JEE Main 2021 August 27, Shift 1 (Physics)

1. Evaluate coefficient of volume expansion, for an ideal gas, for which the equation of state is PT^3 =K, where K is a constant.

(A) $1/T$

(B) $4/T$

(C) $T/5$

(D) $4T$

Ans. (B)

$\gamma = \frac{1}{V}\frac{dV}{dt}$

PT^3 =constant

$\Rightarrow \frac{T}{V}T^3$ = constant

$\Rightarrow T^4 V^{-1} = C$

$\Rightarrow V^{-1}(4T^3 dT) + T^4\left(\frac{-1}{V^2}dV\right) = 0$

$\Rightarrow \frac{dV}{dT} = \frac{4T^3/V}{\frac{T^4}{V^2}} = \frac{4V}{T} \quad \therefore \gamma = \frac{4}{T}$

2. Evaluate the speed in terms of C for an electromagnetic wave, which ia given as $E = sin(50x + 10^{10}t)\frac{V}{m}$. Here C is the speed of light in vacuum.

(A) $\frac{2}{5}C$

(B) $\frac{3}{5}C$

(C) $\frac{2}{3}C$

(D) $\frac{3}{4}C$

Ans. (C)

$\omega = 10^{10}$

$$k = 50$$

Speed= $\frac{\omega}{k} = \frac{10^{10}}{50} = 2 \times 10^8 = \frac{2}{3}C$

3. Find SI unit of E/H. Here E denotes the electric field intensity and H is representing the magnetic intensity.

(A) $Ampere - metre\hat{}2$

(B) $Joule/Ampere^2 - metre$

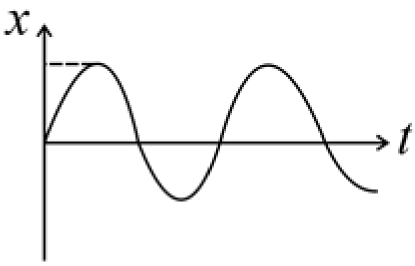
(C) $Ampere/(second)^3$

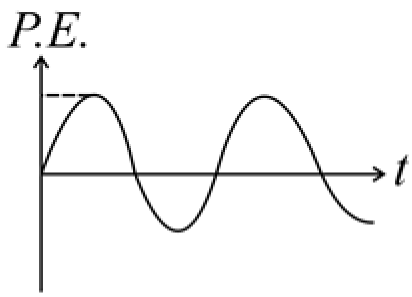
(D) $Joule/(second - Ampere)^2$

Ans. (D)

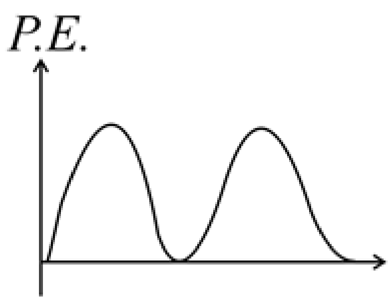
$\frac{E}{H} = \frac{q}{I} = \frac{Fr}{I^2 t} = Joule/(second - Ampere)^2$

4. For a given displacement-time curve of a Simple Harmonic Motion. The potential energy vs time curve will be?

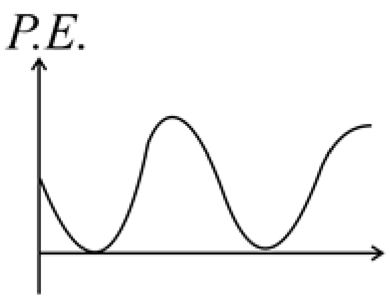




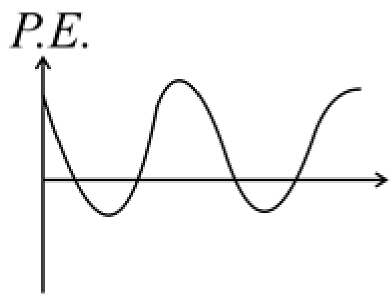
(A)



(B)



(C)



(D)

Ans. (B)

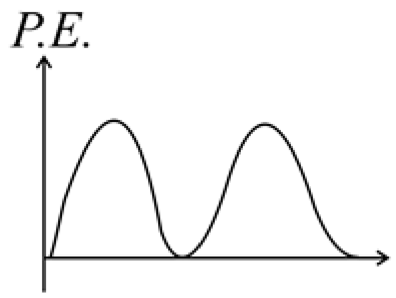
The mathematical equation of the given Simple harmonic motion is given by

$$x = x_0 sin\omega t$$

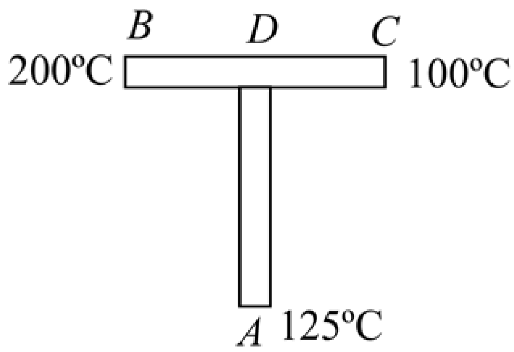
The potential energy of the spring will be

