Jee Main 2020(Sep)
02-Sep-2020 (Evening Shift)

Question Paper, Key and Solutions
Physics

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 in all other cases.

1. In a Young’s double slit experiment, 16 fringes are observed in a certain segment of the screen when light of wavelength 700 nm is used. If the wavelength of light is changed to 400 nm, the number of fringes observed in the same segment of the screen would be:
   1) 28 2) 18 3) 24 4) 30

Key: 1

Sol:

\[ \lambda = 700 \text{nm} \]

\[ Z = 16\beta = \frac{16\lambda D}{d} = \frac{16(700 \text{nm})D}{d} \quad (1) \]

\[ \lambda = 400 \text{nm} \]

\[ Z = n \lambda D \quad (2) \]

From (1) and (2)

\[ 16(700 \text{nm}) \frac{D}{d} = n(400 \text{nm}) \frac{D}{d} \]

\[ n = 28 \]
2. A potentiometer wire PQ of 1 m length is connected to a standard cell \( E_1 \). Another cell \( E_2 \) of emf 1.02 V is connected with a resistance ‘r’ and switch S (as shown in figure). With switch S open, the null position is obtained at a distance of 49 cm from Q. The potential gradient in the potentiometer wire is:

1) 0.02 V/cm  
2) 0.01 V/cm  
3) 0.04 V/cm  
4) 0.03 V/cm

Key: 1

Sol: As switch S is open

→ Under balancing condition, no current flows through secondary circuit.

So \( V_{PJ} = E_2 \)

(Pot. gradient) \( \ell_{PJ} = E_2 \)

\[
\text{(Pot. gradient)} = \frac{1.02 V}{51 \text{ cm}} = 0.02 \frac{V}{\text{cm}}
\]
3. A capillary tube made of glass of radius 0.15 mm is dipped vertically in a beaker filled with methylene iodide (surface tension = 0.05 Nm\(^{-1}\), density = 667 kg m\(^{-3}\)) which rises to height \(h\) in the tube. It is observed that the two tangents drawn from liquid-glass interfaces (from opp. Sides of the capillary) make an angle of 60\(^\circ\) with one another. Then \(h\) is close to \((g = 10 \text{ ms}^{-2})\).

1) 0.049 m  
2) 0.172 m  
3) 0.087 m  
4) 0.137 m

Key: 3

Sol:  
\[ h = \frac{2S \cos \theta}{\rho rg} \]  
\[ \theta = 30^\circ = \frac{2 \times 0.05 \cos 30^\circ}{667 \times 0.15 \times 10^{-3} \times 10} \]

\[ h = 0.087 \text{ m} \]

4. A charge \(Q\) is distributed over two concentric conducting thin spherical shells radii \(r\) and \(R\) \((R > r)\). If the surface charge densities on the two shells are equal, the electric potential at the common centre is:

1) \[ \frac{1}{4\pi\varepsilon_0} \frac{(2R + r)}{(R^2 + r^2)}Q \]

2) \[ \frac{1}{4\pi\varepsilon_0} \frac{(R + r)}{(R^2 + r^2)}Q \]

3) \[ \frac{1}{4\pi\varepsilon_0} \frac{(R + 2r)Q}{2(R^2 + r^2)} \]

4) \[ \frac{1}{4\pi\varepsilon_0} \frac{(R + r)}{2(R^2 + r^2)}Q \]

Key: 2
Sol: \[
\frac{q_1}{4\pi r^2} = \frac{q_2}{4\pi R^2}
\]

\[
\frac{q_1}{q_2} = \frac{r^2}{R^2} \rightarrow (1) \quad \& \quad q_1 + q_2 = Q \rightarrow (2)
\]

\[
q_1 = \frac{r^2}{r^2 + R^2} Q \quad \& \quad q_2 = \frac{R^2}{r^2 + R^2} Q
\]

Pot at common centre \[
V = \frac{Kq_1}{r} + \frac{Kq_2}{R} = \frac{K Q r}{r^2 + R^2} + \frac{KQR}{r^2 + R^2} = \frac{KQ(r + R)}{r^2 + R^2}
\]

5. The height \(h\) at which the weight of a body will be the same as that at the same depth \(h\) from the surface of the earth is (Radius of the earth is \(R\) and effect of the rotation of the earth is neglected):

1) \(\frac{\sqrt{3}R - R}{2}\)
2) \(\frac{\sqrt{5}}{2}R - R\)
3) \(\frac{\sqrt{5}R - R}{2}\)
4) \(\frac{R}{2}\)

Key: 3

Sol: \[
g \text{ at height } h = \frac{GM_e}{(R_e + h)^2}
\]

\[
g \text{ at depth } d = \frac{GM_e}{R_e^2} \left(1 - \frac{d}{R_e}\right)
\]

For Weight to be equal \[
\frac{GM_e}{R_e^2 \left(1 + \frac{h}{R_e}\right)^2} = \frac{GM_e}{R_e^2} \left(1 - \frac{d}{R_e}\right)
\]

Given \(d = h\)

\[
1 = \left(1 + \frac{h}{R_e}\right)^2 \left(1 - \frac{h}{R_e}\right)
\]

\[
h^2 + hR - R^2 = 0 \quad h = \frac{\sqrt{5}R - R}{2}
\]
6. Two uniform circular discs are rotating independently in the same direction around their common axis passing through their centres. The moment of inertia and angular velocity of the first disc are 0.1 kg·m² and 10 rad s⁻¹ respectively while those for the second one are 0.2 kg·m² and 5 rad s⁻¹ respectively. At some instant they get stuck together and start rotating as a single system about their common axis with some angular speed. The kinetic energy of the combined system is:

1) \( \frac{10}{3} \text{ J} \)  
2) \( \frac{2}{3} \text{ J} \)  
3) \( \frac{5}{3} \text{ J} \)  
4) \( \frac{20}{3} \text{ J} \)

Key: 4

Sol: After they stuck, let they move with angular velocity \( \omega \). By conserving Angular momentum

\[ I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega \]

\[ \omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} = \frac{20}{3} \text{ rad / s} \]

KE of system \( = \frac{1}{2}(I_1 + I_2)\omega^2 = \frac{20}{3} \text{ J} \)

7. An inductance coil has a reactance of 100Ω. When an AC signal of frequency 1000 Hz is applied to the coil, the applied voltage leads the current by 45°. The self-inductance of the coil is:

1) \( 6.7 \times 10^{-7} \text{ H} \)  
2) \( 1.1 \times 10^{-1} \text{ H} \)  
3) \( 5.5 \times 10^{-5} \text{ H} \)  
4) \( 1.1 \times 10^{-2} \text{ H} \)

Key: 4

Sol: \( \tan \phi = \frac{X_L}{R} \)

\[ \tan 45^0 = \frac{X_L}{R} \]

\[ X_L = R \]

\[ Z = \sqrt{R^2 + X_L^2} \]
\[ Z = \sqrt{2}X_L \]
\[ 100 = \sqrt{2}X_L \Rightarrow X_L = \frac{100}{\sqrt{2}} \]
\[ \omega L = \frac{100}{\sqrt{2}} \Rightarrow 2\pi fL = \frac{100}{\sqrt{2}} \Rightarrow L = 1.1 \times 10^{-2} \text{H}. \]

8. In the following, digital circuit, what will be the output at ‘Z’, when the inputs (A, B) are (1, 0), (0, 0), (1, 1), (0, 1):

1) 0, 0, 1, 0
2) 0, 1, 0, 0
3) 1, 1, 0, 1
4) 1, 0, 1, 1

Key: 1

Sol:

\[ C = \overline{A}B \]
\[ D = A + B \]
\[ E = CD \]
\[ Z = C + E \]

Now we will check for each input:

- For \( A = 1 \), \( B = 0 \)
  - \( C = 1 \)
  - \( D = 1 \)
  - \( E = 1 \)
  - \( Z = 0 \)

- For \( A = 0 \), \( B = 0 \)
  - \( C = 1 \)
  - \( D = 0 \)
  - \( E = 0 \)
  - \( Z = 0 \)

- For \( A = 1 \), \( B = 1 \)
  - \( C = 1 \)
  - \( D = 1 \)
  - \( E = 0 \)
  - \( Z = 1 \)

- For \( A = 0 \), \( B = 0 \)
  - \( C = 1 \)
  - \( D = 1 \)
  - \( E = 1 \)
  - \( Z = 0 \)
9. In a hydrogen atom the electron makes a transition from \((n + 1)\)th level to the \(n\)th level.

