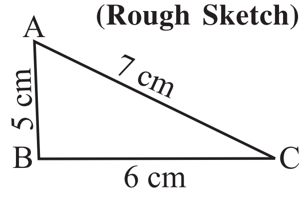
the three lines. See the following example:

EXAMPLE 1 Construct a triangle ABC, given that AB = 5 cm, BC = 6 cm and AC = 7 cm.

SOLUTION

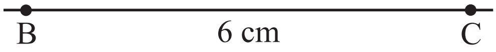
Step 1 First, we draw a rough sketch with given measure, (This will help us in deciding how to proceed) [Fig 10.3(i)].

(Rough Sketch)



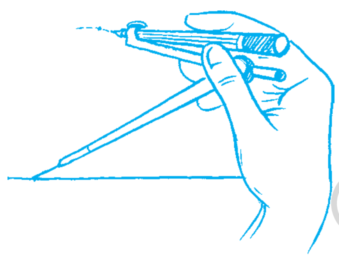
(i)

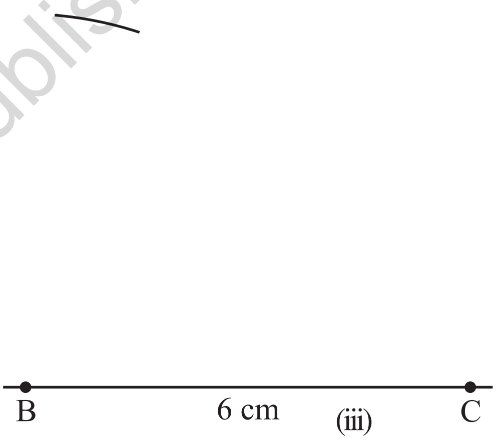
Step 2 Draw a line segment BC of length 6 cm [Fig 10.3(ii)].



(ii)

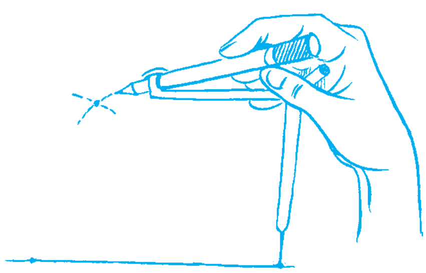
Step 3 From B, point A is at a distance of 5 cm. So, with B as centre, draw an arc of radius 5 cm. (Now A will be somewhere on this arc. Our job is to find where exactly A is) [Fig 10.3(iii)].

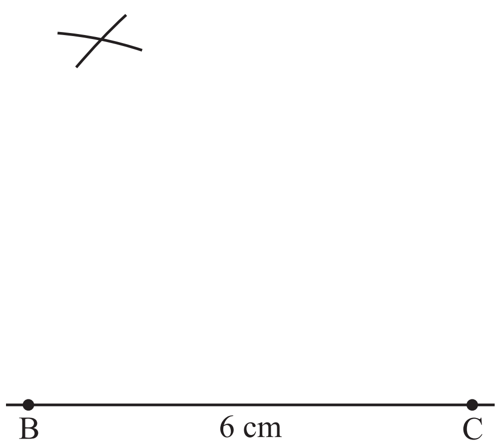




(iii)

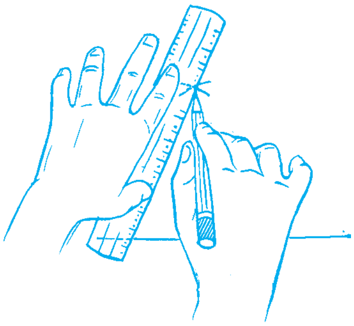
Step 4 From C, point A is at a distance of 7 cm. So, with C as centre, draw an arc of radius 7 cm. (A will be somewhere on this arc, we have to fix it) [Fig 10.3(iv)].