$U = \frac{1}{2}kx^2 = \frac{1}{2}kx_0^2 sin^2\omega t = csin^2\omega t$

Hence, the graph of Potential, P.E. with time will be

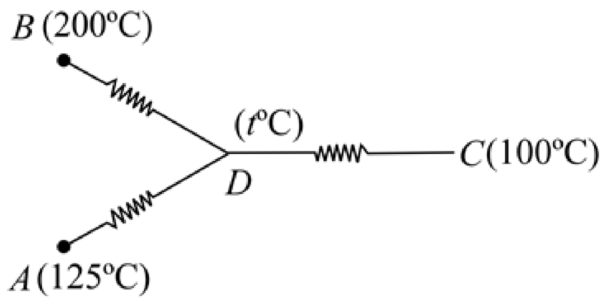


5. Two identical rods BC and AD are arranged as shown in figure. Point D is the midpoint of rod BC.If the area of cross section, length and the thermal conductivity of each rod is A, L and K, then the value of $\frac{L}{KA}$ is given as



$20\ kW^{-1}$. Find the rate of heat transfer through rod AD.

Ans. 1



$$Q_{BD} + Q_{AD} = Q_{DC}$$

$\frac{KA(200-t)}{\frac{l}{2}} + \frac{KA(125-t)}{l} = \frac{(t-100)KA}{\frac{l}{2}}$

$400 - 2t + 125 - t = 2t - 200$

$$725 = 5t$$

$t = 145°C$

$Q_{AD} = \frac{KA(125-145)}{l} = \left|-\frac{20}{\frac{l}{KA}}\right| = \left|-\frac{20}{20}W\right| = 1\ W$

6. Identify which among the following is not unitless.

(A) μ_r

(B) Refractive index

(C) Specific gravity

(D) μ_0

Ans. (D)

7. When F_1 force is acting at an angle of $60°$ from the direction of displacement and when a force F_2 is acting on same block at an angle $45°$ from the direction of displacement. We observe that the work done in both cases are equal. Displacement of the block in both cases are also equal. If $\frac{F_2}{F_1}$ is equal to $\frac{1}{\sqrt{x}}$, find the value of x.

Ans. (2)

$$W_1 = W_2$$

$$F_1 s \cos \cos 45º = F_2 s \cos \cos 60º$$

$\frac{F_1}{F_2} = \frac{1}{\sqrt{2}}$

$$x = 2$$

8. A disc is placed in the y-z plane with its centre at the origin. The uniform surface charge density of disc is σ and radius of disc is R. Find the electric field at a point $(x, 0,0)$

(A) $E_x = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{x}{\sqrt{R^2+x^2}}\right]$

(B) $E_x = \frac{2\sigma}{\epsilon_0}\left[1 + \frac{x}{\sqrt{R^2+x^2}}\right]$

(C) $E_x = \frac{2\sigma}{\epsilon_0}\left[1 - \frac{\sqrt{R^2+x^2}}{x}\right]$

(D) $E_x = \frac{\sigma}{2\epsilon_0}\left[x + \frac{x}{\sqrt{R^2+x^2}}\right]$

The disc can be considered to be a collection of large number of concentric rings. Consider an element of the shape of rings of radius r and of width dr. Electric field due to this ring at P is

$dE = \frac{K\cdot\sigma 2\pi r\cdot dr\cdot x}{(r^2+x^2)^{\frac{3}{2}}}$

Put, $r^2 + x^2 = y^2$

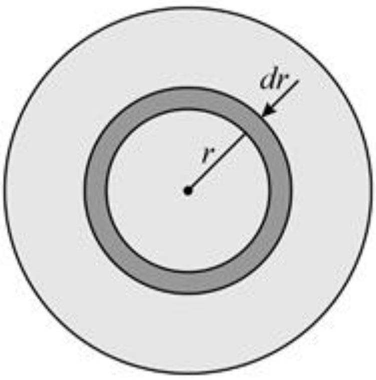
$$2rdr = 2ydy$$

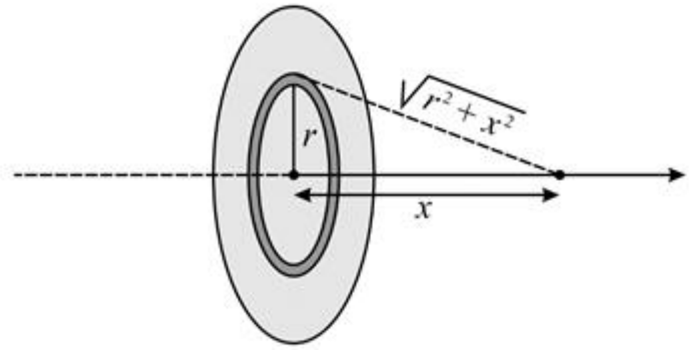
$\therefore dE = \frac{K\cdot\sigma 2\pi y\cdot dy\cdot x}{y^3} = 2\,K\sigma\pi\cdot x\frac{ydy}{y^3}$