If \(n >> 1\), the frequency of radiation emitted is proportional to:

1) \(\frac{1}{n}\)  
2) \(\frac{1}{n^3}\)  
3) \(\frac{1}{n^4}\)  
4) \(\frac{1}{n^2}\)

Key: 2

Sol:

\[
\frac{1}{\lambda} = RZ^2 \left[ \frac{1}{n^2} - \frac{1}{(n+1)^2} \right]
\]

\[
f = \frac{c}{\lambda} = RcZ^2 \left[ \frac{1}{n^2} - \frac{1}{(n+1)^2} \right]
\]

\[
f = RcZ^2 \left[ \frac{2(n+1)}{n^2(n+1)^2} \right]
\]

For \(n>>>>\) \(f \propto \frac{n}{n^4} \Rightarrow f \propto \frac{1}{n^3}\)

10. A particle is moving 5 times as fast as an electron. The ratio of the de-Broglie wavelength of the particle to that of the electron is \(1.878 \times 10^{-4}\). The mass of the particle is close to:

1) \(4.8 \times 10^{-27}\) kg  
2) \(1.2 \times 10^{-28}\) kg  
3) \(9.1 \times 10^{-31}\) kg  
4) \(9.7 \times 10^{-28}\) kg

Key: 4

Sol:

\[
\lambda_p = \frac{h}{M_p V_p}; \quad \lambda_e = \frac{h}{M_e V_e}
\]

\[
\frac{\lambda_p}{\lambda_e} = \frac{M_e V_e}{M_p V_p}
\]

Given \(V_p = 5V_e\)

\[
1.878 \times 10^{-4} = \frac{M_e}{M_p} \frac{1}{5}
\]

\[
M_p = \frac{M_e}{5 \times 1.878 \times 10^{-4}} = 9.69 \times 10^{-28}\) Kg \(\approx 9.7 \times 10^{-28}\) Kg
11. The figure shows a region of length \( l \) with a uniform magnetic field of 0.3T in it and a proton entering the region with velocity \( 4 \times 10^5 \) ms\(^{-1} \) making an angle 60° with the field. If the proton completes 10 revolution by the time it cross the region shown, \( l \) is close to (mass of proton = \( 1.67 \times 10^{-27} \) kg, charge of the proton = \( 1.6 \times 10^{-19} \) C)

1) 0.44 m  
2) 0.22 m  
3) 0.88 m  
4) 0.11 m

Key: 1

Sol: 
\[
\ell = 10\text{ (pitch)} = 10 \left( \frac{2\pi M}{qB} V \cos \theta \right) = \frac{10 \times 2\pi \times 1.67 \times 10^{-27} \times 4 \times 10^5 \cos 60^0}{1.6 \times 10^{-19} \times 0.3}
\]

\[
\ell = 0.44m
\]

12. A 10\( \mu \)F capacitor is fully charged to a potential difference of 50V. After removing the source voltage it is connected to an uncharged capacitor in parallel. Now the potential difference across them becomes 20V. The capacitance of the second capacitor is:

1) 15\( \mu \)F  
2) 10\( \mu \)F  
3) 30\( \mu \)F  
4) 20\( \mu \)F

Key: 1
13. A heat engine is involved with exchange of heat of 1915 J, -40J, +125J and -QJ during one cycle achieving an efficiency of 50.0%. The value of Q is:
1) 400J  
2) 640J  
3) 980J  
4) 40J

Key: 3

Sol: \(\% \eta = \frac{W_{\text{net}}}{Q_{\text{absorbed}}} \times 100\) For closed cycle \(W_{\text{net}} = Q_{\text{net}}\)

\[
50 = \frac{1915 - 40 + 125 - Q}{1915 + 125} \times 100
\]

\[Q = 980J\]

14. In a plane electromagnetic wave, the directions of electric filed and magnetic field are represented by \(\hat{k}\) and \(2\hat{i} - 2\hat{j}\), respectively. What is the unit vector along direction of propagation of the wave.

1) \(\frac{1}{\sqrt{5}}(2\hat{i} + \hat{j})\)  
2) \(\frac{1}{\sqrt{2}}(\hat{j} + \hat{k})\)  
3) \(\frac{1}{\sqrt{2}}(\hat{i} + \hat{j})\)  
4) \(\frac{1}{\sqrt{5}}(\hat{i} + 2\hat{j})\)

Key: 3

Sol: \(\hat{P} = \hat{E} \times \hat{B}\)

Where \(\hat{P}\) = Unit vector along propagation direction

\[
\hat{P} = \hat{k} \times \left(\frac{2\hat{i} - 2\hat{j}}{2\sqrt{2}}\right) = \frac{1}{\sqrt{2}}(\hat{j} + \hat{i})
\]
15. A wire carrying current I is bent in the shape ABCDEFA as shown, where rectangle ABCDA and ADEFA are perpendicular to each other. If the sides of the rectangles are of lengths a and b, then the magnitude and direction of magnetic moment of the loop ABCDEFA is:

1) \( \sqrt{2}abI \), along \( \hat{j} + \frac{\hat{k}}{\sqrt{2}} \)
2) \( abI \), along \( \frac{\hat{j}}{\sqrt{5}} + \frac{2\hat{k}}{\sqrt{5}} \)
3) \( abI \), along \( \frac{\hat{j}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}} \)
4) \( 2abI \), along \( \frac{\hat{j}}{\sqrt{5}} + \frac{2\hat{k}}{\sqrt{5}} \)

Key: 1

Sol: Magnetic moment of loop ABCDA
\[ \mathbf{M}_1 = Iab\mathbf{\hat{k}} \]

Magnetic moment of loop ADEFA
\[ \mathbf{M}_2 = Iab\mathbf{\hat{j}} \]

Net magnetic moment
\[ \mathbf{M}_{\text{net}} = \mathbf{M}_1 + \mathbf{M}_2 \]
\[ \mathbf{M}_{\text{net}} = Iab(\hat{j} + \mathbf{\hat{k}}) \]
\[ |\mathbf{M}_{\text{net}}| = \sqrt{2}Iab \]

Unit vector along magnetic moment direction
\[ \frac{(\hat{j} + \mathbf{\hat{k}})}{\sqrt{2}} \]
16. An ideal gas in a closed container is slowly heated. As its temperature increases, which of the following statements are true?

(A) the mean free path of the molecules decreases
(B) the mean collision time between the molecules decreases.
(C) the mean free path remains unchanged.
(D) the mean collision time remains unchanged.

1) (C) and D  
2) (A) and (B)  
3) (A) and (D)  
4) (B) and (C)

Key: 4

Sol: Mean free path \( \lambda = \frac{1}{\sqrt{2} \pi d^2 \left( \frac{N}{V} \right)} \)

For closed container \( V = \text{constant} \) So \( \lambda = \text{constant} \)

Mean collision time \( = \frac{\lambda}{V} \) as \( V \propto \sqrt{T} \)

So when \( T \uparrow \Rightarrow V \uparrow \Rightarrow \text{time} \downarrow \)

17. If momentum (P), area (A) and time (T) are taken to be the fundamental quantities then the dimensional formula for energy is:

1) \( [PA^{1/2}T^{-1}] \)  
2) \( [P^{1/2}AT^{-1}] \)  
3) \( [P^2AT^{-2}] \)  
4) \( [PA^{-1}T^{-2}] \)

Key: 1

Sol: \( [E] = [P]^a [A]^b [T]^c \)

\( [M^1L^2T^{-2}] = [M^aL^bT^c] \)

\( 1 = a \)  
\( 2 = a + 2b \)  
\( -2 = -a + c \)  
\( 6 = 1/2 \)  
\( c = -1 \)

\( [E] = PA^{1/2}T^{-1} \)

18. When the temperature of a metal wire is increased from \( 0^\circ \text{C} \) to \( 10^\circ \text{C} \), its length increased by 0.02%. The percentage change in its mass density will be closest to:

1) 0.06  
2) 2.3  
3) 0.008  
4) 0.8
19. A small point mass carrying some positive charge on it, is released from the edge of a table. There is a uniform electric field in this region in the horizontal direction. Which of the following options then correctly describe the trajectory of the mass? (Curves are drawn schematically and are not to scale).

Key: 3

Sol: Acceleration of particle in x direction \( a_x = \frac{qE}{M} \)

Acceleration of particle in y direction \( a_y = +g \)

X co-ordinate of particle after time \( t \) \( x = \frac{1}{2} \left( \frac{qE}{M} \right) t^2 \)

Y co-ordinate of particles after time \( t \) \( y = \frac{1}{2} gt^2 \)

\[ \frac{y}{x} = \frac{Mg}{qE} \Rightarrow y = \left( \frac{Mg}{qE} \right) x \]
20. The displacement time graph of a particle executing S.H.M. is given in figure: (sketch is schematic and not to scale)

Which of the following statements is/are true for this motion?

