The squares are identical; the eight triangles inserted are also identical.

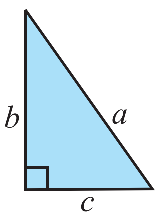
Hence the uncovered area of square A = Uncovered area of square B.

i.e., Area of inner square of square A = The total area of two uncovered squares in square B.

$$a^2 = b^2 + c^2$$

This is Pythagoras property. It may be stated as follows:

> In a right-angled triangle,
> the square on the hypotenuse = sum of the squares on the legs.



Pythagoras property is a very useful tool in mathematics. It is formally proved as a theorem in later classes. You should be clear about its meaning.

It says that for any right-angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the legs.

Draw a right triangle, preferably on a square sheet, construct squares on its sides, compute the area of these squares and verify the theorem practically (Fig 6.26).

If you have a right-angled triangle, the Pythagoras property holds. If the Pythagoras property holds for some triangle, will the triangle be right-angled? (Such problems are known as converse problems). We will try to answer this. Now, we will show that, if there is a triangle such that sum of the squares on two of its sides is equal to the square of the third side, it must be a right-angled triangle.

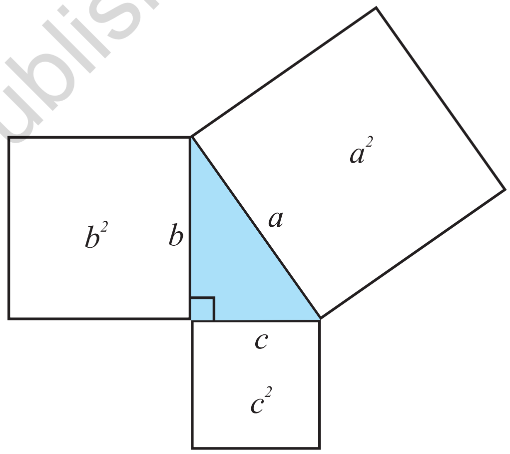


Fig 6.26

Do This

1. Have cut-outs of squares with sides 4 cm, 5 cm, 6 cm long. Arrange to get a triangular shape by placing the corners of the squares suitably as shown in the figure (Fig 6.27). Trace out the triangle formed. Measure each angle of the triangle. You find that there is no right angle at all.

In fact, in this case each angle will be acute! Note that $4^2 + 5^2 \neq 6^2$, $5^2 + 6^2 \neq 4^2$ and $6^2 + 4^2 \neq 5^2$.

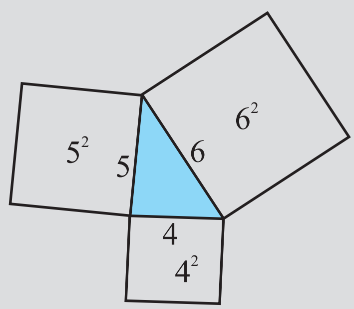


Fig 6.27

2. Repeat the above activity with squares whose sides have lengths 4 cm, 5 cm and 7 cm. You get an obtuse-angled triangle! Note that

$$4^2 + 5^2 \neq 7^2 \text{ etc.}$$

This shows that Pythagoras property holds if and only if the triangle is right-angled. Hence we get this fact:

If the Pythagoras property holds, the triangle must be right-angled.

EXAMPLE 5 Determine whether the triangle whose lengths of sides are 3 cm, 4 cm, 5 cm is a right-angled triangle.

SOLUTION $3^2 = 3 \times 3 = 9$; $4^2 = 4 \times 4 = 16$; $5^2 = 5 \times 5 = 25$

We find $3^2 + 4^2 = 5^2$.

Therefore, the triangle is right-angled.

Note: In any right-angled triangle, the hypotenuse happens to be the longest side. In this example, the side with length 5 cm is the hypotenuse.

EXAMPLE 6 Δ ABC is right-angled at C. If AC = 5 cm and BC = 12 cm find the length of AB.

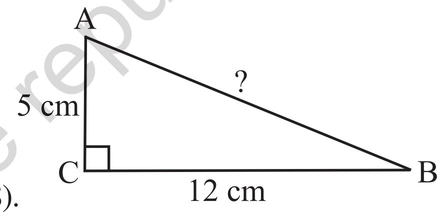


Fig 6.28

SOLUTION A rough figure will help us (Fig 6.28).