Electric field at P due to all rings is along the axis:

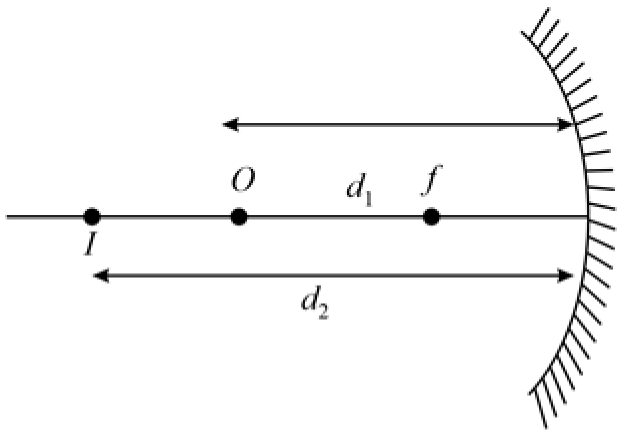
$\therefore E = \int dE \Rightarrow E = 2K\sigma\pi X\ \int_x^{\sqrt{R^2+x^2}} \frac{1}{y^2}dy = 2\,K\rho\pi x\cdot\left[-\frac{1}{y}\right]_x^{\sqrt{R^2+x^2}}$

$= 2\,K\sigma\pi x\left[+\frac{1}{x} - \frac{1}{\sqrt{R^2+x^2}}\right] = 2\,K\sigma\pi\left[1 - \frac{x}{\sqrt{R^2+x^2}}\right] = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{R^2+x^2}}\right]$ along the axis



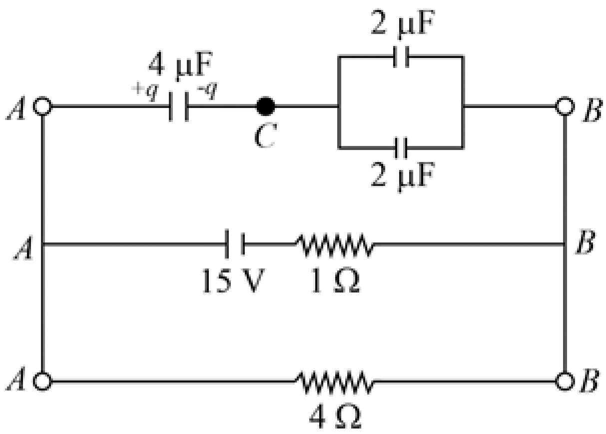


9. The object and its image are at distance d_1 and d_2 away from the Pole of a concave mirror. Calculate radius of curvature of the concave mirror.



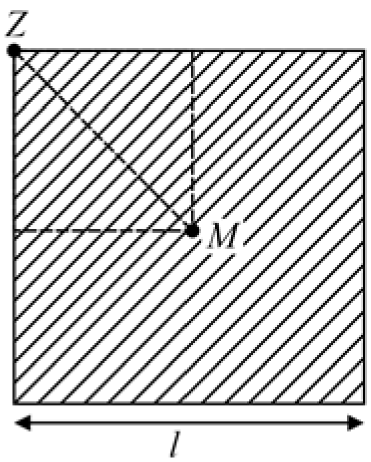
(A) $\frac{d_1 d_2}{2d_1+3d_2}$
(B) $\frac{d_1\, d_2}{d_1-d_2}$
(C) $\frac{2\, d_1\, d_2}{d_1+d_2}$
(D) $\frac{2d_1 d_2}{d_1+2d_2}$
Ans. (C)
$xy = f^2$
$(d_1 - f)(d_2 - f) = f^2$
$f = \frac{d_1 d_2}{d_1+d_2}$
Roc= $2f = \frac{2d_1 d_2}{d_1+d_2}$

10. Find $charge$ on the capacitor connected between points A and C at $t = \infty$



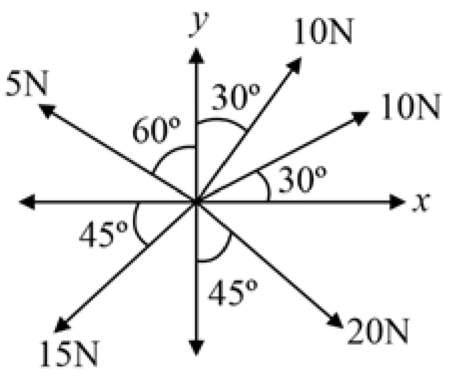
(A) $24\mu C$
(B) $120\mu C$
(C) $10\mu C$
(D) $6\mu C$
Ans. (A)
Using $i = \frac{15}{4+1} = 3\ A$
$\therefore V_{AB} = i \times 4 = 12\ V \quad \therefore V_{AC} = V_{CB} = 6$ Volt
$\therefore q$ on $4\mu F = CV_{AC} = 24\mu C$