(A) The force is zero at \( t = \frac{3T}{4} \)

(B) The acceleration is maximum at \( t = T \)

(C) The speed is maximum at \( t = \frac{T}{4} \)

(D) The P.E. is equal to K.E. of the oscillation at \( t = \frac{T}{2} \)

1) (B), (C) and (D)  
2) (A), (B) and (C)  
3) (A), (B) and (D)  
4) (A) and (D)

Key: 2

Sol: In SHM \( a = -\omega^2 x \)
So when \( x \) is max \( \Rightarrow \) acceleration is maximum
When \( x \) is zero \( \Rightarrow \) acceleration is zero \( \Rightarrow \) Force is zero & velocity is max.
PE & KE are equal when \( x = \pm \frac{A}{\sqrt{2}} \)
This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. e.g. 6.25, 7.00, -0.33, -30, 30.27, -127.30).

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. A square shaped hole of side \( \ell = \frac{a}{2} \) is carved out at a distance \( d = \frac{a}{2} \) from the centre ‘O’ of a uniform circular disk of radius \( a \). If the distance of the centre of mass of the remaining portion from \( O \) is \( -\frac{a}{X} \), value of \( X \) (to the nearest integer) is ______

Key: 23

Sol: CM of remaining portion shifts towards -x axis

\[
\frac{M_{\text{remaining}} \times x = M_{\text{removed}} \times d}{(M_{\text{total}} - M_{\text{removed}}) \times x = M_{\text{removed}} \times d}
\]

\[
x = \frac{M_{\text{removed}} \times d}{M_{\text{total}} - M_{\text{removed}}} = \frac{\sigma \ell^2 d}{\sigma \pi a^2 - \sigma \ell^2} : M_{\text{removed}} = \sigma \ell^2 , M_{\text{total}} = \sigma \left( \pi a^2 \right)
\]

\[
x = \frac{\ell^2 d}{\pi a^2 - \ell^2} = \frac{a}{8\pi - 2}
\]

So \( X \approx 23 \); (-ve sign is due to the reason that CM is on -ve x axis)
22. A particle of mass \( m \) is moving along the x-axis with initial velocity \( \vec{u} \). It collides elastically with a particle of mass \( 10m \) at rest and then moves with half its initial kinetic energy, (see figure). If \( \sin \theta_1 = \sqrt{n} \sin \theta_2 \) then value of \( n \) is

Key: 10

Sol:

\[ \vec{u} \quad \text{10 m} \quad \Rightarrow \quad \text{M} \]

\[ - \frac{1}{2} MV_1^2 = \frac{1}{2} \left( \frac{1}{2} m \vec{u}^2 \right) \Rightarrow V_i = \frac{u}{\sqrt{2}} \]

For conservation of momentum in y – direction
\[ V_i \sin \theta_1 = 10V_2 \sin \theta_2 \rightarrow (1) \]

\[ \rightarrow \text{As collision is elastic KE remains conserved} \]
\[ \frac{1}{2} Mu^2 = \frac{1}{2} MV_1^2 + \frac{1}{2} (10M) V_2^2 \]
\[ \frac{1}{2} Mu^2 = \frac{1}{2} \left( \frac{1}{2} Mu^2 \right) + \frac{1}{2} (10M) V_2^2 \]
\[ \frac{1}{2} Mu^2 = \frac{1}{2} MV_2^2 \]
\[ \frac{u^2}{20} = V_2^2 \Rightarrow V_2 = \frac{u}{\sqrt{20}} \]

Put in equation (1)
\[ \frac{u}{\sqrt{2}} \sin \theta_1 = 10 \cdot \frac{u}{\sqrt{20}} \sin \theta_2 \]
\[ \sin \theta_1 = \sqrt{10} \sin \theta_2 \]
23. An ideal cell of emf 10V is connected in circuit shown in figure. Each resistance is 2Ω. The potential difference (in V) across the capacitor when it is fully charged is

Key: 8

Sol: When capacitor is fully charged no current flows through it & acts as open circuit.

\[ V = 2I_1 + 2I_2 = 0 \]
\[ I_2 = 2I_1 \rightarrow (1) \]

In loop – 3
\[ 10 - 2I_2 - 2I = 0 \]
\[ 10 - 2I_2 - 2(I_1 + I_2) = 0 \]
\[ 10 = 2I_1 + 4I_2 \rightarrow (2) \]

From 1st & 2nd \( I_1 = 1 \text{A} \) & \( I_2 = 2 \text{A} \) So I = 3A

In loop – 2
\[ -V_C + 2I + 2I_1 = 0 \Rightarrow V_C = 2I_1 + 2I = 2 + 6 = 8 \]
24. A light ray enters a solid glass sphere of refractive index $\mu = \sqrt{3}$ at an angle of incidence $60^0$. The ray is both reflected and refracted at the farther surface of the sphere. The angle (in degrees) between the reflected and refracted rays at this surface is _______.

Key: 90°

Sol: At A: $1.\sin 60^0 = \mu \sin r$, $r = 30^0$

At B: $\mu \sin r = 1.\sin i$

$i = 60^0$

So angle between reflected ray & refracted ray at B = 90°

25. A wire of density $9 \times 10^{-3}$ kg cm$^{-3}$ is stretched between two clamps 1m apart. The resulting strain in the wire is $4.9 \times 10^{-4}$. The lowest frequency of the transverse vibrations in the wire is (Young’s modulus of wire $Y = 9 \times 10^{10}$ Nm$^{-2}$), (to the nearest integer), ______.

Key: 35

Sol:

Lowest frequency is fundamental frequency
\[ f_0 = \frac{1}{2\ell} \sqrt{\frac{T}{\mu}} \]
\[ f_0 = \frac{1}{2\ell} \sqrt{\frac{T}{\rho A}} \]
\[ f_0 = \frac{1}{2\ell} \sqrt{\frac{Y\Delta\ell}{\rho \ell}} \]

Now \( \ell = 1 \)
\[ \Delta\ell = 4.9 \times 10^{-4}; \quad Y = 9 \times 10^{10} \]
\[ \rho = 9 \times 10^{-3} \text{ Kg/M}^3 \]
\[ \text{So} = \frac{1}{2} \sqrt{\frac{9 \times 10^{10} \times 4.9 \times 10^{-4}}{9 \times 10^3}} = \frac{1}{2} \sqrt{49 \times 10^2} = \frac{70}{2} = 35 \text{Hz} \]
## CHEMISTRY

**(SINGLE CORRECT ANSWER TYPE)**

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

**Marking scheme:** +4 for correct answer, 0 if not attempted and -1 in all other cases.

1. The number of subshells associated with \(n = 4\) and \(m = -2\) quantum numbers is:
   
<table>
<thead>
<tr>
<th>1) 16</th>
<th>2) 2</th>
<th>3) 8</th>
<th>4) 4</th>
</tr>
</thead>
</table>

   **Key:** 2

   **Sol:**
   
   \(n = 4\) \(\ell = 0,1,2,3\)
   
   But \(m = -2\) only for \(\ell = 2\) & \(\ell = 3\)
   
   \(\therefore\) Only two subshells

2. The molecular geometry of \(\text{SF}_6\) is octahedral. What is the geometry of \(\text{SF}_4\) (including lone pair(s) of electrons, if any)?
   
<table>
<thead>
<tr>
<th>1) Trigonal bipyramidal</th>
<th>2) Pyramidal</th>
</tr>
</thead>
<tbody>
<tr>
<td>3) Square planar</td>
<td>4) Tetrahedral</td>
</tr>
</tbody>
</table>

   **Key:** 1

   **Sol:**
   
   \(\text{SF}_4 = 4\) bond pairs + 1 lone pair
   
   \(\therefore\) 5 hybrid orbitals
   
   \(\Rightarrow\) \(sp^3d\) hybridization
   
   \(\Rightarrow\) Trigonal bipyramidal

3. If your spill a chemical toilet cleaning liquid on your hand, your first aid would be:
   
<table>
<thead>
<tr>
<th>1) vinegar</th>
<th>2) aqueous (\text{NH}_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3) aqueous (\text{NaOH})</td>
<td>4) aqueous (\text{NaHCO}_3)</td>
</tr>
</tbody>
</table>

   **Key:** 4

   **Sol:**
   
   Toilet cleaners are generally acids. Hence mild bases like \(\text{NaHCO}_3\) are used to neutralize their effect.
   