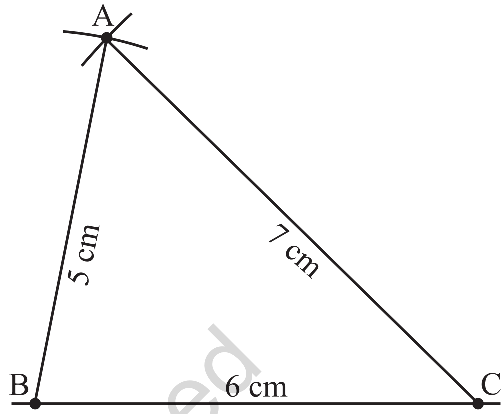




(iv)

Step 5 A has to be on both the arcs drawn. So, it is the point of intersection of arcs. Mark the point of intersection of arcs as A. Join AB and AC. ΔABC is now ready [Fig 10.3(v)].

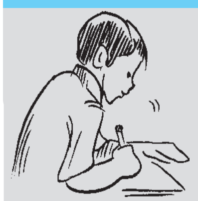




(v)

Fig 10.3 (i) – (v)

Do This



Now, let us construct another triangle DEF such that DE = 5 cm, EF = 6 cm, and DF = 7 cm. Take a cutout of ΔDEF and place it on ΔABC. What do we observe?

We observe that ΔDEF exactly coincides with ΔABC. (Note that the triangles have been constructed when their three sides are given.) Thus, if three sides of one triangle are equal to the corresponding three sides of another triangle, then the two triangles are congruent. This is SSS congruency rule which we have learnt in our earlier chapter.

Think, Discuss and Write

A student attempted to draw a triangle whose rough figure is given here. He drew QR first. Then with Q as centre, he drew an arc of 3 cm and with R as centre, he drew an arc of 2 cm. But he could not get P. What is the reason? What property of triangle do you know in connection with this problem?

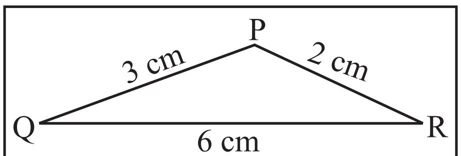
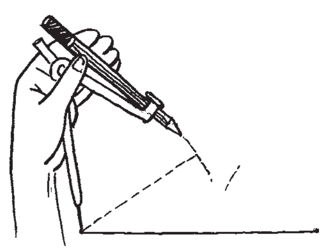
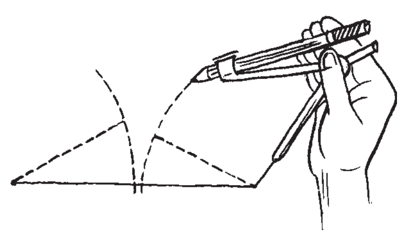


Fig 10.4 Think: Is this right?

Can such a triangle exist? (Remember the property of triangles 'The sum of any two sides of a traingle is always greater than the third side'!)





Exercise 10.2

1. Construct ΔXYZ in which XY = 4.5 cm, YZ = 5 cm and ZX = 6 cm.
2. Construct an equilateral triangle of side 5.5 cm.
3. Draw ΔPQR with PQ = 4 cm, QR = 3.5 cm and PR = 4 cm. What type of triangle is this?
4. Construct ΔABC such that AB = 2.5 cm, BC = 6 cm and AC = 6.5 cm. Measure ∠B.

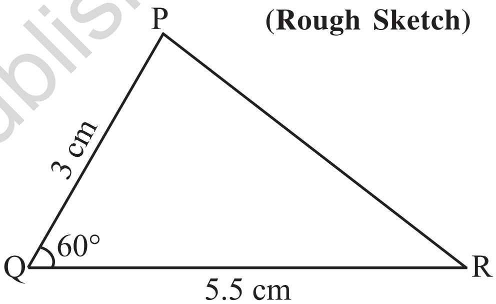
10.5 Constructing a Triangle when the Lengths of Two Sides and the Measure of the Angle Between them are Known. (SAS Criterion)

Here, we have two sides given and the one angle between them. We first draw a sketch and then draw one of the given line segments. The other steps follow. See Example 2.

Example 2 Construct a triangle PQR, given that PQ = 3 cm, QR = 5.5 cm and ∠PQR = 60°.

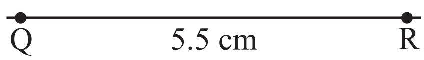
Solution

Step 1 First, we draw a rough sketch with given measures. (This helps us to determine the procedure in construction) [Fig 10.5(i)].