11. Moment of inertia of a uniform square plate about an axis passing through one of its vertices and perpendicular to the plane is equal to? Mass of plate is M and side length is l.
(A) $\frac{M^2}{4}$
(B) $\frac{Ml^2}{3}$
(C) $\frac{2Ml^2}{3}$
(D) $\frac{Ml^2}{6}$
Ans. (C)



$I_Z = I_{Cm} + M\left(\frac{l}{\sqrt{2}}\right)^2$
$= \frac{Ml^2}{6} + \frac{Ml^2}{2} = \frac{4Ml^2}{6} = \frac{2Ml^2}{3}$

12. Calculate the resultant of the given combination of vectors?



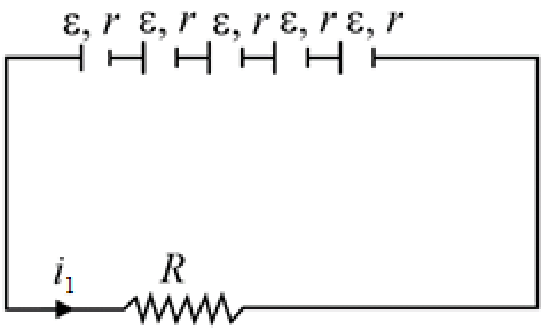
(A) $-15\cdot75i^{\wedge}-9j^{\wedge}$
(B) $10\cdot75i^{\wedge}-18j^{\wedge}$
(C) $-12\cdot75i^{\wedge}+17j^{\wedge}$
(D) $-10\cdot75i^{\wedge}-8j^{\wedge}$
Ans. (A)

Resultant$(\vec{R})=i^{\wedge}(10cos30°+10cos60°-5cos30°-15cos45°+20cos45°)$
$+j^{\wedge}(10sin30°+10sin60°+5sin30°-15sin45°-20cos45°)$
$=-15\cdot75i^{\wedge}-9j^{\wedge}$

13. Five identical cells of emf 5 V and internal resistance 3 Ω are first connected in series and then connected in parallel with an external resistance R respectively. If the current through R is same in both the cases, then the value of R is

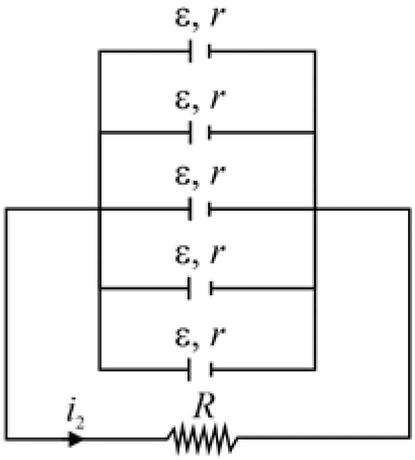
(A) 2 Ω
(B) 2.5 Ω
(C) 3 Ω
(D) 5 Ω
Ans. (C)

When cells are connected in series,



The equivalent emf in series combination, $\varepsilon_1=5\varepsilon$
The equivalent resistance in series combination, $R_1=R+5r$
The current in series combination, $i_1=\frac{5\varepsilon}{R+5r}$

When cells are connected in parallel,

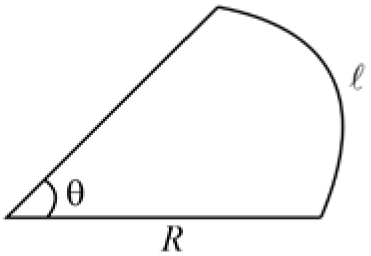


The equivalent cell emf in parallel combination, $\varepsilon_2=\varepsilon$
The equivalent resistance in parallel combination, $R_2=R+r/5$

The current in parallel combination, $i_2 = \frac{\varepsilon}{R+\frac{r}{5}}$

Given,
$i_1 = i_2$
$\frac{5\varepsilon}{R+5r} = \frac{\varepsilon}{R+\frac{r}{5}} \Rightarrow \quad R = r = 3\,\Omega$

14. A huge circular arc of length 4.4 ly subtends an angle $'4\,S'$ at the centre of the circle. How long it would take for a body to complete 4 revolutions if its speed is 8 Au per second? Given: $1\ ly = 9.46 \times 10^{15} m$; $1\ Au = 1.5 \times 10^{11}\ m$
(A) $3.2 \times 10^{7}\ s$
(B) $4.8 \times 10^{8}\ s$
(C) $6.7 \times 10^{8}\ s$
(D) $4.5 \times 10^{10}\ s$
Ans. (D)