   \(\text{NH}_3\) fumes and \(\text{NaOH}\) are corrosive /caustic
4. The major product obtained from $E_2$ – elimination of 3-bromo-2-fluoropentane is:

1) $\text{CH}_3\text{CH}_2\text{CH} = \text{CH} = \text{CH}_2$

2) $\text{CH}_3\text{CH} = \text{CH} = \text{CH}_2$

3) $\text{CH}_3\text{CH}_2\text{C} = \text{CH}_2$

4) $\text{CH}_3\text{C}\text{H}_2\text{CH} = \text{C} = \text{F} = \text{CH}_3$

Key: 4

Sol: Leaving ability order $R - \text{Br} > R - \text{F}$

5. The one that is not expected to show isomerism is:

1) $[\text{Ni} (\text{NH}_3)_2 \text{Cl}_2]$  
2) $[\text{Ni} (\text{NH}_3)_4 (\text{H}_2\text{O})_2]^{2+}$

3) $[\text{Pt} (\text{NH}_3)_2 \text{Cl}_2]$  
4) $[\text{Ni} (\text{en})_3]^{2+}$

Key: 1

Sol: $[\text{Ni} (\text{NH}_3)_2 \text{Cl}_2]$

- $sp^3$ hybridization & Tetrahedral
- No Geometrical isomerism
- No Optical isomerism
6. The major product of the following reaction is

Key: 3

Sol:

+M effect > Hyperconjugation > -M effect
7. Arrange the following labelled hydrogens in decreasing order of acidity:

1) c > b > a > d  
2) c > b > d > a  
3) b > c > d > a  
4) b > a > c > d

Key: 3

Sol:
8. Simplified absorption spectra of three complexes ((i), (ii) and (iii)) of M^{n+} ion are provided below; their \( \lambda_{\text{max}} \) values are marked as A, B and C respectively. The correct match between the complexes and their \( \lambda_{\text{max}} \) values is:

![Absorption Spectrum](image)

(i) \[ \text{M(NCS)}_{6}^{-(6+n)} \]  
(ii) \[ \text{MF}_{6}^{-(6+n)} \]  
(iii) \[ \text{M(NH}_{3})_{6}^{n+} \]  

1) A – (i), B – (ii), C – (iii)  
2) A – (ii), B – (i), C – (iii)  
3) A – (ii), B – (iii), C – (i)  
4) A – (iii), B – (i), C – (ii)

Key: No answer

Sol: Stability of complex \( \frac{1}{\lambda_{\text{absorbed}}} \alpha \)

Strength of ligand \( \text{NH}_3 > \text{NCS}^- > \text{F}^- \)

\( \lambda_{\text{NH}_3} < \lambda_{\text{NCS}^-} < \lambda_{\text{F}^-} \)

9. Match the type of interaction in column A with the distance dependence of their interaction energy in column B:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I) ion – ion</td>
<td>(a) [ \frac{1}{r} ]</td>
</tr>
<tr>
<td>(II) dipole – dipole</td>
<td>(b) [ \frac{1}{r^2} ]</td>
</tr>
<tr>
<td>(III) London dispersion</td>
<td>(c) [ \frac{1}{r^3} ]</td>
</tr>
<tr>
<td>(d) [ \frac{1}{r^6} ]</td>
<td></td>
</tr>
</tbody>
</table>

1) (I) – (a) ; (II) – (b) ; III – (c)  
2) (I) – (b) ; (II) – (d) ; III – (c)  
3) (I) – (a) ; (II) – (b) ; III – (d)  
4) (I) – (a) ; (II) – (c) ; III – (d)
Key: 4

Sol: According to NCERT Pg 138 Part-I chemistry Class 11, London dispersion, interaction energy \( \propto \frac{1}{r^6} \)

Dipole – dipole interaction energy between stationary polar molecules \( \propto \frac{1}{r^6} \)

And from Coulomb’s law; Ion – ion interaction energy \( \propto \frac{1}{r^3} \)

\( \therefore \) (I) – (a) ; (II) – (c) ; III – (d)

10. The size of a raw mango shrinks to a much smaller size when kept in a concentrated salt solution. Which one of the following processes can explain this?

1) Reverse osmosis  
2) Osmosis  
3) Dialysis  
4) Diffusion

Key: 2

Sol: Size of raw mango Shrinks in conc. Salt solution because it is placed in a hypertonic solution. This is due to osmosis.

11. Amongst the following statements regarding adsorption, those that are valid are :

(a) \( \Delta H \) becomes less negative as adsorption proceeds.

(b) On a given adsorbent, ammonia is adsorbed more than nitrogen gas.

(c) On adsorption, the residual force acting along the surface of the adsorbent increases.

(d) With increase in temperature, the equilibrium concentration of adsorbate increases.

1) (b) and (c)  
2) (a) and (b)  
3) (c) and (d)  
4) (d) and (a)

Key: 2

Sol: During adsorption, after some time equilibrium is reached. Hence \( \Delta H \) becomes less negative.

\( \text{NH}_3 \) has more \( T_c \), hence more adsorbed than \( \text{N}_2 \).

Due to adsorption, residual forces decrease.

As \( T \) increases, \( \frac{x}{m} \) decreases. \( \therefore \) (a) & (b) are true.
12. Cast iron is used for the manufacture of:
   1) wrought iron, pig iron and steel
   2) wrought iron and steel
   3) pig iron, scrap iron and steel
   4) wrought iron and pig iron

Key: 2

Sol: Cast iron comes from pig iron and is used to manufacture wrought iron and steel.

13. The correct observation in the following reaction is:

\[
\text{Sucrose} \xrightarrow{\text{Gly cosidic bond}} \text{A} + \text{B} \xrightarrow{\text{Seliwanoff's reagent}} ?
\]

1) Formation of violet colour
2) Gives no colour
3) Formation of blue colour
4) Formation of red colour

Key: 4

Sol: It is used to distinguish aldoses and ketoses
Ketoses given red colour with Seliwanoff reagent [Resorcinol + Conc.HCl]

14. Two elements A and B have similar chemical properties. They don't form solid
hydrogencarbonates, but react with nitrogen to form nitrides. A and B, respectively,
are:

1) Li and Mg   2) Cs and Ba   3) Na and Ca   4) Na and Rb

Key: 1
Sol: IA from solid bicarbonates and don’t readily react with nitrogen to form nitrides, except Li
IIA form nitrides with N₂ but don’t form solid bicarbonates
∴ Ans is Li & Mg

15. An organic compound 'A' (C₅H₁₀O) when treated with conc. HI undergoes cleavage to yield compounds 'B' and 'C'. 'B' gives yellow precipitate with AgNO₃ whereas 'C' tautomerizes to 'D'. 'D' gives positive iodoform test. 'A' could be:

1) ![Structure 1]

2) ![Structure 2]

3) ![Structure 3]

4) ![Structure 4]

Key: 3

Sol:
16. Consider the reaction sequence given below:

\[ \text{Br} + \text{OH}^- \rightarrow \text{OH} + \text{Br}^- \] \hspace{1cm} \text{(1)}

\[ \text{OH}^- + \text{C}_2\text{H}_5\text{OH} \rightarrow \text{CH}_3 + \text{HOH} + \text{Br}^- \] \hspace{1cm} \text{(2)}

Which of the following statements is true:

1) Doubling the concentration of base will double the rate of both the reactions.
2) Changing the concentration of base will have no effect on reaction (2).
3) Changing the base from OH\(^-\) to OR\(^-\) will have no effect on reaction (2).
4) Changing the concentration of base will have no effect on reaction (1).

Key: 4

Sol: Reaction (1) rate = \( K [R - X] \)

Reaction (2) rate = \( K [R - X] [OH^-] \)
17. The results given in the below table were obtained during kinetic studies of the following reaction:

\[ 2A + B \rightarrow C + D \]

<table>
<thead>
<tr>
<th>Experiment</th>
<th>[A] / molL(^{-1})</th>
<th>[B]/molL(^{-1})</th>
<th>Initial rate/ molL(^{-1})min(^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.1</td>
<td>0.1</td>
<td>(6.00 \times 10^{-3})</td>
</tr>
<tr>
<td>II</td>
<td>0.1</td>
<td>0.2</td>
<td>(2.40 \times 10^{-2})</td>
</tr>
<tr>
<td>III</td>
<td>0.2</td>
<td>0.1</td>
<td>(1.20 \times 10^{-2})</td>
</tr>
<tr>
<td>IV</td>
<td>X</td>
<td>0.2</td>
<td>(7.20 \times 10^{-2})</td>
</tr>
<tr>
<td>V</td>
<td>0.3</td>
<td>Y</td>
<td>(2.88 \times 10^{-1})</td>
</tr>
</tbody>
</table>

X and Y in the given table are respectively:

1) 0.4, 0.4  
2) 0.3, 0.4  
3) 0.4, 0.3  
4) 0.3, 0.3

Key: 2

Sol: If rate law is \( R = K [A]^a [B]^b \)

From expt. I & II:

\[ \frac{R_2}{R_1} = \frac{K [A_2]^a [B_2]^b}{K [A_1]^a [B_1]^b} \]

\[ \frac{2.4 \times 10^{-2}}{6 \times 10^{-3}} = \left( \frac{0.1}{0.1} \right)^a \left( \frac{0.2}{0.1} \right)^b \]

\[ 4 = 2^b \Rightarrow b = 2 \]

respectively from expt. III & I:

\[ \frac{R_3}{R_1} = \frac{K [A_3]^a [B_3]^b}{K [A_1]^a [B_1]^b} \]

\[ \frac{1.2 \times 10^{-2}}{6 \times 10^{-3}} = \left( \frac{0.2}{0.1} \right)^a \left( \frac{0.1}{0.1} \right)^b \]

\[ 2 = 2^a \Rightarrow a = 1 \]

\[ \Rightarrow \frac{R_4}{R_1} = \left( \frac{x}{0.1} \right)^2 \left( \frac{0.2}{0.1} \right)^2 \]
Two compounds A and B with same molecular formula (C₃H₆O) undergo Grignard’s reaction with methylmagnesium bromide to give products C and D. Products C and D show following chemical tests.