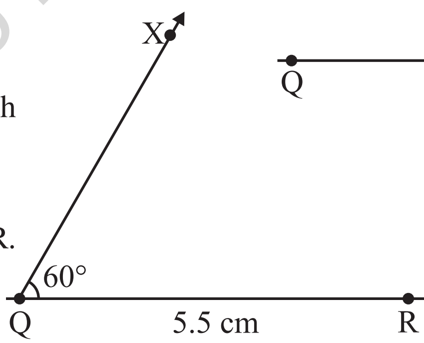
(i)

Step 2 Draw a line segment QR of length 5.5 cm [Fig 10.5(ii)].



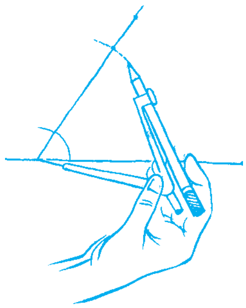
(ii)

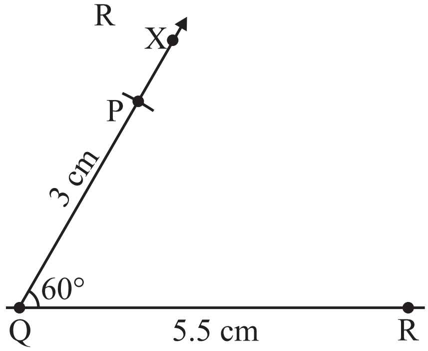
Step 3 At Q, draw QX making 60° with QR. (The point P must be somewhere on this ray of the angle) [Fig 10.5(iii)].



(iii)

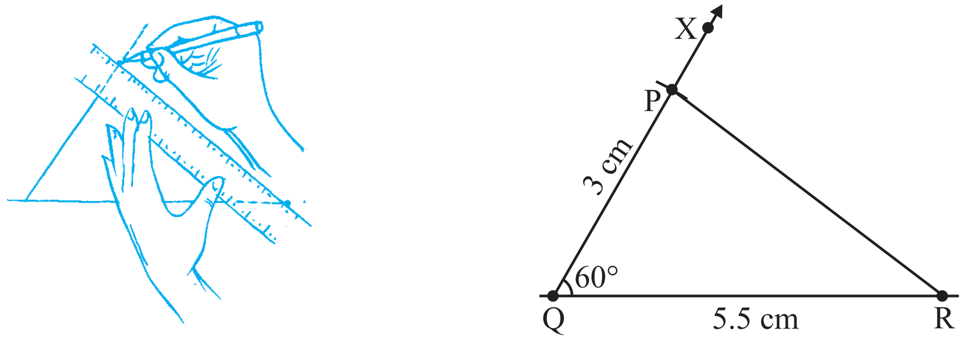
Step 4 (To fix P, the distance QP has been given).
With Q as centre, draw an arc of radius 3 cm. It cuts QX at the point P [Fig 10.5(iv)].





(iv)

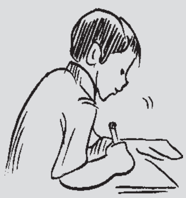
Step 5 Join PR. ΔPQR is now obtained (Fig 10.5(v)).



(v)

Fig 10.5 (i) – (v)

DO THIS



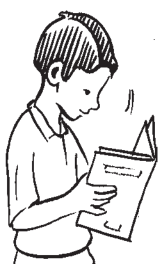
Let us now construct another triangle ABC such that AB = 3 cm, BC = 5.5 cm and m∠ABC = 60°. Take a cut out of ΔABC and place it on ΔPQR. What do we observe? We observe that ΔABC exactly coincides with ΔPQR. Thus, if two sides and the included angle of one triangle are equal to the corresponding two sides and the included angle of another triangle, then the two triangles are congruent. This is SAS congruency rule which we have learnt in our earlier chapter. (Note that the triangles have been constructed when their two sides and the angle included between these two sides are given.)

THINK, DISCUSS AND WRITE

In the above construction, lengths of two sides and measure of one angle were given. Now study the following problems:

In ΔABC, if AB = 3cm, AC = 5 cm and m∠C = 30°. Can we draw this triangle? We may draw AC = 5 cm and draw ∠C of measure 30°. CA is one arm of ∠C. Point B should be lying on the other arm of ∠C. But, observe that point B cannot be located uniquely. Therefore, the given data is not sufficient for construction of ΔABC.