1 S = 4.843×10^{-6}Radian

$\theta = 4\,S = 4 \times 4.843 \times 10^{-6} = 1.94 \times 10^{-5}$ Rad
$l = 4.4\ ly = 4.4 \times 9.46 \times 10^{15}$m

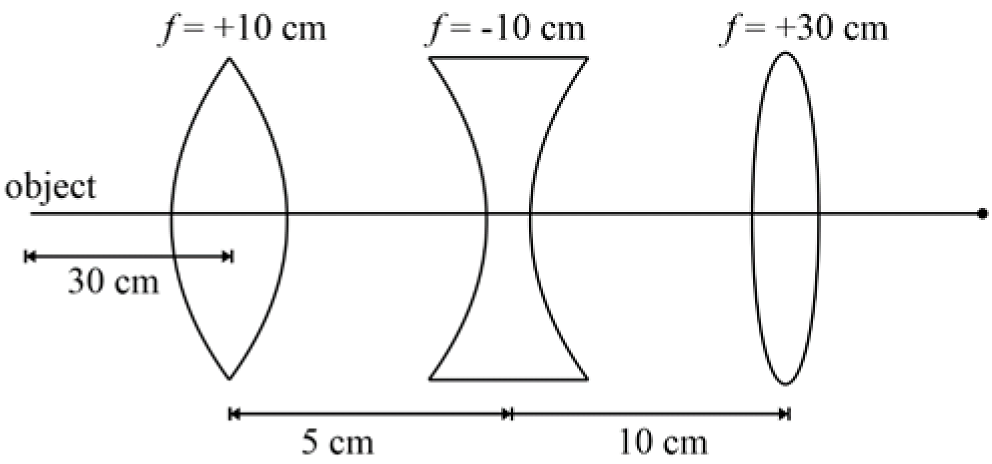
Length of Arc, $l = R\theta$

$$4.4 \times 9.46 \times 10^{15} = R\theta$$

$$4.4 \times 9.46 \times 10^{15} = R \times 1.94 \times 10^{-5}$$

$R = 2.1455 \times 10^{21}$meter
Speed $= 8\ Au = 8 \times 1.5 \times 10^{11}\ ms^{-1} = 12 \times 10^{11}\ ms^{-1}$
4 revolution means distance $= 4 \times 2\pi R$ metre
time$= \frac{distance}{speed} = \frac{4\times 2\pi R}{12\times 10^{11}}$; time $= \frac{8\times 3.14\times 2.1455\times 10^{21}}{12\times 10^{11}} \Rightarrow 4.5 \times 10^{10}$ sec

15. For the given arrangement, what will be the distance of the final image from the third lens having focal length f= 30 cm?



(A) *30*
(B) 45
(C) ∞
(D) 70
Ans. (A)

(1) Using Lens formula for 1st lens having focal length, f_1=10

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$\frac{1}{v}-\frac{1}{-30}=\frac{1}{10}$

$\frac{1}{v}=\frac{1}{10}-\frac{1}{30}$; $\frac{1}{v}=\frac{3-1}{30}$; $v=15\ cm$

Therefore, the object distance for 2nd lens, u_2 = 15-5=10 cm

$\frac{1}{v}-\frac{1}{10}=\frac{1}{-10}$

$$v=\infty$$

Parallel rays will be formed by the 2nd lens which will converge at the focal length of the 3rd lens.
Hence, the final image distance from 3rd lens, $v=+30\ cm$

16. Common Emitter transistor is used in which region as an amplifier.
(A) Cut off region
(B) Active region
(C) Saturation region
(D) Cut and saturation
Ans. (B)
In an active region, with a small variation in input voltage, the output voltage will drastically vary. So, the transistor is used as an amplifier in the active region.

17. Initial number of active nuclei in a radioactive element is 2×10^{15}. If half-life of radioactive element is 1 min. What will be the number of active nuclei after 30 sec? $[e^{-0.35}=0.70]$
(A) 1.4×10^{15}
(B) 2.1×10^{13}
(C) 0.7×10^{14}
(D) 0.7×10^{16}
Ans. (A)
If T is the half-life, then the decay constant is given by, $\lambda=\frac{\ln 2}{T}$

Using Radioactive decay law,
The number of nuclei remaining after time,t is

$N = N_0 e^{-\lambda t}$

$N = N_0 e^{\frac{\ln 2}{T} t}$

$N = N_0 e^{\frac{30 \ln 2}{60}}$

N= $2 \times 10^{15} e^{-0.35} = 1.4 \times 10^{15}$

18. When n identical resistance is connected in series with battery of emf $20\ V$ and internal resistance $10\ \Omega$, current in circuit is i_s.When the resistances are connected in parallel combination across the same battery, then current in circuit is i_p. If $i_p = 20$ is then number of resistances is? Resistance of each resistor is $10\ \Omega$.