<table>
<thead>
<tr>
<th>Test</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ceric ammonium nitrate</td>
<td>Positive</td>
<td>Positive</td>
</tr>
<tr>
<td>Test</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lucas Test</td>
<td>Turbidity obtained after five minutes</td>
<td>Turbidity obtained immediately</td>
</tr>
<tr>
<td>Iodoform Test</td>
<td>Positive</td>
<td>Negative</td>
</tr>
</tbody>
</table>

C and D respectively are:

1) C = \[\text{Structure Image}\]

2) C = \[\text{Structure Image}\]

3) C = \[\text{Structure Image}\]

4) C = \[\text{Structure Image}\]

Key: 2

Sol:
Secondary alcohol it shows +ve haloform test gives turbidity after 5 minutes.

Tertiary alcohol shows -ve haloform test & gives turbidity immediately.

19. The shape/structure of \([\text{XeF}_5]^-\) and \(\text{XeO}_3\text{F}_2\), respectively, are:

1) octahedral and square pyramidal
2) pentagonal planar and trigonal bipyramidal
3) trigonal bipyramidal and trigonal bipyramidal
4) trigonal bipyramidal and pentagonal planar

Key: 2

Sol: \([\text{XeF}_5]^-\) has 2 lone pairs + 5 bond pairs = 7 hybrid orbitals

\[ \Rightarrow \text{sp}^3\text{d}^3 \text{ hybridization with 2 lone pairs} \]

\[ \Rightarrow \text{Pentagonal planar}. \]

\(\text{XeO}_3\text{F}_2\) has zero lone pairs + 5 bond pairs = 5 hybrid orbitals

\[ \Rightarrow \text{sp}^3\text{d} \text{ hybridization} \]

\[ \Rightarrow \text{trigonal bipyramidal} \]

20. Three elements X, Y and Z are in the 3rd period of the periodic table. The oxides of X, Y and Z, respectively, are basic, amphoteric and acidic. The correct order of the atomic numbers of X, Y and Z is:

1) \(X < Y < Z\)  
2) \(Y < X < Z\)  
3) \(X < Z < Y\)  
4) \(Z < Y < X\)

Key: 1
Sol: Oxide of x → basic
⇒ IA or IIA

Oxide of y → amphoteric ⇒ IIIA

Oxide of z → acidic
⇒ IVA or VA or VIA or VIIA

Atomic number order is x < y < z

(or)

As we move across the period, acidic nature of oxides increases.

∴ x < y < z

(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. e.g. 6.25, 7.00, -0.33, -0.30, 30.27, -127.30).

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. For the disproportionation reaction \(2\text{Cu}^+ (aq) \rightleftharpoons \text{Cu} (s) + \text{Cu}^{2+} (aq)\) at 298 K, In \(K\) (where \(K\) is the equilibrium constant) is \(144 \times 10^{-1}\).

Given

\((E^{0}_{\text{Cu}^{2+}/\text{Cu}^+} = 0.16\text{V})\)

\((E^{0}_{\text{Cu}^+/\text{Cu}} = 0.52\text{V})\)

\((\frac{RT}{F} = 0.025)\)

Key: 144

Sol:

\(2\text{Cu}^+ \rightleftharpoons \text{Cu} (s) + \text{Cu}^{2+} (aq)\)

\(E^{0}_{\text{cell}} = E^{0}_{\text{cell Cu}^+} + E^{0}_{\text{cell Cu}^{2+}} = 0.52 + (-0.16) = 0.36\text{V}\)

\(E^{0}_{\text{cell}} = \frac{RT}{nF} \ln K\)

\(0.36 = \frac{0.025}{1} \ln K\)

⇒ \(\ln K = 14.4 = 144 \times 10^{-1}\)

∴ Ans is 144
22. The ratio of the mass percentages of 'C & H' and 'C & 0' of a saturated acyclic organic compound 'X' are 4 : 1 and 3 : 4 respectively. Then, the moles of oxygen gas required for complete combustion of two moles of organic compound 'X' is ______

Key: 5

Sol:

\[
\begin{align*}
\text{C} & : \text{H} : \text{O} \\
\text{Mass ratio} & = 4 : 1 : \frac{16}{3} \\
\text{Mole ratio} & = \frac{4}{12} : \frac{1}{3} : \frac{16}{3} \\
& = 1 : 3 : 1 \\
\end{align*}
\]

\[\therefore \text{empirical formula} = \text{CH}_3\text{O}\]

\[\therefore \text{molecular formula possibility} = \text{C}_2\text{H}_6\text{O}_2 \text{ (diol)}\]

\[\text{C}_2\text{H}_6\text{O}_2 + \frac{5}{2}\text{O}_2 \rightarrow 2\text{CO}_2 + 3\text{H}_2\text{O}\]

\[\therefore 2 \text{ moles of organic compound requires 5 moles of O}_2\].

23. The heat of combustion of ethanol into carbon dioxids and water is -327 kcal at constant pressure. The heat evolved (in cal) at constant volume and 270°C (if all gases behave ideally) is (R=2 cal mol\(^{-1}\) K\(^{-1}\)) ___

Key: -324600

Sol:

\[\text{C}_2\text{H}_5\text{OH} \rightarrow 2\text{CO}_2 + 3\text{H}_2\text{O}\]

\[\Delta H_{\text{comb}} = -327 \text{ Kcal}\]

\[\Delta U = \Delta H - \Delta n_gRT\]

\[= -327 - \left(\frac{-1}{2} \times 2 \times 300\right) = -327 + 0.6\]

\[= -326.4 \text{ Kcal} = -326400 \text{cal}\]
24. The oxidation states of transition metal atoms in $\text{K}_2\text{Cr}_2\text{O}_7$, $\text{KMnO}_4$ and $\text{K}_2\text{FeO}_4$, respectively, are $x$, $y$ and $z$. The sum of $x$, $y$ and $z$ is ______

Key: 19

Sol: $\text{K}_2\text{Cr}_2\text{O}_7$ $\quad \text{Cr} = +6$
$\text{KMnO}_4$ $\quad \text{Mn} = +7$
$\text{K}_2\text{FeO}_4$ $\quad \text{Fe} = +6$
$\therefore$ Ans $= 6 + 7 + 6 = 19$

25. The work function of sodium metal is $4.41 \times 10^{-19}$ J. If photons of wavelength 300 nm are incident on the metal, the kinetic energy of the ejected electrons will be ($h = 6.63 \times 10^{-34}$ J s; $c = 3 \times 10^8$ m/s) ______ $\times 10^{-21}$ J.

Key: 222

Sol: $\text{KE} = \frac{hc}{\lambda} - W$
$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{300 \times 10^{-9}} - 4.41 \times 10^{-19} = 6.63 \times 10^{-19} - 4.41 \times 10^{-19} = 2.22 \times 10^{-19}$
$= 222 \times 10^{-21}$ J
$\therefore$ Ans $= 222$
MATHEMATICS
(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 in all other cases.

1. Let \( f(x) \) be a quadratic polynomial such that \( f(-1) + f(2) = 0 \). If one of the roots of \( f(x) = 0 \) is 3, then its other root lies in:

1) (-3, -1)  
2) (1, 3)  
3) (0, 1)  
4) (-1, 0)

Key: 4

Sol: Let, \( f(x) = A(x - 3)(x - \alpha), (A \neq 0) \)

As \( x = 3 \) is given as one root, we can assume \( x = \alpha \) be the other real root.