Now, try to construct ΔABC if AB = 3cm, AC = 5 cm and m∠B = 30°. What do we observe? Again, ΔABC cannot be constructed uniquely. Thus, we can conclude that a unique triangle can be constructed only if the lengths of its two sides and the measure of the included angle between them is given.



EXERCISE 10.3

1. Construct ΔDEF such that DE = 5 cm, DF = 3 cm and m∠EDF = 90°.
2. Construct an isosceles triangle in which the lengths of each of its equal sides is 6.5 cm and the angle between them is 110°.
3. Construct ΔABC with BC = 7.5 cm, AC = 5 cm and m∠C = 60°.

10.6 Constructing a Triangle when the Measures of two of its Angles and the Length of the Side included between Them is Given. (ASA Criterion)

As before, draw a rough sketch. Now, draw the given line segment. Make angles on the two ends. See the Example 3.

Example 3 Construct ΔXYZ if it is given that XY = 6 cm, m∠ZXY = 30° and m∠XYZ = 100°.

Solution

Step 1 Before actual construction, we draw a rough sketch with measures marked on it. (This is just to get an idea as how to proceed) [Fig 10.6(i)].

Step 2 Draw XY of length 6 cm.

Step 3 At X, draw a ray XP making an angle of 30° with XY. By the given condition Z must be somewhere on the XP.

Step 4 At Y, draw a ray YQ making an angle of 100° with YX. By the given condition, Z must be on the ray YQ also.

Step 5 Z has to lie on both the rays XP and YQ. So, the point of intersection of the two rays is Z.

ΔXYZ is now completed.