By Pythagoras property,

$$AB^2 = AC^2 + BC^2$$

$$= 5^2 + 12^2 = 25 + 144 = 169 = 13^2$$

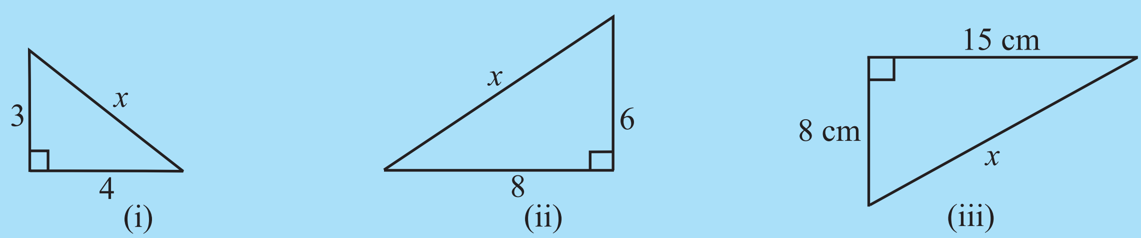
or $AB^2 = 13^2$. So, $AB = 13$

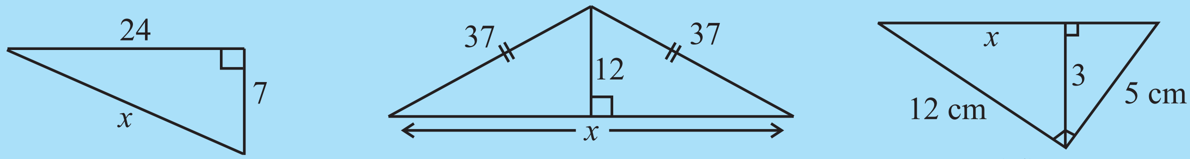
or the length of AB is 13 cm.

Note: To identify perfect squares, you may use prime factorisation technique.

TRY THESE

Find the unknown length x in the following figures (Fig 6.29):

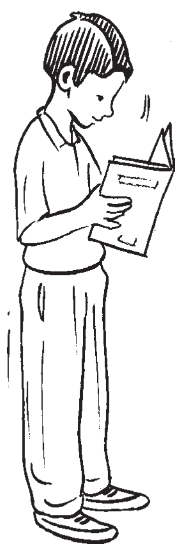




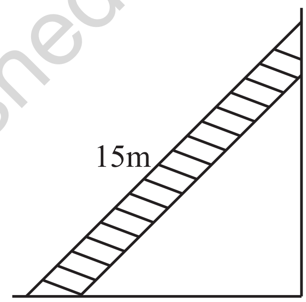
(iv) (v) (vi)

Fig 6.29

EXERCISE 6.5



1. PQR is a triangle, right-angled at P. If PQ = 10 cm and PR = 24 cm, find QR.
2. ABC is a triangle, right-angled at C. If AB = 25 cm and AC = 7 cm, find BC.
3. A 15 m long ladder reached a window 12 m high from the ground on placing it against a wall at a distance *a*. Find the distance of the foot of the ladder from the wall.



4. Which of the following can be the sides of a right triangle?

   (i) 2.5 cm,6.5 cm, 6 cm.

   (ii) 2 cm, 2 cm, 5 cm.

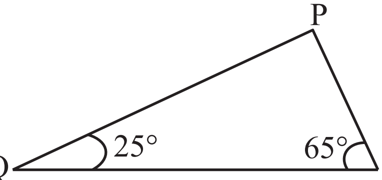
   (iii) 1.5 cm, 2cm, 2.5 cm.

   In the case of right-angled triangles, identify the right angles.
5. A tree is broken at a height of 5 m from the ground and its top touches the ground at a distance of 12 m from the base of the tree. Find the original height of the tree.
6. Angles Q and R of a ΔPQR are 25° and 65°. Write which of the following is true:

   (i) $PQ^2 + QR^2 = RP^2$

   (ii) $PQ^2 + RP^2 = QR^2$

   (iii) $RP^2 + QR^2 = PQ^2$



7. Find the perimeter of the rectangle whose length is 40 cm and a diagonal is 41 cm.
8. The diagonals of a rhombus measure 16 cm and 30 cm. Find its perimeter.

Think, Discuss and Write

1. Which is the longest side in the triangle PQR, right-angled at P?
2. Which is the longest side in the triangle ABC, right-angled at B?
3. Which is the longest side of a right triangle?
4. 'The diagonal of a rectangle produce by itself the same area as produced by its length and breadth'– This is Baudhayan Theorem. Compare it with the Pythagoras property.

Do This

Enrichment activity

There are many proofs for Pythagoras theorem, using 'dissection' and 'rearrangement' procedure. Try to collect a few of them and draw charts explaining them.

What have We Discussed?

1. The **six elements** of a triangle are its **three angles** and the **three sides**.
2. The line segment joining a vertex of a triangle to the mid point of its opposite side is called a **median** of the triangle. A triangle has 3 medians.
3. The perpendicular line segment from a vertex of a triangle to its opposite side is called an **altitude** of the triangle. A triangle has 3 altitudes.
4. An **exterior angle** of a triangle is formed, when a side of a triangle is produced. At each vertex, you have two ways of forming an exterior angle.
5. A property of exterior angles:

   The measure of any exterior angle of a triangle is equal to the sum of the measures of its interior opposite angles.
6. The angle sum property of a triangle:

   The total measure of the three angles of a triangle is 180°.
7. A triangle is said to be **equilateral,** if each one of its sides has the same length.

   In an equilateral triangle, each angle has measure 60°.
8. A triangle is said to be **isosceles,** if atleast any two of its sides are of same length.

   The non-equal side of an isosceles triangle is called its **base**; the base angles of an isosceles triangle have equal measure.
9. Property of the lengths of sides of a triangle:

   The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

   The difference between the lengths of any two sides is smaller than the length of the third side.

This property is useful to know if it is possible to draw a triangle when the lengths of the three sides are known.

10. In a right angled triangle, the side opposite to the right angle is called the **hypotenuse** and the other two sides are called its **legs**.

11. **Pythagoras property**:

In a right-angled triangle,

the square on the hypotenuse = the sum of the squares on its legs.

If a triangle is not right-angled, this property does not hold good. This property is useful to decide whether a given triangle is right-angled or not.