(A) 40
(B) 20
(C) 9
(D) 36

Ans. (B)

$i_p = 20 i_s$

$\frac{20}{\left(\frac{10}{n}+10\right)} = 20\left(\frac{20}{10n+10}\right)$

$\frac{20n}{10+10n} = 20\left(\frac{20}{10n+10}\right)$

$$n = 20$$

19. Calculate the range of Line of Sight (LOS) Communication. The height of transmission tower is $380\ m$ and that of receiver tower is $800\ m$. Take radius of earth equal to $6400\ km$.

(A) $190\ km$
(B) $101\ km$
(C) $171\ km$
(D) $300\ km$

Ans. (C)

Range = $\sqrt{2Rh_1} + \sqrt{2Rh_2} = \sqrt{2 \times 6400 \times 380} + \sqrt{2 \times 6400 \times 800} \approx 171\ km$

20. In Millikan's oil drop experiment what will be the terminal velocity of an uncharged drop of radius $2.0 \times 10^5\ m$ and density of oil is$1.2 \times 10^3\ kg\ m^{-3}$. Take viscosity of liquid $= 1.8 \times 10^{-5} N\ s\ m^{-2}$and
neglect the buoyant force due to air

(A) $1.9 \times 10^{-2}\ ms^{-1}$
(B) $9.9 \times 10^{-2}\ ms^{-1}$
(C) $7.9 \times 10^{-2}\ ms^{-1}$
(D) $5.9 \times 10^{-2}\ ms^{-1}$

Ans. (D)

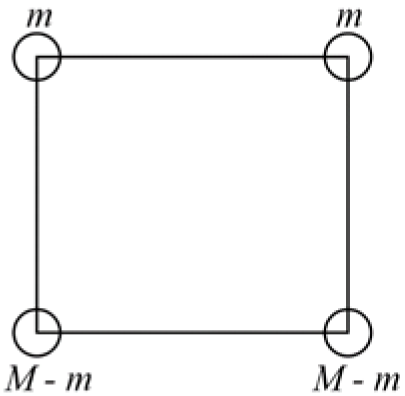
At terminal velocity, velocity will not further change, so the drop will be in equilibrium.

Viscous force on drop= Weight of drop

$$6\pi\eta r v_T = mg$$

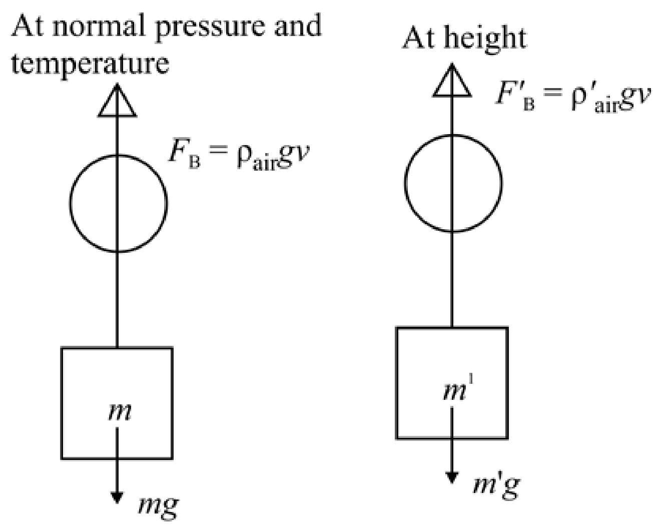
$v_T = \frac{\frac{4}{3}\pi r^3 \cdot \rho \cdot g}{6\pi\eta r} = \frac{2}{9}\frac{r^2 \rho g}{\eta} = \frac{2}{9} \times \frac{\left(2\times10^{-5}\right)^2 \times 1.2\times10^3\times10}{1.8\times10^{-5}} = 5.9 \times 10^{-2} ms^{-1}$

21. A particle of mass $2M$ is divided into four masses of mass $m, m, M-m$ and $M-m$. After that they are arranged on vertex of square of side a. Evaluate the ratio M/m, for which potential energy of this system is maximum/minimum:
(A) 4
(B) $1/2$
(C) 2
(D) $1/3$
Ans. (C)
Solution:
Potential energy of four particle system,
$U = \frac{Gm^2}{a} + \frac{G(M-m)^2}{a} + \frac{2Gm(M-m)}{a} + \frac{2Gm(M-m)}{\sqrt{2}a}$



For maxima or minima of potential energy, $\frac{dU}{dm} = 0$
$\frac{dU}{dm} = \frac{G}{a}[2m - 2(M-m) + (M-2m)2 + \sqrt{2}(M-2m)] = 0$
$\Rightarrow \left(4m - 4m - 2\sqrt{2}m\right) + \left(-2M + 2M + \sqrt{2}M\right) = 0;$
$\Rightarrow \sqrt{2}\,M = 2\sqrt{2}m$
$\Rightarrow \frac{M}{m} = 2$