So \( f(-1) + f(2) = 0 \)

\[ \Rightarrow A(4(\alpha + 1) + (\alpha - 2)) = 0 \]

\[ \Rightarrow 5\alpha + 2 = 0 \]

\[ \Rightarrow \alpha = -\frac{2}{5} \in (-1,0) \]

2. The imaginary part of \( \left(3 + 2\sqrt{-54}\right)^{1/2} - \left(3 - 2\sqrt{-54}\right)^{1/2} \) can be:

1) \( \sqrt{6} \)  
2) \( 6 \)  
3) \( -\sqrt{6} \)  
4) \( -2\sqrt{6} \)

Key: 4

Sol: The expression can be written as,

\[ \sqrt{3 + 2\sqrt{54i} - \sqrt{3 - 2\sqrt{54i}}} \]

\[ = \sqrt{3^2 + (\sqrt{6i})^2 + 2 \times 3 \times \sqrt{6i} - \sqrt{3^2 + (\sqrt{6i})^2 - 2 \times 3 \times \sqrt{6i}}} \]

\[ = (\pm (3 + \sqrt{6i})) - (\pm (3 - \sqrt{6i})) = \pm 2\sqrt{6i} \text{ or } \pm 6 \]

Hence imaginary part = \(-2\sqrt{6}\)
3. \[ \lim_{x \to 0} \left( \tan \left( \frac{\pi}{4} + x \right) \right)^{1/x} \] is equal to:

1) \( e^2 \) 2) \( e \) 3) 1 4) 2

Key: 1

Sol: \[ \lim_{x \to 0} \left( \frac{\tan \frac{\pi}{4} + \tan x}{1 - \tan \frac{\pi}{4} \cdot \tan x} \right)^{1/x} \]

\[ \lim_{x \to 0} \left( 1 + \tan x \right) \frac{1}{\tan x} = e^1 = e \]

\[ \lim_{x \to 0} \left( 1 - \tan x \right) \frac{1}{\tan x} = e^{-1} = e^{-2} \]

4. Let \( A = \{ X = (x, y, z)^T : PX = 0 \text{ and } x^2 + y^2 + z^2 = 1 \} \), where \( P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix} \), then the set \( A \):

1) contains more than two elements. 2) contains exactly two elements.
3) is an empty set. 4) is a singleton.

Key: 2

Sol: \( X = (x, y, z)^T = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \)

\[ PX = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \]

\[ \Rightarrow x + 2y + z = 0 \to (1) \]
\[ -2x + 3y - 4z = 0 \to (2) \]
\[ x + 9y - z = 0 \to (3) \]

Simultaneously Solving (1) & (2)

\[ \frac{x}{-11} = \frac{y}{2} = \frac{z}{7} = K \]
Clearly the above solution also satisfies equation (3) 
Now \( x^2 + y^2 + z^2 = 1 \)

\[ \Rightarrow 121K^2 + 4K^2 + 49K^2 = 1 \Rightarrow K^2 = \frac{1}{174} \]

\[ \Rightarrow K = \pm \frac{1}{\sqrt{174}} \]

So the set \( A \) contains exactly two elements.

5. For some \( \theta \in \left(0, \frac{\pi}{2}\right)\), if the eccentricity of the hyperbola, \( x^2 - y^2 \sec^2 \theta = 10 \) is \( \sqrt{5} \) times the eccentricity of the ellipse, \( x^2 \sec^2 \theta + y^2 = 5 \), then the length of the latus rectum of the ellipse, is:

1) \( \frac{4\sqrt{5}}{3} \)  
2) \( \sqrt{30} \)  
3) \( \frac{2\sqrt{5}}{3} \)  
4) \( 2\sqrt{6} \)

Key: 1

Sol: Eccentricity of hyperbola \( \frac{x^2}{10} - \frac{y^2}{10\cos^2 \theta} = 1 \) is

\[ e_1 = \sqrt{1 + \frac{10\cos^2 \theta}{10}} = \sqrt{1 + \cos^2 \theta} \]

Eccentricity of ellipse \( \frac{x^2}{5\cos^2 \theta} + \frac{y^2}{5} = 1 \) is

\[ e_2 = \sqrt{1 - \frac{5\cos^2 \theta}{5}} \quad \text{(as } 5\cos^2 \theta < 5 \forall \theta \in \left(0, \frac{\pi}{2}\right)\text{)} \]

Given \( e_1 = \sqrt{5}e_2 \Rightarrow e_1^2 = 5e_2^2 \)

\[ 1 + \cos^2 \theta = 5 - 5\cos^2 \theta \Rightarrow \cos^2 \theta = \frac{2}{3} \]

So latus rectum of ellipse has length

\[ \frac{2b^2}{a} = \frac{2 \times 5\cos^2 \theta}{\sqrt{5}} = 2\sqrt{5} \times \frac{2}{3} = \frac{4\sqrt{5}}{3} \]
6. Let $E^C$ denote the complement of an event $E$. Let $E_1, E_2$ and $E_3$ be any pairwise independent events with $P(E_1) > 0$ and $P(E_1 \cap E_2 \cap E_3) = 0$. Then

$P(E_2^C \cap E_3^C / E_1)$ is equal to:

1) $P(E_2^C) + P(E_3)$
2) $P(E_3) - P(E_2^C)$
3) $P(E_3^C) - P(E_2)$
4) $P(E_3^C) - P(E_2^C)$

Key: 3

Sol: $P\left(\frac{E_2^C \cap E_3^C}{E_1}\right) = \frac{P(E_1 \cap E_2^C \cap E_3^C)}{P(E_1)}$

Now using Venn diagram, the shaded part is $E_1 \cap E_2^C \cap E_3^C$

Can be expressed as

$P(E_1 \cap E_2^C \cap E_3^C) = P(E_1) - P(E_1 \cap E_2) - P(E_1 \cap E_3) + P(E_1 \cap E_2 \cap E_3)$

$= P(E_1) - P(E_1).P(E_2) - P(E_1).P(E_3) + 0$

(As $E_1, E_2$ & $E_1, E_3$ are pair wise independent)

So $P\left(\frac{E_2^C \cap E_3^C}{E_1}\right) = \frac{P(E_1) - P(E_1).P(E_2) - P(E_1).P(E_3)}{P(E_1)}$

$= (1 - P(E_3)) - P(E_2) = P(E_3^C) - P(E_2)$

7. Let $n > 2$ be an integer. Suppose that there are $n$ Metro stations in a city located along a circular path. Each pair of stations is connected by a straight track only. Further, each pair of nearest stations is connected by blue line, whereas all remaining pairs of stations are connected by red line. If the number of red lines is 99 times the number of blue lines, then the value of $n$ is:

1) 199  2) 201  3) 200  4) 101

Key: 2
Sol: Considering the n stations to be the n vertices of a convex polygon of n slides inscribed within the circular path, nearest stations connecting lines are the sides of the polygon and are colored line. Any other stations are connected along diagonals of the polygon colored red.

No. of red lines = No. of diagonals = \(^nC_2\) – n
No. of blue lines = No. of sides = n.

\[\frac{n(n-1)}{2} = 100n \Rightarrow n - 1 = 200 \Rightarrow n = 201\]

8. If a curve \(y = f(x)\), passing through the point \((1, 2)\), is the solution of the differential equation, \(2x^2\frac{dy}{dx} = (2xy + y^2)\), then \(f\left(\frac{1}{2}\right)\) is equal to:

1) \(\frac{1}{1 + \log_e 2}\)  
2) \(-\frac{1}{1 + \log_e 2}\)  
3) \(\frac{1}{1 - \log_e 2}\)  
4) \(1 + \log_e 2\)

Key: 1

Sol: \(2x^2\frac{dy}{dx} = (2xy + y^2)\)

\[2x^2\frac{dy}{dx} - 2xydx = y^2dx\]

\[2x(xdy - ydx) = y^2dx\]

\[-(ydx - xdy) = \frac{dx}{2x}\]

\[-\frac{x}{y} = \frac{1}{2} \ln|x| + C\]

Putting \(x = 1, y = 2\) we get \(c = -1/2\)

So putting \(x = 1/2\)

\[-\frac{1}{2y} = \frac{1}{2} \ln\frac{1}{2} - \frac{1}{2}\]

\[y = \frac{1}{1 + \log_e 2}\]
9. If the equation \( \cos^4 \theta + \sin^4 \theta + \lambda = 0 \) has real solutions for \( \theta \), then \( \lambda \) lies in the interval:

1) \( \left( -\frac{5}{4}, -1 \right) \)  
2) \( \left( -\frac{1}{2}, -\frac{1}{4} \right) \)  
3) \( \left[ -\frac{3}{2}, -\frac{5}{4} \right] \)  
4) \( \left[ -1, -\frac{1}{2} \right] \)

Key: 4

Sol: The equation is \( \cos^4 \theta + \sin^4 \theta = -\lambda \)

