} \)

Ans:- (2)

Let

A \( (x, 0) \) and B \( (-x, 0) \)
C \( (-x, x^2 - 1) \)
D \( (x, x^2 - 1) \)

Area of rectangle \( ABCD \) 
\( \text{Area} = (2x)(1-x^2) \)

\[
\frac{dA}{dx} = 2(1-x^2) - 4x^2
\]
\[
\frac{dA}{dx} = 0 \Rightarrow x^2 = \frac{1}{3}
\]
\[
\frac{d^2A}{dx^2} = -\frac{4}{3}x < 0 \quad \text{at} \quad x = \frac{1}{\sqrt{3}}
\]

Then, \( \text{Area} = \frac{4}{3\sqrt{3}} (1 - \frac{1}{3}) \)
\[
= \frac{4}{\sqrt{3}} \text{ sq. unit}
\]
Q:- There are 6 multiple choice questions in a paper. Each having 4 options, of which only one is correct. In how many ways a person can solve exactly four correct questions, if he attempted all 6 questions.

(1) 134  (2) 135  (3) 136  (4) 137

Ans:- (2)

Sol:- Each question has 4 options in which only one is correct.

Hence no. of ways for 4 correct questions = \( \binom{6}{4} \times 4^4 \times 3^2 \)

= 15 \times 9

= 135.