22. A balloon carries a total load of mass185 kg at normal pressure and temperature of $27°C$. What load with the balloon be able to carry on rising to a height at which the temperature is $-7°C$ and the barometer pressure is 45 cm of Hg. (Assume the volume remains constant)
(A) 165.46 kg
(B) 123.54 kg
(C) 225.12 kg
(D) 150.50 kg
Ans. (B)



For equilibrium of balloon at normal pressure and temperature,

$\rho_{air} g V = mg$

$\rho_{air} V = 185 \ldots (1)$

For equilibrium of balloon at pressure of $45\ cm$ of Hg and the temperature of $-7°C$,

$$\rho'_{air} g V = m'g$$

$\rho_{air}\left(\frac{P'}{P} \times \frac{T}{T'}\right) V = m'$

From eq. (1),

$185\left[\frac{45}{76} \times \frac{300}{266}\right] = m'$

$$m = 123.54\ kg$$

23. Source and observer are moving towards each other. The speed of source and observer are $36\ km\ h^{-1}$ and $72\ km\ h^{-1}$ respectively. Find frequency observed by observer (speed of sound $V = 330\ ms^{-1}$). Frequency of sound produced by source is $1120\ Hz$.

(A) $1270\ Hz$

(B) $1400\ Hz$

(C) $1325\ Hz$

(D) $1225\ Hz$

Ans. (D)

$V_S = 36$ km/hr $= 10$ m/s $\qquad V_0 = 72$ km/hr $= 20$ m/s

cg

$f' = f_0\left(\frac{V+V_0}{V-V_S}\right)$

$\Rightarrow f' = 1120\left(\frac{330+20}{330-10}\right) = 1120 \times \frac{35}{32} = 1225\ Hz$

24. If the intensity of light is increased in photoelectric effect, then

(A) Kinetic energy of photoelectron increases

(B) Momentum of photon increases

(C) Frequency increases

(D) Number of photons increases

Ans. (D)

In photoelectric effect, the intensity of light is directly proportional to the number of photons.

$N = \frac{IA}{h\nu}$

25. In a circuit, current is given by, $i = \sqrt{72}\, sin\left(\frac{6t}{T}\right) + 8$. Find rms current in the circuit?

(A) 10

(B) 6

(C) 8

(D) $\sqrt{72}$

Ans. (A)

$i^2 = 72sin^2\left(\frac{6t}{T}\right) + 64 + 16\sqrt{72}sin\left(\frac{6t}{T}\right)$

$< i^2 >= \frac{72}{2} + 64;\ \ i_{rms} = \sqrt{< i^2 >} = \sqrt{100} = 10$

26. If velocity of a body having x displacement is given by $v = \sqrt{100 + 16x}$. The acceleration of the body is:

(A) 10 m/s^2

(B) 16 m/s^2

(C) 20 m/s^2

(D) 8 m/s^2

Ans. (D)

$$v = \sqrt{100 + 16x}$$
$$\frac{dv}{dx} = \frac{1}{2\sqrt{100 + 16x}}$$

$a = v\frac{dv}{dx}$

$a = \sqrt{100 + 16x} \times \frac{1}{2\sqrt{100+16x}} \times 16$

$$a = 8\, m/s^2$$

27. A conducting wire of length is used to form two setups of coils. The length of coil is $24a$. First in the form of square of side a and second in the form of equilateral triangle of side a. Find ratio of magnetic moment of coil in two cases?

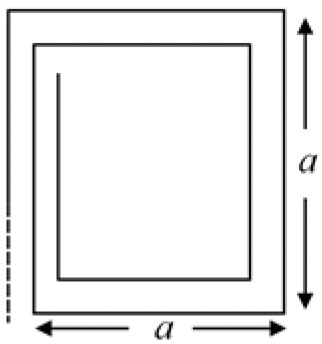
(A) $\sqrt{\frac{3}{7}}$

(B) $\sqrt{5}$

(C) $\sqrt{3}$

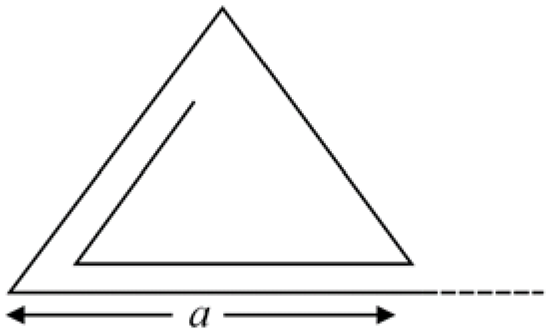
(D) $\sqrt{\frac{5}{3}}$

Ans. (C)