Now \( \cos^4 \theta + \sin^4 \theta \)

\[ = \left( \cos^2 \theta + \sin^2 \theta \right)^2 - 2 \sin^2 \theta \cos^2 \theta = 1 - \frac{\sin^2 2\theta}{2} \]

\[ 0 \leq \sin^2 2\theta \leq 1 \ \forall \theta \in \mathbb{R} \]

So, \( \frac{1}{2} \leq 1 - \frac{\sin^2 2\theta}{2} \leq 1 \)

Hence for real solutions

\[ \frac{1}{2} \leq -\lambda \leq 1 \Rightarrow \lambda \in \left[ -1, -\frac{1}{2} \right] \]

10. Let \( f : \mathbb{R} \to \mathbb{R} \) be a function which satisfies \( f(x + y) = f(x) + f(y) \) \( \forall x, y \in \mathbb{R} \). If \( f(1) = 2 \)

and \( g(n) = \sum_{k=1}^{(n-1)} f(k), n \in \mathbb{N} \) then the value of \( n \), for which \( g(n) = 20 \), is:

1) 9  
2) 5  
3) 20  
4) 4

Key: 4

Sol: Putting \( x = y = 0 \), we get \( f(0) = 0 \)

\[ f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \to 0} \frac{f(x) + f(h) - f(x)}{h} \]

\[ = f'(0) \]

Integrating \( f(x) = xf'(0) + C \)

As \( f(0) = 0 \Rightarrow C = 0 \)

Now given \( f(1) = f'(0) = 2 \)

So \( f'(0) = 2 \) Hence \( f(x) = 2x \)

\[ g(n) = \sum_{K=1}^{n-1} 2K = 2 \times \frac{(n-1)n}{2} = n^2 - n \]

So, \( n^2 - n = 20 \Rightarrow n = 5 \) (as \( n \in \mathbb{N} \))
11. Which of the following is a tautology?

1) \((p \rightarrow q) \land (q \rightarrow p)\)
2) \((\neg q) \lor (p \land q) \rightarrow q\)
3) \((\neg p) \land (p \lor q) \rightarrow q\)
4) \((q \rightarrow p) \lor \neg (p \rightarrow q)\)

Key: 3

Sol: (1) \((p \rightarrow q) \land (q \rightarrow p) \equiv p \leftrightarrow q\), not a tautology

(2) \((\neg q) \lor (p \land q) \rightarrow q\) clearly if q is false, irrespective of P the statement is False, (not a tautology)

3) \((\neg p) \land (p \lor q) \rightarrow q\)

\(= ((\neg p \land p) \lor (\neg p \land q)) \rightarrow q\)

\(= (\neg p \land q) \rightarrow q\) (as \(\neg p \land p\) is contradiction)

\(\neg (\neg p \land q) \lor q\)

\(= p \lor \neg q \lor q\)

tautology (as \(q \lor \neg q\) is a tautology)

4) \((\neg (p \rightarrow q) \lor (q \rightarrow p) \equiv (p \rightarrow q) \rightarrow (q \rightarrow p)\)

If p is false and q is true, the statement is false (not a tautology)

12. Let \(f : (-1, \infty) \rightarrow \mathbb{R}\) be defined by \(f(0) = 1\) and \(f(x) = \frac{1}{x} \log_e (1 + x), x \neq 0\). Then the function f:

1) increases in \((-1, 0)\) and decreases in \((0, \infty)\)

2) increases in \((-1, \infty)\)

3) decreases in \((-1, 0)\) and increases in \((0, \infty)\)

4) decreases in \((-1, \infty)\)

Key: 4
Sol: \( f(x) = \frac{\log_e(1+x)}{x} \)

Differentiating w.r.t \( f'(x) = \frac{x \times \frac{1}{1+x} - \log_e(1+x)}{x^2} = \frac{x - (1+x)\ln(1+x)}{x^2(1+x)} \)

as \( x \in (-1, \infty) \) so denominator is always positive

Now let \( g(x) = x - (1+x)\ln(1+x) \)

\( g'(x) = 1 - \ln(1+x) - 1 = -\ln(1+x) \)

So as \( x \in (-1,0), g'(x) > 0 \) and as \( x \in (0, \infty), g'(x) < 0 \)

So maximum value of \( g(x) \) is at \( x = 0 \)

\( g(0) = 0 \)

So \( g(x) \leq 0 \forall x \in (-1, \infty) \)

Hence \( f'(x) \leq 0 \forall x \in (-1, \infty) \)

So \( f(x) \) is decreasing \( \forall x \in (-1, \infty) \)

13. A plane passing through the point \((3, 1, 1)\) contains two lines whose direction ratios are 1, -2, 2 and 2, 3, -1 respectively. If this plane also passes through the point 
\((\alpha, -3, 5)\), then \( \alpha \) is equal to:

1) 10  2) -5  3) 5  4) -10

Key: 3

Sol: Direction ratio along the normal to the plane = \[
\begin{vmatrix}
i & j & k \\
1 & -2 & 2 \\
2 & 3 & -1
\end{vmatrix} = -4i + 5j + 7k
\]

So eqn of plane passing through \((3, 1, 1)\) is 
\(-4(x - 3) + 5(y - 1) + 7(z - 1) = 0\)

\(\Rightarrow 4x - 5y - 7z = 0\)

Putting \(y = -3, z = 5\) we get \(x = 5\)

So, \(\alpha = 5\)

14. Consider a region \( R = \{(x, y) \in \mathbb{R}^2 : x^2 \leq y \leq 2x\} \). If a line \( y = \alpha \) divides the area of region \( R \) into two equal parts, then which of the following is true?

1) \( \alpha^3 - 6\alpha^2 + 16 = 0 \)  2) \( 3\alpha^2 - 8\alpha + 8 = 0 \)
3) \( 3\alpha^2 - 8\alpha^{3/2} + 8 = 0 \)  4) \( \alpha^3 - 6\alpha^{3/2} - 16 = 0 \)
Key: 3

Sol: Equating the bounded area above and below the line $y = \alpha$

\[
\int_{\alpha}^{4} \left(\sqrt{y} - \frac{y}{2}\right) \, dy = \int_{0}^{\alpha} \left(\sqrt{y} - y^2\right) \, dy
\]

So, \[
\frac{2y^{3/2}}{3} - \frac{y^2}{4}\bigg|_{\alpha}^{4} = \frac{2y^{3/2}}{3} - \frac{y^2}{4}\bigg|_{0}^{\alpha} \Rightarrow 3\alpha^2 - 8\alpha^{3/2} + 8 = 0
\]

15. Let $a, b, c \in \mathbb{R}$ be all non-zero and satisfy $a^3 + b^3 + c^3 = 2$. If the matrix

\[
A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}
\]

satisfies $A^T A = I$, then a value of $abc$ can be:

1) $-\frac{1}{3}$  
2) $\frac{2}{3}$  
3) $3$  
4) $\frac{1}{3}$

Key: 4

Sol: $A^T A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix} \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

So comparing elements $a^2 + b^2 + c^2 = 1$, $ab + bc + ca = 0$

Now $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \rightarrow (1)$

and $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) = 1 + 2 \times 0 = 1$

So $a + b + c = \pm 1$

Putting in (1) $a^3 + b^3 + c^3 - 3abc = \pm (1 - 0)$

$\Rightarrow 3abc = 1$ or $3$  
$\Rightarrow abc = \frac{1}{3}$ or $1$
16. If the sum of first 11 terms of an A.P., \(a_1, a_2, a_3, \ldots\) is 0 \((a_1 \neq 0)\), then the sum of the A.P., \(a_1, a_3, a_5, \ldots, a_{23}\) is \(ka_1\), where \(k\) is equal to:

1) \(-\frac{121}{10}\)  
2) \(\frac{121}{10}\)  
3) \(-\frac{72}{5}\)  
4) \(\frac{72}{5}\)

Key: 3

Sol: Let \(d\) be the common difference of the A.P.