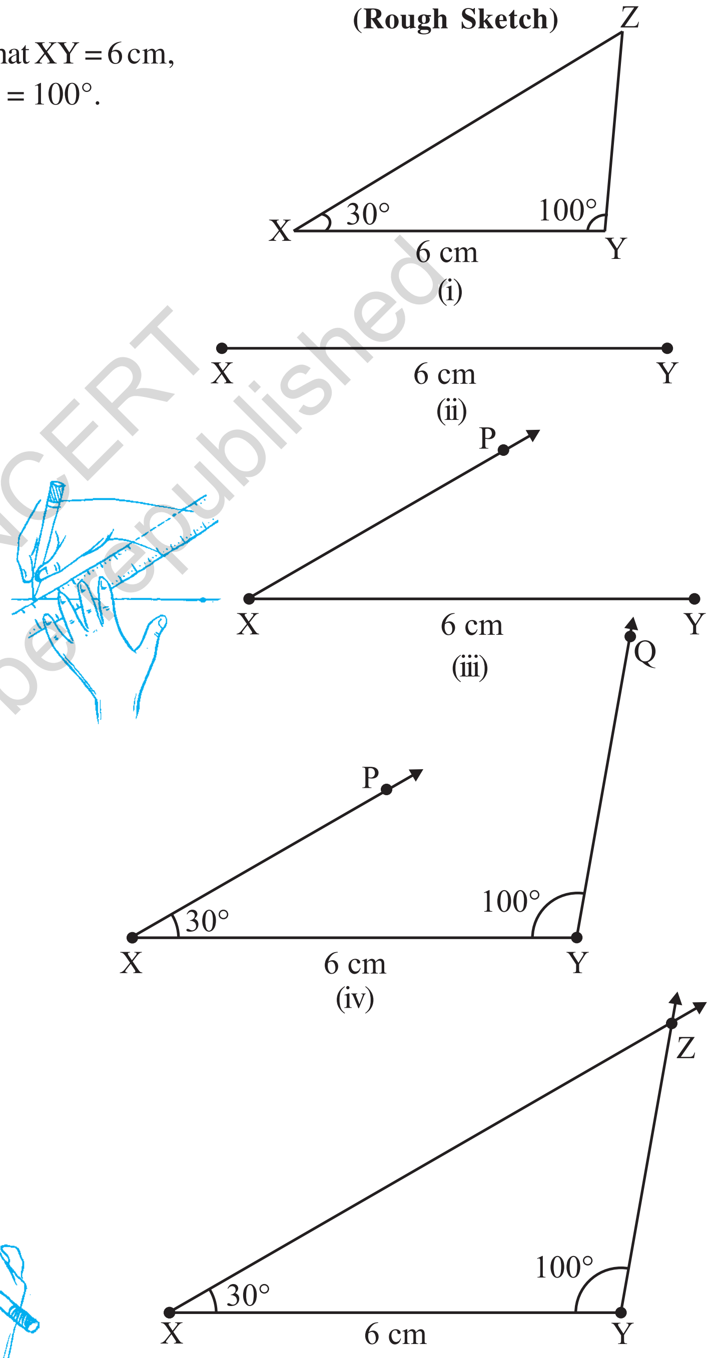


Fig 10.6 (i) – (v)

Do This

Now, draw another ΔLMN, where m∠NLM = 30°, LM = 6 cm and m∠NML = 100°. Take a cutout of ΔLMN and place it on the ΔXYZ. We observe that ΔLMN exactly coincides with ΔXYZ. Thus, if two angles and the included side of one triangle are equal to the corresponding two angles and the included side of another triangle, then the two triangles are congruent. This is ASA congruency rule which you have learnt in the earlier chapter. (Note that the triangles have been constructed when two angles and the included side between these angles are given.)

Think, Discuss and Write

In the above example, length of a side and measures of two angles were given. Now study the following problem:

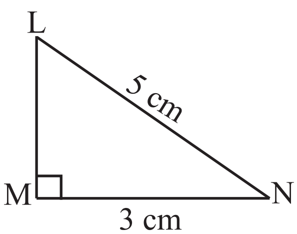
In ΔABC, if AC = 7 cm, m∠A = 60° and m∠B = 50°, can you draw the triangle? (Angle-sum property of a triangle may help you!)

Exercise 10.4

1. Construct ΔABC, given m∠A = 60°, m∠B = 30° and AB = 5.8 cm.
2. Construct ΔPQR if PQ = 5 cm, m∠PQR = 105° and m∠QRP = 40°.
   (Hint: Recall angle-sum property of a triangle).
3. Examine whether you can construct ΔDEF such that EF = 7.2 cm, m∠E = 110° and m∠F = 80°. Justify your answer.

10.7 Constructing a Right-Angled Triangle when the Length of One Leg and its Hypotenuse are Given (RHS Criterion)

(Rough Sketch)



(i)

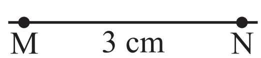
Here it is easy to make the rough sketch. Now, draw a line as per the given side. Make a right angle on one of its points. Use compasses to mark length of side and hypotenuse of the triangle. Complete the triangle. Consider the following:

Example 4 Construct ΔLMN, right-angled at M, given that LN = 5 cm and MN = 3 cm.

Solution

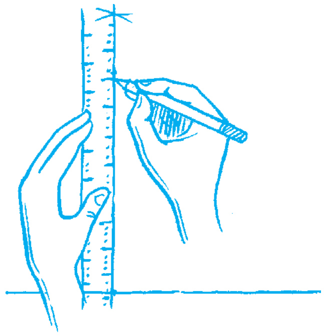
Step 1 Draw a rough sketch and mark the measures. Remember to mark the right angle [Fig 10.7(i)].

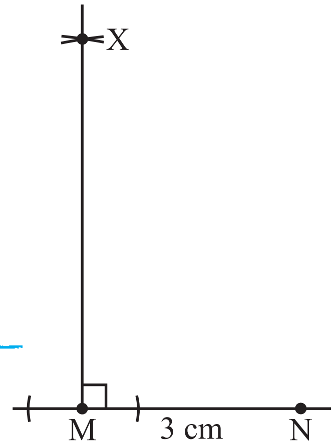
Step 2 Draw MN of length 3 cm. [Fig 10.7(ii)].