From length conservation, $4an_1 = 24a$

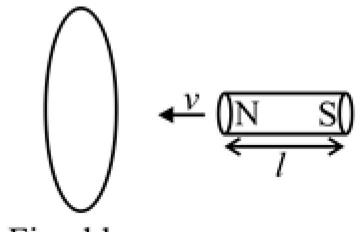
Thus $n_1 = 6$ turns

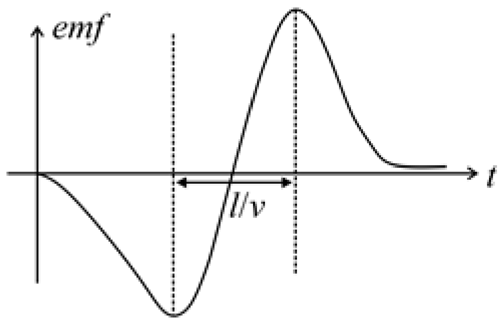


Again, $3an_2 = 24a \Rightarrow n_2 = 8$ turns

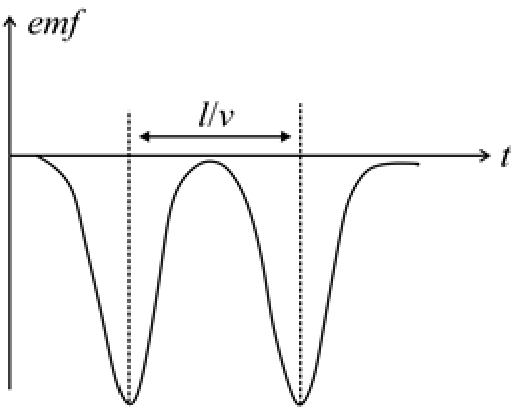
$\frac{M_1}{M_2} = \frac{n_1 i A_1}{n_2 i A_2} = \frac{6 \times a^2}{8 \times a^2 \times \frac{\sqrt{3}}{4}} = \frac{\sqrt{3}}{1}$

28. A Bar magnet moving with velocity 'v' towards a fixed conducting circular loop find the graph of emf $v/s\ \ t$. Consider anticlockwise emf as positive emf where the observer is right of south pole of magnet.

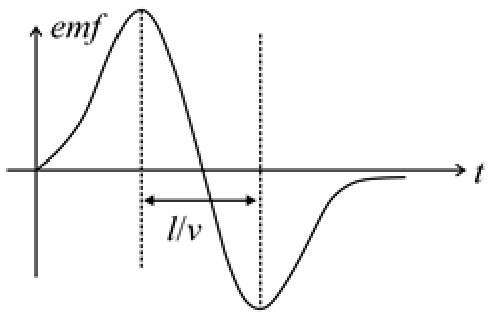




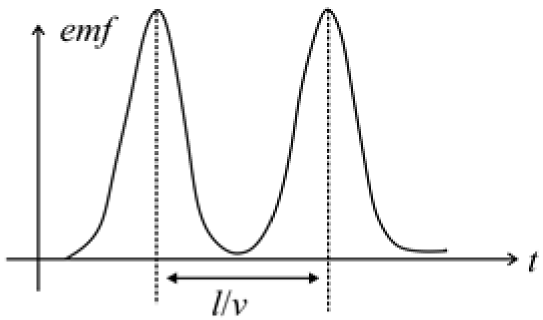
(A)



(B)



(C)

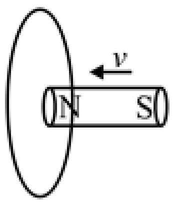


(D)

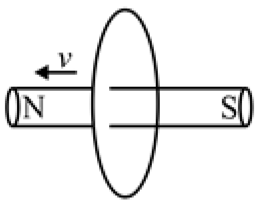
Ans. (c)

Magnetic flux will increase as the magnet will approach towards the loop. Rate of increment of flux also increases so emf increases with time

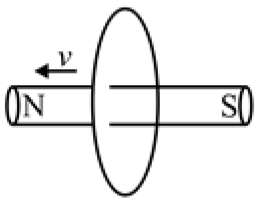
At this moment emf is maximum,



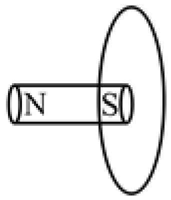
Magnet is in middle of coil, at this moment emf is equal to zero



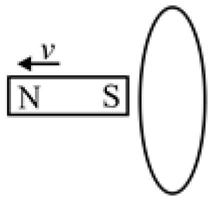
Now again, flux will start changing with the negative polarity.

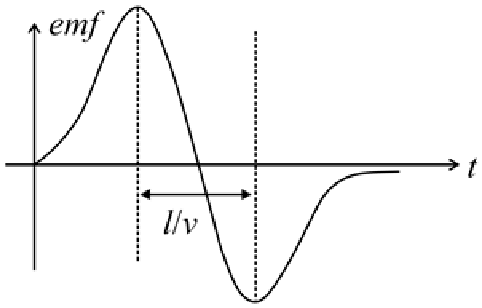


At this moment emf is again maximum,



emf again become zero





Ans. (3)