So \(\frac{11}{2} [2a_1 + 10d] = 0\)

\(\Rightarrow a_1 = -5d\)

Now \(a_1 + a_3 + a_5 + \ldots + a_{23}\)

\(= \frac{12}{2} [2a_1 + 11 \times 2d] = 6 \left(2a_1 - \frac{22a_1}{5}\right) = -\frac{72}{5} a_1\)

17. The area (in sq.units) of an equilateral triangle inscribed in the parabola \(y^2 = 8x\), with one of its vertices on the vertex of this parabola, is:

1) \(128\sqrt{3}\)  
2) \(256\sqrt{3}\)  
3) \(192\sqrt{3}\)  
4) \(64\sqrt{3}\)

Key: 3

Sol:

Let \(\triangle OAB\) is an equilateral triangle. \(A(2t_1^2, 4t_1), B(2t_1^2, -4t_1)\)

Now \(\angle AOP = 30^\circ\)

So \(\tan 30^\circ = \frac{AP}{OP} = \frac{4t_1}{2t_1^2} \Rightarrow t_1 = 2\sqrt{3}\)

SO \(AB = 8t_1 = 16\sqrt{3}\)

Hence area of \(\triangle OAB = \frac{\sqrt{3}}{4} \times (16\sqrt{3})^2 = 192\sqrt{3}\) sq units
18. The set of all possible values of $\theta$ in the interval $(0, \pi)$ for which the points $(1, 2)$ and $(\sin \theta, \cos \theta)$ lie on the same side of the line $x + y = 1$ is:

1) $\left(0, \frac{\pi}{2}\right)$  
2) $\left(0, \frac{\pi}{4}\right)$  
3) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$  
4) $\left(0, \frac{3\pi}{4}\right)$

Key: 1

Sol: $(\sin \theta, \cos \theta)$ lies on the unit circle $x^2 + y^2 = 1$.

So as per the figure $(\sin \theta, \cos \theta)$ must lie in the shaded arc of the circle.

So clearly $\theta \in \left(0, \frac{\pi}{2}\right)$

19. Let $s$ be the sum of the first 9 terms of the series:

\[
\left\{x + ka\right\} + \left\{x^2 + (k + 2)a\right\} + \left\{x^3 + (k + 4)a\right\} + \left\{x^4 + (k + 6)a\right\} + ..... \text{ where } a \neq 0 \text{ and } x \neq 1. \text{ If } S = \frac{x^{10} - x + 45a(x - 1)}{x - 1}, \text{ then } k \text{ is equal to:}
\]

1) -5  
2) 1  
3) 3  
4) -3

Key: 4

Sol: 

\[
S = \left(x + x^2 + x^3 + ....... \text{to 9 terms}\right) + a\left(k + k + 2 + k + 4 + ......... \text{to 9 terms}\right)
\]

\[
= \frac{x(x^9 - 1)}{x - 1} + a \times \frac{9}{2}[2K + 8 \times 2]
\]

\[
= \frac{x^{10} - x + 9a(k + 8)(x - 1)}{x - 1}
\]

So comparing $9a(k + 8) = 45a$, Hence $k = -3$
20. The equation of the normal to the curve \( y = (1 + x)^2 + \cos^2 \left( \sin^{-1} x \right) \) at \( x = 0 \) is:

1) \( y = 4x + 2 \)  
2) \( 2y + x = 4 \)  
3) \( x + 4y = 8 \)  
4) \( y + 4x = 2 \)

Key: 3

Sol:  
\( y = (1 + x)^2 + \cos^2 \left( \sin^{-1} x \right) \)
\( y = e^{2y \ln(1+x)} + (1 - x^2), \) putting \( x = 0, y = 2 \)

Differentiating w.r.t \( x, \)

\[
\frac{dy}{dx} = e^{2y \ln(1+x)} \left( \frac{2y}{1+x} + 2 \frac{dy}{dx} \ln(1+x) \right) - 2x
\]

Putting \( x = 0, y = 2 \)

\[
\frac{dy}{dx} = 4
\]

So equation of normal is

\[
y - 2 = -\frac{1}{4} (x - 0)
\]

\[
x + 4y - 8 = 0
\]

(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place) (e.g. 6.25, 7.00, -0.33, -0.30, 30.27, -127.30).  
Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. Let \([t]\) denote the greatest integer less than or equal to \( t \). Then the value of

\[
\int_1^2 \left[ 2x - [3x] \right] \, dx
\]

is ______

Key: 1.00

Sol:  
\[
\int_1^2 \left[ 2x - 3[x]\right] \, dx = \frac{2}{3} \int_1^2 \left[ 3x - x \right] \, dx
\]

\[
= \left[ \frac{3x}{2} - x \right]_1^2 = \frac{2}{3} \left[ x - \{x\} \right]_1^2 \quad \text{(As \( \{x\} < 1 \ \forall x \in \mathbb{R} \))}
\]

\[
\int_1^2 x \, dx - 3 \int_0^{1/3} 3x \, dx = \left( 2 - \frac{1}{2} \right) - \frac{9}{2} \left( \frac{1}{9} - 0 \right) = 1.00
\]
22. Let the position vectors of points ‘A’ and ‘B’ be \( \mathbf{i} + \mathbf{j} + \mathbf{k} \) and \( 2\mathbf{i} + \mathbf{j} + 3\mathbf{k} \), respectively.

A point ‘P’ divides the line segment AB internally in the ratio \( \lambda : 1(\lambda > 0) \). If O is the origin and \( \overrightarrow{OB}.\overrightarrow{OP} - 3|\overrightarrow{OA} \times \overrightarrow{OP}|^2 = 6 \), then \( \lambda \) is equal to ______

Key: 0.80

Sol: \( \overrightarrow{OP} = \frac{\lambda \mathbf{b} + \mathbf{a}}{\lambda + 1} \), where \( \overrightarrow{OA} = \mathbf{a}, \overrightarrow{OB} = \mathbf{b} \)

So \( \overrightarrow{OB}.\overrightarrow{OP} - 3|\overrightarrow{OA} \times \overrightarrow{OP}|^2 = 6 \) \( \Rightarrow \mathbf{b} \cdot \left( \frac{\lambda \mathbf{b} + \mathbf{a}}{\lambda + 1} \right) - 3|\mathbf{a} \times \left( \frac{\lambda \mathbf{b} + \mathbf{a}}{\lambda + 1} \right)|^2 = 6 \)

\( \Rightarrow \frac{\mathbf{a} \cdot \mathbf{b} + \lambda |\mathbf{b}|^2}{\lambda + 1} - \frac{3\lambda^2}{(\lambda + 1)^2} |\mathbf{a} \times \mathbf{b}|^2 = 6 \)

\( \frac{6 + 14\lambda}{\lambda + 1} - \frac{3\lambda^2}{(\lambda + 1)^2} \times 6 = 6 \)

\( \Rightarrow \frac{8\lambda}{\lambda + 1} = \frac{18\lambda^2}{(\lambda + 1)^2} \Rightarrow 4(\lambda + 1) = 9\lambda \)

\( \lambda = \frac{4}{5} = 0.80 \)

23. If \( y = \sum_{k=1}^{6} k \cos^{-1}\left\{ \frac{3}{5} \cos kx - \frac{4}{5} \sin kx \right\} \), then \( \frac{dy}{dx} \) at \( x = 0 \) is ________

Key: 91.00

Sol: Let \( \cos \alpha = \frac{3}{5}, \sin \alpha = \frac{4}{5} \)

\( y = \sum_{k=1}^{6} K \cos^{-1}\left( \cos Kx.\cos \alpha - \sin Kx.\sin \alpha \right) \)

\( = \sum_{k=1}^{6} K.\cos^{-1}\left( \cos(Kx + \alpha) \right) = x \sum_{i=1}^{6} K^2 + \alpha \sum_{i=1}^{6} K \)

\( \frac{dy}{dx} = \sum_{i=1}^{6} K^2 = \frac{6 \times 7 \times 13}{6} = 91.00 \)
24. For a positive integer \( n \), \( \left( 1 + \frac{1}{x} \right)^n \) is expanded in increasing powers of \( x \). If three consecutive coefficients in this expansion are in the ratio, \( 2 : 5 : 12 \), then \( n \) is equal to 

Key: 118.00

Sol: Given \( \binom{n}{r} : \binom{n}{r-1} : \binom{n}{r-2} = 2 : 5 : 12 \)

So \( \frac{\binom{n}{r}}{\binom{n}{r-1}} = \frac{2}{5} \)

\( \Rightarrow (n - r + 1)5 = 2r \rightarrow (1) \)

\( \frac{\binom{n}{r-1}}{\binom{n}{r-2}} = \frac{5}{12} \)

\( \Rightarrow (n - r + 2)12 = 5(r - 1) \rightarrow (2) \)

Solving (1) and (2) \( n = 118.00 \)

25. If the variance of the terms in an increasing A.P., \( b_1, b_2, b_3, \ldots, b_{11} \) is 90, then the common difference of this A.P. is 

Key: 3.00

Sol: \( \text{Var}(b_1, b_2, b_3, \ldots, b_{11}) = \text{Var}(0, d, 2d, \ldots, 10d) \) (\( d = \text{common difference} \))

\( \text{So} \sum \frac{d^2(1^2 + 2^2 + \ldots + 10^2)}{11} - \left( \frac{\sum d(0+1+\ldots+10)}{11} \right)^2 = 90 \)

\( \Rightarrow 35d^2 - 25d^2 = 90 \)

\( \Rightarrow d = 3 \) (as increasing A.P.)

Ans: 3.00

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