(ii)

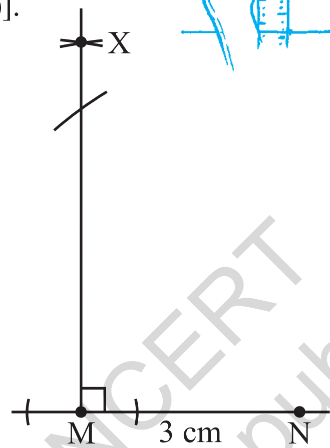
Step 3 At M, draw MX ⊥ MN. (L should be somewhere on this perpendicular) [Fig 10.7(iii)].





(iii)

Step 4 With N as centre, draw an arc of radius 5 cm. (L must be on this arc, since it is at a distance of 5 cm from N) [Fig 10.7(iv)].

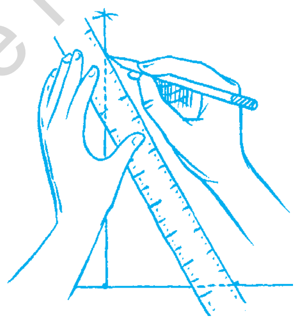


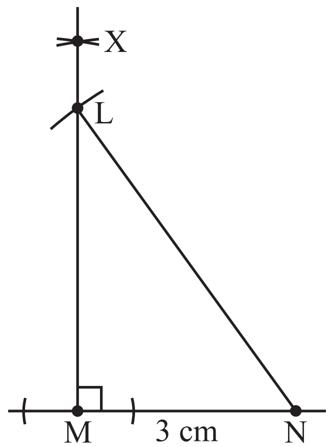
(iv)

Step 5 L has to be on the perpendicular line MX as well as on the arc drawn with centre N. Therefore, L is the meeting point of these two.

ΔLMN is now obtained.

[Fig 10.7 (v)]





(v)

Fig 10.7 (i) – (v)

Exercise 10.5

1. Construct the right angled ΔPQR, where m∠Q = 90°, QR = 8cm and PR = 10 cm.
2. Construct a right-angled triangle whose hypotenuse is 6 cm long and one of the legs is 4 cm long.
3. Construct an isosceles right-angled triangle ABC, where m∠ACB = 90° and AC = 6 cm.

Miscellaneous questions

Below are given the measures of certain sides and angles of triangles. Identify those which cannot be constructed and, say why you cannot construct them. Construct rest of the triangles.

	Triangle	Given measurements		
1.	ΔABC	m∠A = 85°;	m∠B = 115°;	AB = 5 cm.
2.	ΔPQR	m∠Q = 30°;	m∠R = 60°;	QR = 4.7 cm.
3.	ΔABC	m∠A = 70°;	m∠B = 50°;	AC = 3 cm.
4.	ΔLMN	m∠L = 60°;	m∠N = 120°;	LM = 5 cm.
5.	ΔABC	BC = 2 cm;	AB = 4 cm;	AC = 2 cm.
6.	ΔPQR	PQ = 3.5 cm.;	QR = 4 cm.;	PR = 3.5 cm.
7.	ΔXYZ	XY = 3 cm;	YZ = 4 cm;	XZ = 5 cm
8.	ΔDEF	DE = 4.5cm;	EF = 5.5cm;	DF = 4 cm.

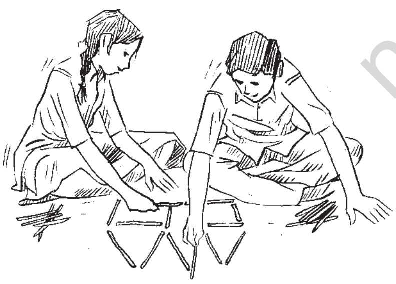
What have We Discussed?

In this Chapter, we looked into the methods of some ruler and compasses constructions.

1. Given a line *l* and a point not on it, we used the idea of 'equal alternate angles' in a transversal diagram to draw a line parallel to *l*.

We could also have used the idea of 'equal corresponding angles' to do the construction.

2. We studied the method of drawing a triangle, using indirectly the concept of congruence of triangles.



The following cases were discussed:

(i) SSS: Given the three side lengths of a triangle.

(ii) SAS: Given the lengths of any two sides and the measure of the angle between these sides.

(iii) ASA: Given the measures of two angles and the length of side included between them.

(iv) RHS: Given the length of hypotenuse of a right-angled triangle and the length of one of its legs.